第7章 位移法
(Displacement Method)

§7-1 位移法的基本概念

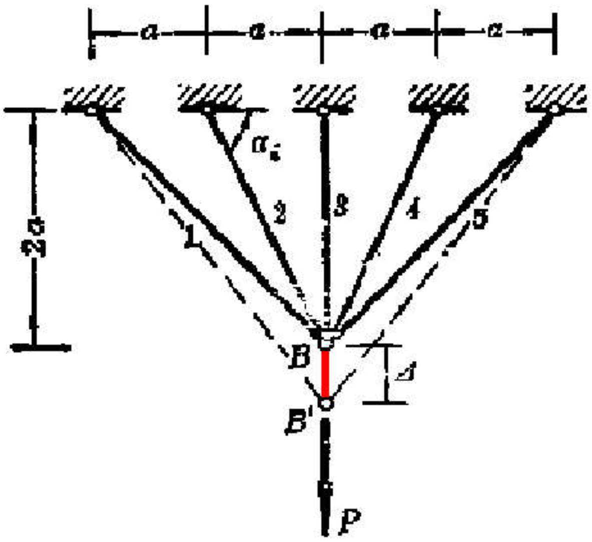
（1）体系的几何组成分析；

（2）静定结构的内力分析和位移计算；

（3）超静定结构的内力分析和位移计算

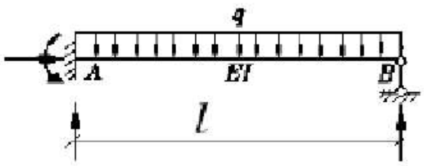
—— 力法。

1、位移法简例

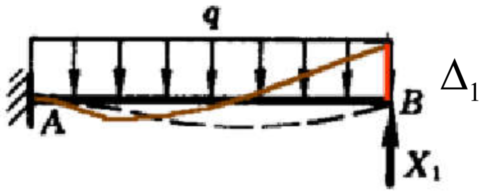


位移法基本未知量----结点位移.

回顾力法的思路：



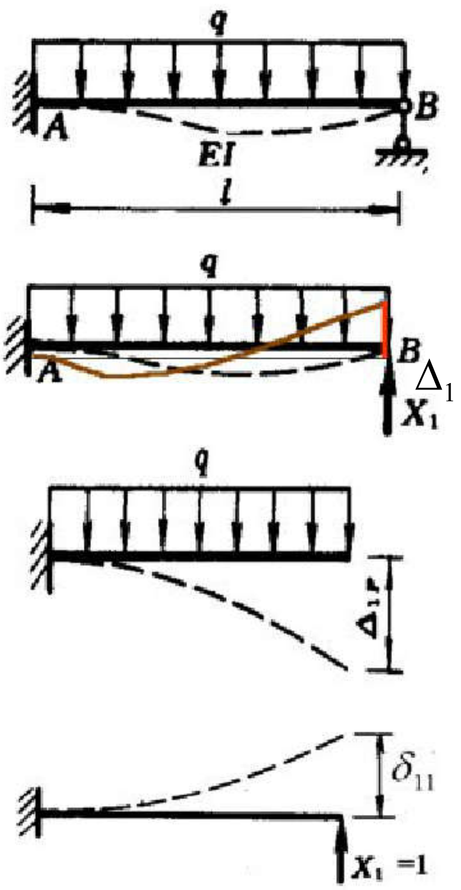
基本结构



基本体系

X_1 ⇦ 力法基本未知量

$\Delta_1 = 0$ ⇦ 变形协调条件

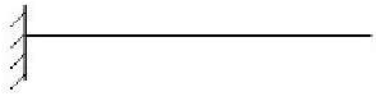


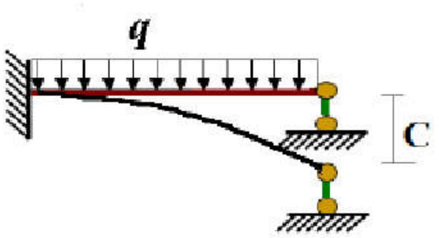
$$\Delta_1 = \Delta_{1X_1} + \Delta_{1P} = 0$$

$$\delta_{11} \cdot \boldsymbol{X}_1 + \Delta_{1\boldsymbol{P}} = 0$$

力法典型方程

核心是化未知为已知





$$\delta_{11}X_1 + \Delta_{1P} = -C$$

$$\delta_{11} = \frac{l^3}{3EI}$$

$$\Delta_{1P} = -\frac{ql^4}{8EI}$$

$$X_1 = -3EIC / l^3 + \frac{3ql}{8}$$

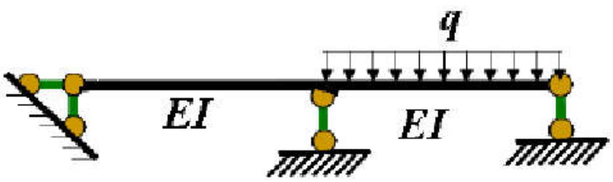
力法求解荷载、温度改变和支座移动作用下的单跨超静定梁。

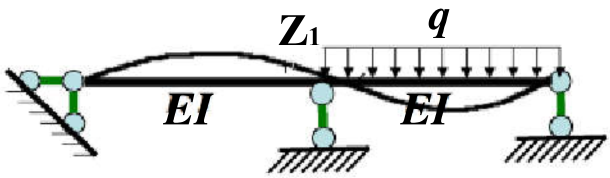
2、位移法的思路

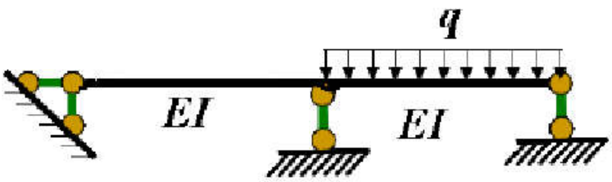
基本假设：

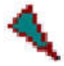
(1)结点位移都很小；

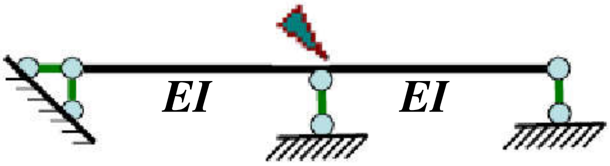
(2)忽略受弯杆的轴向变形（杆长不变）。



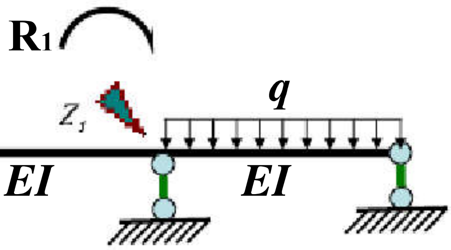




 ----刚臂,限制转动的约束

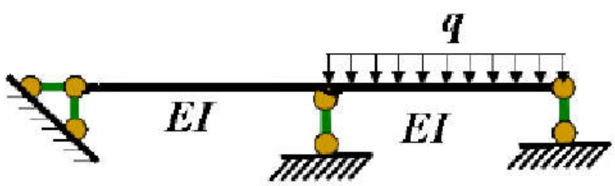


基本结构

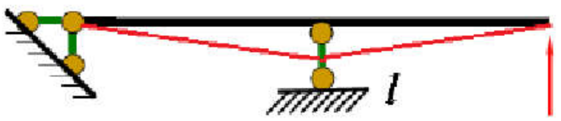


基本体系

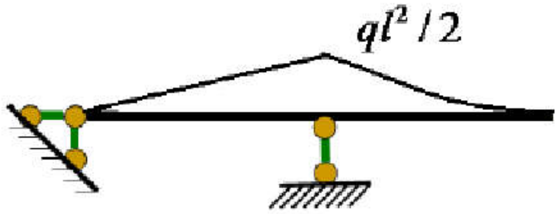
$R_1=0$



$$\delta_{11}X_1 + \Delta_{1P} = 0$$

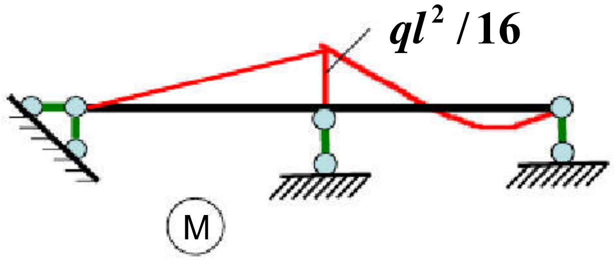


$$\delta_{11} = \frac{2l^3}{3EI}$$



$$\Delta_{1P} = -\frac{7ql^4}{24EI}$$

$$X_1 = \frac{7}{16}ql$$

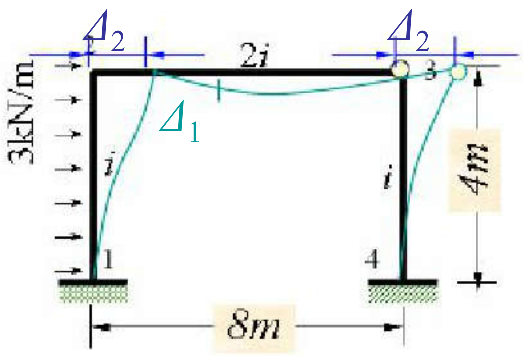


3、位移法要点

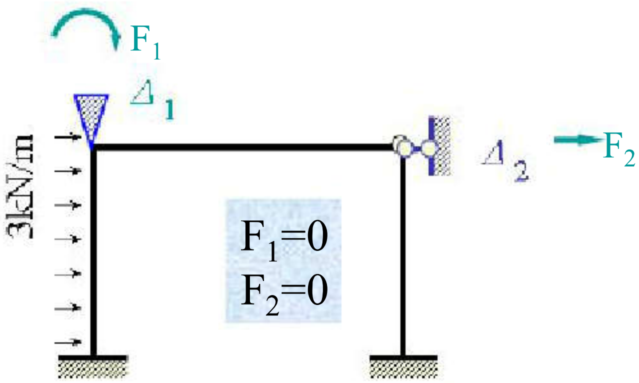
位移法基本未知量----结点位移.

位移法的基本结构----单跨梁.

位移法的基本方程----平衡方程.



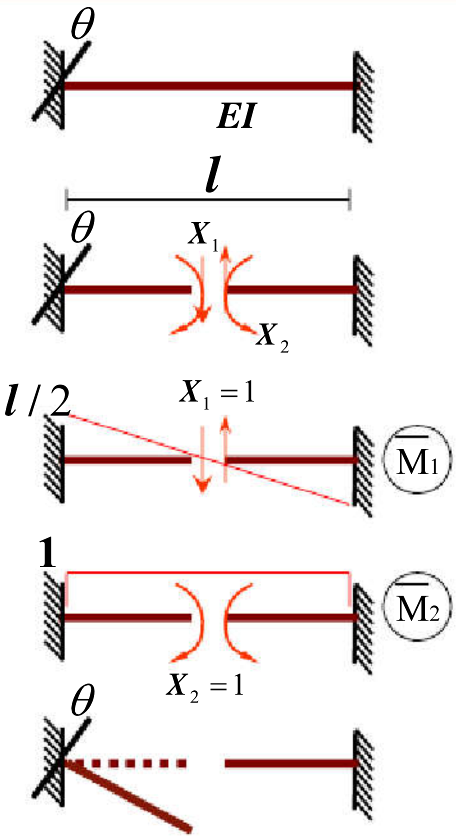
Δ2
Δ2
2i
3kN/m
Δ1
i
i
3
4m
1
4
8m



F1
Δ1
3kN/m
Δ2
F2
F1=0
F2=0

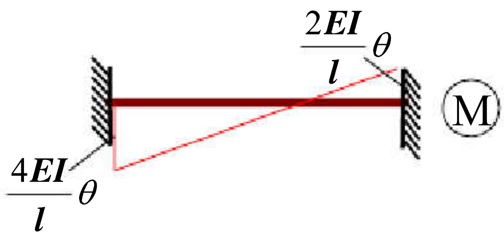
§ 7-2 等截面杆件的刚度方程

求作图示梁由于支座移动引起的M图。



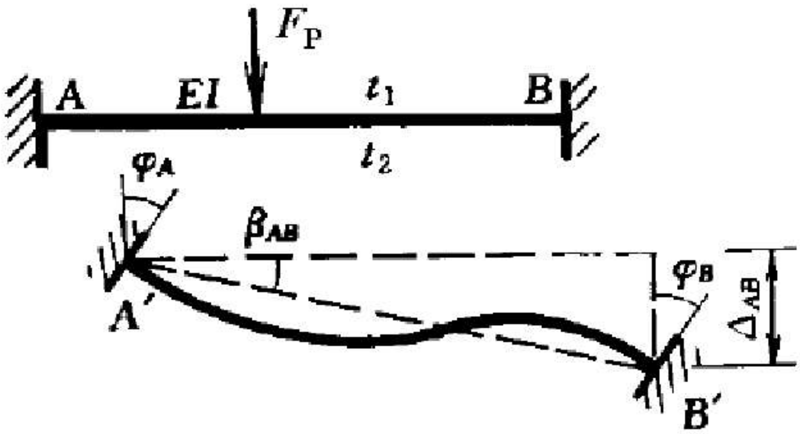
解：

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1C} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \Delta_{2C} = 0 \end{cases}$$



$\frac{4EI}{l}$ $\frac{2EI}{l}$ 形常数

转角位移方程(刚度方程)



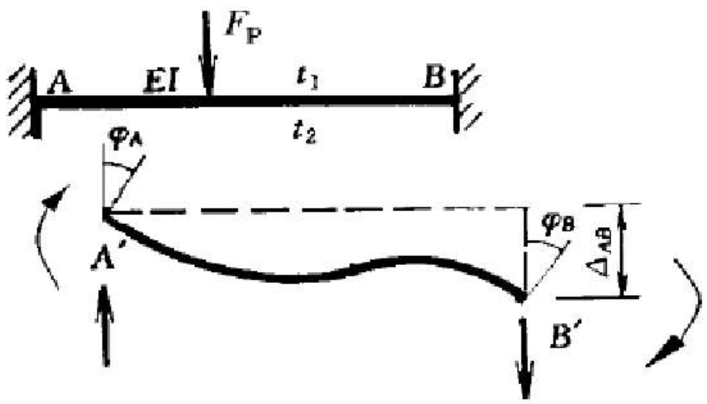
在线性小变形条件下，由叠加原理求解

基本假设：
(1)结点位移都很小；
(2)忽略受弯杆的轴向变形（杆长不变）。

正负号规定：

(1)杆端转角φ_A、φ_B ，侧移Δ_{AB}都以顺时针为正；

(2)杆端弯矩、剪力以顺时针为正。



两端固定

$$\begin{cases} M_{AB} = 4i\varphi_A + 2i\varphi_B - \dfrac{6i}{l}\Delta_{AB} + M_{AB}^F \\ M_{BA} = 4i\varphi_B + 2i\varphi_A - \dfrac{6i}{l}\Delta_{AB} + M_{BA}^F \end{cases}$$

转角位移方程(刚度方程) P279

Slope-Deflection (Stiffness) Equation

其中： $i = \dfrac{EI}{l}$ 杆件的线刚度。

M_{AB}^F，M_{BA}^F 为由荷载和温度变化引起的杆端弯矩，称为固端弯矩。

$$\begin{cases} M_{AB} = 4i\varphi_A + 2i\varphi_B - \dfrac{6i}{l}\Delta_{AB} + M_{AB}^F \\ M_{BA} = 4i\varphi_B + 2i\varphi_A - \dfrac{6i}{l}\Delta_{AB} + M_{BA}^F \end{cases}$$

$$M_{AB} = 3i\varphi_A - \frac{3i}{l}\Delta_{AB} + M_{AB}^F$$

$$M_{AB} = i\varphi_A + M_{AB}^F$$
$$M_{BA} = -i\varphi_A + M_{BA}^F$$

§ 7-3 无侧移刚架

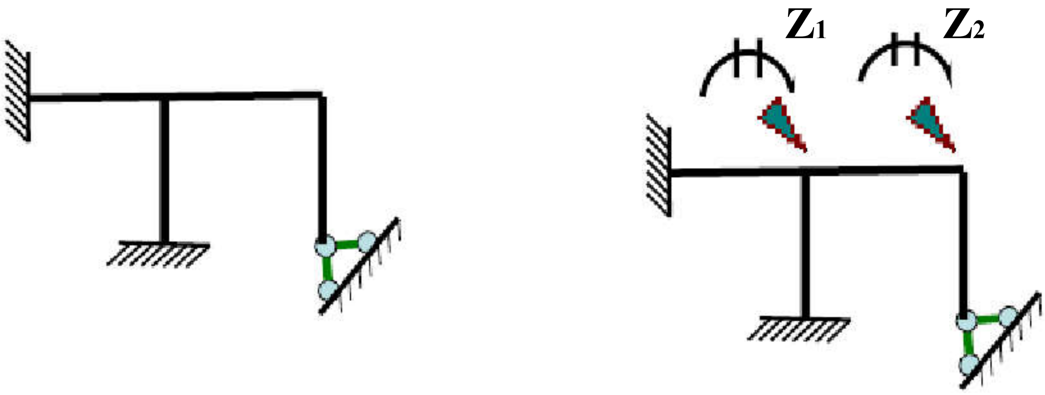
基本未知量:独立的 结点位移。
包括刚结点角位移和独立的线位移。

基本结构:增加附加约束后,使得原结构的结点不能发生位移的结构。

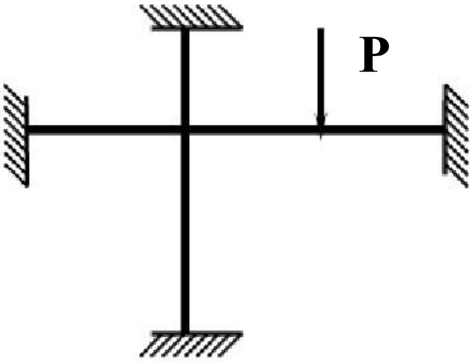
基本体系:有荷载、结点位移作用的基本结构。

1、基本未知量:独立的 结点位移.包括角位移和线位移

1.无侧移结构(刚架与梁不计轴向变形)



基本未知量为所有刚结点的转角
基本结构为在所有刚结点上加刚臂后的结构

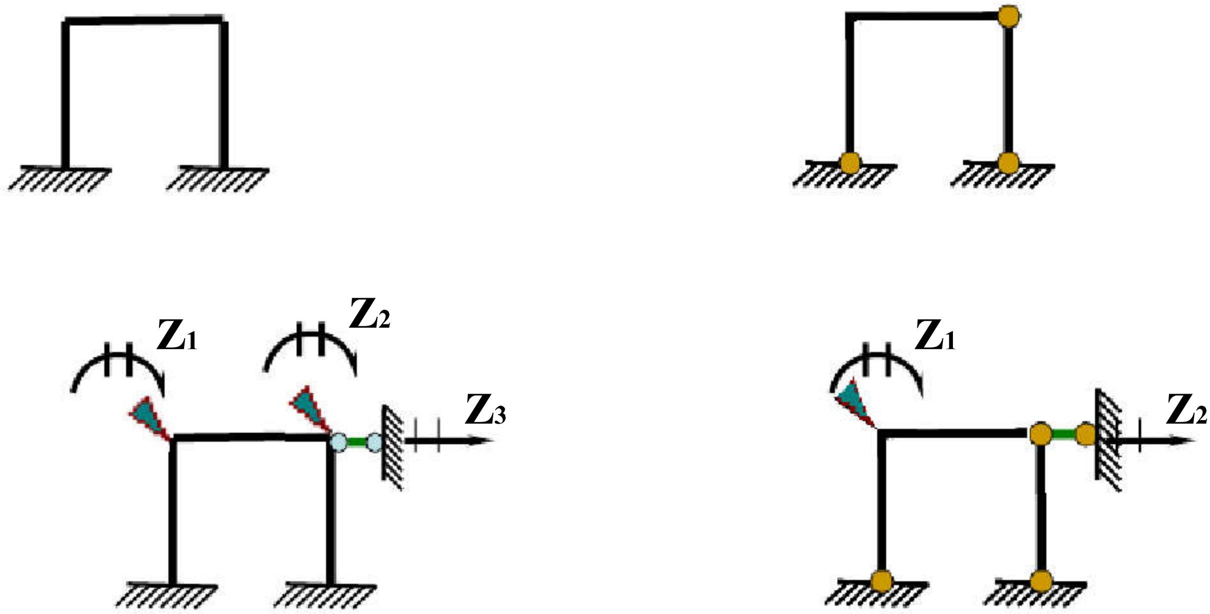


力法计算几个基本未知量 **9个**

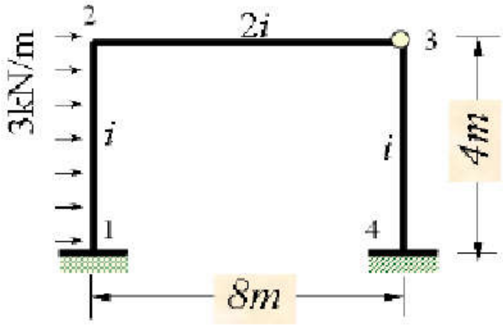
位移法计算 **1个**

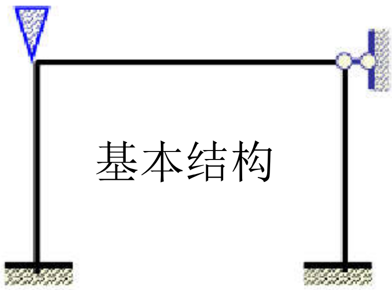
刚结点角位移

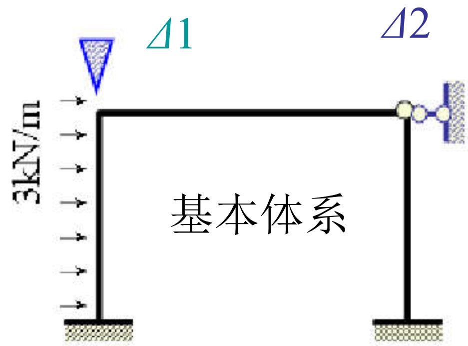
2.有侧移结构(刚架与梁不计轴向变形)



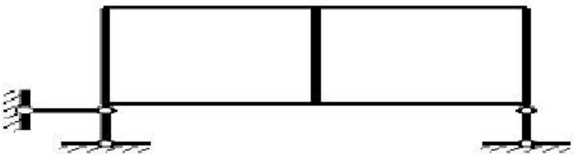
7.3.2基本体系



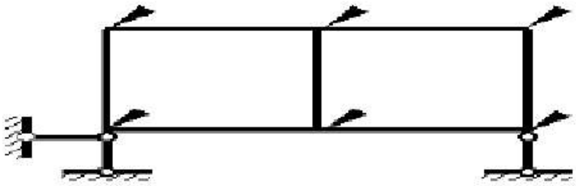




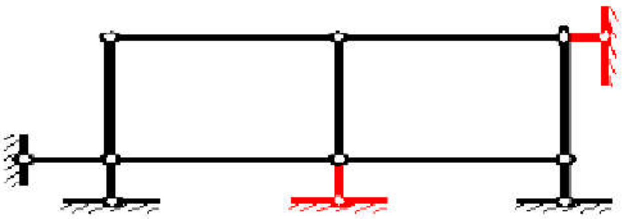
未知数确定举例

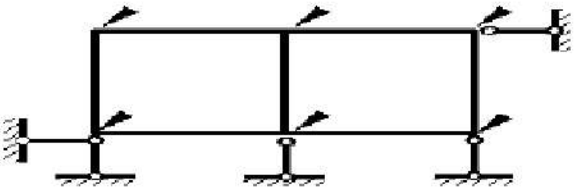


待分析结构

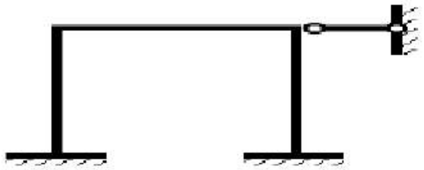


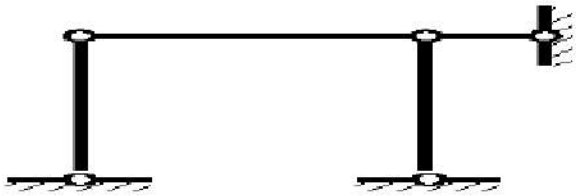
六个转角未知量



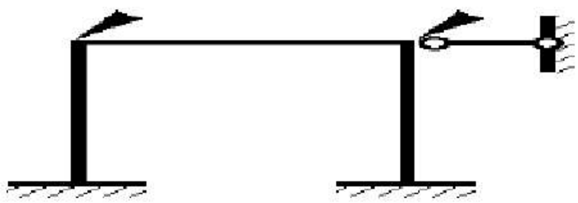


$n=8$

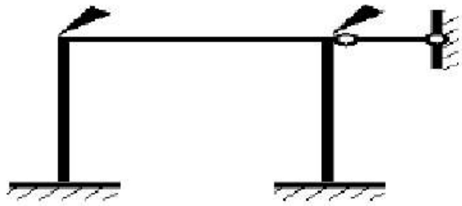




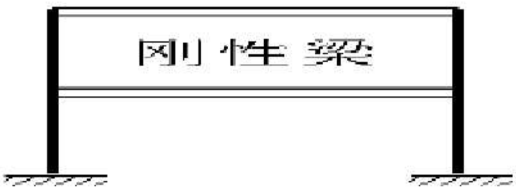
线 位 移 数0



转 角 数2

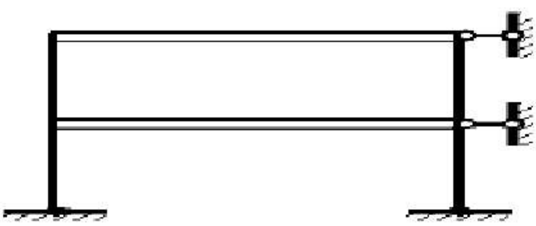


$n=2$



待分析结构

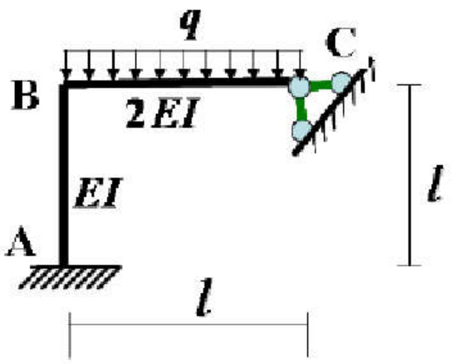
不计轴向变形时，虽有刚结点,但横梁不能转动,因此转角未知量为零。

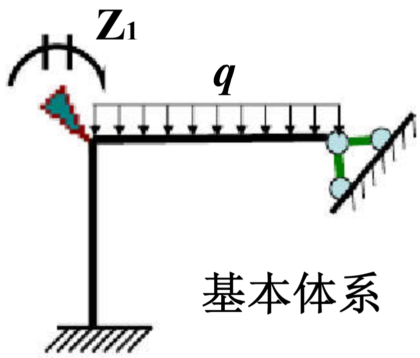


线位移数2

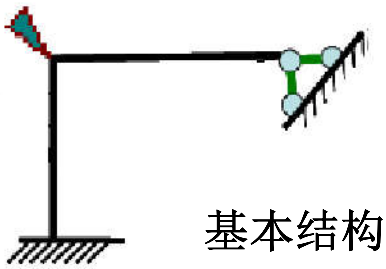
7.3.3计算示例

作图示刚架弯矩图



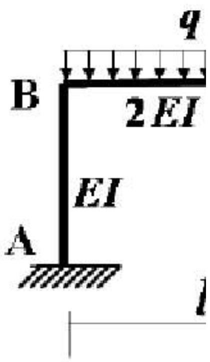


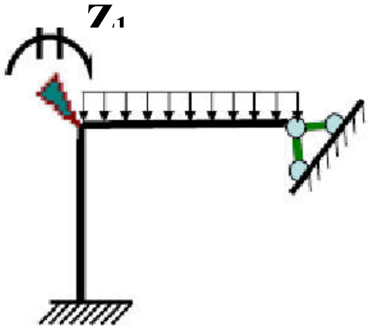
基本体系

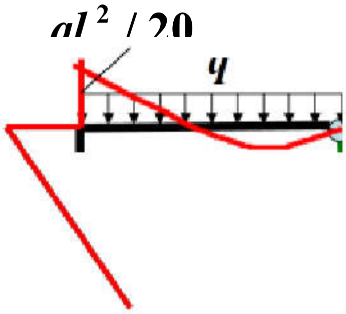


基本结构

例：作图示刚架弯矩图





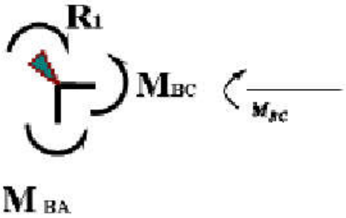


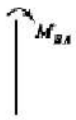
解：M_{AB} —

$M_{BA} = 4\times$ —

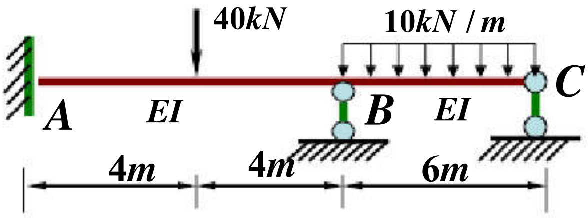
$M_{BC} = 3\times$ —— −

$R_1 = 0$

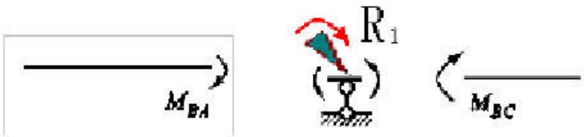


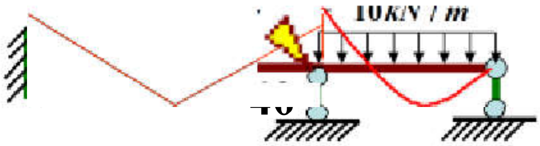


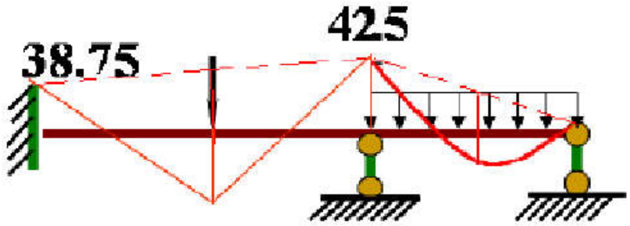
例.计算图示梁,作弯矩图



解：$M \quad = 2\times\frac{EI}{\quad}\times Z - \frac{1}{\quad}\times 40\times 8$

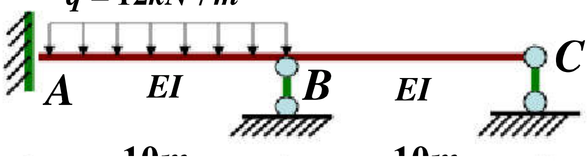






M 图kNm

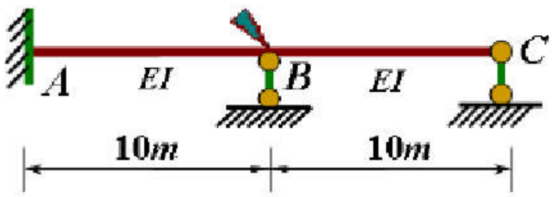
例.计算图示梁,作弯矩图

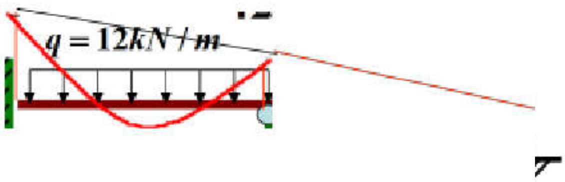


解

$M \quad = 2\times \frac{EI}{} \times Z \quad \frac{1}{} \times 12 \times 10^2$

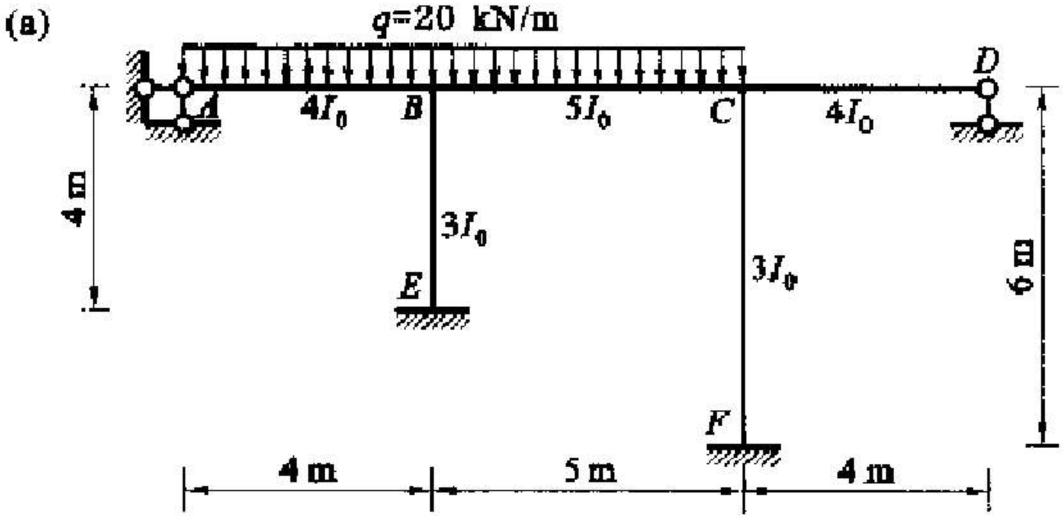
M



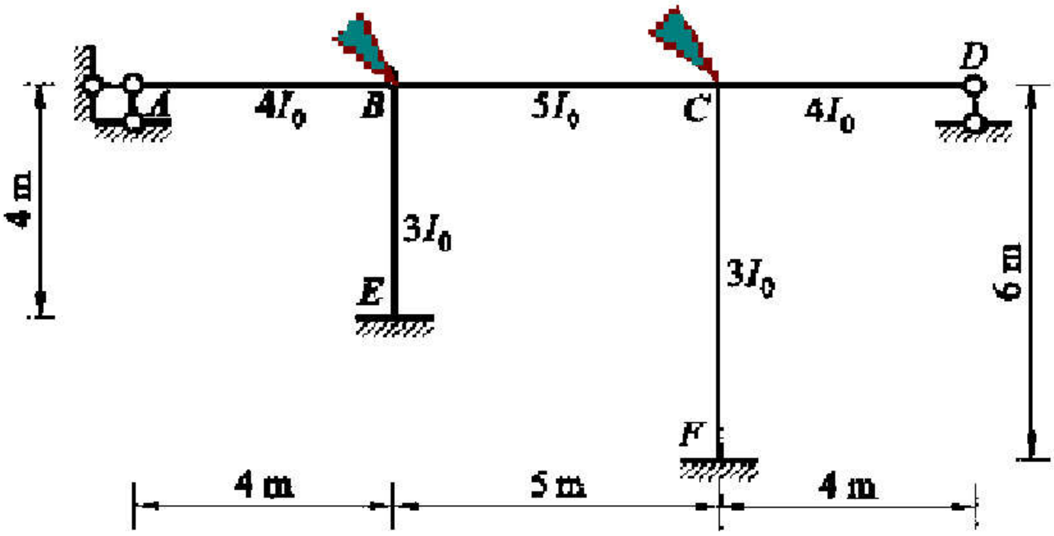


M 图KNm

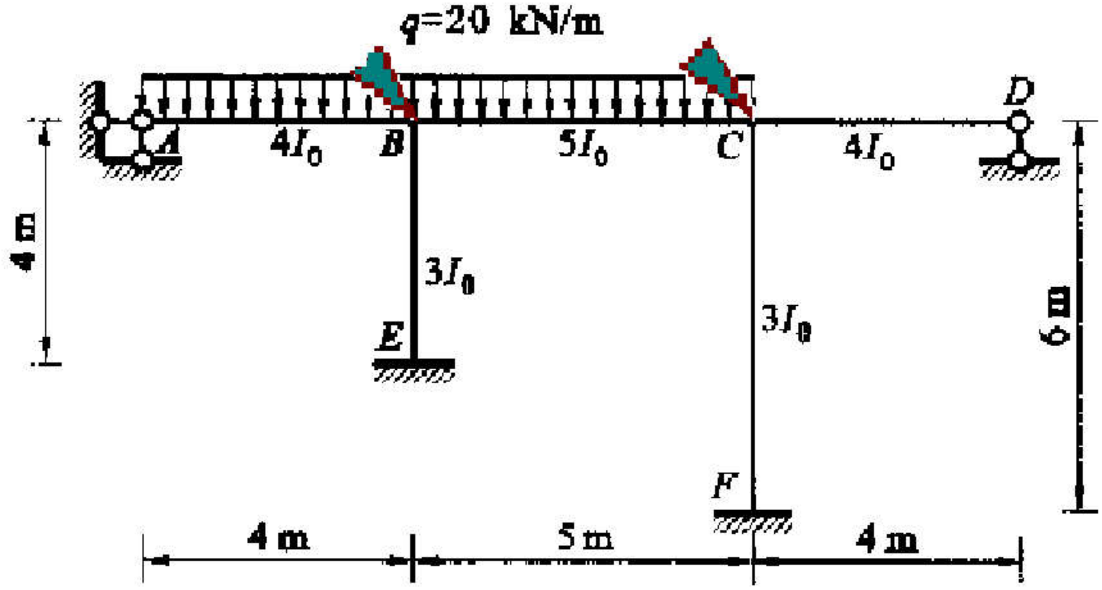
书例：作图示刚架弯矩图



基本结构



基本体系



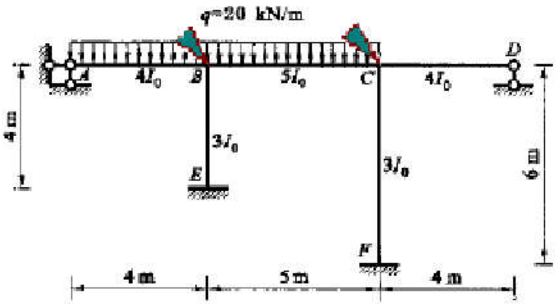
$$M_{BA}^{F} = \frac{1}{8} q l^{2} = \frac{20 \times 4^{2}}{8} = 40 kNm$$

$$M_{BC}^{F} = -\frac{1}{12} q l^{2} = -\frac{20 \times 5^{2}}{12} = -41.7 kNm = -M_{CB}^{F}$$

$$i_{BA} = \frac{4EI_0}{4} = i \qquad i_{\mathrm{BC}} = \frac{5EI_0}{5} = i$$

$$i_{\mathrm{CD}} = \frac{4EI_0}{4} = i \qquad i_{\mathrm{BE}} = \frac{3EI_0}{4} = \frac{3}{4} i$$

$$i_{\mathrm{CF}} = \frac{3EI_0}{6} = \frac{1}{2} i$$



$$M_{\mathrm{BA}} = 3 i_{BA} \theta_B + M_{BA}^{F} = 3 \mathrm{i} \theta_B + 40$$

$$M_{\mathrm{BC}} = 4 i_{BC} \theta_B + 2 i_{BC} \theta_C + M_{BC}^{F} = 4 \mathrm{i} \theta_B + 2 \mathrm{i} \theta_C - 41.7$$

$$M_{\mathrm{CB}} = 2 i_{BC} \theta_B + 4 i_{BC} \theta_C + M_{CB}^{F} = 2 \mathrm{i} \theta_B + 4 \mathrm{i} \theta_C + 41.7$$

$$M_{\mathrm{CD}} = 3 i_{CD} \theta_C = 3 \mathrm{i} \theta_C$$

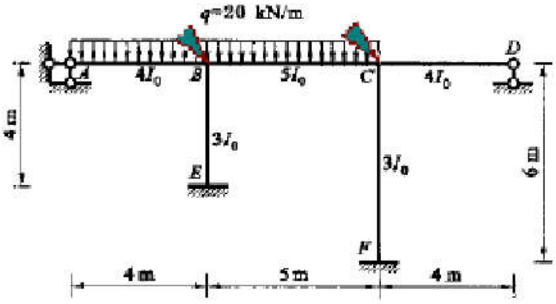
$$M_{\mathrm{BE}} = 4 i_{BE} \theta_B = 3 \mathrm{i} \theta_B \qquad M_{\mathrm{EB}} = 2 i_{BE} \theta_B = 1.5 \mathrm{i} \theta_B$$

$$M_{\mathrm{CF}} = 4 i_{\mathrm{CF}} \theta_{\mathrm{C}} = 2 \mathrm{i} \theta_{\mathrm{C}} \qquad M_{\mathrm{FC}} = 2 i_{\mathrm{CF}} \theta_{\mathrm{C}} = \mathrm{i} \theta_{\mathrm{C}}$$

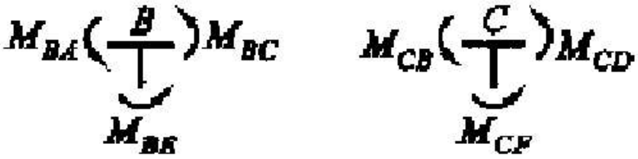
$$M_{BA} + M_{BC} + M_{BE} = 0$$
$$M_{CB} + M_{CD} + M_{CF} = 0$$

$$i\theta_B = 1.15$$
$$i\theta_C = -4.89$$

$$10i\theta_B + 2i\theta_C - 1.7 = 0$$
$$2i\theta_B + 9i\theta_C + 41.7 = 0$$



$$M_{BA} = 3i_{BA}\theta_B + M_{BA}^F = 3i\theta_B + 40$$
$$M_{BC} = 4i_{BC}\theta_B + 2i_{BC}\theta_C + M_{BC}^F = 4i\theta_B + 2i\theta_C - 41.7$$
$$M_{CB} = 2i_{BC}\theta_B + 4i_{BC}\theta_C + M_{CB}^F = 2i\theta_B + 4i\theta_C + 41.7$$
$$M_{CD} = 3i_{CD}\theta_C = 3i\theta_C$$
$$M_{BE} = 4i_{BE}\theta_B = 3i\theta_B \qquad M_{EB} = 2i_{BE}\theta_B = 1.5i\theta_B$$
$$M_{CF} = 4i_{CF}\theta_C = 2i\theta_C \qquad M_{FC} = 2i_{CF}\theta_C = i\theta_C$$



$M_{BA} = 43.5$

$M_{BC} = -46.9$

$M_{CB} = 24.5$

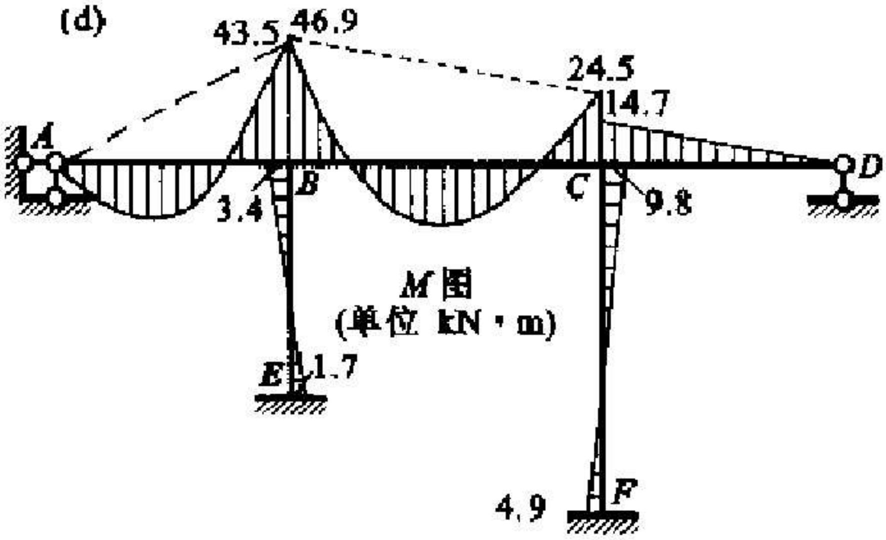
$M_{CD} = -14.7$

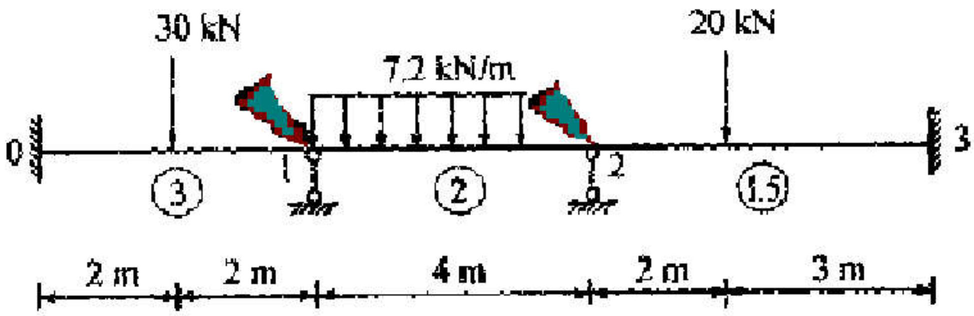
$M_{BE} = 3.4$

$M_{EB} = 1.7$

$M_{CF} = -9.8$

$M_{FC} = -4.9$



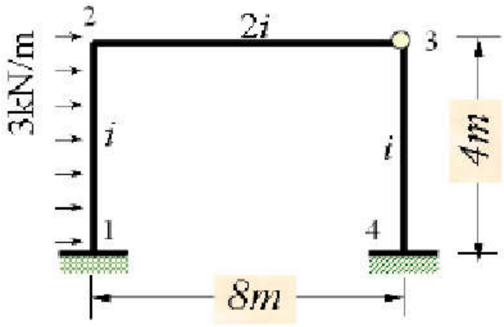


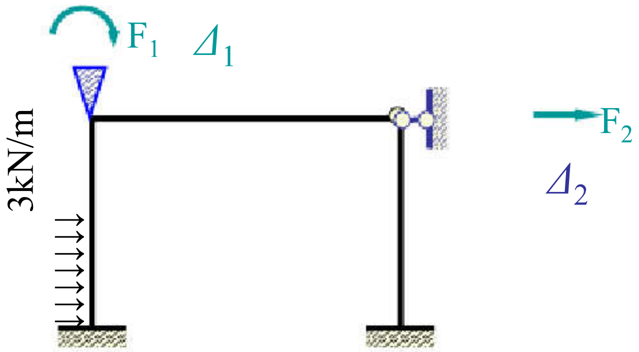
$$M_{01}=2\times 3\theta_1-\frac{1}{8}\times 4\times 30 \qquad M_{10}=4\times 3\theta_1+\frac{1}{8}\times 4\times 30$$

$$M_{12}=4\times 2\theta_1+2\times 2\theta_2-\frac{1}{12}\times 7.2\times 4^2 \qquad M_{21}=2\times 2\theta_1+4\times 2\theta_2+\frac{1}{12}\times 7.2\times 4^2$$

$$M_{23}=4\times 1.5\theta_2-\frac{20\times 2\times 3^2}{5^2} \qquad M_{32}=2\times 3\theta_2+\frac{20\times 2^2\times 3}{5^2}$$

§ 7-4 有侧移刚架的计算





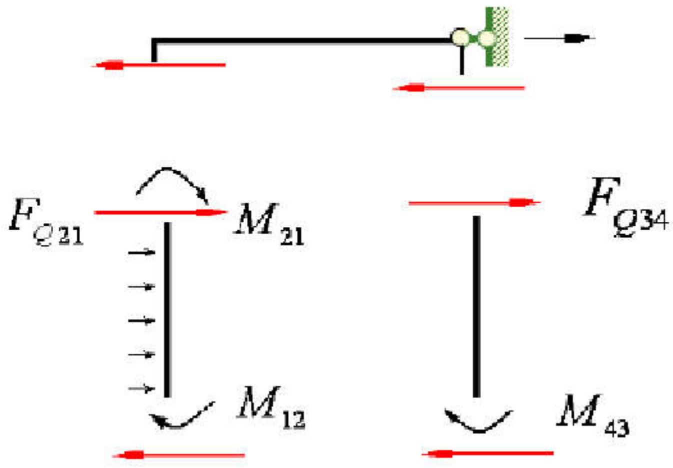
$$M_{12} = 2i \times \Delta_1 - \frac{6i}{4} \times \Delta_2 - \frac{1}{12} \times 3 \times 4^2 \qquad M_{43} = -\frac{3i}{4} \times \Delta_2$$

$$M_{21} = 4i \times \Delta_1 - \frac{6i}{4} \times \Delta_2 - \frac{1}{12} \times 3 \times 4^2 \qquad M_{23} = 3 \times 2i \times \Delta_1$$

$$F_1 = M_{21} + M_{23} = 0$$

$$10i \times \Delta_1 - 1.5i \times \Delta_2 + 4 = 0 \qquad (1)$$

$$F_2 = F_{Q21} + F_{Q34} = 0$$



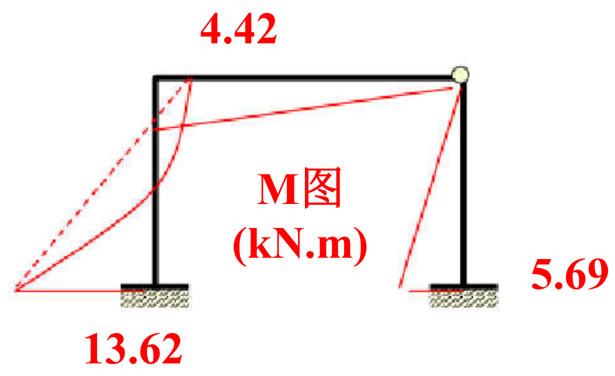
$$F_{Q21} = -\frac{M_{12} + M_{21} + \frac{1}{2}ql^2}{l}$$

$$F_{Q34} = -\frac{M_{43}}{l}$$

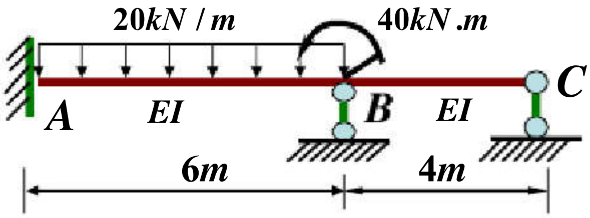
$$-\frac{M_{12} + M_{21} + M_{43}}{l} - \frac{1}{2}ql = 0$$

$$-1.5i\Delta_1 + \frac{15}{16}i\Delta_2 - 6 = 0 \qquad （2）$$

联合求解（1）、（2）得　　$\Delta_1=0.737/i$，$\Delta_2=7.58/i$


4.42
M图
(kN.m)
5.69
13.62

例.计算图示梁,作弯矩图



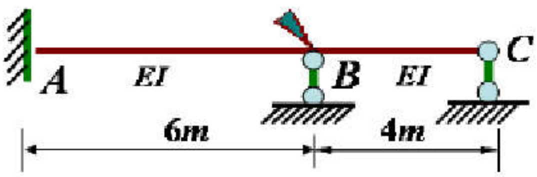
$$M_{AB}=2\times\frac{EI}{6}\times Z_1-\frac{1}{12}\times 20\times 6^2$$

$$M_{BA}=4\times\frac{EI}{6}\times Z_1+\frac{1}{12}\times 20\times 6^2$$

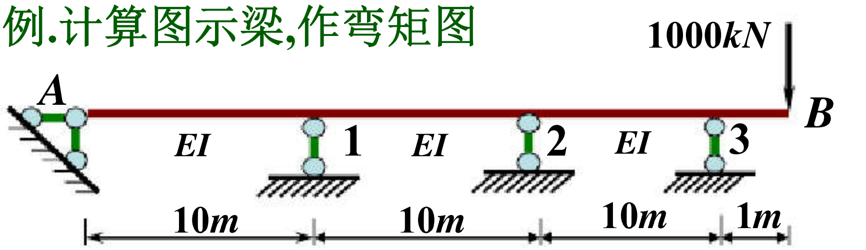
$$M_{BC}=3\times\frac{EI}{4}\times Z_1$$

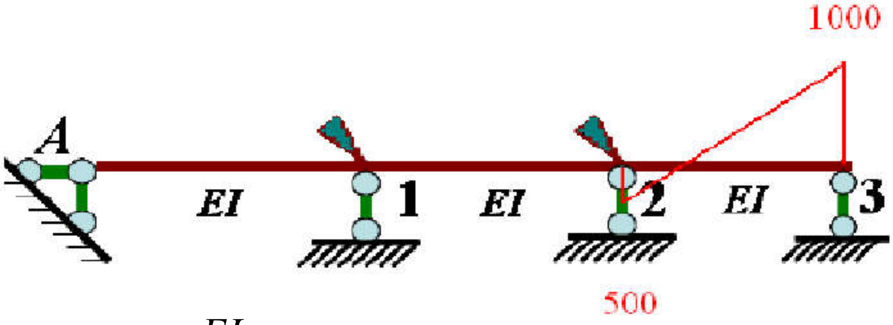
$$M_{BC}+M_{BA}+40=0$$

$$Z_1=-\frac{1200}{17}$$



例.计算图示梁,作弯矩图



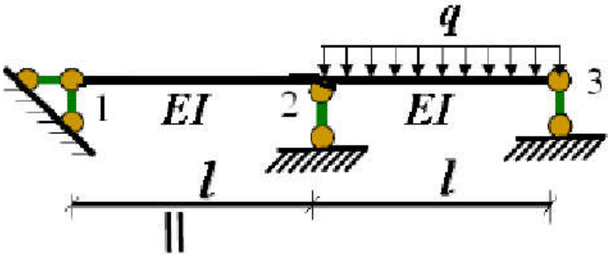


$$M_{1A}=3\times\frac{EI}{10}\times Z_1$$

$$M_{12}=4\times\frac{EI}{10}\times Z_1+2\times\frac{EI}{10}\times Z_2 \qquad M_{21}=2\times\frac{EI}{10}\times Z_1+4\times\frac{EI}{10}\times Z_2$$

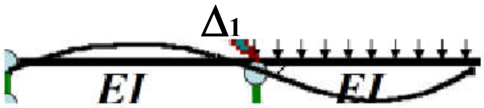
$$M_{23}=3\times\frac{EI}{10}\times Z_2+\frac{1}{2}\times 1000 \qquad M_{32}=1000$$

§7-5 位移法方程的基本体系



$$M_{21} = 3i\Delta_1 \quad M_{23} = 3i\Delta_1 - ql^2/8$$

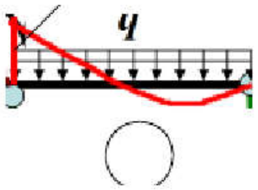
$$M_{21} + M_{23} = 6i\Delta_1 - ql^2/8 = 0$$

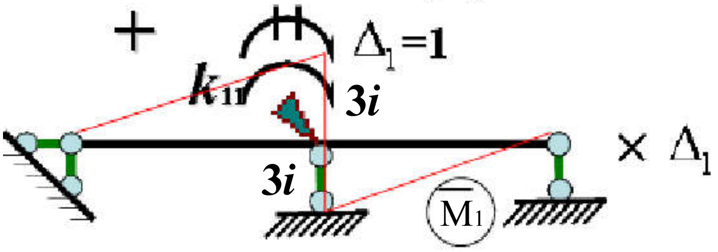


$$k_{11}\Delta_1 + F_{1P} = 0$$

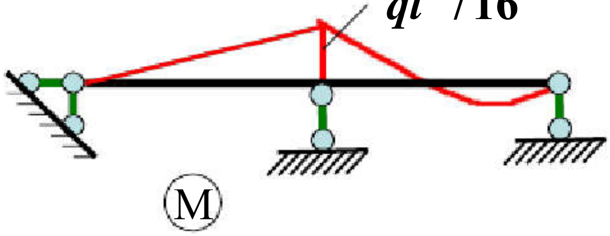
---位移法典型方程

F

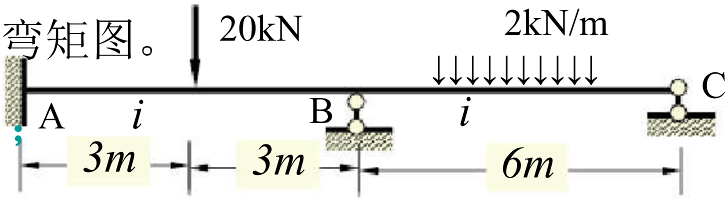




$$M = \bar{M}_1\Delta_1 + M_P$$



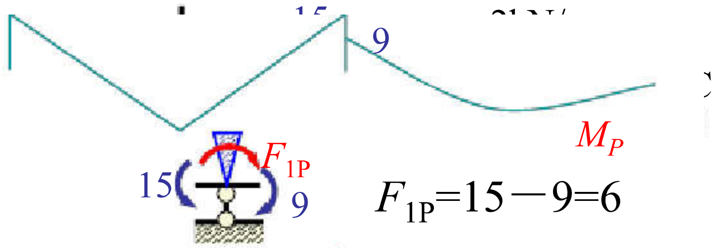
用位移法解图示连续梁作弯矩图。



1）确定基本未知量 $\Delta_1=\theta_B$；

2）确定位移法基本体系；

3）建

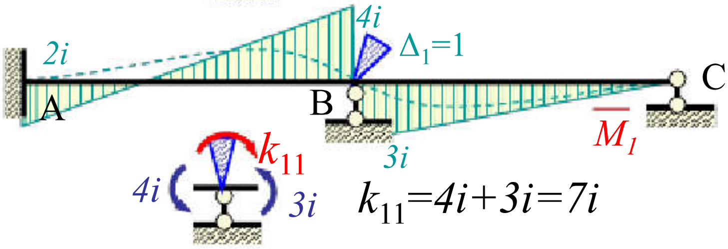


4）画

数

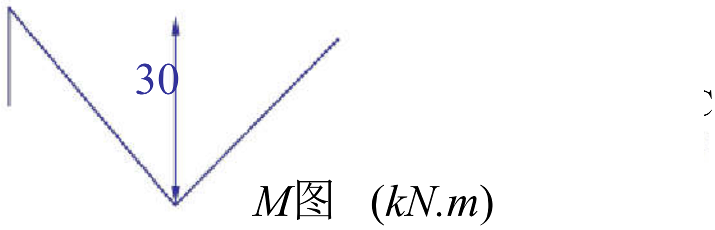
5）解方程，求基本未知量；

$$\Delta_1=-\frac{F_{1P}}{k_{11}}=-\frac{6}{7i}$$

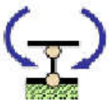


6）按 $M=\sum\overline{M}_i\cdot\Delta_i+M_P$

叠加最后弯矩图

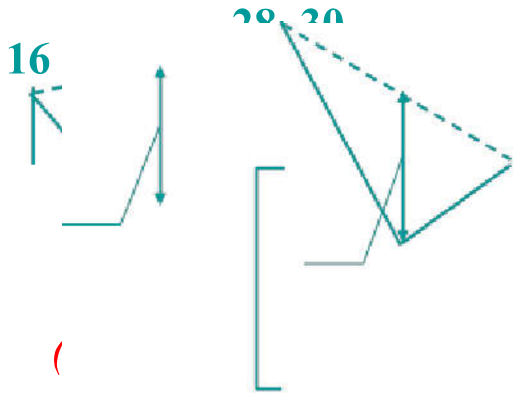


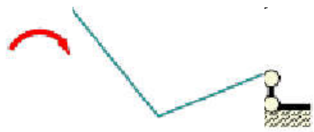
7）校核平衡条件

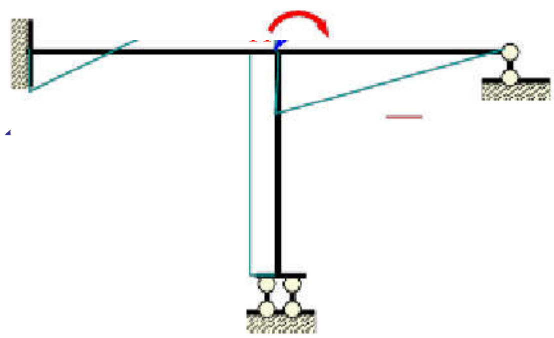
11.57  11.57

$\sum M_B=0$

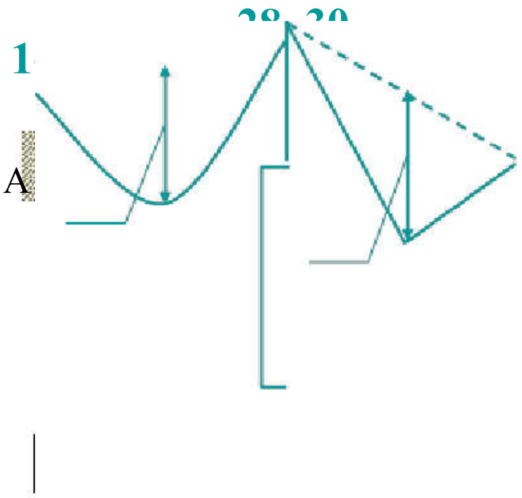
- 位移法求解过程
- **1)**确定基本体系和基本未知量
- **2)**建立位移法方程
- **3)**作单位弯矩图和荷载弯矩图
- **4)**求系数和自由项
- **5)**解方程
- **6)**作弯矩图



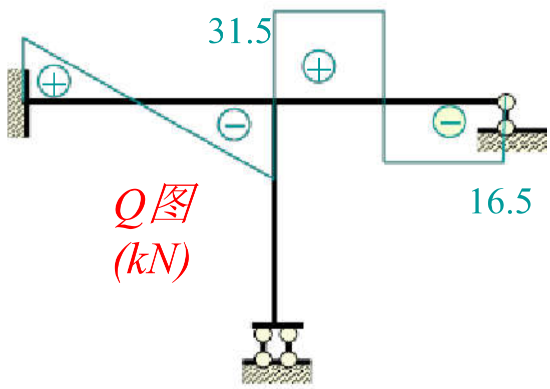




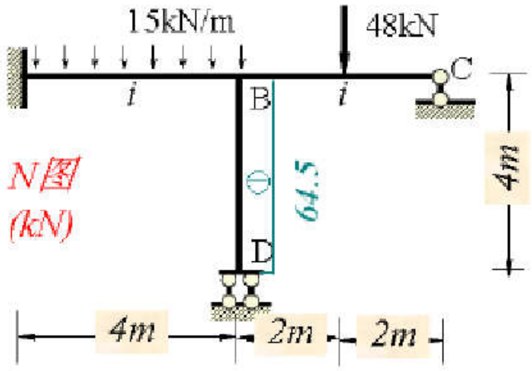
$$F_{1P} = 0$$

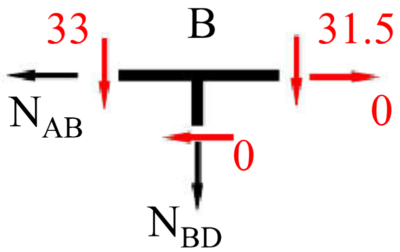


图：



•由已知的Q图求轴力，作轴力图：

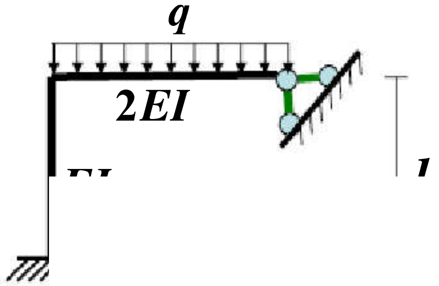


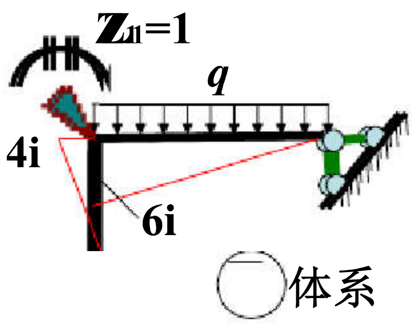


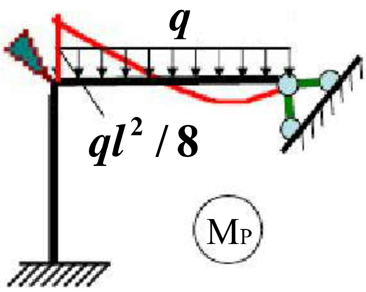
$\sum X=0$　　$N_{AB}=0$

$\sum Y=0$　　$N_{BD}=-64.5$

作图示刚架弯矩图







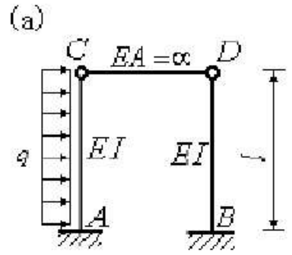
$r_{11}=10i$

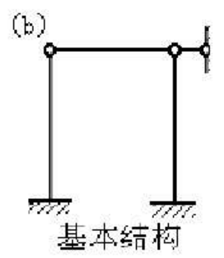
$R_{1P}=-ql^2/8$

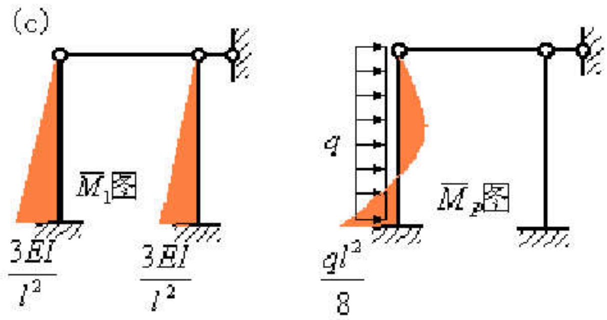
$Z_1=ql^2/80i$

$M=\bar{M}_1 Z_1+M_P$

例　用位移法计算排架，作**M**图







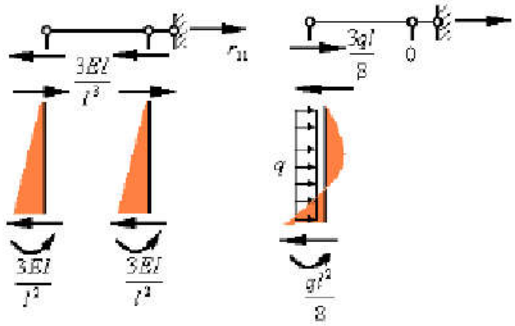
$$r_{11}Z_1 + R_{1P} = 0$$

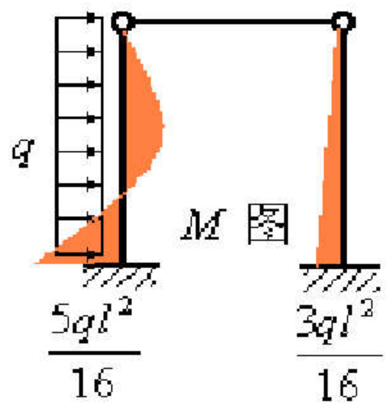
$$r_{11} = \frac{6EI}{l^3}$$

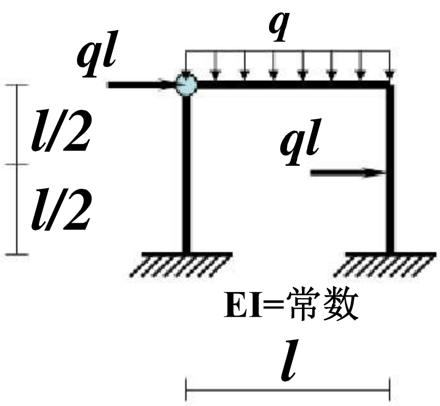
$$R_{1P} = -\frac{3ql}{8},$$

$$Z_1 = \frac{ql^4}{16EI}$$

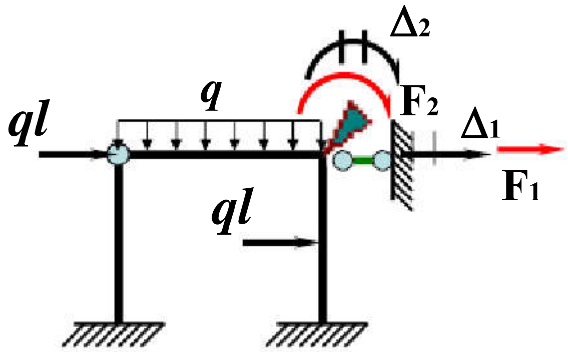
$$M = \overline{M}_1 Z_1 + M_P$$

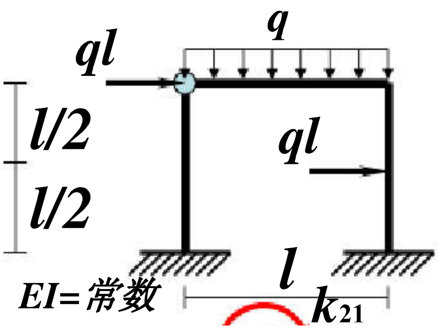






$$\begin{cases} F_1=0 \\ F_2=0 \end{cases}$$





$$F_1 = k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0$$

$$F_2 = k_{21}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0$$

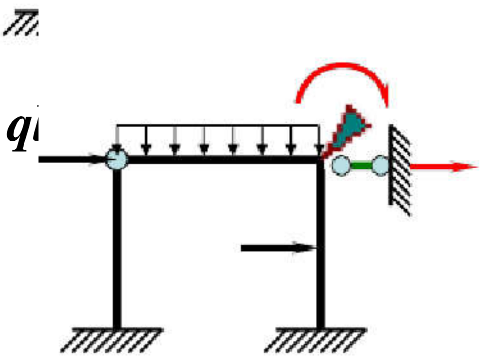
---位移法典型方程

k_{ij} ($i=j$) 主系数>0

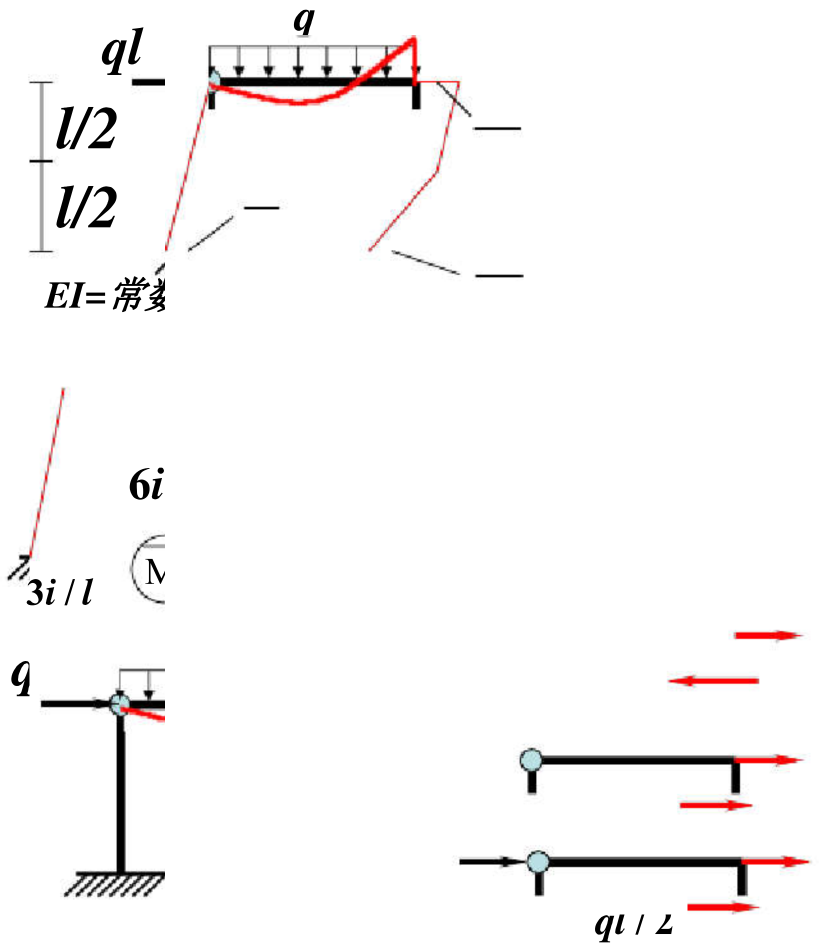
k_{ij} ($i\neq j$) 副系数

$k_{ij}=k_{ji}$ 反力互等

F_{iP} 荷载系数
自由项



刚度系数,
体系常数



$+F_{1P}=0$

$+F_{2P}=0$

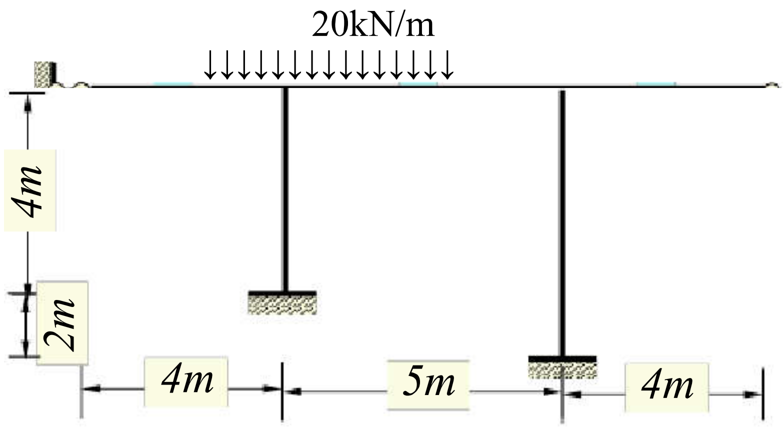
$=-6i/l$

$=-6i/l$

$=ql^2/4$

Δ_2+M_P

例：求结点位移,**E=c**

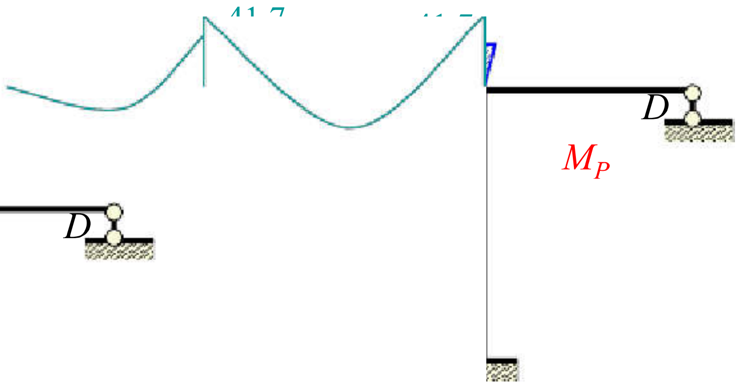


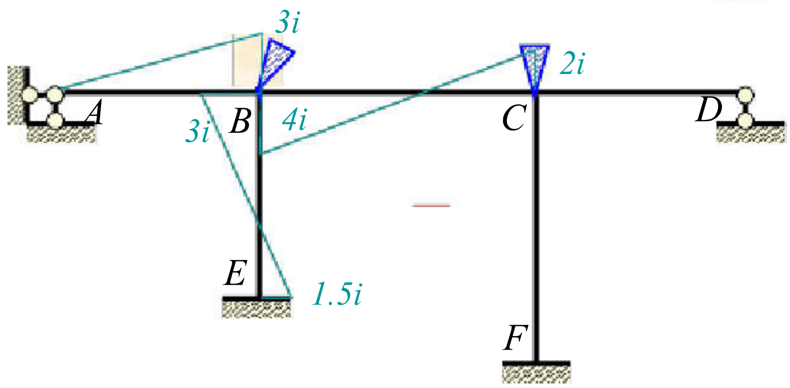
解：$k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0$

$k_{21}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0$

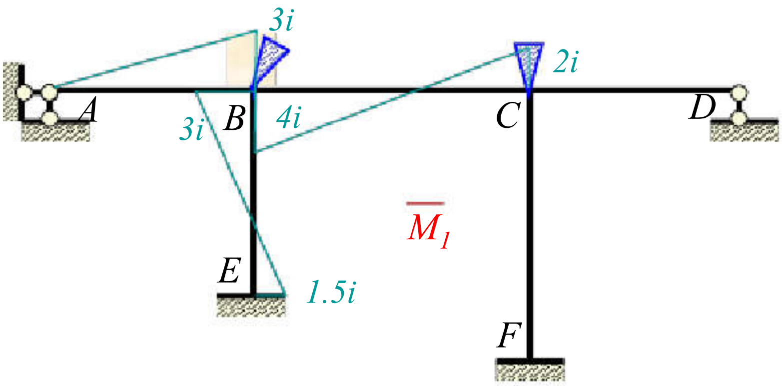
$k_{11}=4i+3i+3i=10i$

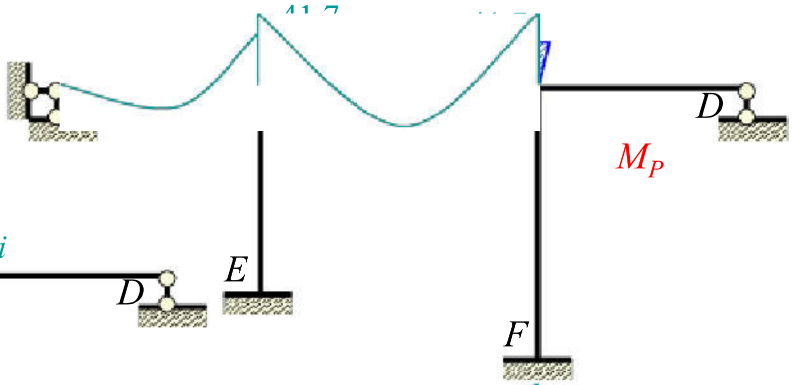
$k_{21}=2i$

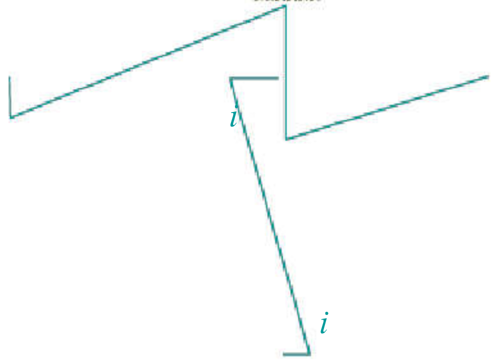




$$10i\Delta_1 + 2i\Delta_2 - 1.7 = 0$$
$$2i\Delta_1 + 9i\Delta_2 + 41.7 = 0$$



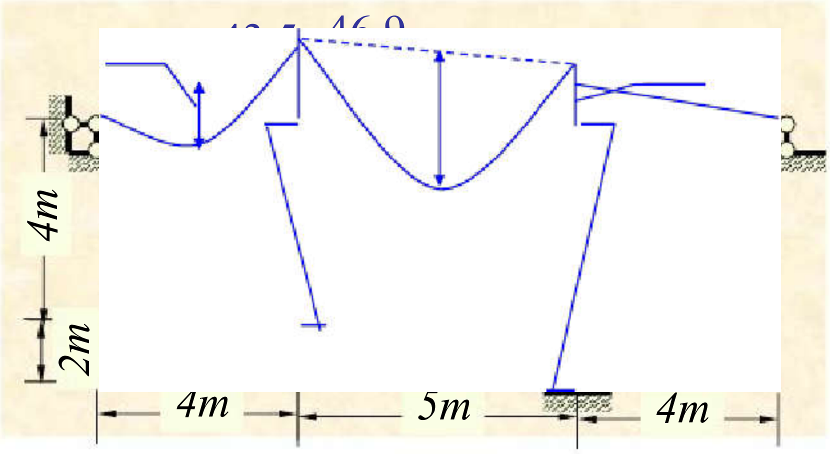




$$\Delta_1 = 1.15/i$$
$$\Delta_2 = -4.89/i$$

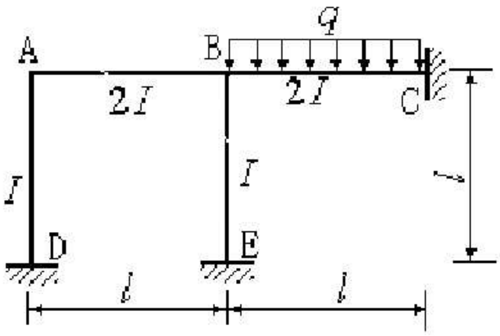
$k_{22}=4i+3i+2i= 9i$

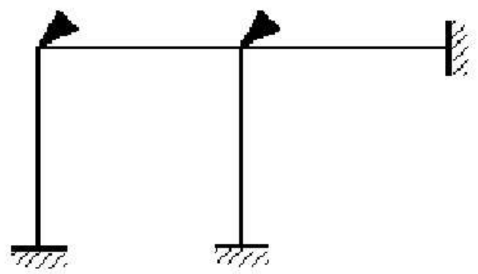
$k_{21}=2i$



46.9
4m
2m
4m
5m
4m

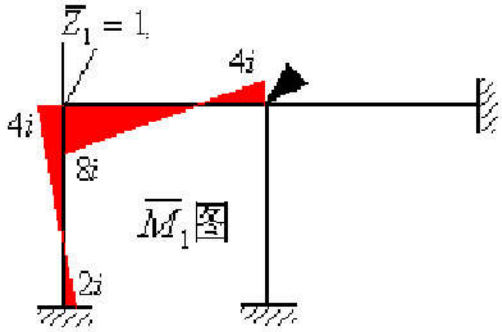
例：求结点位移,E=c

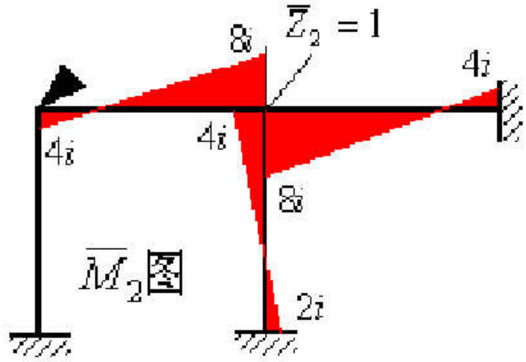


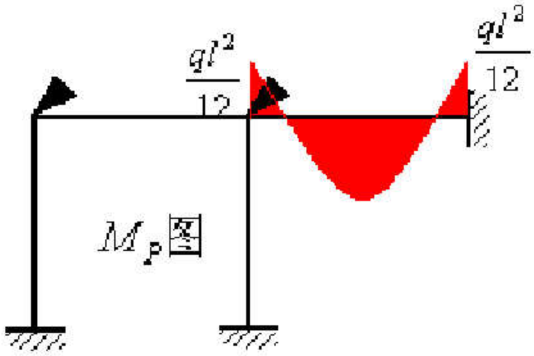


基本结构

$$\left.\begin{aligned} r_{11}Z_1 + r_{12}Z_2 + R_{1P} &= 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2P} &= 0 \end{aligned}\right\}$$







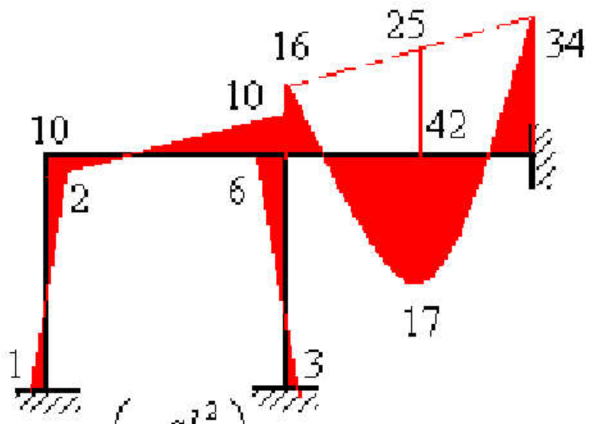
$$\left.\begin{aligned} r_{11}Z_1 + r_{12}Z_2 + R_{1P} &= 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2P} &= 0 \end{aligned}\right\}$$

$r_{11} = 12i$, $r_{12} = r_{21} = 4i$, $r_{22} = 20i$

$R_{1P} = 0$, $R_{2P} = -\dfrac{ql^2}{12}$

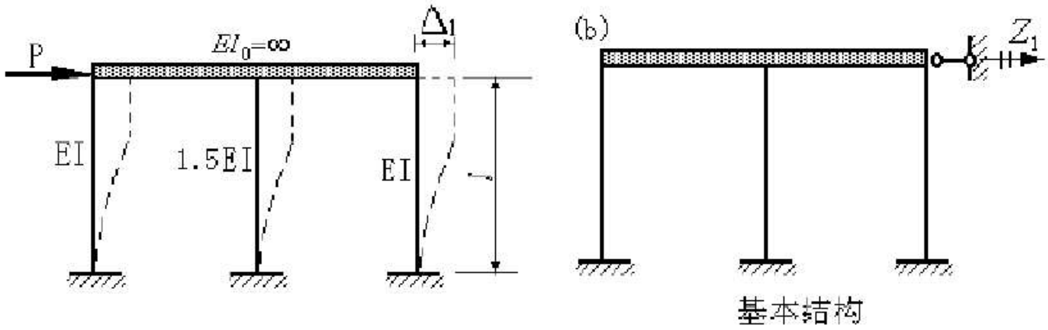
$$Z_1 = -\frac{ql^2}{672i}, \quad Z_2 = \frac{3ql^2}{672i}$$

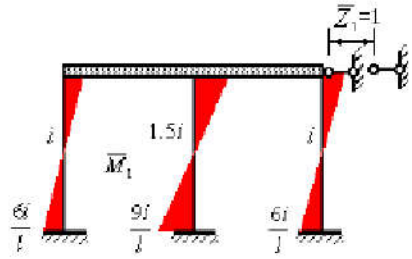
$$M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$$



$M\left(\times\dfrac{ql^2}{336}\right)$

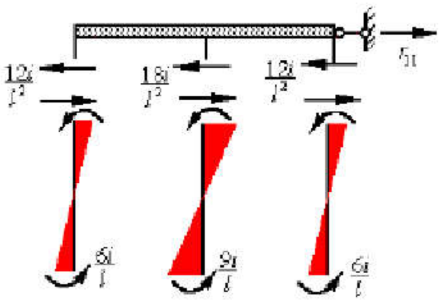
例　用位移法计算横梁刚度无穷大的刚架，绘弯矩图。
E=常数。



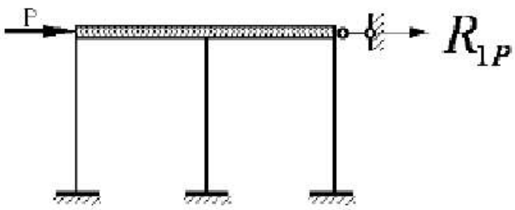


$$r_{11} = \frac{12i}{l^2} + \frac{18i}{l^2} + \frac{12i}{l^2}$$

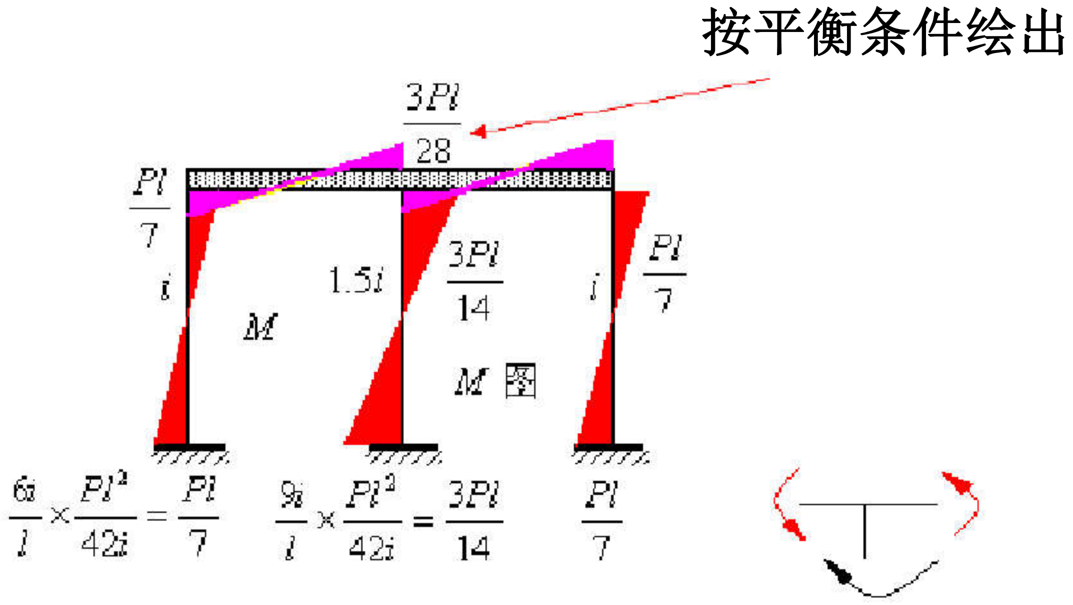
$$R_{1P} = -P$$



$$Z_1 = \frac{P}{\frac{12i}{l^2} + \frac{18i}{l^2} + \frac{12i}{l^2}} = \frac{Pl^2}{42i}$$

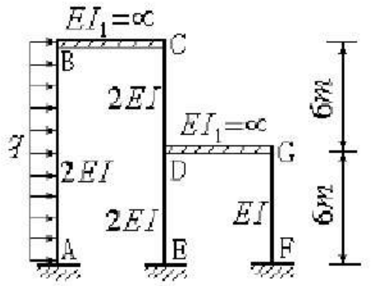


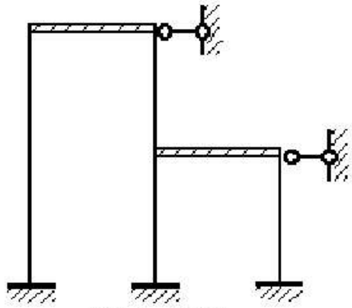
$$M=\overline{M}_1 Z_1+M_P=\overline{M}_1 Z_1+0=\overline{M}_1 Z_1$$



例　试求位移法方程中的自由项与系数

横梁刚度无穷大，两个线位移

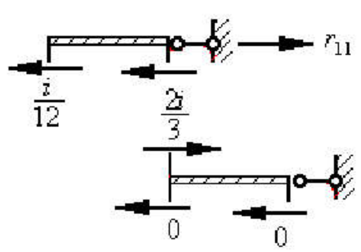


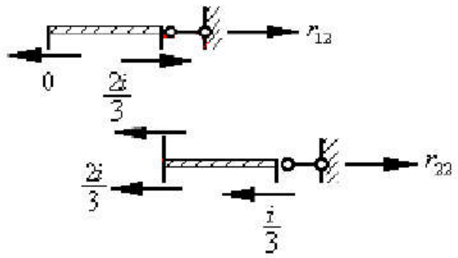

基本结构

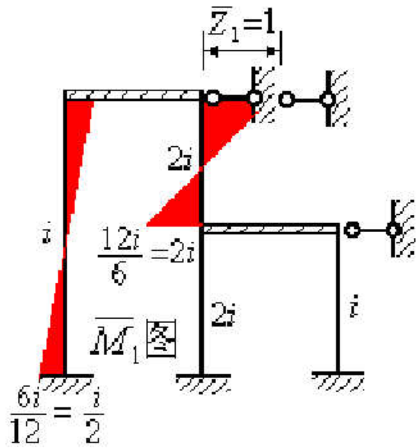
$$i=\frac{EI}{6}$$

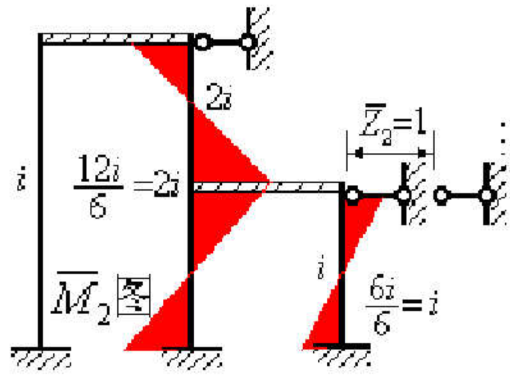
$$\left.\begin{aligned} r_{11}Z_1+r_{12}Z_2+R_{1P}=0 \\ r_{21}Z_1+r_{22}Z_2+R_{2P}=0 \end{aligned}\right\}$$

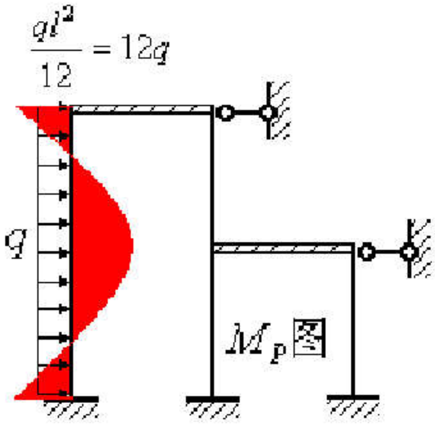
$$r_{11}=\frac{i}{12}+\frac{2i}{3}=\frac{9i}{12},r_{12}=r_{21}=-\frac{2i}{3},r_{22}=\frac{2i}{3}+\frac{2i}{3}+\frac{i}{3}=\frac{5i}{3},$$

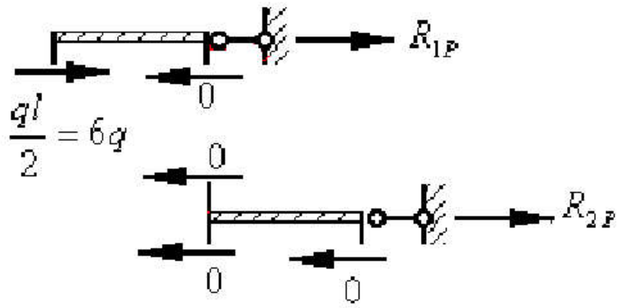






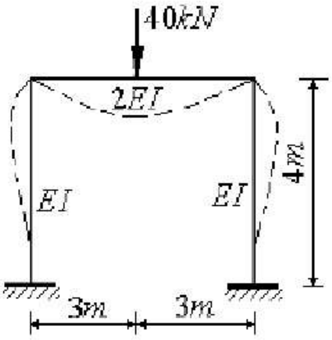


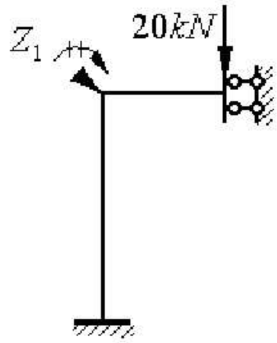


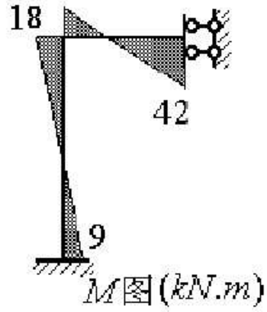


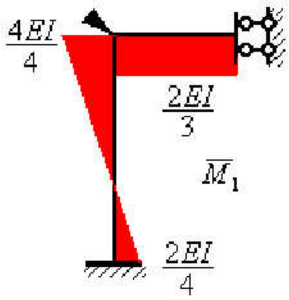
$$R_{1P} = -6q, \qquad R_{2P} = 0$$

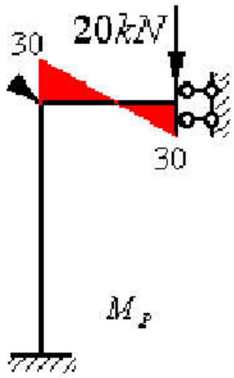
§ 7-6 对称结构的计算









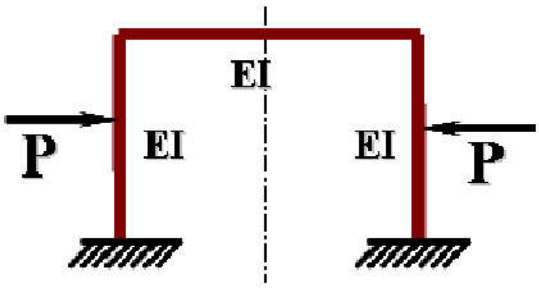


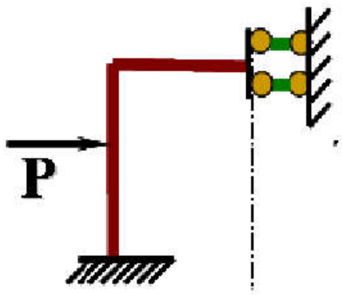
$$r_{11}Z_1 + R_{1P} = 0$$

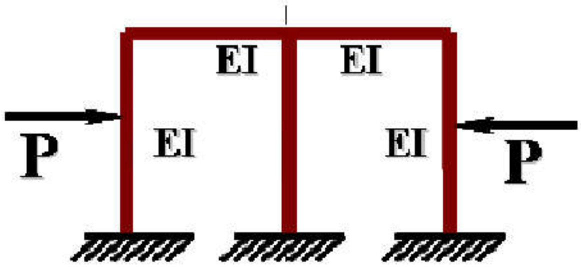
$$r_{11} = \frac{5EI}{3}$$

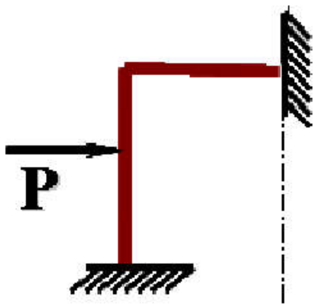
$$R_{1P} = -30kN \cdot m$$

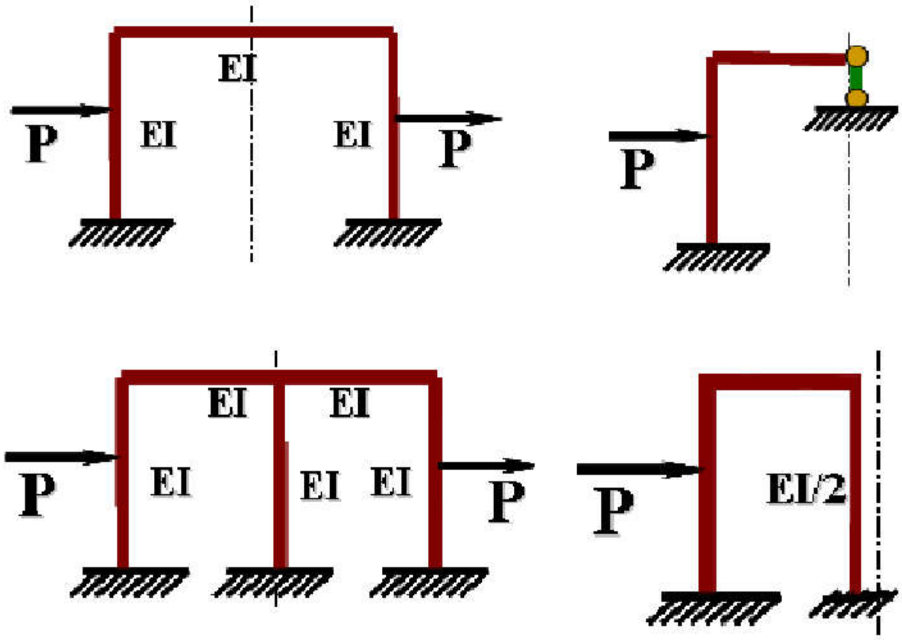
$$Z_1 = \frac{90}{5EI}$$


EI
P
EI
EI
P


P


EI
EI
P
EI
EI
P


P

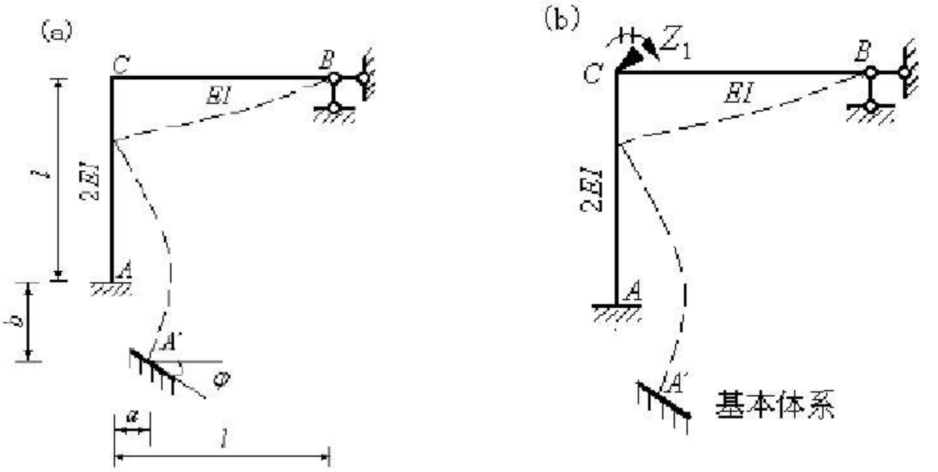


P303书例

§7-7 支座移动和温度变化的计算

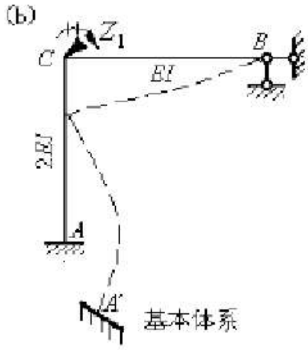
图示刚架的**A**支座产生了水平位移**a**、竖向位移**b=4a**

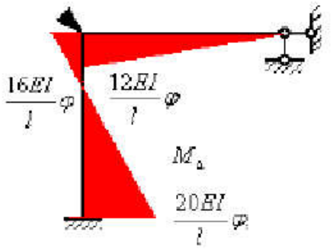
转角 $\varphi = a / l$ ，试绘其弯矩图。



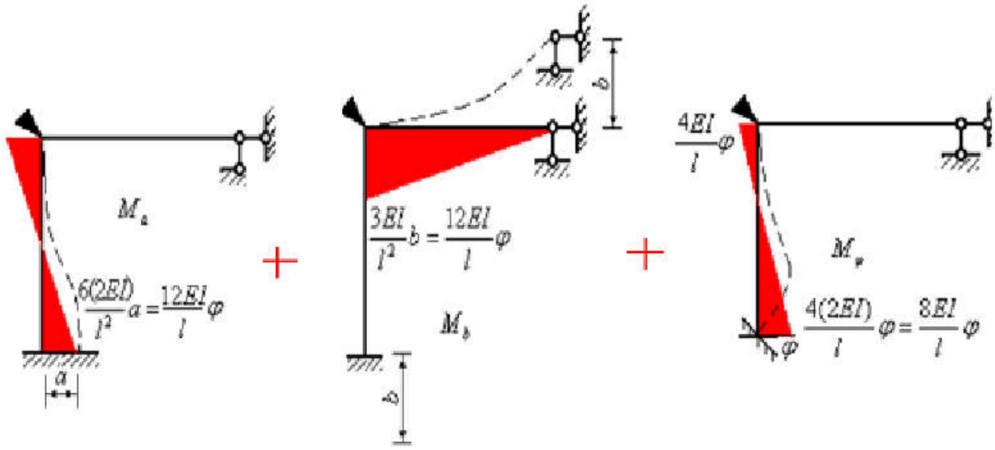
$$r_{11}Z_1 + R_{1\Delta} = 0$$

$$\varphi = \frac{a}{l}$$ **b=4a**



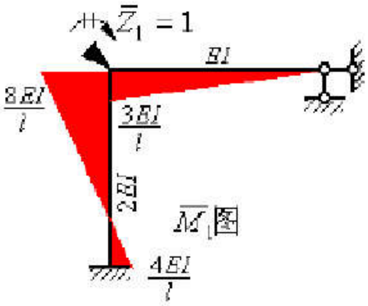


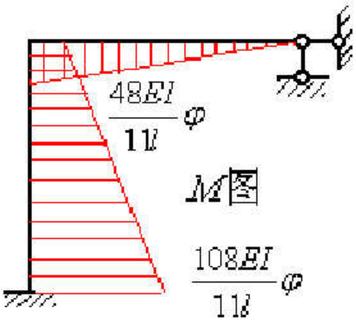
$$R_{1\Delta} = \frac{16EI}{l}\phi + \frac{12EI}{l}\phi = \frac{28EI}{l}\phi$$



$$r_{11}=\frac{3EI}{l}+\frac{8EI}{l}=\frac{11EI}{l}$$

$$Z_1=-\frac{28}{11}\varphi$$



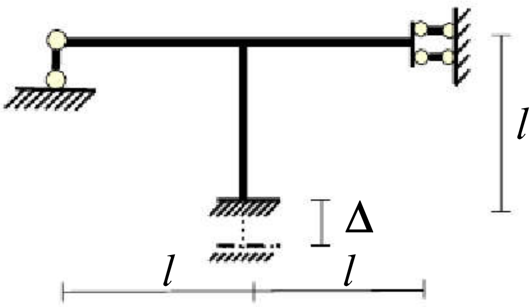


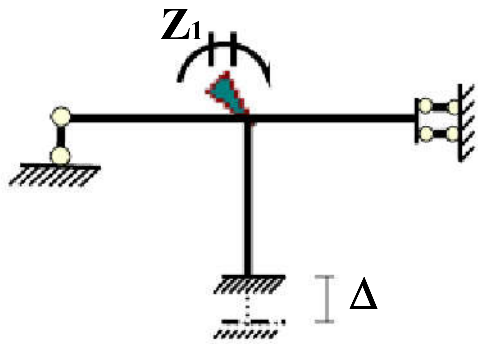
$$M=\overline{M}_1 Z_1+M_P$$

由结果可见：内力与EI大小有关

作M图，

EI=常数





解:

$R_1=0$

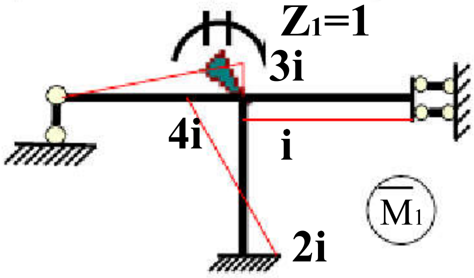
$r_{11}Z_1+R_{1C}=0$

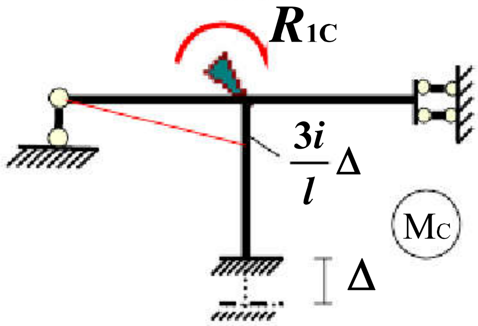
$r_{11}=8i$

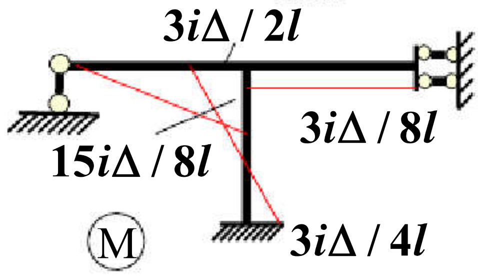
$R_{1c}=-3i\Delta/l$

$Z_1=3\Delta/8l$

$M=\overline{M}_1Z_1+M_c$

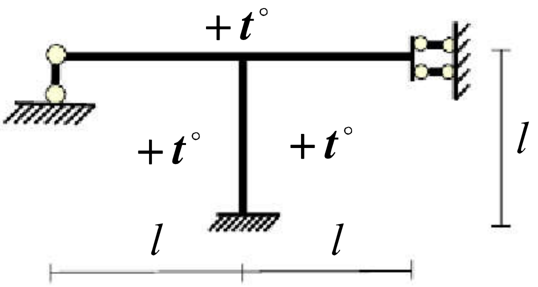


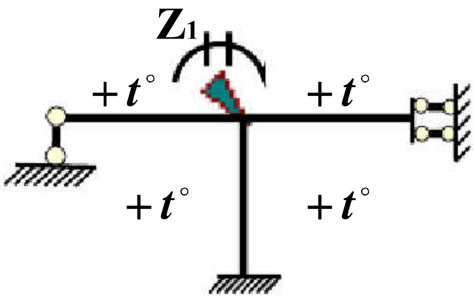




作M图，

EI=常数





解:

$R_1=0$

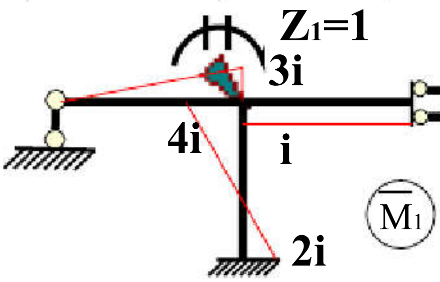
$r_{11}Z_1+R_{1t}=0$

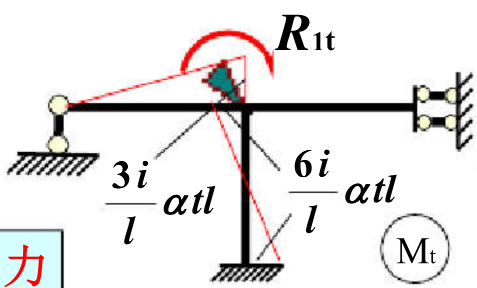
$r_{11}=8i$

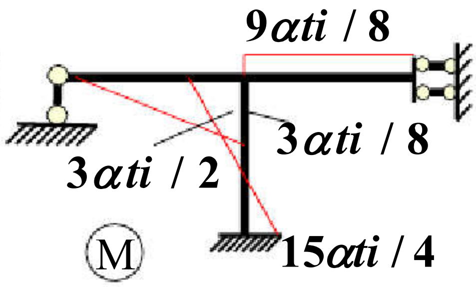
$R_{1t}=9i\alpha t$

$Z_1=-9\alpha t/8$

$M=\overline{M}_1Z_1+M_t$



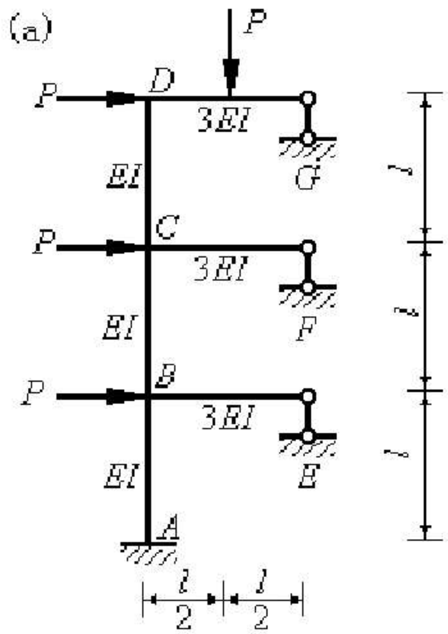




温度变化引起的内力与EI大小有关

剪力静定杆

（单跨多层刚架的计算）（了解）

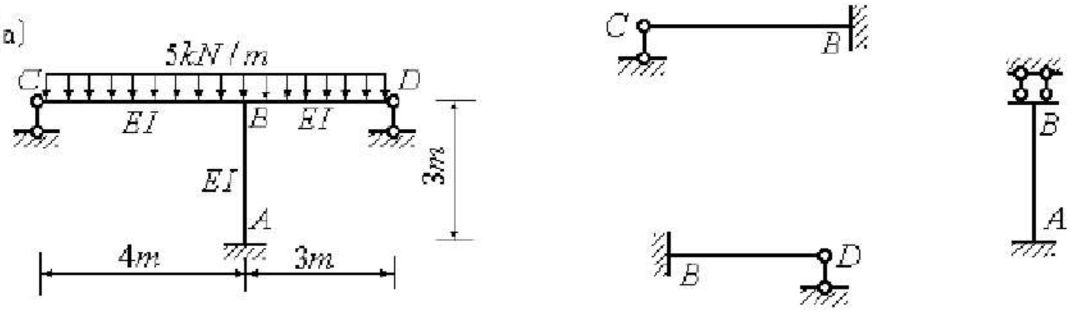


本题特点是：

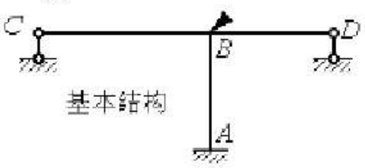
（1）　柱AB为剪力静定柱。

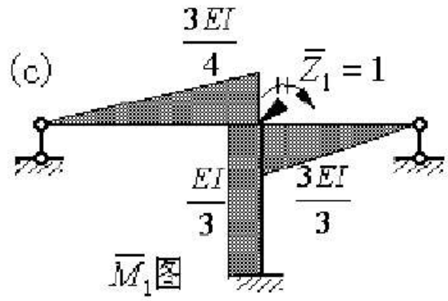
（2）　横梁为无侧移杆。

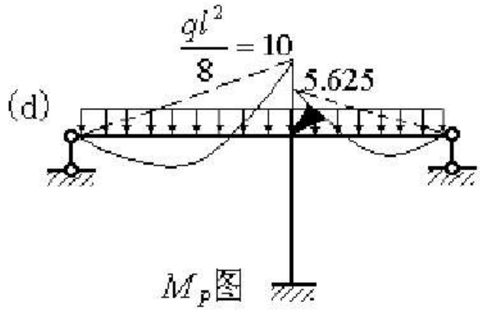
(a)



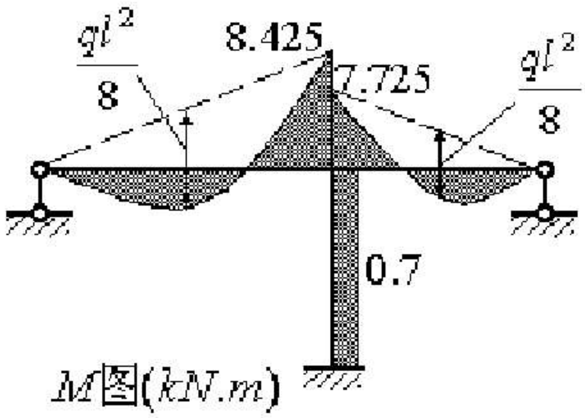
(b)





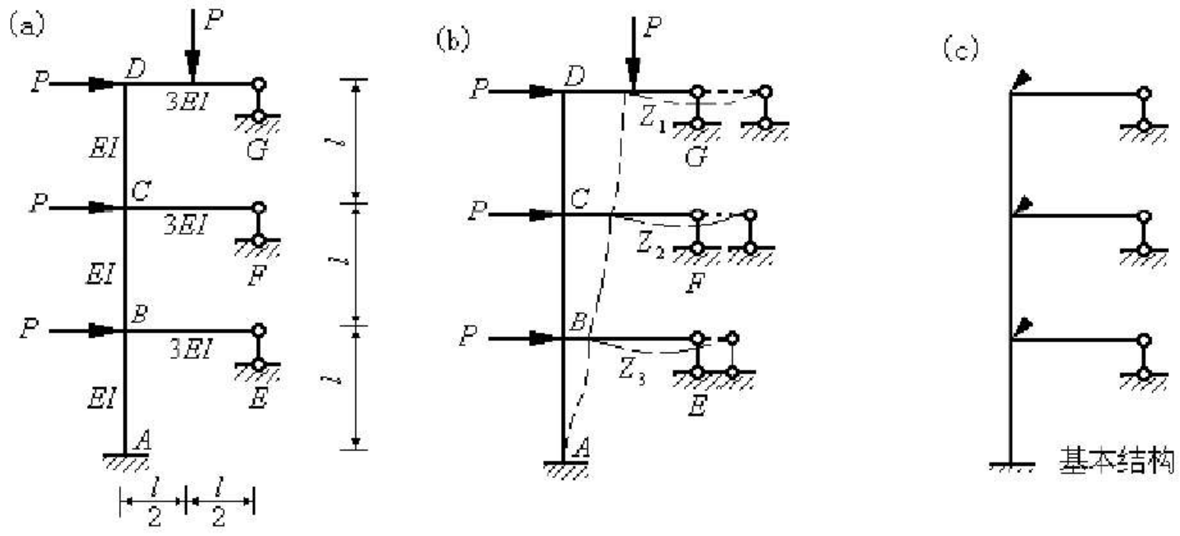


$$r_{11} = \frac{3EI}{4} + \frac{3EI}{3} + \frac{EI}{3} = \frac{(9+12+4)EI}{12} = 2.08$$



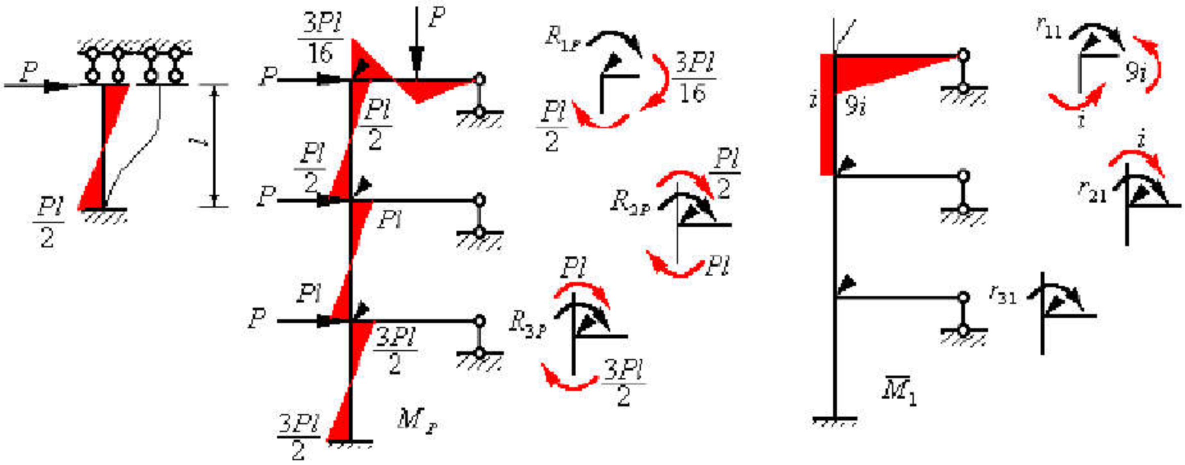
$$R_{1P} = 4.375$$

$$Z_1 = -2.1$$



$$\left.\begin{aligned} r_{11}Z_1 + r_{12}Z_2 + r_{13}Z_3 + R_{1P} &= 0 \\ r_{21}Z_1 + r_{22}Z_2 + r_{23}Z_3 + R_{2P} &= 0 \\ r_{31}Z_1 + r_{32}Z_2 + r_{33}Z_3 + R_{3P} &= 0 \end{aligned}\right\}$$

剪力静定柱
无侧移梁



$$R_{1P} = -\frac{3Pl}{16} - \frac{Pl}{2} = -\frac{11Pl}{16}$$

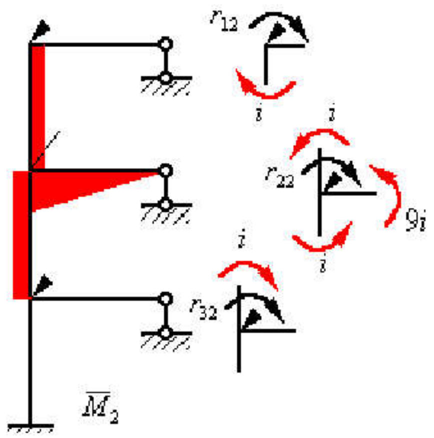
$$R_{2P} = -\frac{Pl}{2} - Pl = -\frac{3Pl}{2}$$

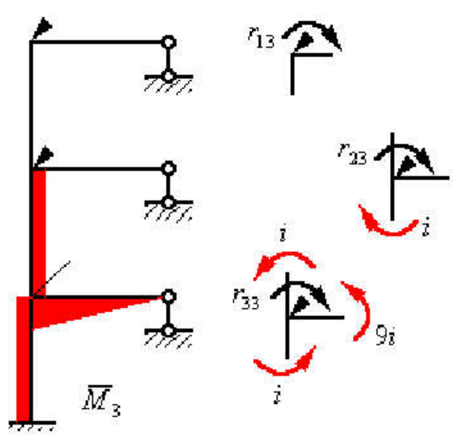
$$R_{3P} = -Pl - \frac{3Pl}{2} = -\frac{5Pl}{2}$$

$$r_{11} = 9i + i = 10i$$

$$r_{21} = -i$$

$$r_{31} = 0$$





$$r_{12} = -i$$

$$r_{22} = 9i + i + i = 11i$$

$$r_{32} = -i$$

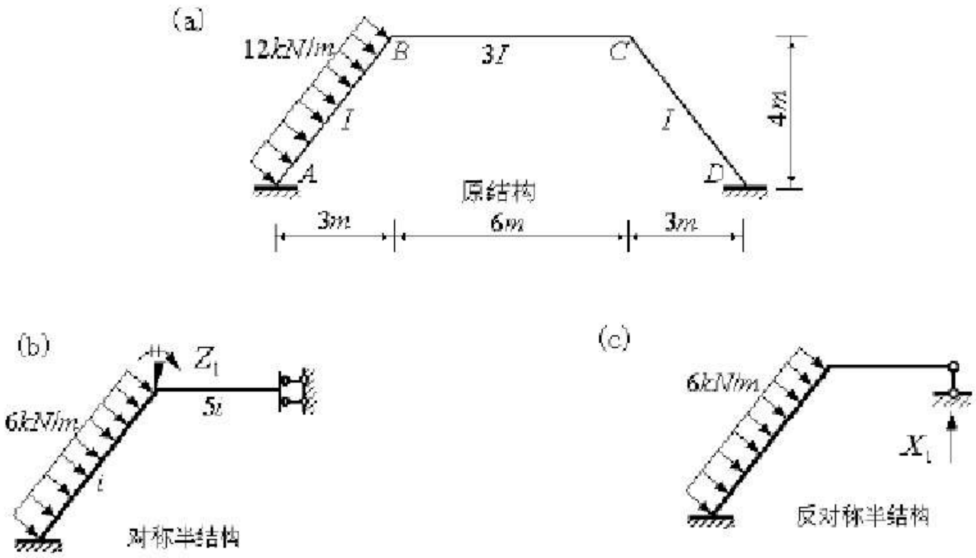
$$r_{13} = 0$$

$$r_{23} = -i$$

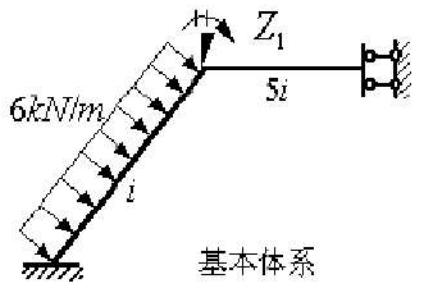
$$r_{33} = 9i + i + i = 11i$$

联合法

用位移法计算图a所示结构，绘制弯矩图。E=常数。



这种求解同一问题时，联合应用力法、位移法的方法，称为联合法。



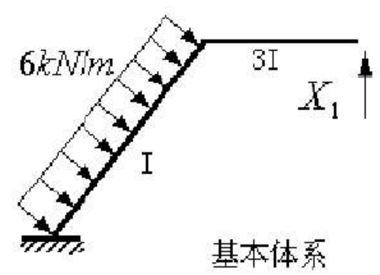
$$r_{11}Z_1 + R_{1P} = 0$$

$$r_{11} = 5i + 4i = 9i$$

$$R_{1P} = \frac{ql^2}{12} = \frac{6kN/m \times 5^2}{12} = 12.5kN \cdot m$$

$$Z_1 = -\frac{R_{1P}}{r_{11}} = -\frac{12.5kN \cdot m}{9i} = -\frac{1.3889kN \cdot m}{i}$$

$$M = \overline{M}_1 Z_1 + M_P$$



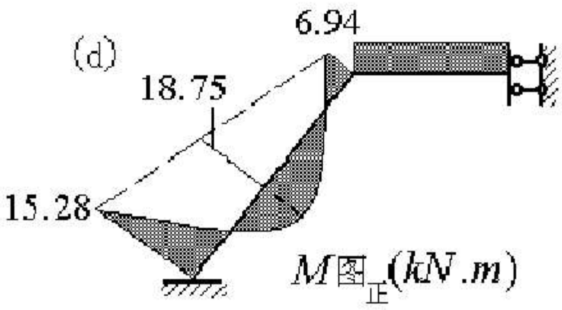
$$\delta_{11}X_1 + \Delta_{1P} = 0$$

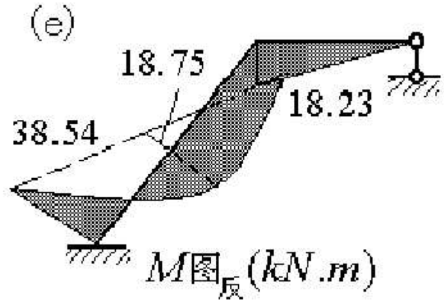
$$\delta_{11} = 108m^3$$

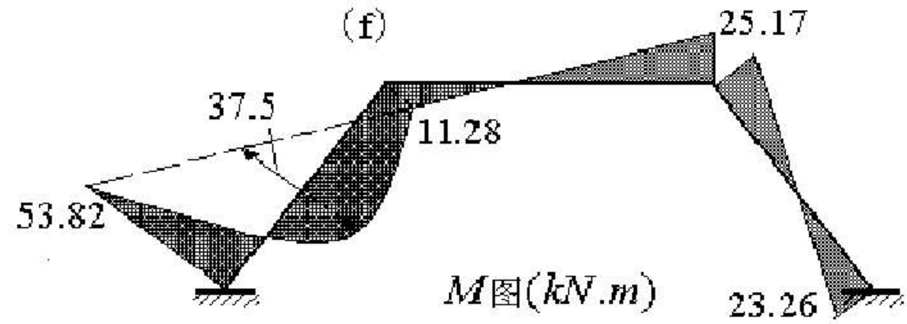
$$\Delta_{1P} = -656.25kN \cdot m^3$$

$$X_1 = -\frac{\Delta_{1P}}{\delta_{11}} = \frac{656.25kN \cdot m^3}{108m^3} = 6.0764kN$$

$$M = \overline{M}_1 X_1 + M_P$$

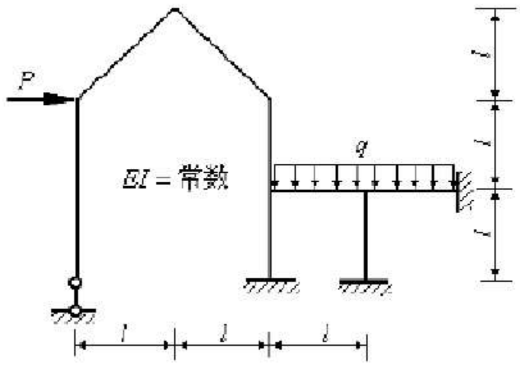


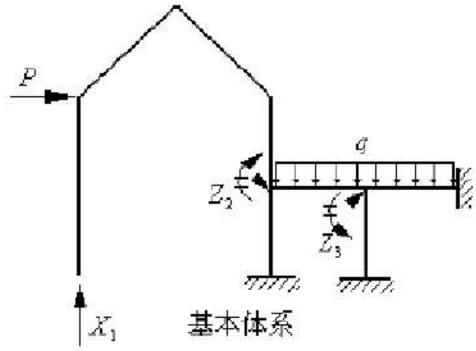




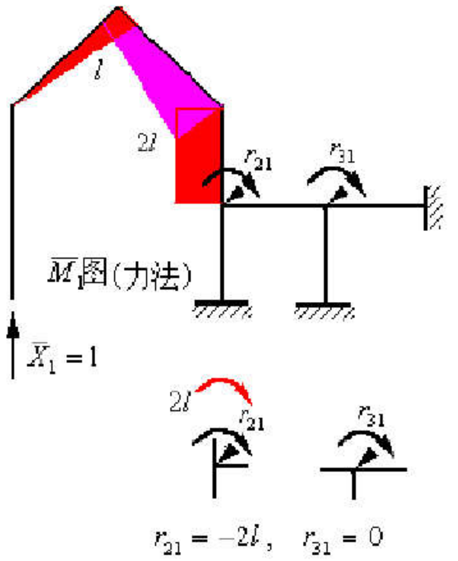
注意点：用联合法求解对称结构时，每个半结构的计算简图的求解是很方便的，但从半结构的结果，利用对称性和进行叠加时必须细心，否则将前功尽弃。

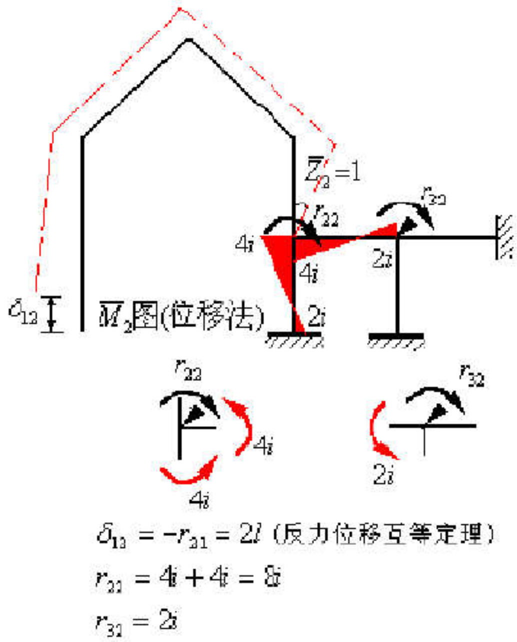
混合法



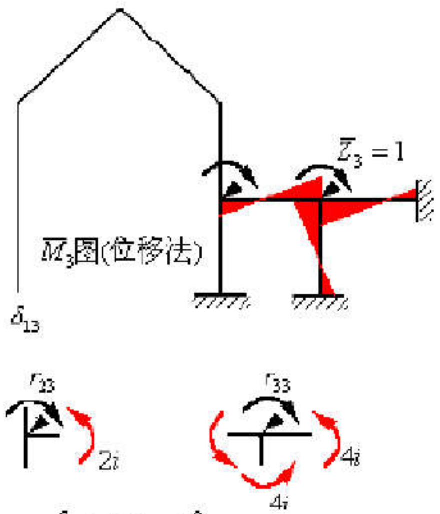


分析：左边“主厂房”部分一次超静定，但独立位移有 5个。由边“附属厂房”部分独立位移只有 2个，而超静定次数为六次。　　如果左边部分以力作未知量，右边部分以位移作未知量，混合用两类未知量的总未知量只有 3个，如图所示。下面说明混合法解题思路





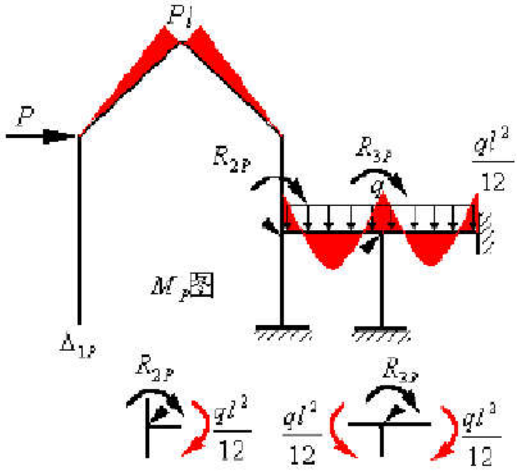
此例说明,解决问题不能墨守成规，要深刻理解和掌握力学概念、原理和方法，在此基础上灵活应用知识，才能既好又省地解决问题。



$\delta_{13}=-r_{31}=0$

$r_{33}=4i+4i+4i=12i$

$r_{33}=2i$



$\Delta_{1P}=\int \overline{M}_1 M_P \frac{dx}{EI}$

$R_{2P}=-\frac{ql^2}{12}$

$R_{3P}=0$

$$\left.\begin{aligned}\delta_{11}X_1+\delta_{12}Z_2+\delta_{13}Z_3+\Delta_{1P}&=0\\ r_{21}X_1+r_{22}Z_2+r_{23}Z_3+R_{2P}&=0\\ r_{31}X_1+r_{32}Z_2+r_{33}Z_3+R_{3P}&=0\end{aligned}\right\}$$