第8章 渐近法及其他算法简介

§8-1 力矩分配法的基本概念

力法、位移法：精确，求解方程。

力矩分配法是基于位移法，逐步逼近精确解的近似方法。

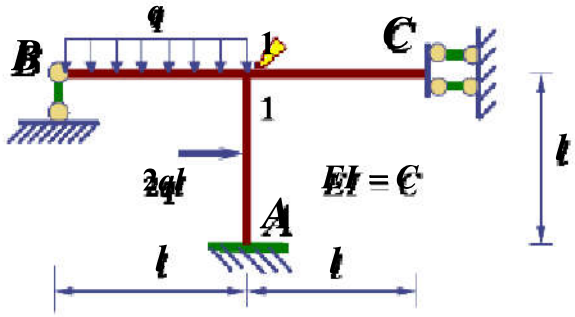
单独使用时只能用于无侧移（无线位移）的结构。

1.名词解释

$$M_{1B}=3i\theta+ql^2/8$$

$$M_{1A}=4i\theta+ql^2/4$$

$$M_{1C}=i\theta$$



（1）转动刚度：使*AB*杆的*A*端产生单位转动，在*A*端所需施加的杆端弯矩称为*AB*杆A端的转动刚度，记作S_{AB}。

对等直杆，S_{AB}与线刚度及*B*端的支承条件有关。

*A*端一般称为近端（本端），

*B*端一般称为远端（它端）。

$S_{1B}=3i$　远端铰支

$S_{1A}=4i$　远端固定

$S_{1C}=i$　远端定向支承

$$M_{1B}=3i\theta+ql^2/8$$

$$M_{1A}=4i\theta+ql^2/4$$

$$M_{1C}=i\theta$$

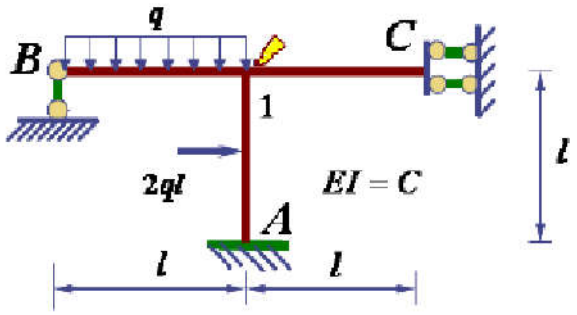
$$M_{1B}^F=ql^2/8$$

$$M_{1A}^F=2ql\cdot l/8$$

$$M_{1C}^F=0$$

$$M_1^u=M_{1B}^F+M_{1A}^F+M_{1C}^F=3ql^2/8$$

M_1^u ---不平衡力矩，顺时针为正



$$8i\theta+3ql^2/8=0$$

$$8i\theta+M_1^u=0$$

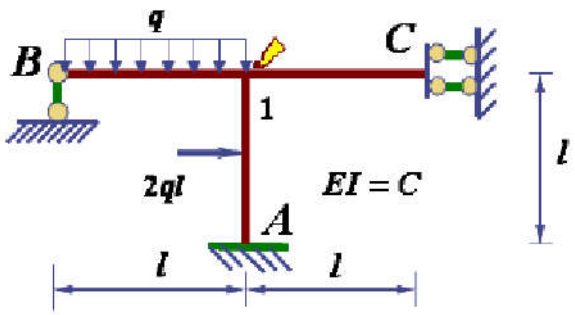
$$\theta=-M_1^u/8i$$

$$\theta=-\frac{M_1^u}{S_{1B}+S_{1A}+S_{1C}}$$

$$k_{11}\Delta+F_{1P}=0$$

$$M_{1B}=3i\theta+ql^2/8 \quad =3i\ \frac{-M_1^u}{8i}+ql^2/8$$

$$=\frac{3i}{8i}(-M_1^u)+ql^2/8$$

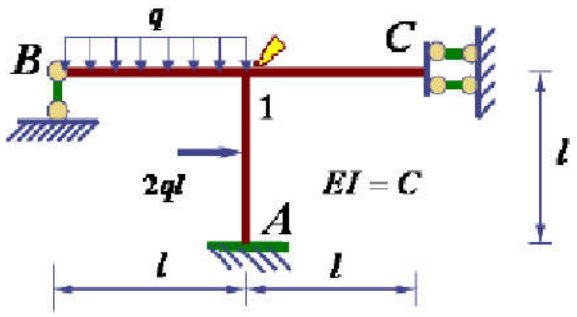


$$=\frac{S_{1B}}{S_{1B}+S_{1A}+S_{1C}}(-M_1^u)+ql^2/8$$

$$=\mu_{1B}(-M_1^u)+ql^2/8 \quad =M_{1B}^d+M_{1B}^F$$

M_{1B}^d 分配弯矩

$$\mu_{1B}=\frac{S_{1B}}{S_{1B}+S_{1A}+S_{1C}}$$ （2）分配系数



$$M_{1A}=4i\theta+M_{1A}^{F}=\frac{S_{1A}}{S_{1A}+S_{1B}+S_{1C}}(-M_1^{u})+M_{1A}^{F}=\mu_{1A}(-M_1^{u})+M_{1A}^{F}$$

$$M_{1C}=i\theta=\frac{S_{1C}}{S_{1A}+S_{1B}+S_{1C}}(-M_1^{u})=\mu_{1C}(-M_1^{u})$$

一个结点上的各杆端分配系数总和恒等于**1**。

$$M_{A1} = 2i\theta - ql^2/4$$
$$= CM_{1A}^{d} + M_{A1}^{F}$$
$$= M_{A1}^{C} + M_{A1}^{F}$$

M_{A1}^{C} 传递弯矩

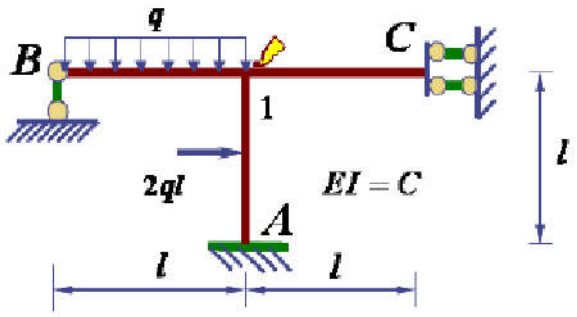
$$C = \frac{\text{结点位移引起的远端弯矩}}{\text{结点位移引起的近端弯矩}}$$

远端固定: C=1/2

远端铰支: C=0

远端定向支承: C=－1

与远端支承情况有关



（3）传递系数

$$M_{B1} = 0$$
$$M_{A1} = 2i\theta - ql^2/4$$
$$M_{C1} = -i\theta$$

2.基本运算（ 单结点的力矩分配）

固定状态:

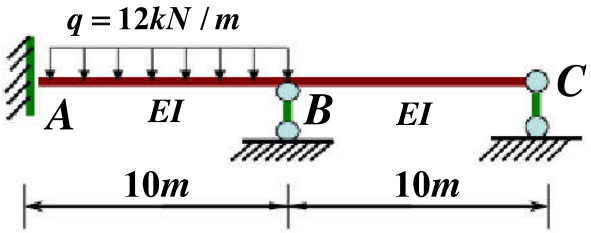
$$M_{AB}^{F} = -ql^2/12 = -100kN.m$$

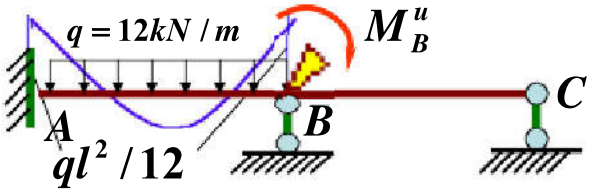
$$M_{BA}^{F} = 100kN.m$$

$$M_{BC}^{F} = M_{CB}^{F} = 0$$

$$\mu_{BA} = \frac{4}{4+3} = 0.571$$

$$\mu_{BC} = \frac{3}{4+3} = 0.429$$



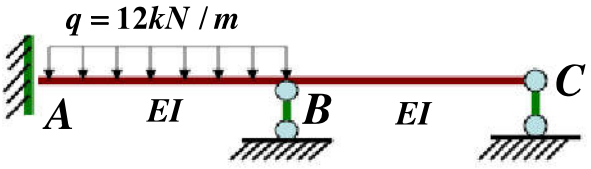


固定状态:

$$M_{AB}^{F} = -ql^2/12 = -100kN.m$$

$$M_{BA}^{F} = 100kN.m$$

$$M_{BC}^{F} = M_{CB}^{F} = 0$$

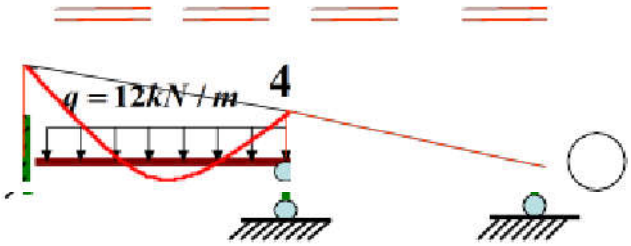


μ		0.571	0.429	
M^F	−100	100	0	0

$$\mu_{BA} = \frac{4}{} \quad -0.571$$

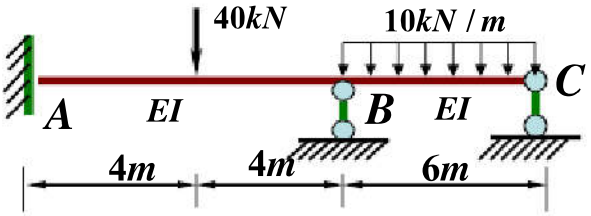
$$\mu_{BC} = \text{——}$$

放松:



最终杆端弯矩

例1.计算图示梁,作弯矩图



解：$S_{BA}=4\times\frac{EI}{8}=0.5EI$

$S_{BC}=3\times\frac{EI}{6}=0.5EI$

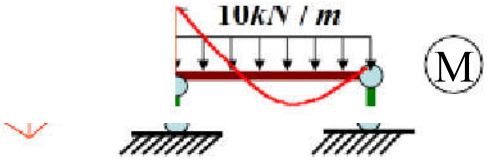
$\mu_{BA}=\frac{0.5EI}{(0.5+0.5)EI}=0.5$

$\mu_{BC}=$

μ		0.5	0.5	
M^F	−40	40	−45	0
分配	1.25 ←	2.5	2.5 →	0

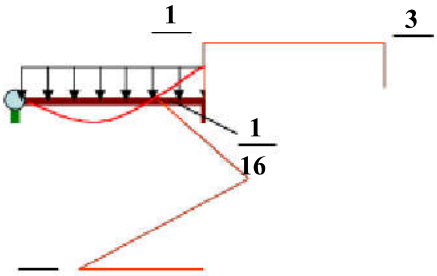
$M_{AB}^F=$

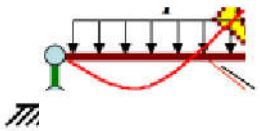
$M_{BC}^F=-\frac{}{8}\times$



例

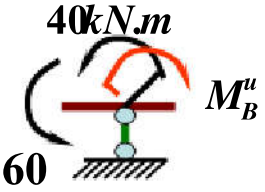
解



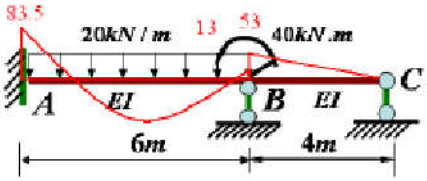


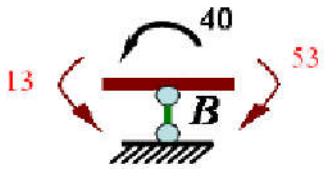
$ql^2/4$

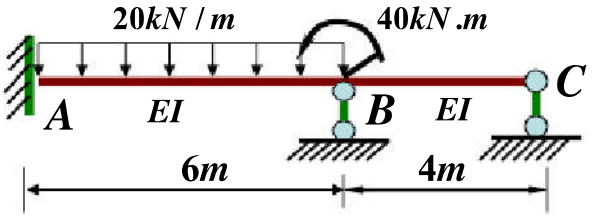
求不平衡力矩

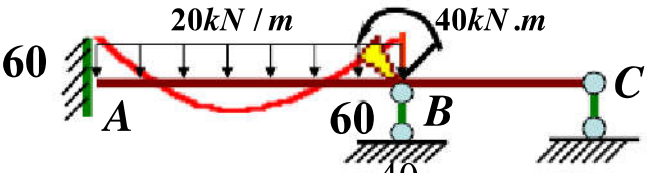


$$M_B^u = 60 + 40 = 100kN.m$$





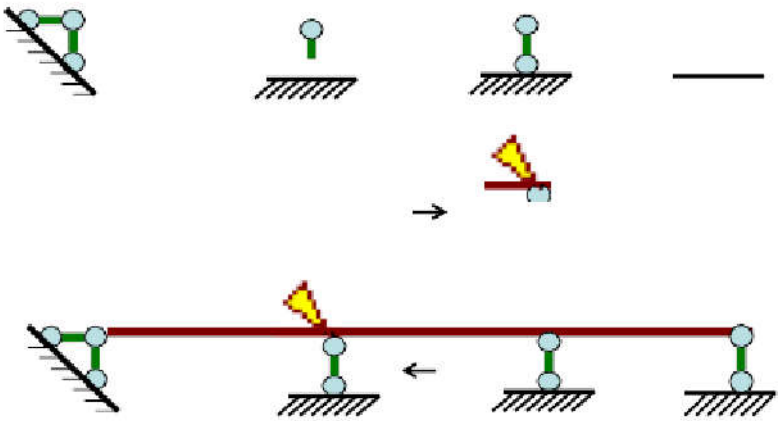




40

μ		8/17	9/17
M^F	−60	60	
分配传递	−23.5	← −47	−53
M	−83.5	13	−53

§



μ

$M_{1A}^{F} = ql^2/8 = 150$

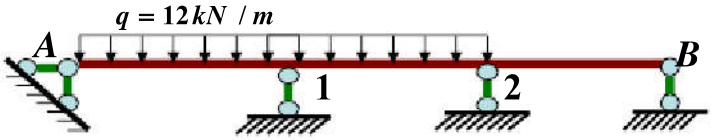
$M_{12}^{F} = -ql^2/12 = -100$

$S_{21} = 4i$ $\quad S_{2B} = 3i$

$\mu_{21} = 0.571$ $\quad \mu_{2B} = 0.429$

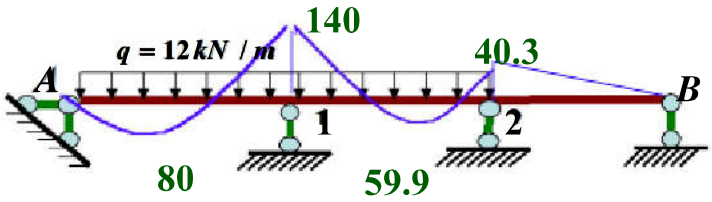
$S_{12} = 4i$ $\quad S_{1A} = 3i$

$\mu_{12} = 0.571$ $\quad \mu_{1A} = 0.429$



μ		0.429	0.571	0.571	0.429	
M^F	0	150	-100	100	0	0
分配传递			-28.6 ←	-57.1	-42.9 →	0
	0 ←	-9.2	-12.2 →	-6.1		
			1.8 ←	3.5	2.6 →	0
		-0.8	-1.0			
M	0	140	-140	40.3	-40.3	0

$M(kN.m)$



作剪力图

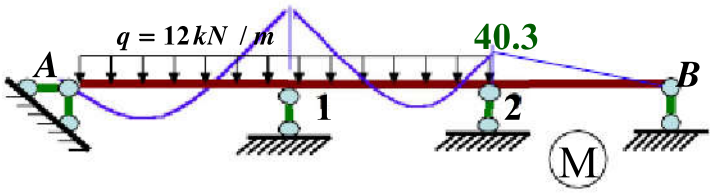
$$\sum M_A = 0$$

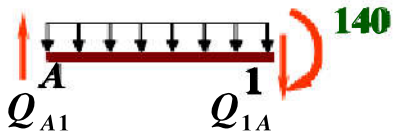
$$Q_{1A} \times 10 + 140 + 12 \times 10 \times 5 = 0$$

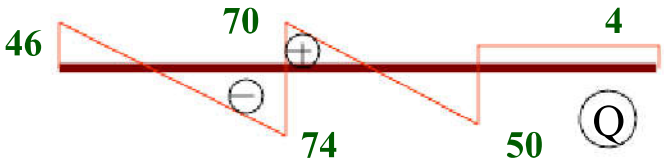
$$Q_{1A} = -74\,kN$$

$$\sum F_y = 0$$

$$Q_{A1} = 46\,kN$$



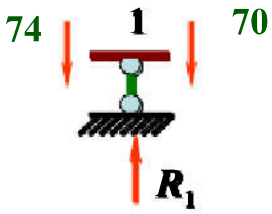




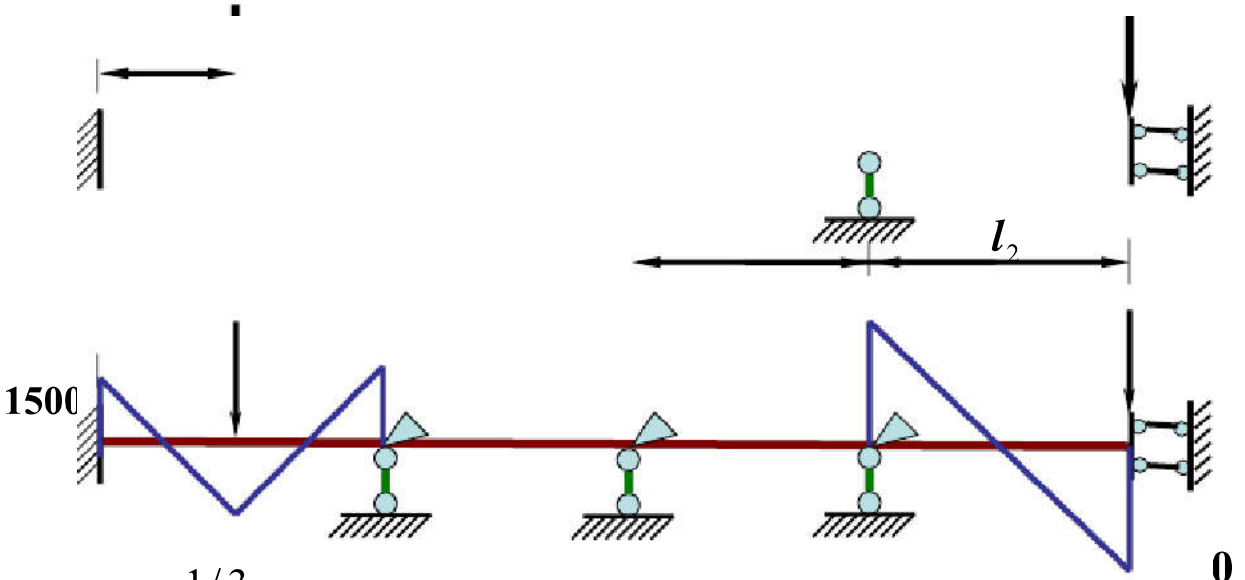
求反力

$$\sum F_y = 0$$

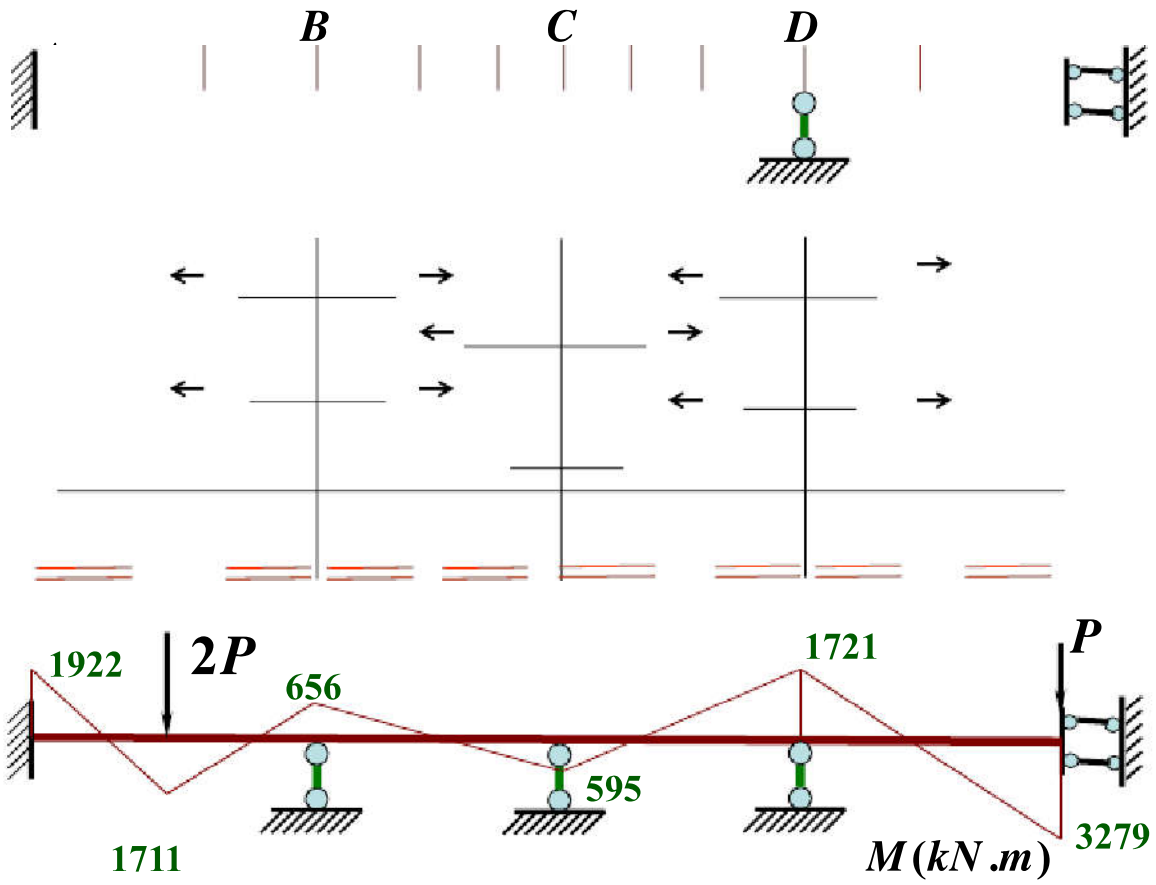
$$R_1 = 74 + 70 = 144\,(kN)(\uparrow)$$



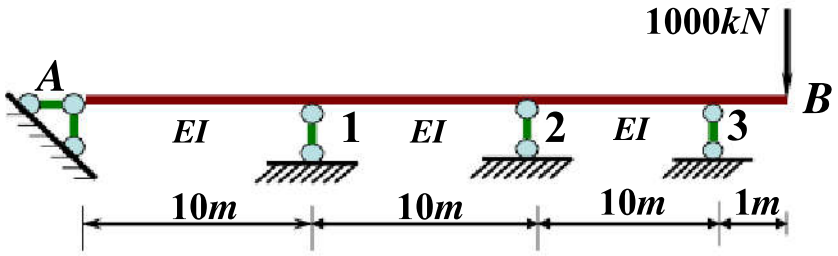
例：试求作图示连续梁的M图。EI等于常数，l_1=6 m，l_2=5 m，P=1000kN。（只计算二轮）



$$\mu_{BA}=\frac{1/3}{1/3+1/5}=0.675$$


B
C
D
1922
2P
656
1721
P
595
1711
3279
M(kN.m)

力矩分配法计算图示连续梁，作弯矩图。



$$\mu_{1A}=\frac{3}{3+4} \qquad \mu_{12}=\frac{4}{3+4}$$

$$\mu_{21}=\frac{4}{4+4} \qquad \mu_{23}=\frac{4}{4+4}$$

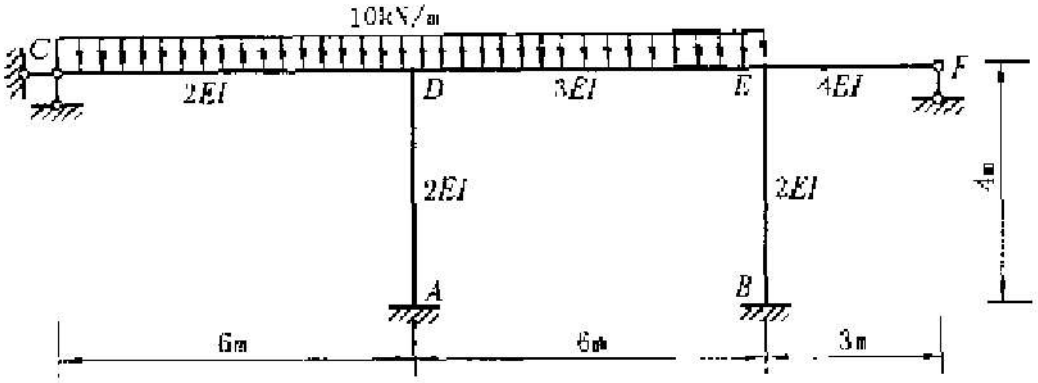
$$\mu_{32}=\frac{4}{4+0} \qquad \mu_{23}=\frac{0}{4+0}$$

$$M_{1A}^{F}=0 \quad M_{12}^{F}=0$$

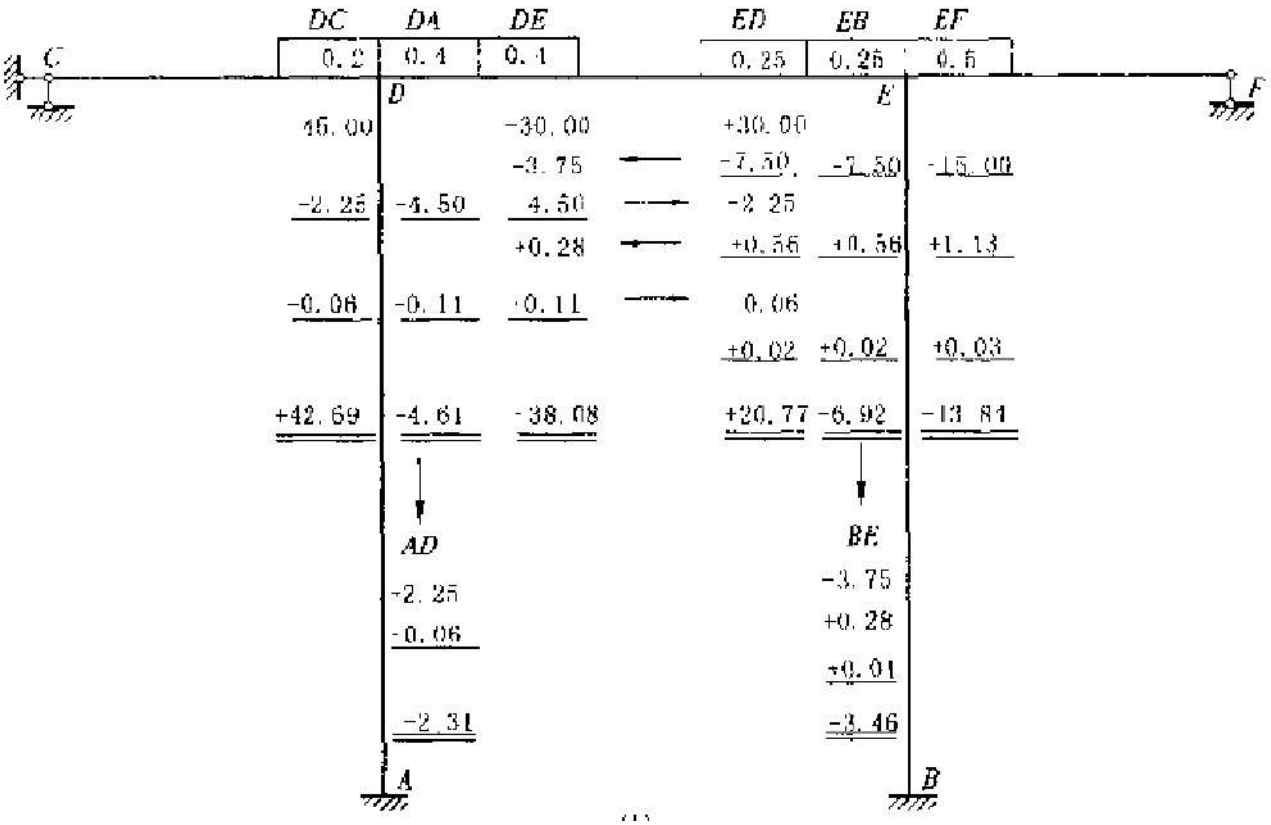
$$M_{21}^{F}=0 \quad M_{23}^{F}=0$$

$$M_{32}^{F}=0 \quad M_{3B}^{F}=-1000\text{kNm}$$

书例

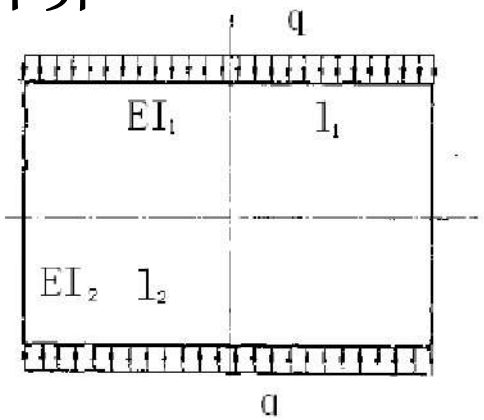


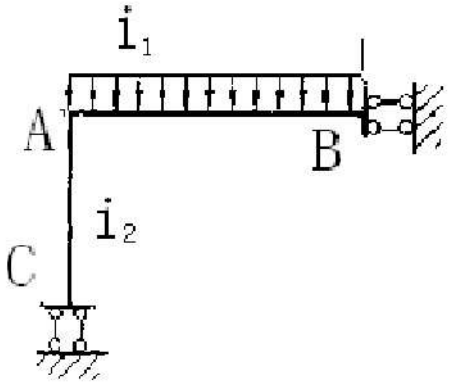
(a)


DC
DA
DE
ED
EB
EF
0.2
0.4
0.4
0.25
0.25
0.5
C
D
E
F
16.00
-30.00
+30.00
-3.75
-7.50
-7.50
-15.00
-2.25
-4.50
4.50
-2.25
+0.28
+0.56
+0.56
+1.13
-0.06
-0.11
-0.11
0.06
+0.02
+0.02
+0.03
+42.69
-4.61
-38.08
+20.77
-6.92
-13.84
AD
+2.25
-0.06
-2.31
BE
-3.75
+0.28
+0.01
-3.46
A
B

§8-3 对称结构的计算

求图示地下工程中的矩形衬砌在上部土压力作用下的弯矩图。假设底面反力均匀。

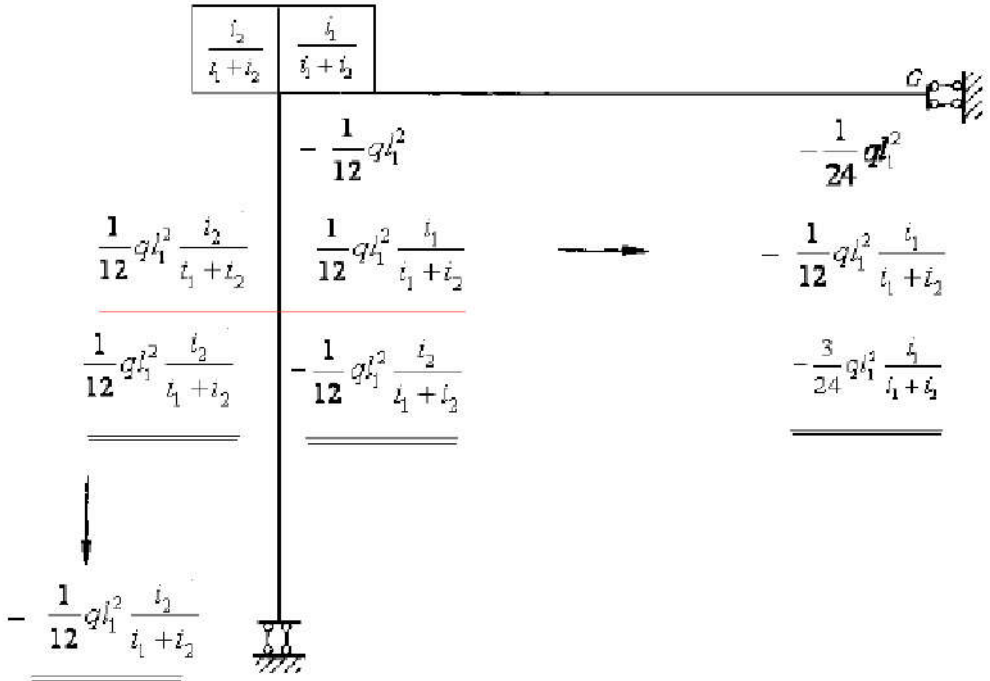


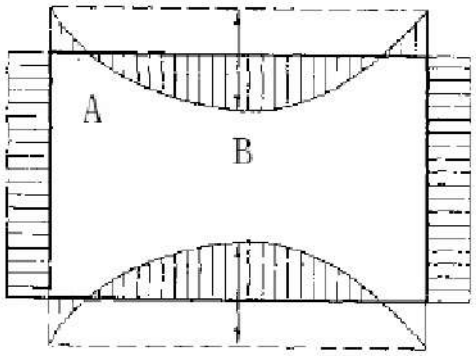


$$M_{AB}^{F}=-\frac{1}{3}q(l_1/2)^2=-\frac{1}{12}ql_1^2$$

$$M_{BA}^{F}=-\frac{1}{6}q(l_1/2)^2=-\frac{1}{24}ql_1^2$$

$$\mu_{AB}=\frac{i_1}{i_1+i_2}\qquad \mu_{AC}=\frac{i_2}{i_1+i_2}$$


$\frac{i_2}{i_1+i_2}$
$\frac{i_1}{i_1+i_2}$
C
$-\frac{1}{12}ql_1^2$
$-\frac{1}{24}ql_1^2$
$\frac{1}{12}ql_1^2\frac{i_2}{i_1+i_2}$
$\frac{1}{12}ql_1^2\frac{i_1}{i_1+i_2}$
$-\frac{1}{12}ql_1^2\frac{i_1}{i_1+i_2}$
$\frac{1}{12}ql_1^2\frac{i_2}{i_1+i_2}$
$-\frac{1}{12}ql_1^2\frac{i_2}{i_1+i_2}$
$-\frac{3}{24}ql_1^2\frac{i_1}{i_1+i_2}$
$-\frac{1}{12}ql_1^2\frac{i_2}{i_1+i_2}$



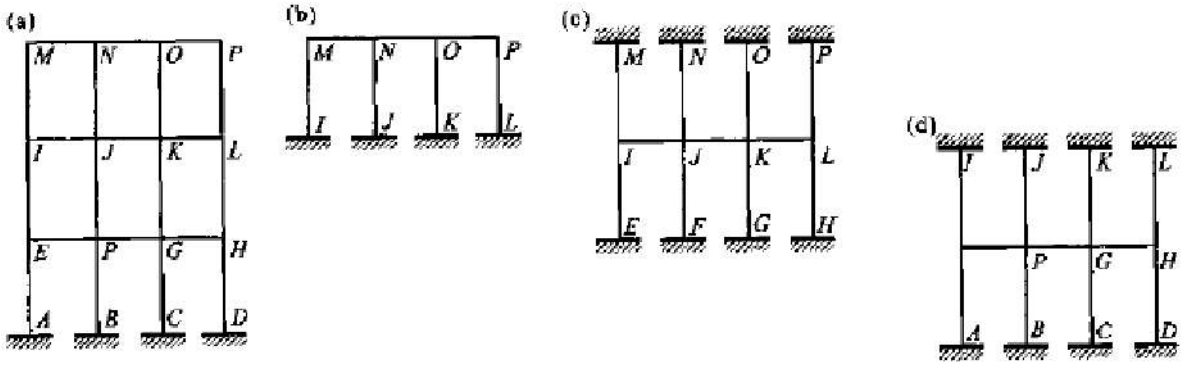
$$M_{AB} = -\frac{1}{12} q l_1^2 \frac{i_2}{i_1 + i_2}$$

$$i_2 \to 0 \qquad M_{AB} \to 0$$

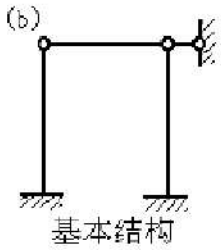
$$i_2 \to \infty \qquad M_{AB} = -\frac{1}{12} q l_1^2$$

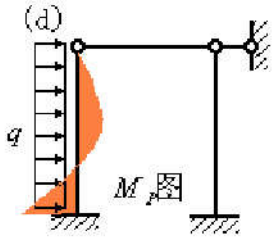
§8-6 近似法

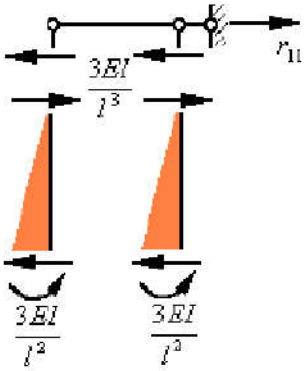
1.分层法（竖向荷载）

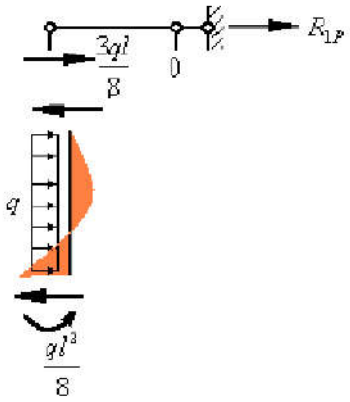


2. 剪力分配法

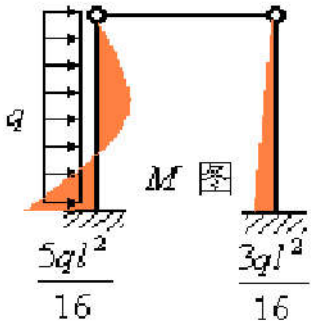






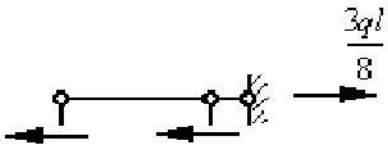


$$M = \overline{M}_1 Z_1 + M_P$$



产生单位侧移时的杆端剪力：抗侧移刚度**D(**抗剪刚度）

$$r_{11}Z_1 + R_{1P} = 0 \qquad r_{11} = D_{CA} + D_{DB} \qquad Z_1 = \frac{-R_{1P}}{D_{CA} + D_{DB}} = \frac{ql^4}{16EI}$$

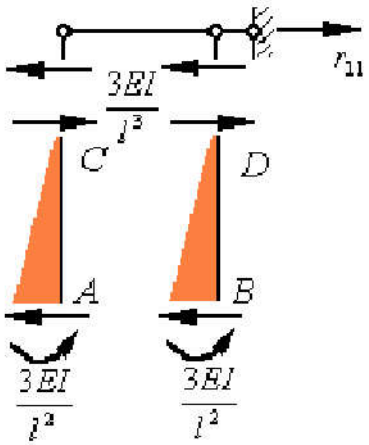


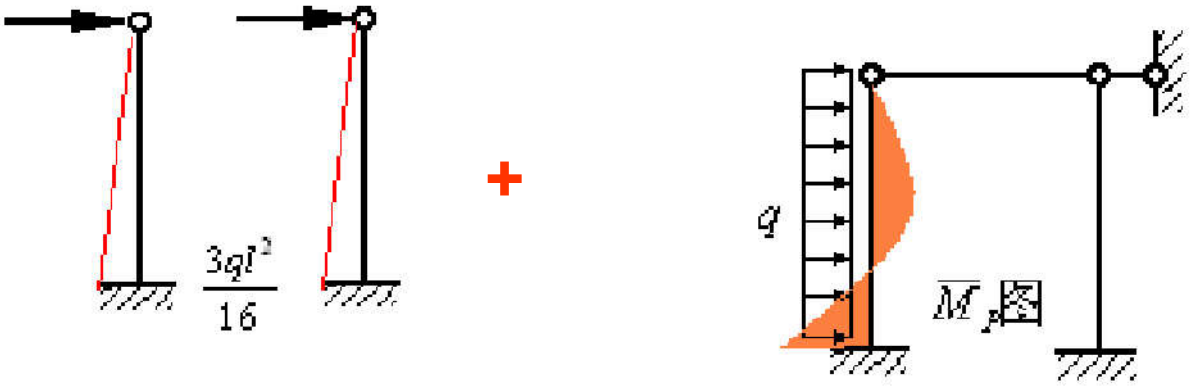
$$R_{1P} = -\frac{3ql}{8}$$

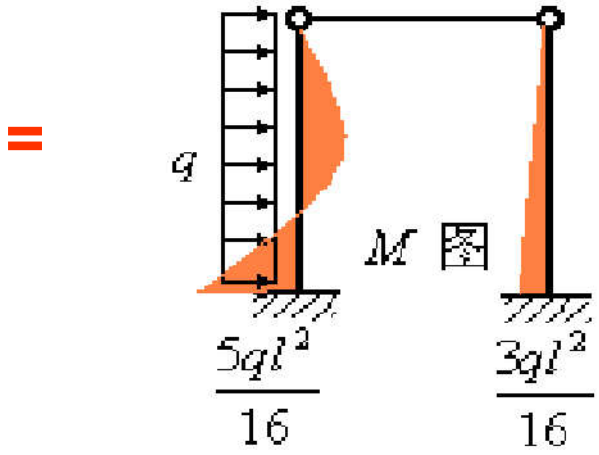
$$Q_{CA} = \frac{3EI}{l^3} \times Z_1 = D_{CA} \times \frac{-R_{1P}}{D_{CA} + D_{DB}}$$

$$= \frac{D_{CA}}{D_{CA} + D_{DB}} \times (-R_{1P})$$

$$= \mu_{CA} \times (-R_{1P}) = \frac{3ql}{16}$$





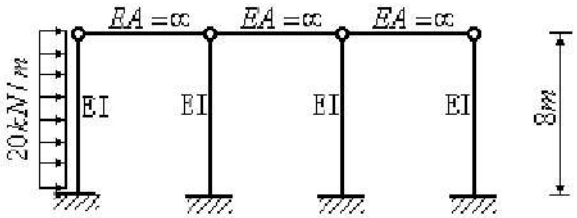


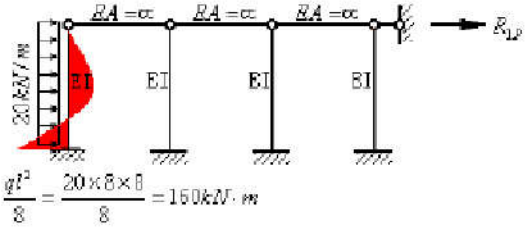
反弯点法

剪力分配法

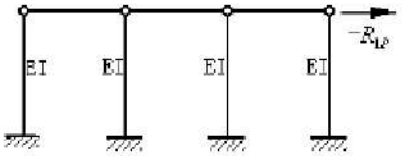
扩展

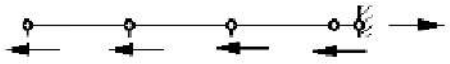
$R_{1P} = -60kN$



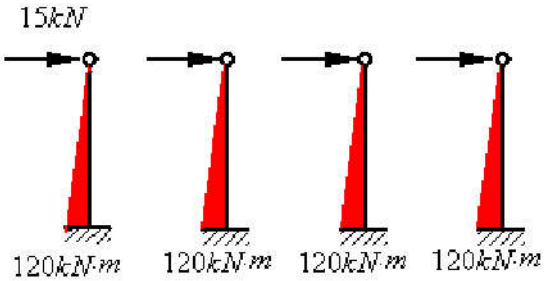


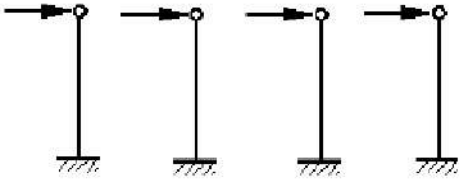
$\frac{ql^2}{8} = \frac{20\times8\times8}{8} = 160kN\cdot m$

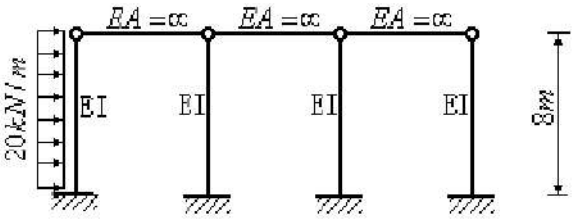


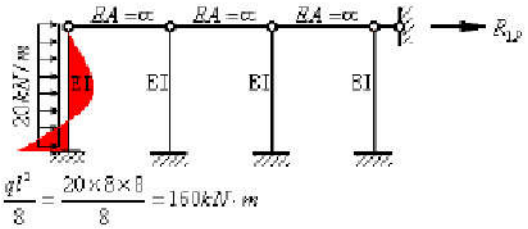


$\frac{3ql}{8}\times\frac{1}{4} = \frac{3\times20\times8}{8}\times\frac{1}{4} = 15kN$



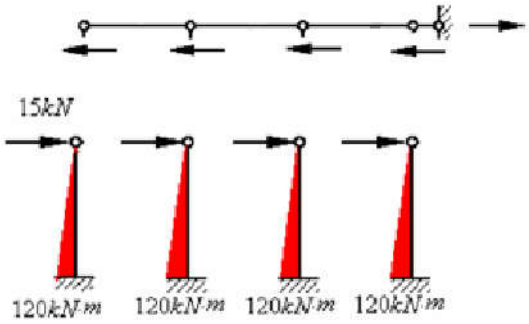




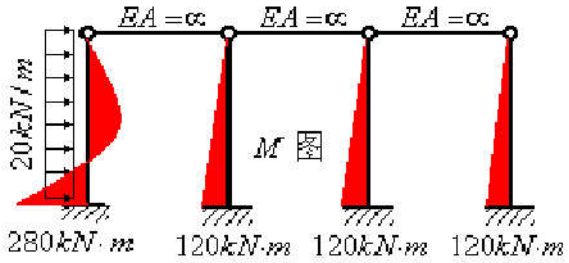


$$\frac{ql^2}{8}=\frac{20\times8\times8}{8}=160kN\cdot m$$

+

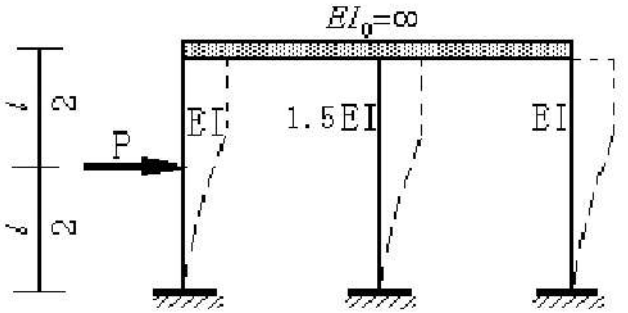


=

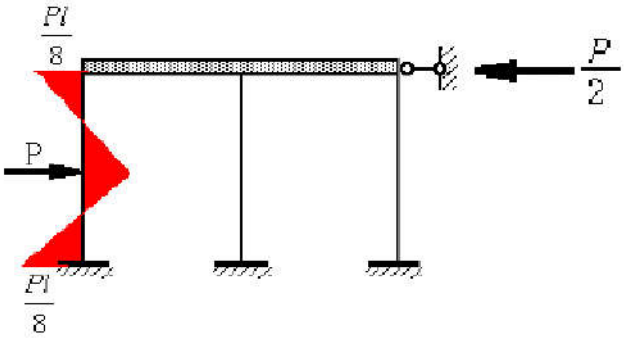


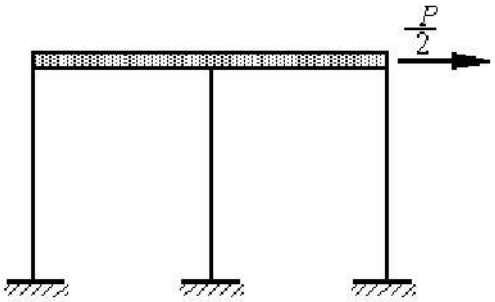
例　用剪力分配法计算横梁刚度无穷大的刚架，绘弯矩图。

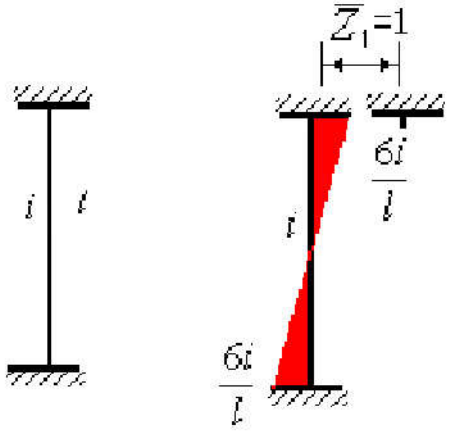
E=常数。

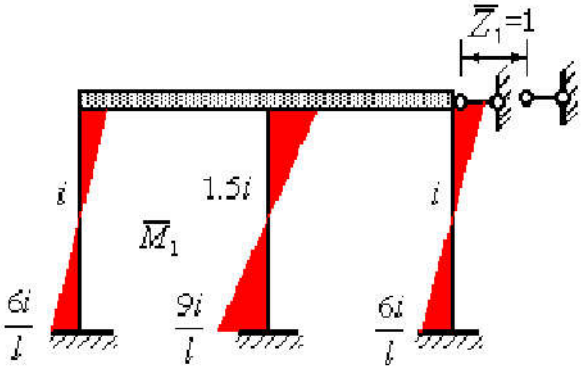


原结构



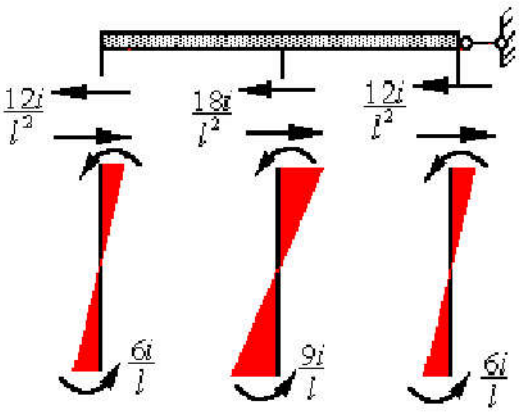






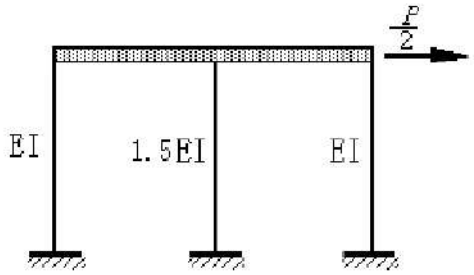
$$D=\frac{12EI}{l^3}$$

$$\sum D_i=\sum \frac{12EI_i}{l^3}$$



$$\sum D_i = \sum \frac{12EI_i}{l^3}$$

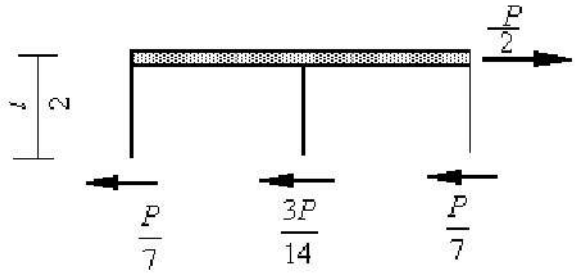
$$\mu_i = \frac{D_i}{\sum D_i} = \frac{I_i}{\sum I_i} = \frac{i_i}{\sum i_i}$$

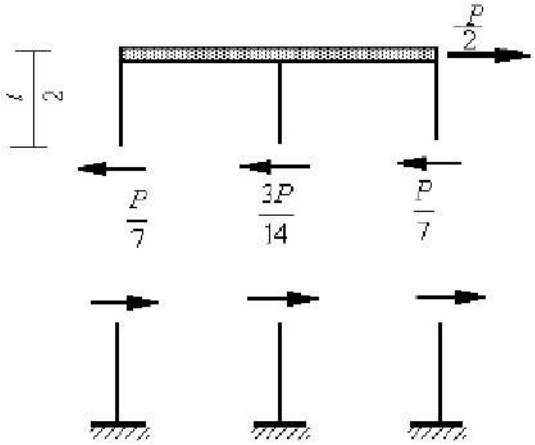


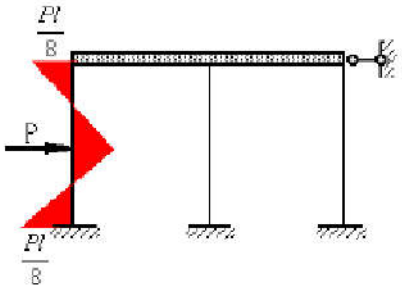
$$1+1+1.5=3.5$$

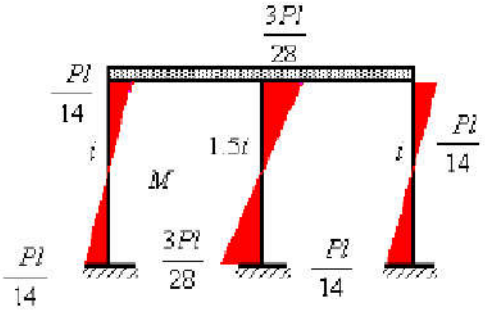
$$Q_1 = \mu_1 \times \frac{P}{2} = \frac{1}{3.5} \times \frac{P}{2} = \frac{P}{7} = Q_3$$

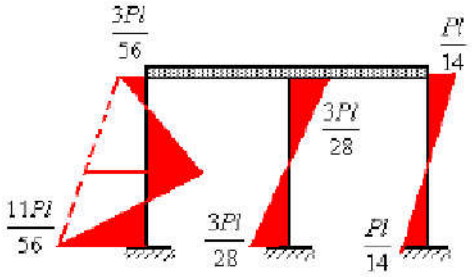
$$Q_2 = \frac{1.5}{3.5} \times \frac{P}{2} = \frac{3P}{14}$$

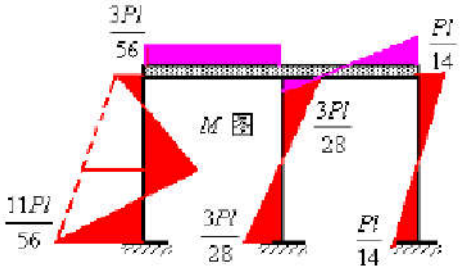



P/2
2
P/7
3P/14
P/7


Pl/8
P
Pl/8

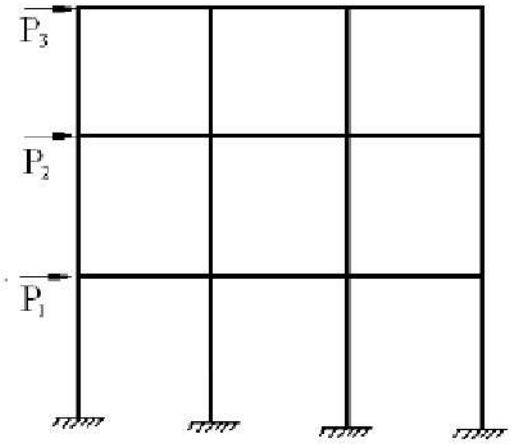

3Pl/28
Pl/14
i
1.5i
i
M
Pl/14
3Pl/28
Pl/14
Pl/14

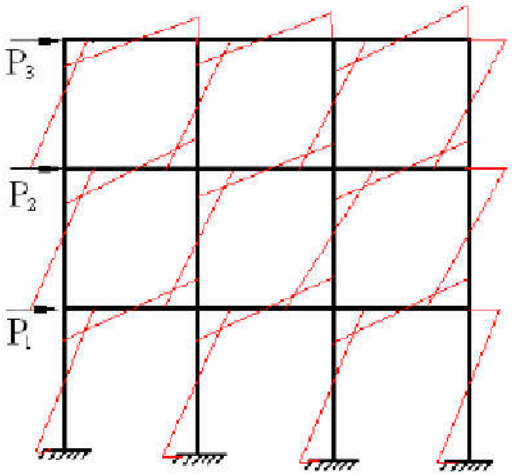

3Pl/56
Pl/14
3Pl/28
11Pl/56
3Pl/28
Pl/14

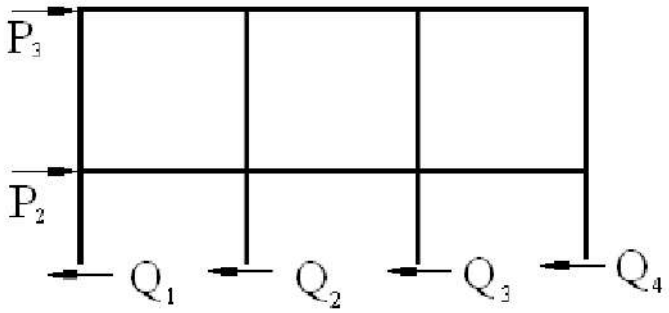

3Pl/56
Pl/14
M
3Pl/28
11Pl/56
3Pl/28
Pl/14

3 反弯点法（水平力）P

图示多层多跨刚架，设横梁的线刚度EI=∞，在水平结点荷载作用下，各结点只有水平线位移而无转角。刚架各杆的弯矩图全部为直线，各柱有一弯矩为零的点，称为反弯点，均在柱高的中点。如果能计算出反弯点处的剪力值，则各柱端弯矩即可求出，进而可算出梁端弯矩，这就是反弯点法的思路。







$$Q_i = \frac{12EI_i}{h_i^3}\Delta_j = D_i\Delta_j$$

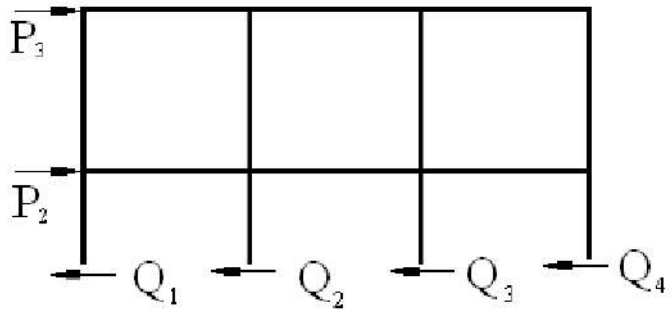
$$D_i = \frac{12EI_i}{h_i^3}$$

$$Q_1 + Q_2 + Q_3 + Q_4 = P_2 + P_3$$

$$(D_1 + D_2 + D_3 + D_4)\Delta_j = P_2 + P_3$$

$$\Delta_j \sum D_i = \sum P_j \qquad \Delta_j = \frac{\sum P_j}{\sum D_i}$$

$$Q_i == D_i \frac{\sum P_j}{\sum D_i} = \frac{D_i}{\sum D_i} \sum P_j = \mu_i \sum P_j$$



$$Q_i == D_i \frac{\sum P_j}{\sum D_i} = \frac{D_i}{\sum D_i} \sum P_j = \mu_i \sum P_j$$

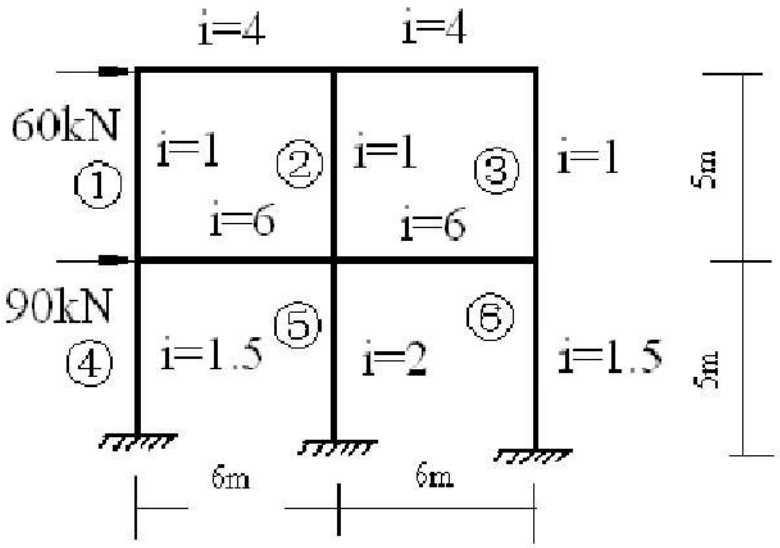
$$\mu_i = \frac{D_i}{\sum D_i} = \frac{I_i}{\sum I_i} = \frac{i_i}{\sum i_i}$$

剪力分配系数

柱端弯矩 $M_i = Q_i \times \frac{h_i}{2}$

梁端弯矩由结点平衡条件确定，按梁的刚度分配柱端弯短求得。

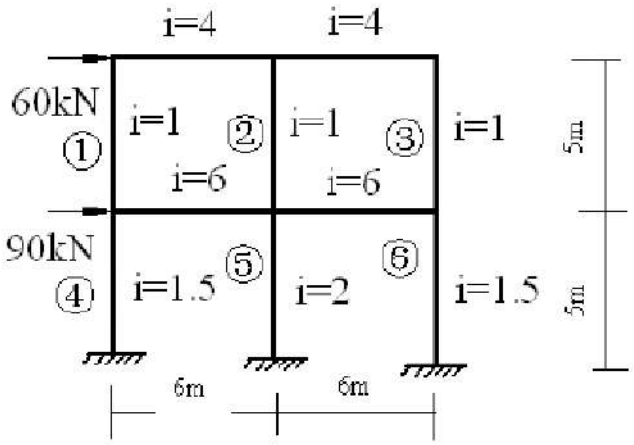
例 用反弯点法计算图示刚架，并绘弯短图。



解　**(1)** 计算剪力分配系数

对于同层柱等高，剪力分配系数可简化为按各柱的线刚度进行分配，即

$$\mu_i = \frac{i_i}{\sum i_i}$$



顶层：

$$\mu_1 = \frac{i_1}{\sum i_i} = \frac{1}{3} = \mu_2 = \mu_3$$

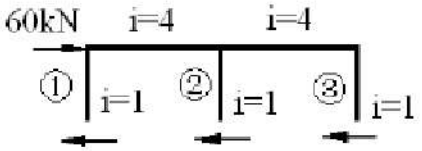
底层：

$$\mu_5 = \frac{i_5}{\sum i_i} = \frac{2}{1.5+2+1.5} = 0.4$$

$$\mu_4 = \frac{i_4}{\sum i_i} = \frac{1.5}{1.5+2+1.5} = 0.3 = \mu_6$$

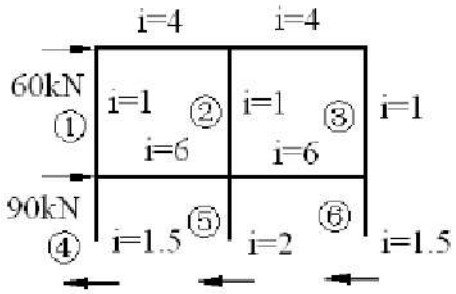
(2)计算各柱剪力

顶层：$Q_1 = \mu_1 \sum P_j = \frac{1}{3} \times 60 = 20kN = Q_2 = Q_3$



底层：$Q_4 = \mu_4 \sum P_j = 0.3 \times (60 + 90) = 45kN = Q_6$

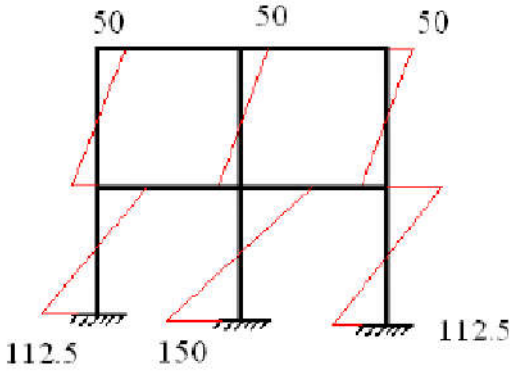
$Q_5 = \mu_5 \sum P_j = 0.4 \times (60 + 90) = 60kN$

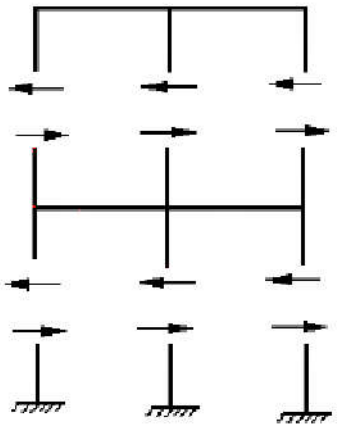


(3)计算柱端弯矩

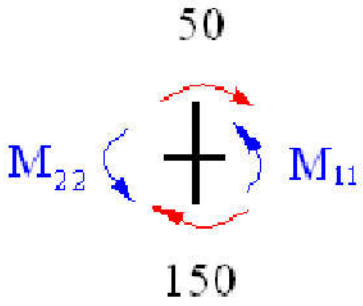
$$M_1 = 20 \times \frac{5}{2} = 50kNm = M_2 = M_3$$

$$M_4 = 45 \times \frac{5}{2} = 112.5kNm = M_6 \qquad M_5 = 60 \times \frac{5}{2} = 150kNm$$

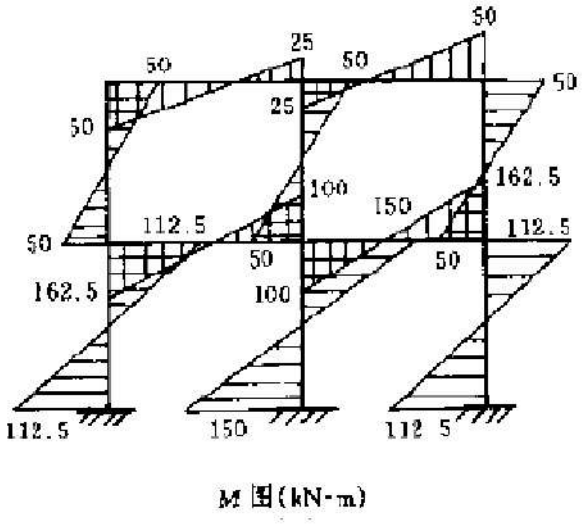




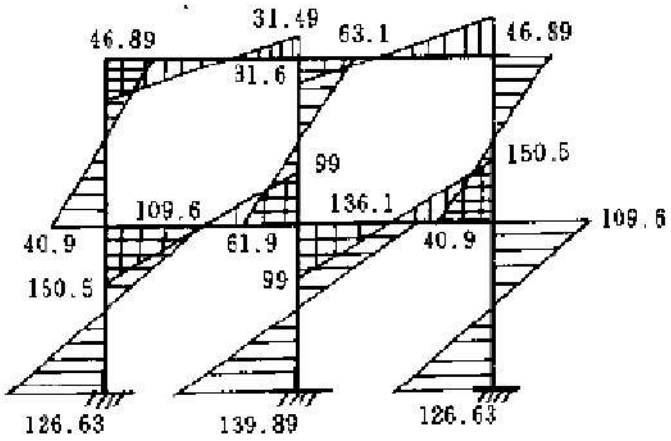
(4)计算梁端弯矩



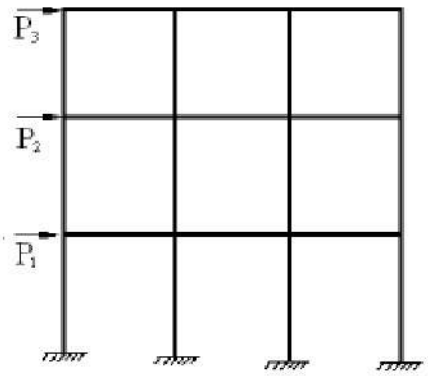
$$M_{11}=\frac{6}{6+6}\times(50+150)=100kNm=M_{22}$$



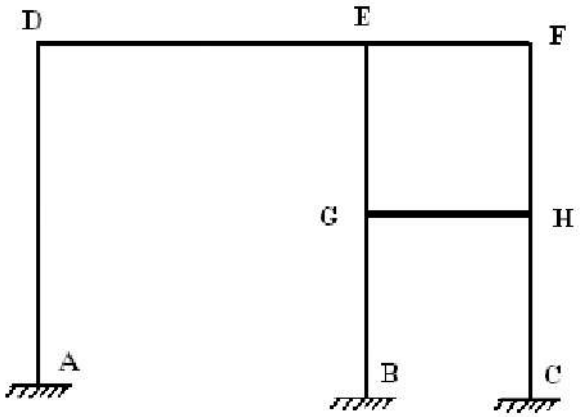
M 图(kN-m)



位移法解



简式框架



复式框架