第四章　连续系统的频域分析（5）

常用信号的傅里叶变换

$$\delta(t) \longleftrightarrow 1$$

$$e^{-\alpha t}u(t) \longleftrightarrow \frac{1}{\alpha + j\omega}$$

$$e^{-\alpha|t|} \longleftrightarrow \frac{2\alpha}{\alpha^2 + \omega^2}$$

$$u(t) \longleftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\mathrm{sgn}(t) \longleftrightarrow \frac{2}{j\omega}$$

$$g_\tau(t) \longleftrightarrow \tau\, Sa(\frac{\omega\tau}{2})$$

$$\cos\omega_0 t \longleftrightarrow \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$$

$$\sin\omega_0 t \longleftrightarrow j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

$$\delta(t - t_0) \longleftrightarrow e^{-j\omega t_0}$$

$$\delta'(t) \longleftrightarrow j\omega$$

$$1 \longleftrightarrow 2\pi\delta(\omega)$$

$$e^{-j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega - \omega_0)$$

周期信号的傅里叶级数

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(n\Omega t)+\sum_{n=1}^{\infty} b_n\sin(n\Omega t) \ =\sum_{n=-\infty}^{\infty} F_n e^{jn\Omega t}$$

$$F_n=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-jn\Omega t}dt$$

非周期信号的傅里叶变换

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$$

$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

4.5 傅里叶变换的性质

■线性

$$\sum_{i=1}^{n} a_i f_i(t) \longleftarrow - - \longrightarrow \sum_{i=1}^{n} a_i F_i(\omega)$$

■奇偶虚实性

实偶函数←--→实偶函数	实奇函数←--→虚奇函数
虚偶函数←--→虚偶函数	虚奇函数←--→实奇函数

■对称性

$$F(jt) \longleftarrow - - \longrightarrow 2\pi f(-\omega)$$

■ 尺度变换特性

$$f(at) \longleftarrow - - \longrightarrow \frac{1}{|a|} F(\frac{\omega}{a})$$

■ 时移特性

$$f(t-t_0) \longleftarrow - - \longrightarrow F(\omega) e^{-j\omega t_0}$$

■ 频移特性

$$f(t) e^{j\omega_0 t} \longleftarrow - - \longrightarrow F(\omega - \omega_0)$$

4.5 傅里叶变换的性质

- 时域微分特性 $\frac{df(t)}{dt} \longleftarrow\!-\!-\!\longrightarrow j\omega F(\omega)$ $\frac{d^n f(t)}{dt^n} \longleftarrow\!-\!-\!\longrightarrow (j\omega)^n F(\omega)$
- 时域积分特性 $\int_{-\infty}^{t} f(\tau)d\tau \longleftarrow\!-\!-\!\longrightarrow \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
- 频域微分特性 $(-jt)^n f(t) \longleftarrow\!-\!-\!\longrightarrow F^n(j\omega)$
- 频域积分特性 $\pi f(0)\delta(t) + \frac{f(t)}{-jt} \longleftarrow\!-\!-\!\longrightarrow F^{(-1)}(j\omega)$
- 时域卷积定理 $f_1(t) * f_2(t) \longleftarrow\!-\!-\!\longrightarrow F_1(j\omega)F_2(j\omega)$
- 频域卷积定理 $f_1(t) \cdot f_2(t) \longleftarrow\!-\!-\!\longrightarrow \frac{1}{2\pi} F_1(j\omega) * F_2(j\omega)$
- **Paseval定理** $E = \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$ $P = \lim_{T\to\infty} \frac{|F(j\omega)|^2}{T}$

13. 相关定理

相关函数:某信号与其另一时延τ的信号之间的相似程度,定义为:

$$R_{12}(\tau)=\int_{-\infty}^{\infty}f_1(t)f_2(t-\tau)dt=\int_{-\infty}^{\infty}f_1(t+\tau)f_2(t)dt=f_1(t)*f_2(-t)$$

$$R_{21}(\tau)=\int_{-\infty}^{\infty}f_2(t)f_1(t-\tau)dt=\int_{-\infty}^{\infty}f_2(t+\tau)f_1(t)dt=f_1(-t)*f_2(t)$$

可见: $R_{12}(\tau)=R_{21}(-\tau)$

若$f_1(t)$和$f_2(t)$是同一函数,则称为自相关函数

$$R(\tau)=\int_{-\infty}^{\infty}f(t)f(t-\tau)dt=f(t)*f(-t)$$

$$R(\tau)=R(-\tau)$$

13. 相关定理

若 $f_1(t) \longleftrightarrow F_1(j\omega) \quad f_2(t) \longleftrightarrow F_2(j\omega)$

则 $R_{12}(\tau) \longleftrightarrow F_1(j\omega)F_2{}^*(j\omega)$

$$R_{21}(\tau) \longleftrightarrow F_1(j\omega)F_2{}^*(j\omega)$$

证明: $R_{12}(\tau) = f_1(t) * f_2(-t)$

$$FT[R_{12}(\tau)] = FT[f_1(t)]FT[f_2(-t)]$$

$$= F_1(j\omega)F_2{}^*(j\omega)$$

同理: $R_{21}(\tau) \longleftrightarrow F_1(j\omega)F_2{}^*(j\omega)$

$$R(\tau) \longleftrightarrow |F(j\omega)|^2$$

14. Paseval定理

信号的能量定义为在时间(-∞,+∞) 区间上信号的能量,用字母E表示

$$E \overset{def}{=} \lim_{T\to\infty}\int_{-T}^{T} |f(t)|^2\, dt \quad = \int_{-\infty}^{\infty} f^2(t)dt$$

$$E = \int_{-\infty}^{\infty} f^2(t)dt = \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega)e^{j\omega t}d\omega\right]dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\left[\int_{-\infty}^{+\infty} f(t)e^{j\omega t}dt\right]d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)F^*(j\omega)d\omega \quad = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(j\omega)\right|^2 d\omega \quad = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{E}(j\omega)d\omega$$

$$R(\tau) \longleftrightarrow \quad \mathcal{E}(j\omega)$$

14. Paseval定理

信号的功率定义为在时间(-∞,+∞) 区间上信号的平均功率,用字母P表示

$$P \overset{def}{=} \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} | f(t) |^2 \, dt \quad = \frac{1}{T}\int_{-\infty}^{\infty} f^2(t)dt$$

$$E = \int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 \, d\omega$$

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|F(j\omega)|^2}{T} d\omega \qquad = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{P}(j\omega)d\omega$$

14. Paseval定理

若f (t) 是功率有限信号,则称为自相关函数

$$R(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)f(t-\tau)dt \quad =\lim_{T\to\infty}\frac{1}{T}\left[f_T(\tau)*f_T(-\tau)\right]$$

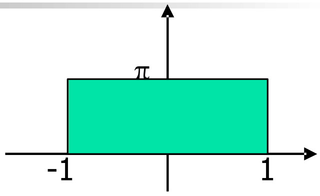
$$FT[R(\tau)]=FT\left\{\lim_{T\to\infty}\frac{1}{T}\left[f_T(\tau)*f_T(-\tau)\right]\right\}$$

$$FT[R(\tau)]=\lim_{T\to\infty}\frac{1}{T}\left|F_T(j\omega)\right|^2$$

$$\mathcal{FT}[R(\tau)]=\mathcal{P}(j\omega)$$

$$R(\tau)=\mathcal{FT}^{-1}[\mathcal{P}(j\omega)]$$

利用能量等式 $\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$



试求下列积分的值

(1) $\int_{-\infty}^{\infty} [\frac{\sin t}{t}]^2 dt$

(2) $\int_{-\infty}^{\infty} [\frac{1}{1+x^2}]^2 dx$

$$g_\tau(t) \longleftrightarrow \tau Sa(\frac{\omega\tau}{2})$$

$$g_2(t) \longleftrightarrow 2Sa(\omega)$$

$$Sa(t) \longleftrightarrow 2\pi\frac{1}{2}g_2(\omega)$$

$$\int_{-\infty}^{\infty} [\frac{\sin t}{t}]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-1}^{1} (\pi)^2 d\omega = \pi$$

利用能量等式 $\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$

(1) $\int_{-\infty}^{\infty} [\frac{\sin t}{t}]^2 dt$

(2) $\int_{-\infty}^{\infty} [\frac{1}{1+x^2}]^2 dx$

$$e^{-a|t|} \longleftrightarrow \frac{1}{a^2+\omega^2}$$

$$e^{-|t|} \longleftrightarrow \frac{1}{1+\omega^2}$$

$$\frac{1}{1+t^2} \longleftrightarrow 2\pi e^{-|\omega|}$$

$$\int_{-\infty}^{\infty} [\frac{\sin t}{t}]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} (2\pi e^{-|\omega|})^2 d\omega$$

$$= 2\pi \times 2\int_{0}^{\infty} e^{-2\omega} d\omega = 4\pi \times (-\frac{1}{2}) e^{-2\omega}\Big|_0^{\infty} = 2\pi$$

试求下列非周期信号的频谱

$$f(t)=e^{-2t}\cos\omega_0 t\varepsilon(t)$$

$$f(t)=e^{-2|t|}\cos\omega_0 t\varepsilon(t)$$

$$f(t)=\sin^2\omega_0 t\cdot\varepsilon(t)$$

(5) $f(t)\delta(t-a)$

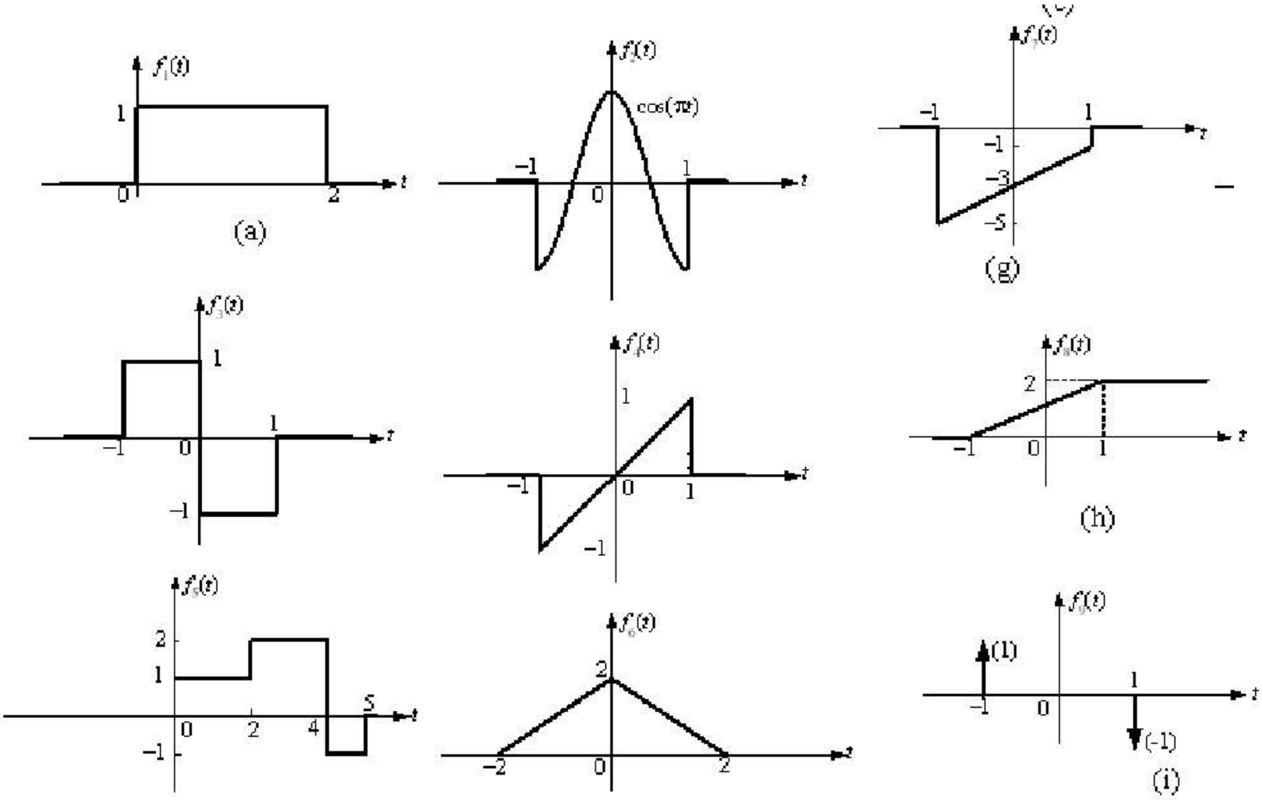
(6) $e^{-at}\varepsilon(-t)$

(7) $f(t)*\delta(\frac{t}{a}-b)$

(8) $(t-2)f(t)$

$$FT[g_1(t)] = Sa\frac{\omega}{2}$$

试求下列非周期信号的频谱



试求下列频谱对应的非周期信号

(1) $F(j\omega)=\dfrac{j\omega}{(j\omega+2)^2}$

(2) $F(j\omega)=\dfrac{4\sin(2\omega-2)}{2\omega-2}+\dfrac{4\sin(2\omega+2)}{2\omega+2}$

(3) $F(j\omega)=\dfrac{1}{j\omega(j\omega+2)}+2\pi\delta(\omega)$

(4) $F(j\omega)=\dfrac{d}{d\omega}[4\cos(3\omega)\dfrac{\sin 2\omega}{\omega}]$

(5) $F(j\omega)=\dfrac{2\sin\omega}{\omega(j\omega+1)}$

(6) $F(j\omega)=\dfrac{4\sin^2\omega}{\omega^2}$

试求下列非周期信号的频谱

$$f(t)=e^{-2t}\varepsilon(t)\cdot\sin\pi t$$

$$f(t)=\frac{2\sin\pi t}{\pi t}\cdot\frac{2\sin\pi t}{\pi t}$$

$$f(t)=\int_{-\infty}^{t}\frac{\sin\pi x}{\pi x}dx$$

$$f(t)=\frac{d}{dt}[\frac{\sin\pi t}{\pi t}\cdot\frac{\sin 2\pi t}{\pi t}]$$

$$f(t)=e^{-3|t-2|}$$

$$f(t)=\frac{d}{dt}[te^{-2t}\sin t\varepsilon(t)]$$

$$f(t)=e^{-2t+1}\varepsilon(\frac{t-4}{2})$$

试求下列非周期信号的频谱

$$f(t)=e^{-jt}\delta(t-2)$$

$$f(t)=e^{-3(t-1)}\delta'(t-1)$$

$$f(t)=\text{sgn}(t^2-9)$$

$$f(t)=e^{-2t}\varepsilon(t+1)$$

$$f(t)=\varepsilon(\frac{1}{2}t-1)$$

$$\delta(t)\longleftrightarrow 1$$

$$\delta(t-2)\longleftrightarrow e^{-j2\omega}$$

$$e^{-jt}\delta(t-2)\longleftrightarrow e^{-j2(\omega+1)}$$

试求下列非周期信号的频谱

$$f(t)=e^{-jt}\delta(t-2)$$

$$f(t)=e^{-3(t-1)}\delta'(t-1)$$

$$f(t)=\mathrm{sgn}(t^2-9)$$

$$f(t)=e^{-2t}\varepsilon(t+1)$$

$$f(t)=\varepsilon(\frac{1}{2}t-1)$$

$$\delta(t)\ \longleftrightarrow\ 1$$

$$\delta'(t)\ \longleftrightarrow\ j\omega$$

$$e^{-3t}\delta'(t)=e^{-3t}\Big|_{t=0}\delta'(t)-(e^{-3t})'\Big|_{t=0}\delta(t)$$

$$=\delta'(t)+3\delta(t)$$

$$e^{-3t}\delta'(t)\ \longleftrightarrow\ j\omega+3$$

$$e^{-3(t-1)}\delta'(t-1)\ \longleftrightarrow\ (j\omega+3)e^{-j\omega}$$

试求下列非周期信号的频谱

$$f(t)=e^{-jt}\delta(t-2)$$

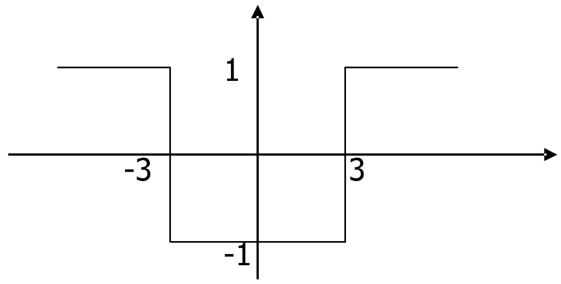
$$f(t)=e^{-3(t-1)}\delta'(t-1)$$

$$f(t)=\text{sgn}(t^2-9)$$

$$f(t)=e^{-2t}\varepsilon(t+1)$$

$$f(t)=\varepsilon(\frac{1}{2}t-1)$$

$$t^2-9<0 \quad \Rightarrow \quad -3<t<3$$



$$\text{sgn}(t^2-9)=1-2g_6(t)$$

$$g_6(t) \quad \longleftrightarrow \quad 6Sa(3\omega)$$

$$\text{sgn}(t^2-9) \quad \longleftrightarrow \quad 2\pi\delta(\omega)-12Sa(3\omega)$$

试求下列非周期信号的频谱

$$f(t)=e^{-jt}\delta(t-2)$$

$$f(t)=e^{-3(t-1)}\delta'(t-1)$$

$$f(t)=\text{sgn}(t^2-9)$$

$$f(t)=e^{-2t}\varepsilon(t+1)$$

$$f(t)=\varepsilon(\frac{1}{2}t-1)$$

$$e^{-2t}\varepsilon(t) \longleftrightarrow \frac{1}{2+j\omega}$$

$$e^{-2(t+1)}\varepsilon(t+1) \longleftrightarrow \frac{e^{+j\omega}}{2+j\omega}$$

$$e^{-2t}\varepsilon(t+1) \longleftrightarrow \frac{e^{2+j\omega}}{2+j\omega}$$

试求下列非周期信号的频谱

$$f(t)=e^{-jt}\delta(t-2)$$

$$f(t)=e^{-3(t-1)}\delta'(t-1)$$

$$f(t)=\mathrm{sgn}(t^2-9)$$

$$f(t)=e^{-2t}\varepsilon(t+1)$$

$$f(t)=\varepsilon(\frac{1}{2}t-1)$$

$$f(t)\longleftrightarrow F(j\omega)$$

$$f(at-b)\longleftrightarrow\frac{1}{|a|}e^{-j\frac{b}{a}\omega}F(j\frac{\omega}{a})$$

$$\varepsilon(t)\longleftrightarrow\pi\delta(\omega)+\frac{1}{j\omega}$$

$$\varepsilon(\frac{1}{2}t-1)\longleftrightarrow 2e^{-j2\omega}[\pi\delta(2\omega)+\frac{1}{j2\omega}]$$

$$\varepsilon(\frac{1}{2}t-1)\longleftrightarrow\pi\delta(\omega)+\frac{e^{-j2\omega}}{j\omega}$$

若已知 $f(t) \longleftrightarrow F(j\omega)$，试求下列函数的频谱

(2) $(t-2)f(t)$

(3) $t\dfrac{df(t)}{dt}$

(4) $f(1-t)$

(7) $\displaystyle\int_{-\infty}^{1-\frac{1}{2}t} f(\tau)d\tau$

$$f(t) \longleftrightarrow F(j\omega)$$

$$(-jt)f(t) \longleftrightarrow \frac{d[F(j\omega)]}{d\omega}$$

$$t\,f(t) \longleftrightarrow j\frac{d[F(j\omega)]}{d\omega}$$

$$(t-2)f(t) = t\,f(t) - 2f(t)$$

$$\longleftrightarrow j\frac{d[F(j\omega)]}{d\omega} - 2F(j\omega)$$

若已知 $f(t) \longleftrightarrow F(j\omega)$,试求下列函数的频谱

(2) $(t-2)f(t)$

(3) $t\dfrac{df(t)}{dt}$

(4) $f(1-t)$

(7) $\int_{-\infty}^{1-\frac{1}{2}t} f(\tau)d\tau$

$$f(t) \longleftrightarrow F(j\omega)$$

$$\frac{df(t)}{dt} \longleftrightarrow j\omega F(j\omega)$$

$$t\,f(t) \longleftrightarrow j\frac{d[F(j\omega)]}{d\omega}$$

$$t\frac{df(t)}{dt} \longleftrightarrow j\frac{d[j\omega F(j\omega)]}{d\omega}$$

$$=-F(j\omega)-\omega\frac{d[F(j\omega)]}{d\omega}$$

若已知 $f(t) \longleftrightarrow F(j\omega)$,试求下列函数的频谱

(2) $(t-2)f(t)$

(3) $t\dfrac{df(t)}{dt}$

(4) $f(1-t)$

(7) $\int_{-\infty}^{1-\frac{1}{2}t} f(\tau)d\tau$

$$f(t) \longleftrightarrow F(j\omega)$$

$$f(at-b) \longleftrightarrow \frac{1}{|a|}e^{-j\frac{b}{a}\omega}F(j\frac{\omega}{a})$$

$$f(1-t) \longleftrightarrow e^{-j\omega}F(-j\omega)$$

若已知 $f(t) \longleftrightarrow F(j\omega)$,试求下列函数的频谱

(2) $(t-2)f(t)$

(3) $t\dfrac{df(t)}{dt}$

(4) $f(1-t)$

(7) $\int_{-\infty}^{1-\frac{1}{2}t} f(\tau)d\tau$

$$= g(1-\frac{1}{2}t)$$

$$f(t) \longleftrightarrow F(j\omega)$$

$$\int_{-\infty}^{t} f(\tau)d\tau \longleftrightarrow \pi F(0)\delta(\omega)+\frac{F(j\omega)}{j\omega}$$

$$g(t)=\int_{-\infty}^{t} f(\tau)d\tau$$

$$f(at-b) \longleftrightarrow \frac{1}{|a|}e^{-j\frac{b}{a}\omega}F(j\frac{\omega}{a})$$

$$g(1-\frac{1}{2}t) \longleftrightarrow 2e^{-j2\omega}[\pi F(0)\delta(2\omega)+\frac{F(-j2\omega)}{-j2\omega}]$$

$$=\pi F(0)\delta(\omega)-\frac{1}{j\omega}e^{-j2\omega}F(-j2\omega)$$

试题1

若已知 $f(t) \longleftrightarrow F(j\omega)$,试求下列函数的频谱

(1) $f(t)=\dfrac{d}{dt}[te^{-2t}\sin t\varepsilon(t)]$

(2) $f(t)=e^{-2t+1}\varepsilon(\dfrac{t-4}{2})$

(3) $e^{-2t}\varepsilon(t)\cdot\sin\pi t$

(4) $e^{jt}f(3-2t)$

(5) $\dfrac{df(t)}{dt}*\dfrac{1}{\pi t}$

4.6 周期信号的傅立叶变换

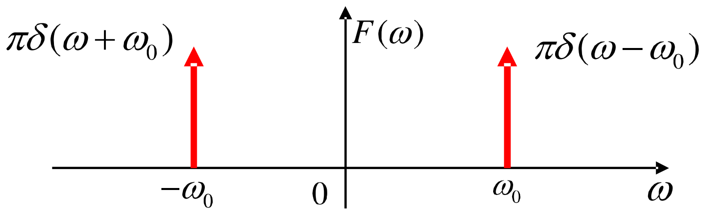
一、正余弦函数的傅里叶变换

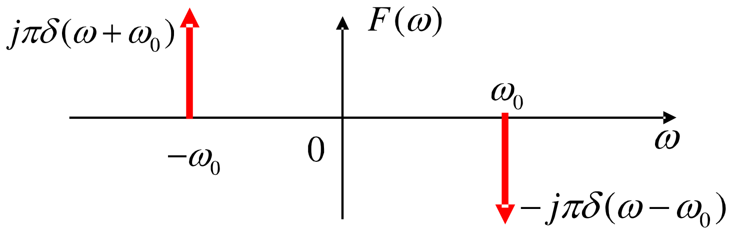
$$1 \longleftarrow - - \longrightarrow 2\pi\delta(\omega)$$

$$e^{-j\omega_0 t} \longleftarrow - - \longrightarrow 2\pi\delta(\omega-\omega_0)$$

$$\cos\omega_0 t \longleftarrow - - \longrightarrow \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$

$$\sin\omega_0 t \longleftarrow - - \longrightarrow j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$





二、一般周期函数的傅里叶变换

若 $$f(t)=\sum_{n=-\infty}^{\infty}F_n.e^{jn\Omega t}$$

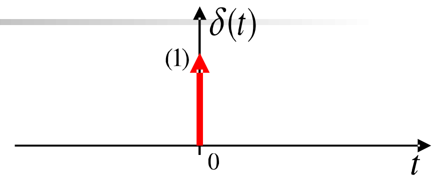
则 $$FT[f(t)]=2\pi\sum_{n=-\infty}^{\infty}F_n\delta(\omega-n\Omega)$$

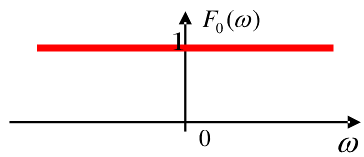
证明： $$FT[f(t)]=FT[\sum_{n=-\infty}^{\infty}F_n.e^{jn\Omega t}]$$

$$=\sum_{n=-\infty}^{\infty}F_n.FT[e^{jn\Omega t}]$$

$$=2\pi\sum_{n=-\infty}^{\infty}F_n\delta(\omega-n\Omega)$$

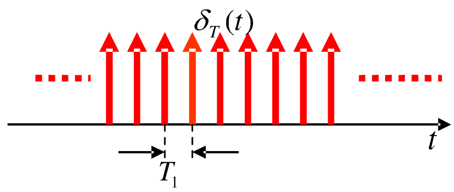
单位冲激序列的FS





$$\delta_T(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)=\sum_{n=-\infty}^{\infty}F_n.e^{jn\Omega t}=\frac{1}{T}\sum_{n=-\infty}^{\infty}e^{jn\Omega t}$$

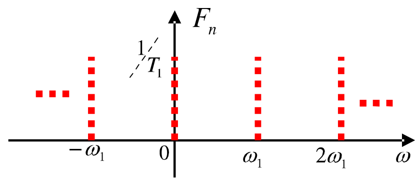
$$F_n=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\delta_T(t).e^{-jn\Omega t}dt=\frac{1}{T}$$



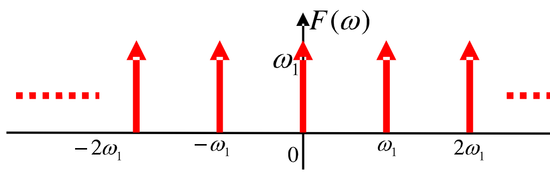
单位冲激序列的FT

$$FT[f(t)]=FT[\delta_T(t)]=\frac{1}{T}\sum_{n=-\infty}^{\infty}FT[e^{jn\omega t}]$$

$$=2\pi\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\Omega)$$

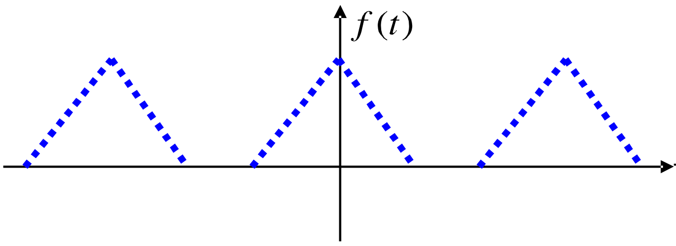


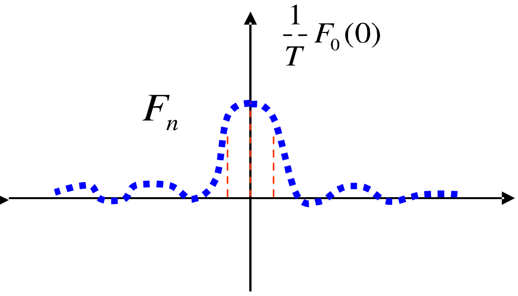
$$F(\omega)=FT[\delta_T(t)]=\Omega\sum_{n=-\infty}^{\infty}\delta(\omega-n\Omega)$$



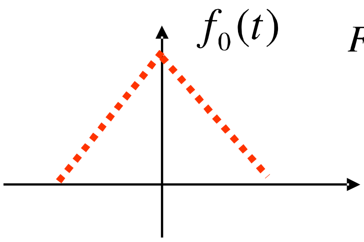
- 由一些冲激组成离散频谱
- 位于信号的谐频处 $(0, \pm\omega_1, 2\omega_1, \cdots)$
- 大小不是有限值，而是无穷小频带内有无穷大的频谱值
- 周期信号不满足绝对可积条件
- 引入冲激信号后，冲激的积分是有意义的
- 在以上意义下，周期信号的傅立叶变换是存在的
- 周期信号的频谱是离散的，其频谱密度， 即傅立叶变换是一系列冲激.

三、傅立叶系数与傅立叶变换

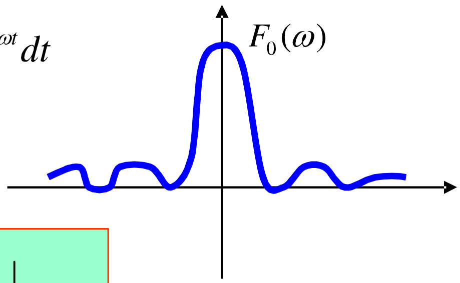




$$F_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t).e^{-jn\Omega t}dt$$

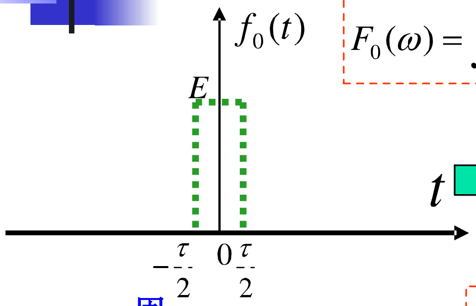


$$F_0(\omega) = \int_{-\frac{T}{2}}^{\frac{T}{2}} f_0(t).e^{-j\omega t}dt$$

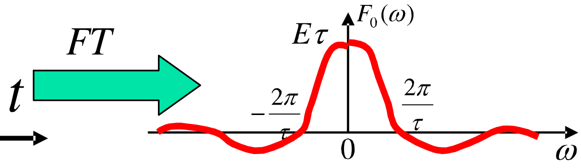


$$F_n = \frac{1}{T_1}F_0(\omega)\ \Big|_{\omega=n\omega_1}$$

周期矩形脉冲的FS和FT



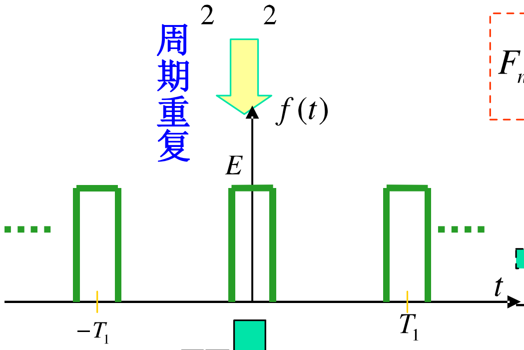
$$F_0(\omega)=\int_{-T_1/2}^{T_1/2} f_0(t).e^{-j\omega t}dt$$



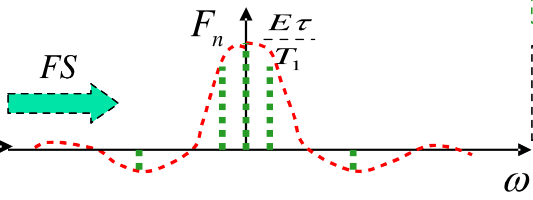
$$F_0(\omega)=E\tau Sa\left(\frac{\omega\tau}{2}\right)$$

$$F_n=\frac{1}{T_1}F_0(\omega)\Big|_{\omega=n\omega_1}$$

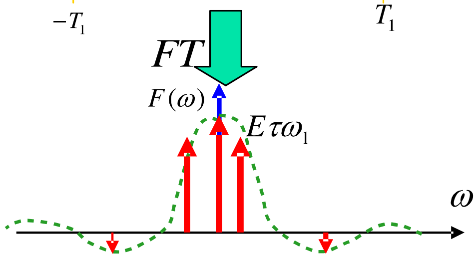
$$F_n=\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} f(t).e^{-jn\omega_1 t}dt$$



$$F_n=\frac{1}{T_1}F_0(\omega)\Big|_{\omega=n\omega_1}=\frac{E\tau}{T_1}Sa\left(\frac{n\omega_1\tau}{2}\right)$$



$$f(t)=\frac{E\tau}{T_1}\sum_{n=-\infty}^{\infty}Sa\left(\frac{n\omega\tau_1}{2}\right).e^{jn\omega_1\tau}$$



$$F(\omega)=E\tau\omega_1\sum_{n=-\infty}^{\infty}Sa\left(\frac{n\omega_1\tau}{2}\right)\delta(\omega-n\omega_1)$$

一个周期为T的周期信号f(t),已知其指数形式的傅里叶系数为Fn,求下列周期信号的傅里叶系数

$$f_1(t) = f(t - t_0) \qquad F_1(j\omega) = e^{-j\omega t_0} F(j\omega) \qquad F_{1n} = e^{-jn\Omega t_0} F_n$$

$$f_2(t) = f(-t) \qquad F_2(j\omega) = F(-j\omega) \qquad F_{2n} = F_{-n}$$

$$f_3(t) = \frac{df(t)}{dt} \qquad F_3(j\omega) = j\omega F(j\omega) \qquad F_{3n} = jn\Omega F_n$$

$$f_4(t) = f(at) \qquad F_4(j\omega) = \frac{1}{|a|} F(j\frac{\omega}{a}) \qquad F_{4n} = F_n$$