7.9 电光效应

The electrooptic effect

7.9.1 Kerr 效应和 Pockel′s 效应

1. 电光效应现象

定义： 因外加电场使介质的光学性质（折射率）发生变化的现象。

$$n = n(E) - n_o = \alpha E + \beta E^2 + \cdots$$

线性电光效应　Pockel's effect　$\Delta n = \alpha E - (7\text{-}161)$

非线性电光效应　Kerr effect　$\Delta n = \beta E^2 --- (7-162)$

7.9.2 电光张量 Electrooptic tensor

1. 定义

Following convention

$$\frac{1}{n_x^2}=(\frac{1}{n^2})_1,\quad \frac{1}{n_y^2}=(\frac{1}{n^2})_2,\quad \frac{1}{n_z^2}=(\frac{1}{n^2})_3$$

$$\frac{1}{n_{yz}^2}=(\frac{1}{n^2})_4,\quad \frac{1}{n_{xz}^2}=(\frac{1}{n^2})_5,\quad \frac{1}{n_{xy}^2}=(\frac{1}{n^2})_6,$$

The equation of the index ellipsoid in the presence of E is

$$(\frac{1}{n^2})_1 x^2+(\frac{1}{n^2})_2 y^2+(\frac{1}{n^2})_3 z^2+2(\frac{1}{n^2})_4 yz+2(\frac{1}{n^2})_5 xz+2(\frac{1}{n^2})_6 xy=1 \quad (7\text{-}164)$$

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1$$

if $x, y, z \parallel$ *principal dielectric axes*, $E = 0$,

must reduce to $$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (7-163)$$

therefore $$\left.\left(\frac{1}{n^2}\right)_4\right|_{E=0} = \left.\left(\frac{1}{n^2}\right)_5\right|_{E=0} \quad \left.\left(\frac{1}{n^2}\right)_6\right|_{E=0} = 0$$

$$\left.\left(\frac{1}{n^2}\right)_1\right|_{E=0} = \frac{1}{n_x^2}, \quad \left.\left(\frac{1}{n^2}\right)_2\right|_{E=0} = \frac{1}{n_y^2}, \quad \left.\left(\frac{1}{n^2}\right)_3\right|_{E=0} = \frac{1}{n_z^2},$$

$$\left(\frac{1}{n^2}\right)_1\Big|_{E=0}=\frac{1}{n_x^2},\quad \left(\frac{1}{n^2}\right)_2\Big|_{E=0}=\frac{1}{n_y^2},\quad \left(\frac{1}{n^2}\right)_3\Big|_{E=0}=\frac{1}{n_z^2},$$

$$\left(\frac{1}{n^2}\right)_4\Big|_{E=0}=\left(\frac{1}{n^2}\right)_5\Big|_{E=0}\left(\frac{1}{n^2}\right)_6\Big|_{E=0}=0$$

The linear change in the coefficients $\left(\frac{1}{n^2}\right)_i \quad i=1,2,\cdots,6$

due to E is defined by $$\Delta\left(\frac{1}{n^2}\right)_i=\sum_{j=1}^{3} r_{ij}E_j \qquad (7-165)$$

$j=1,2,3$，1表示x，2表示y，3表示z

$\Delta(\frac{1}{\ }) = \sum^{3} r\ E$ (7 - 165) $i = 1, 2, 3$, 1表示x, 2表示y, 3表示z

(7 –

exp

matrix form as

$$\begin{bmatrix} \vdots \\ \Delta(\frac{1}{n^2})_6 \end{bmatrix} = \begin{bmatrix} \ & r_{12} & r_{13} \\ \ & r_{22} & r_{23} \\ & \vdots & \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (7\text{-}166)$$

The 6×3 *matrix with elements* r_{ij} *is called the electooptic tensor.*

2. *The form*

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{i=1}^{3} r_{ij} E_j$$

(A) *centrosymmetric crystals*
(*possesing an inversion symmetry*)

$$r_{ij} \equiv 0$$

(B) 一般情况，独立张量元少于18

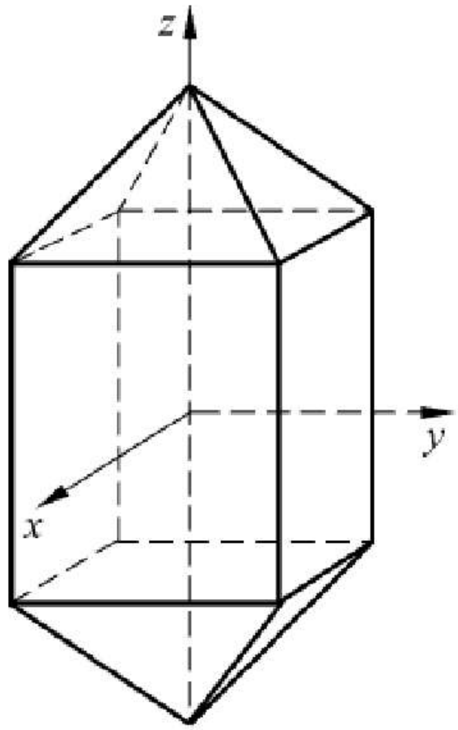
(1) Quantum Electronics
Introduction to Optoelectronics
等著作有各种点群的电光张量形式。

（2）给出的电光张量形式相应于特定的坐标系

7.9.3 KDP晶体的线性电光效应

KH_2PO_4 Kalium dihydrogen phosphate

1. 电光张量形式和折射率椭球



KDP(KH_2PO_4，磷酸二氢钾) 晶体是水溶液培养的一种人工晶体，属四方晶系 。在不加外电场情况下是一单轴晶体 ，属于 $\bar{4}2m$ 类晶体。

点群和矩阵

- **KDP 的线性电光张量的点群和矩阵形式为**

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \bullet & \cdot & \cdot \\ \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet \end{bmatrix} \text{或} \left[r_{ij} \right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

在外加电场E时，

$$\left(\frac{1}{n^2}\right)_i \bigg|_{E\neq 0} = \left(\frac{1}{n^2}\right)_i \bigg|_{E=0}, \qquad i = 1,2,3$$

电光张量

$$\bar{4}2m\,,\ (2\| \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \quad \cdots \quad (7\text{-}167)$$

$$(\frac{1}{n^2})_1 x^2 + (\frac{1}{n^2})_2 y^2 + (\frac{1}{n^2})_3 z^2 + 2(\frac{1}{n^2})_4 yz + 2(\frac{1}{n^2})_5 xz + 2(\frac{1}{n^2})_6 xy = 1 \quad \text{(7-164)}$$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

$$\Delta(\frac{1}{n^2})_i = \sum_{i=1}^{3} r_{ij} E_j$$

$$\left[r_{ij}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

在存在E时，$(\frac{1}{n^2})_{1,2,3}\Big|_{E\neq 0} = (\frac{1}{n^2})_{1,2,3}\Big|_{E=0}$

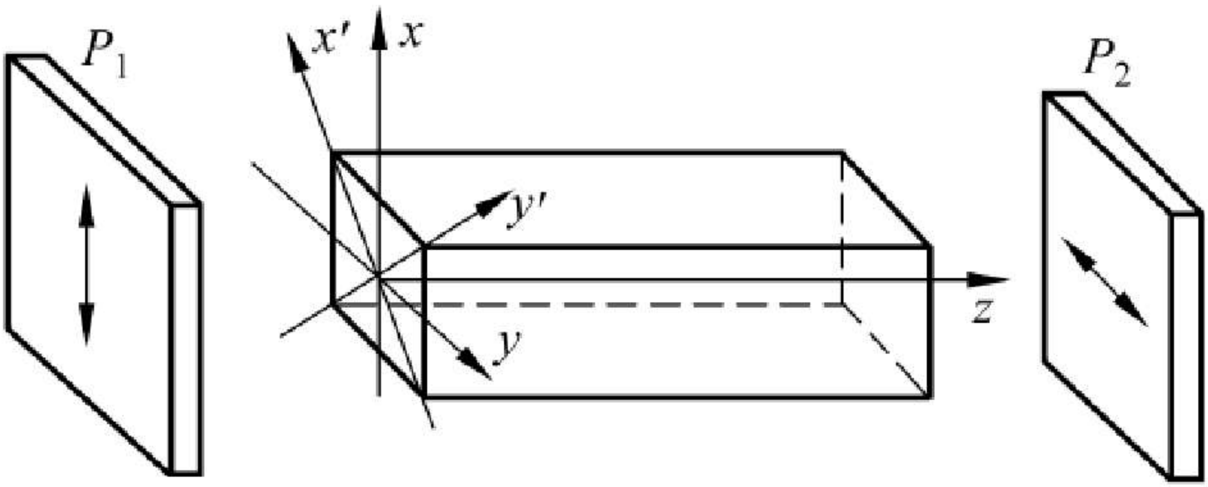
但会新增x, y, z的交叉项，折射率椭球变成

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x \cdot yz + 2r_{41}E_y \cdot xz + 2r_{63}E_z \cdot xy = 1$$

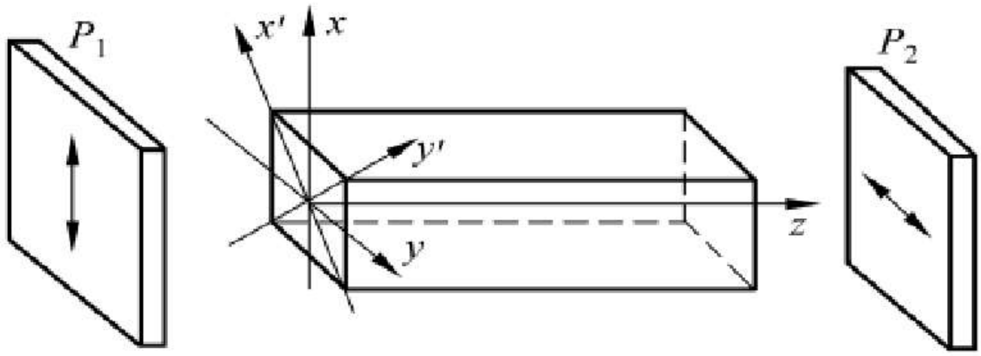
$$\cdots \quad \text{(7-170)}$$

7.9.4 KDP的纵向Pockel's效应

定义：当外加电场 **E** 的方向与光的传播方向平行时，它所产生的线性电光效应称为纵向 **Pockel's** 效应 。



$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}+2r_{41}E_x\cdot yz+2r_{41}E_y\cdot xz+2r_{63}E_z\cdot xy=1 \ldots \text{(7-170)}$$



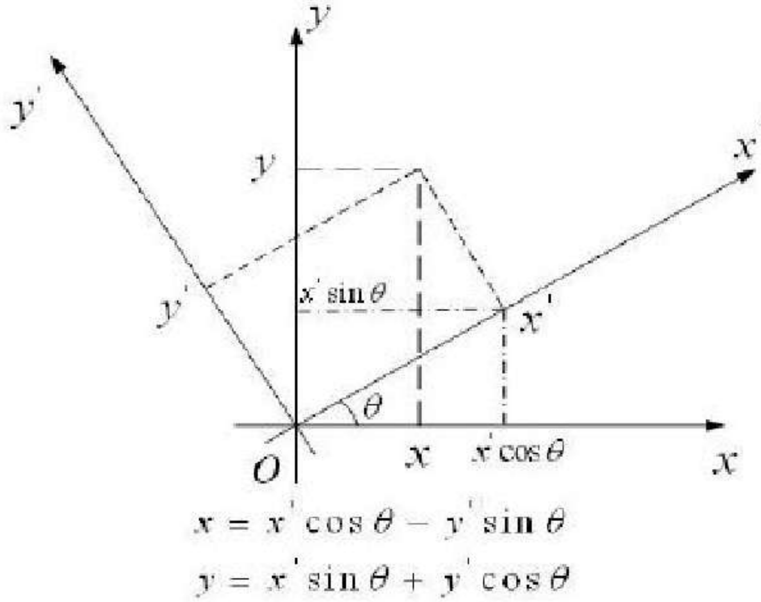
$\because \quad \vec{E} \parallel Z$

$\therefore \quad E_x = E_y = 0,$ (7-170) 变成

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}+2r_{63}E_z\cdot xy=1 \cdots \quad (7-171)$$

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}+2r_{63}E_z\cdot xy=1$$

使坐标系绕 Z 轴旋转 $\pi/4$ ，即作变换



$$\left.\begin{aligned} x &= x'\cos(\pi/4)-y'\sin(\pi/4) \\ y &= x'\sin(\pi/4)-y'\cos(\pi/4) \\ z &= z' \end{aligned}\right\} \quad (7\text{-}172)$$

(7-171) 变成 $$(\frac{1}{n_o^2}+r_{63}E_z)x'^2+(\frac{1}{n_o^2}-r_{63}E_z)y'^2+\frac{1}{n_e^2}z'^2=1 \quad (7\text{-}173)$$

$$(\frac{1}{n_o^2}+r_{63}E_z)x'^2+(\frac{1}{n_o^2}-r_{63}E_z)y'^2+\frac{1}{n_e^2}z'^2=1$$

令 $$\frac{1}{n_o^2}+r_{63}E_z=\frac{1}{n_x'^2}$$

$$\Rightarrow \frac{1}{n_x'^2}-\frac{1}{n_x^2}=r_{63}E_z=\Delta\ (\frac{1}{n_x^2})=-2\frac{\Delta n_x}{n_x^3}=-2\frac{\Delta n_x}{n_o^3}$$

$$\therefore \quad \Delta n_x=-\frac{1}{2}n_o^3 r_{63}E_z \qquad \text{(7-176)}$$

同理 $$\Delta n_y=\frac{1}{2}n_o^3 r_{63}E_z$$

$$\Delta n_y=0$$

$$(\frac{1}{n_o^2}+r_{63}E_z)x'^2+(\frac{1}{n_o^2}-r_{63}E_z)y'^2+\frac{1}{n_e^2}z'^2=1$$

$$\Delta n_x=-\frac{1}{2}n_o^3r_{63}E_z \quad \Delta n_y=\frac{1}{2}n_o^3r_{63}E_z$$

在新坐标系下的主折射率为

$$\left.\begin{aligned} n_{x'}&=n_o-\frac{1}{2}n_o^3r_{63}E_z\\ n_{y'}&=n_o+\frac{1}{2}n_o^3r_{63}E_z\\ n_{z'}&=n_e \end{aligned}\right\} \quad (7\text{-}177)$$

折射率椭球方程为

$$\frac{x'^2}{n_{x'}^2}+\frac{y'^2}{n_{y'}^2}+\frac{z'^2}{n_e^2}=1 \quad (7\text{-}178)$$

x'^2 y'^2 z'^2

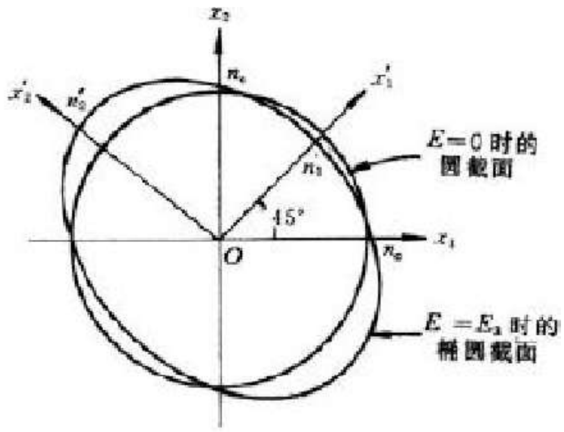


图 5-2 折射率椭球与 x_1Ox_3 面的交线

n

n

注

*

$\Delta n_y > 0$； $\vec{E}$沿z轴负向时，$\Delta n_x > 0$，$\Delta n_y < 0$。

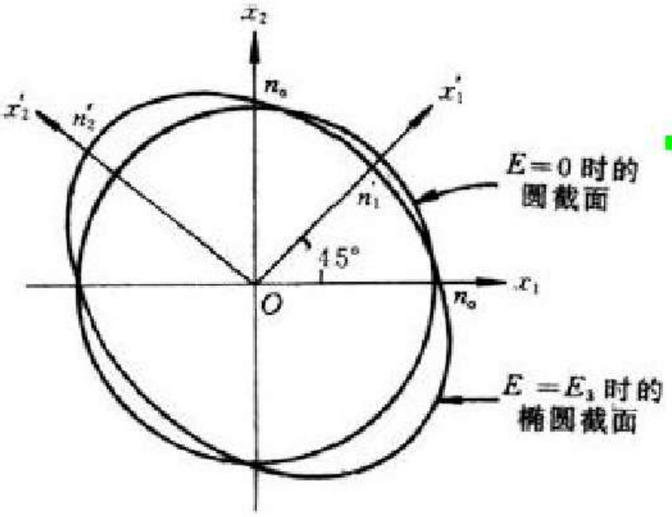
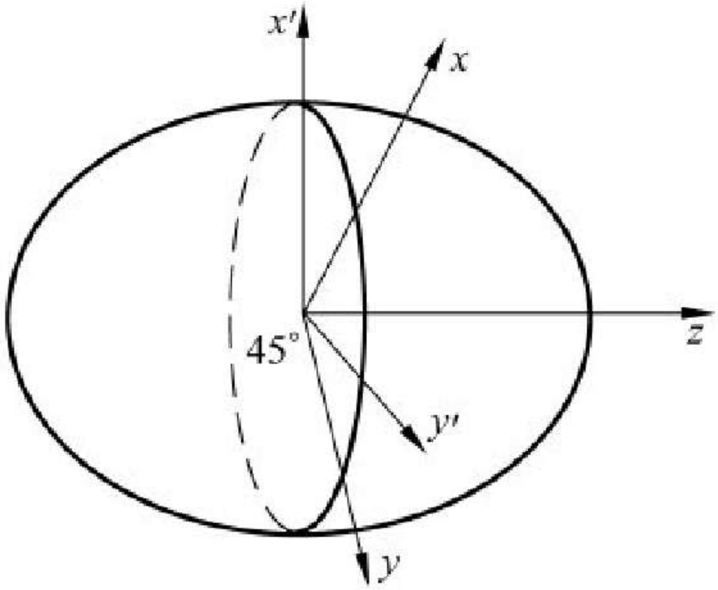


图 5－2 折射率椭球与 x_1Ox_2 面的交线

$$n_{x'}' = n_x - n_o^3 r_{63} E_z / 2$$

$$n_{y'}' = n_y + n_o^3 r_{63} E_z / 2$$



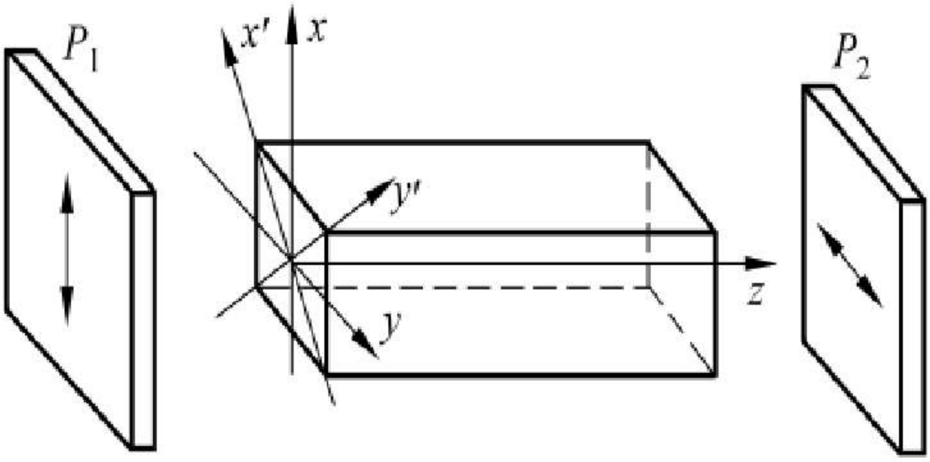
电光延迟

- **KDP 电 光 延 迟 所 产生的相位差为**

$$n_x' = n_o - n_o^3 r_{63} E_z / 2$$

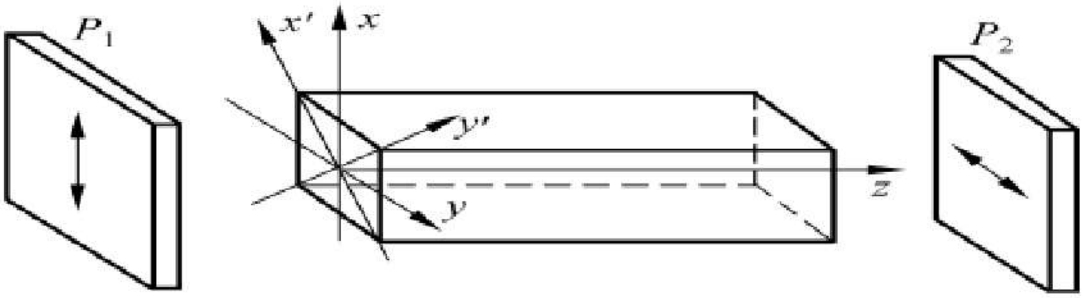
$$n_y' = n_o + n_o^3 r_{63} E_z / 2$$

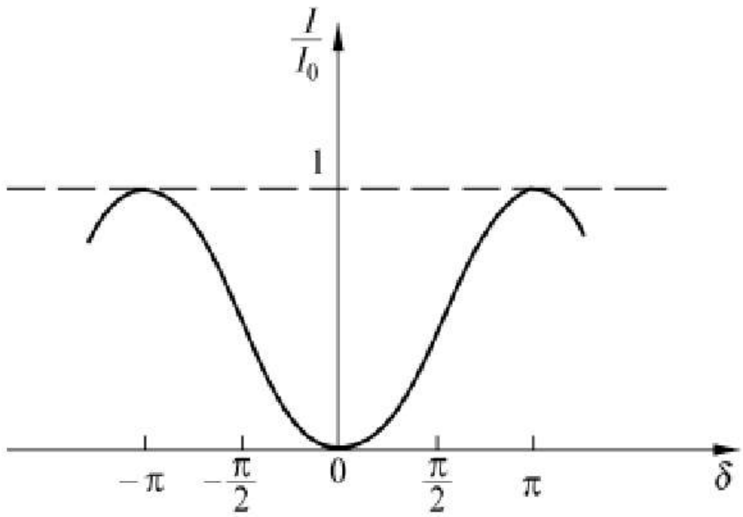
$$\Delta n = n_y' - n_x' = n_o^3 r_{63} E_z$$



$$\delta = \frac{2\pi}{\lambda}\left(n_{y'} - n_{x'}\right)d = \frac{2\pi}{\lambda} n_o^3 r_{63}\, d \cdot E = \frac{2\pi}{\lambda} \cdot n_o^3 r_{63} V \qquad \textbf{(7-180)}$$

P_2透射光强





$$I = I_0 \sin^2 \frac{\delta}{2} = I_0 \sin^2 \left(\frac{\pi}{\lambda} n_o^3 r_{63} V \right)$$

装置的透射率随外加电场地变化如图**7-60**所示

V_π

$$\delta = \frac{2\pi}{\lambda} \cdot n_o^3 r_{63} V$$

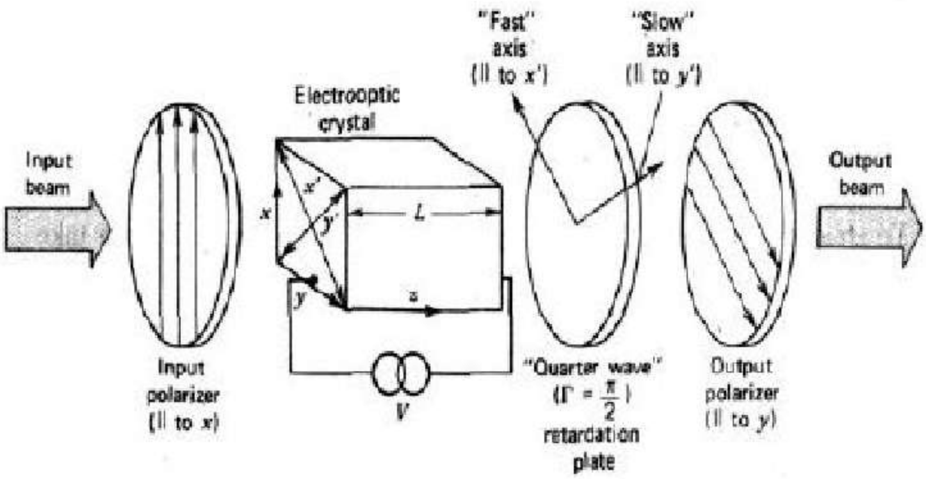


Figure 14.4 A typical electrooptic amplitude modulator. The total retardation Γ is the sum of the fixed retardation bias ($\Gamma_B = \pi/2$) introduced by the "quarter-wave" plate and that caused by the electrooptic crystal.

相应于$\delta=\pi$的电压

$$V_\pi = \frac{\lambda}{2n_o^3 r_{63}} \quad ..(7-182)$$

* 纵向通光劣势V_π高达几千伏

■ 电极材料应能透光 **ITO**（***Indium Tin Oxide***）

7.9.5 KDP晶体的横向Pockel's效应

(光的传播方向与KDP晶体z轴垂直)

344 *The Modulation of Optical Radiation*

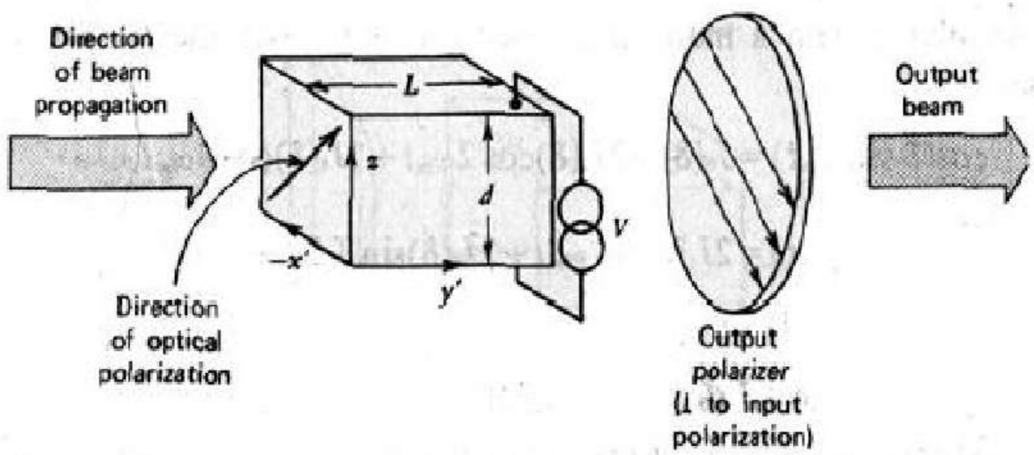
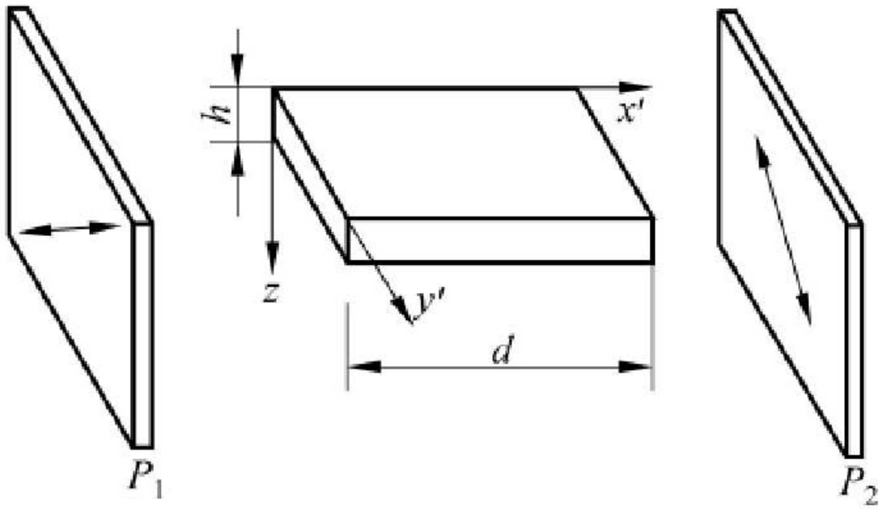


Figure 14.7 A transverse electrooptic amplitude modulator using a KH_2PO_4(KDP) crystal in which field is applied normal to the direction of propagation.

1. KDP的正方形截面的两边分别与x'、y'轴平行，x'轴与光的传播方向平行，外加电场$\boldsymbol{E}$加在 z 轴方向。

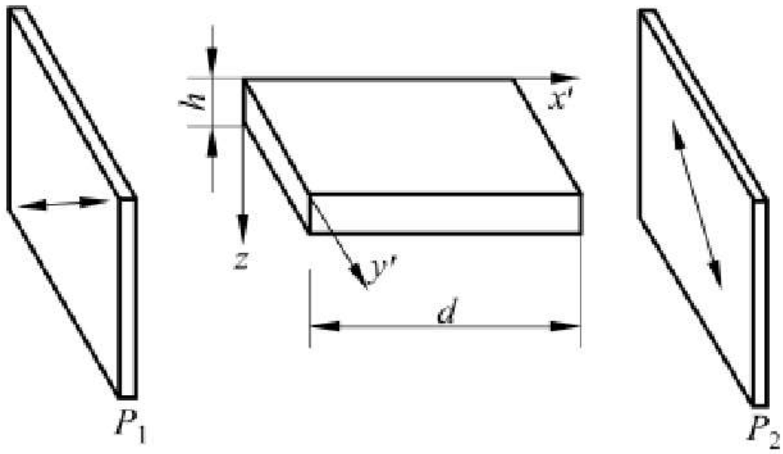


2. 场强与相互作用距离分开控制 $\longrightarrow$ 降低V_π

通过晶体产生位相差

$$n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z$$

$$n'_z = n_e$$

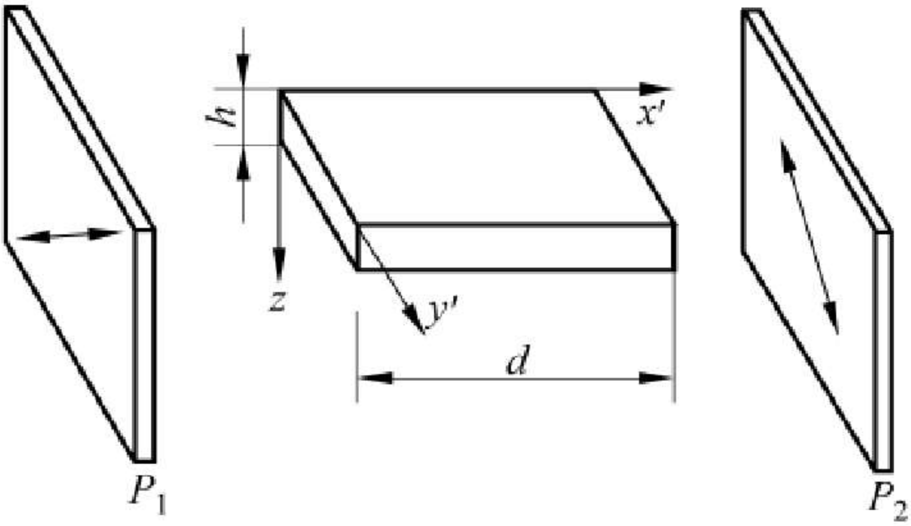


$$\delta = \frac{2\pi}{\lambda}\left(n_{y'} - n'_z\right)d = \frac{2\pi}{\lambda} d[(n_o - n_e) - \frac{1}{2} n_o^3 r_{63} E_z]$$

$$= \frac{2\pi}{\lambda}\left|n_e - n_o\right| d + \frac{\pi}{\lambda} n_o^3 r_{63}\left(\frac{d}{h}\right)V \qquad \textbf{(7-184)}$$

降低V_π

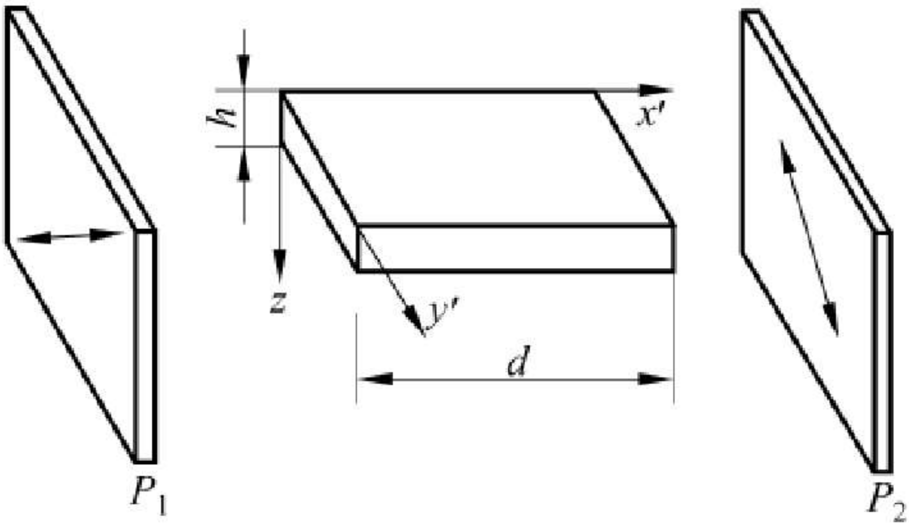
$$\delta = \frac{2\pi}{\lambda}\left|n_e - n_o\right| d + \frac{\pi}{\lambda} n_o^3 r_{63} \left(\frac{d}{h}\right) V$$



- 第二项表线性电光效应，**提高 *d/h* ，可以降低半波电压**

讨论

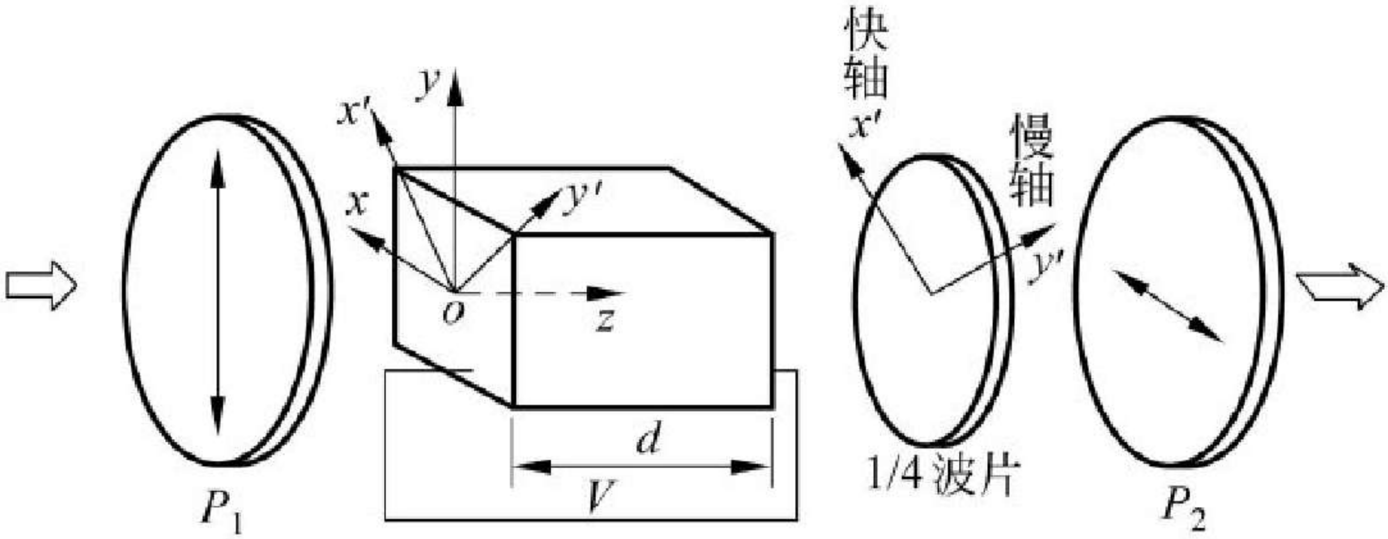
$$\delta = \frac{2\pi}{\lambda}\left|n_e - n_o\right| d + \frac{\pi}{\lambda} n_o^3 r_{63}\left(\frac{d}{h}\right)V$$



■第一项 $\frac{2\pi}{\lambda}(n_o - n_e)L$ 表自然双折射，对温度控制的要求很严格。

7.9.6 电光效应的应用

1. 电光调制



光透过率为

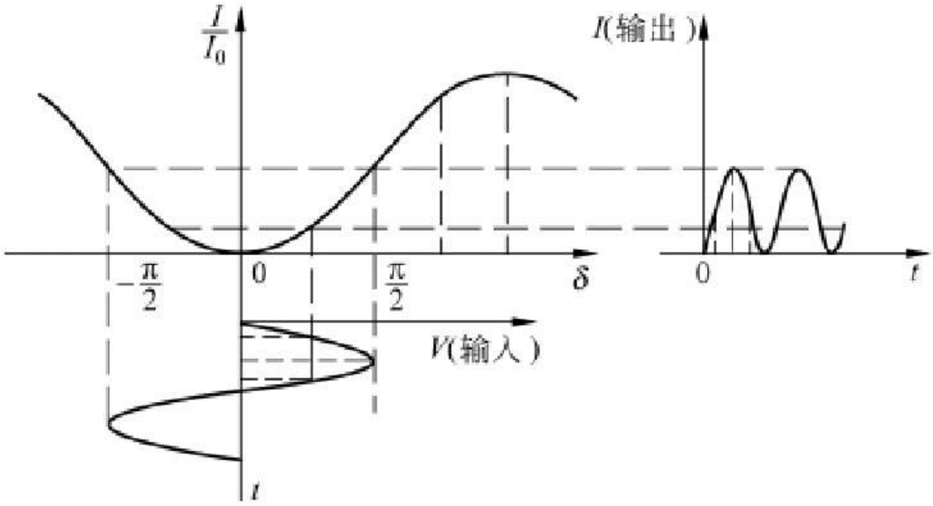
$$\frac{I}{I_o} = \sin^2\frac{\delta}{2} \quad (7\text{-}185)$$

光路中未插入1/4波片时

$$I/I_0 = \sin^2(\delta/2)$$

$$\delta = \pi(V/V_\pi)$$

$$\frac{I}{I_o} = \sin^2\left(\frac{\pi}{2}\frac{V}{V_\pi}\right)$$



加正弦信号 $V = V_0 \sin(\omega_m t)$ 则 $\frac{I}{I_o} = \sin^2\left[\frac{\pi}{2}\frac{V_0}{V_\pi}\sin(\omega_m t)\right]$

(7-189)

插入波片

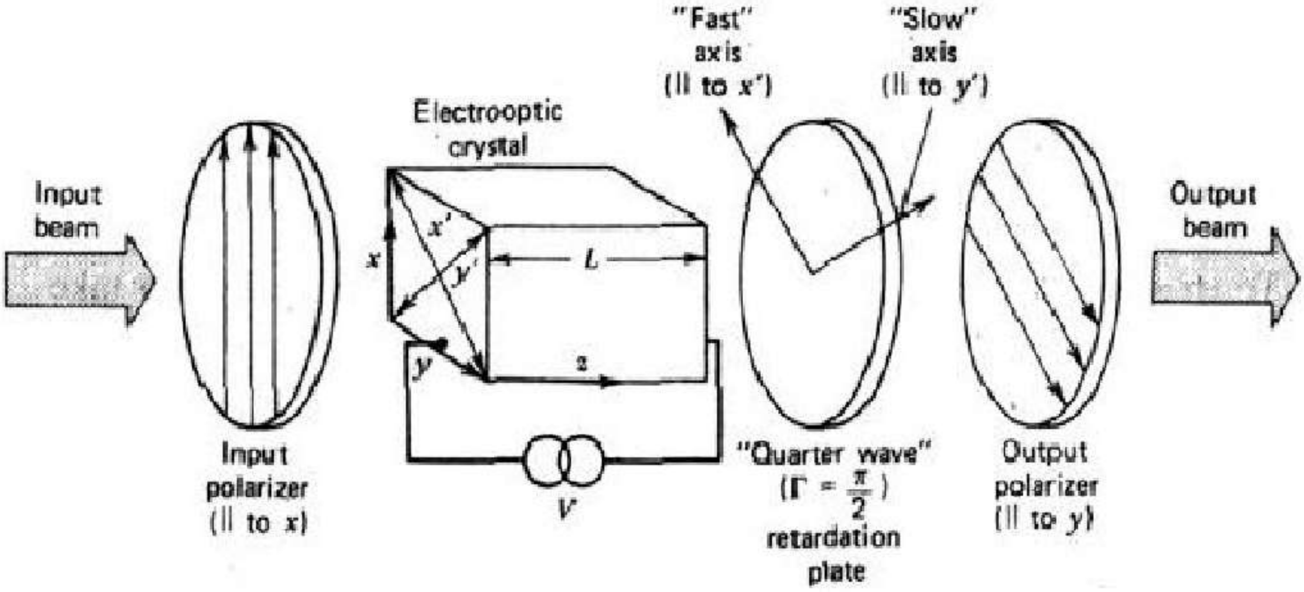


Figure 14.4 A typical electrooptic amplitude modulator. The total retardation Γ is the sum of the fixed retardation bias ($\Gamma_B = \pi/2$) introduced by the "quarter-wave" plate and that caused by the electrooptic crystal.

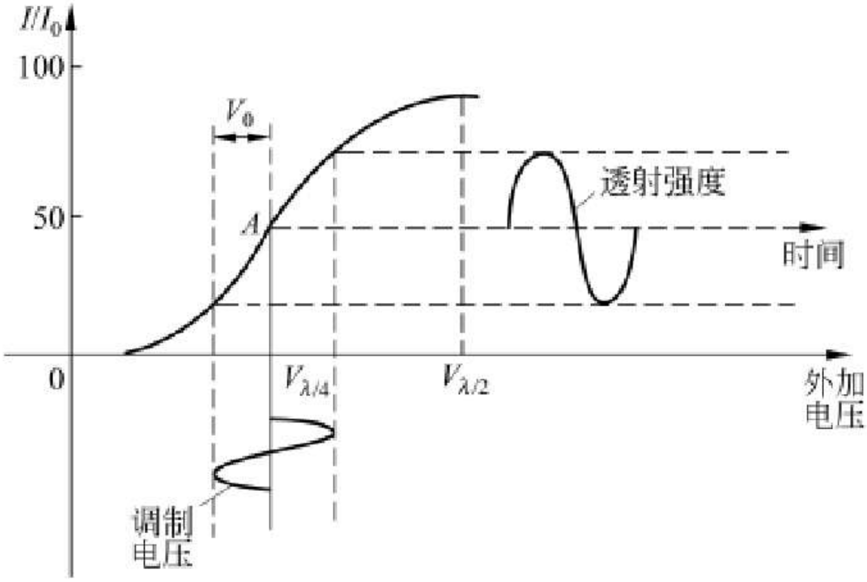
插入 $\lambda/4$ 波片后

$$I/I_0=\sin^2\left[(\pi/2)(V_0/V_\pi)\sin(\omega_m t)\right]$$

$$\frac{I}{I_o}=\sin^2\left[\frac{\pi}{4}+\frac{\pi}{2}\frac{V_0}{V_\pi}\sin(\omega_m t)\right]$$

$$\Rightarrow \frac{I}{I_o}\approx\frac{1}{2}+\frac{\pi}{2}\frac{V_0}{V_\pi}\sin(\omega_m t)$$

(7-189)



7.10 声光效应

Photoelastic Effect

弹光效应：应力或应变使介质光学性质发生变化的现象。

声光效应：超声波引起的弹光效应。

7.10.1 声光衍射

Diffraction of light by acoustic waves

超声波－纵波－

疏密波－光栅

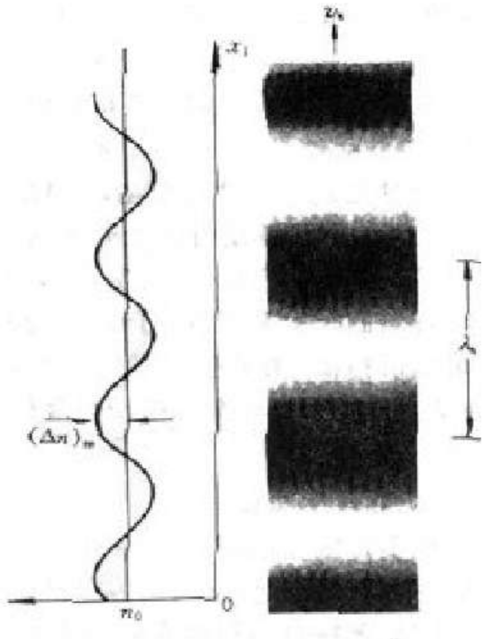
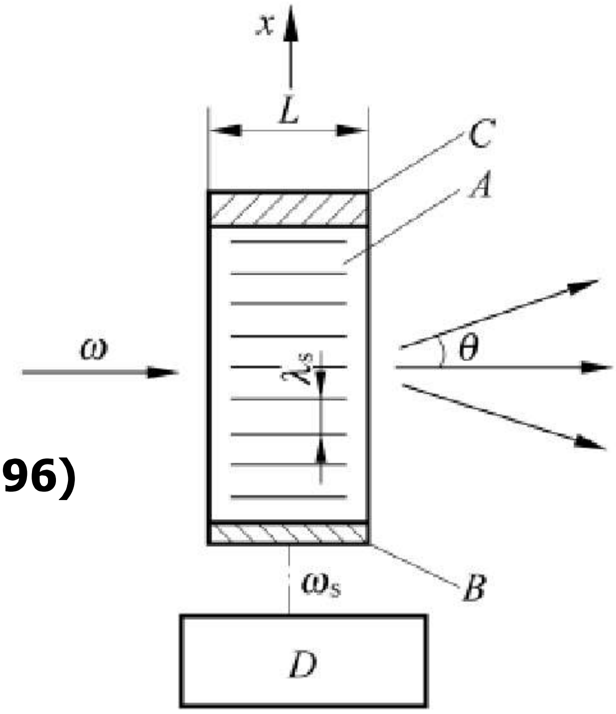


图 5－12 超声行波

折射率变化

- 弹性应变将引起折射率在时间和空间上的周期变化

$$\Delta\left(1/n^2\right)=ps=(-2/n^3)\Delta n$$



$$\Delta n=-n_o^3 ps/2$$

$$=-n_o^3 ps_0\sin\left(k_s x-\omega_s t\right)/2$$

$$=-\left(\Delta n\right)_M \sin\left(k_s x-\omega_s t\right) \qquad \textbf{(7-196)}$$

$$s=s_0\sin\left(k_s x-\omega_s t\right)/2$$

为应变张量. p 为声光系数

两类衍射

- 衡量的两类衍射的参量是 $$Q = 2\pi l \frac{\lambda}{\lambda_s^2}$$ **(7-197)**

$Q \ll 1$ 拉曼-奈斯(Raman—Nath)型衍射

$Q \gg 1$ 布拉格(Bragg) 衍射

7.10.2 拉曼-奈斯声光衍射-薄光栅

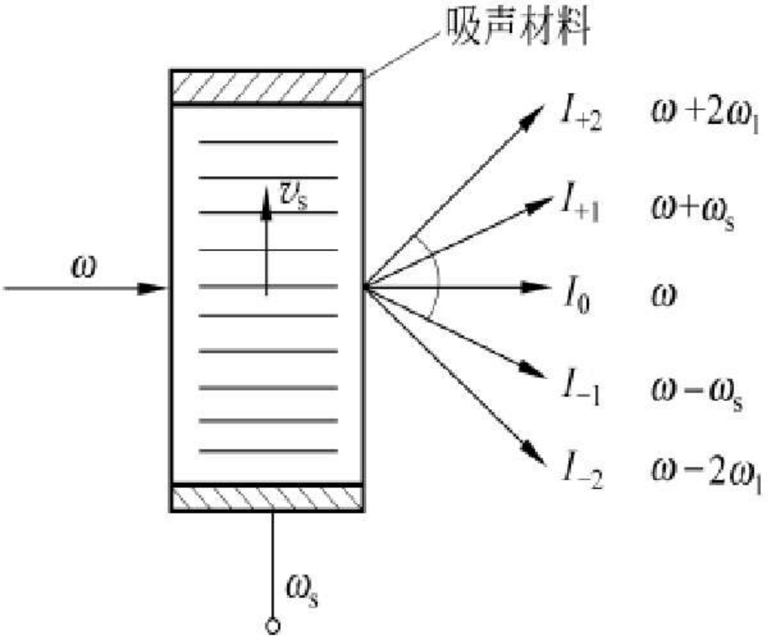
各级衍射极大方向角为 θ

$$d--\lambda_s$$

$$\lambda_s \sin\theta = m\lambda$$

$$\sin\theta = m \cdot \lambda/\lambda_s$$

$$m = 0, \pm 1, \pm 2, \cdots \quad \textbf{(7-198)}$$

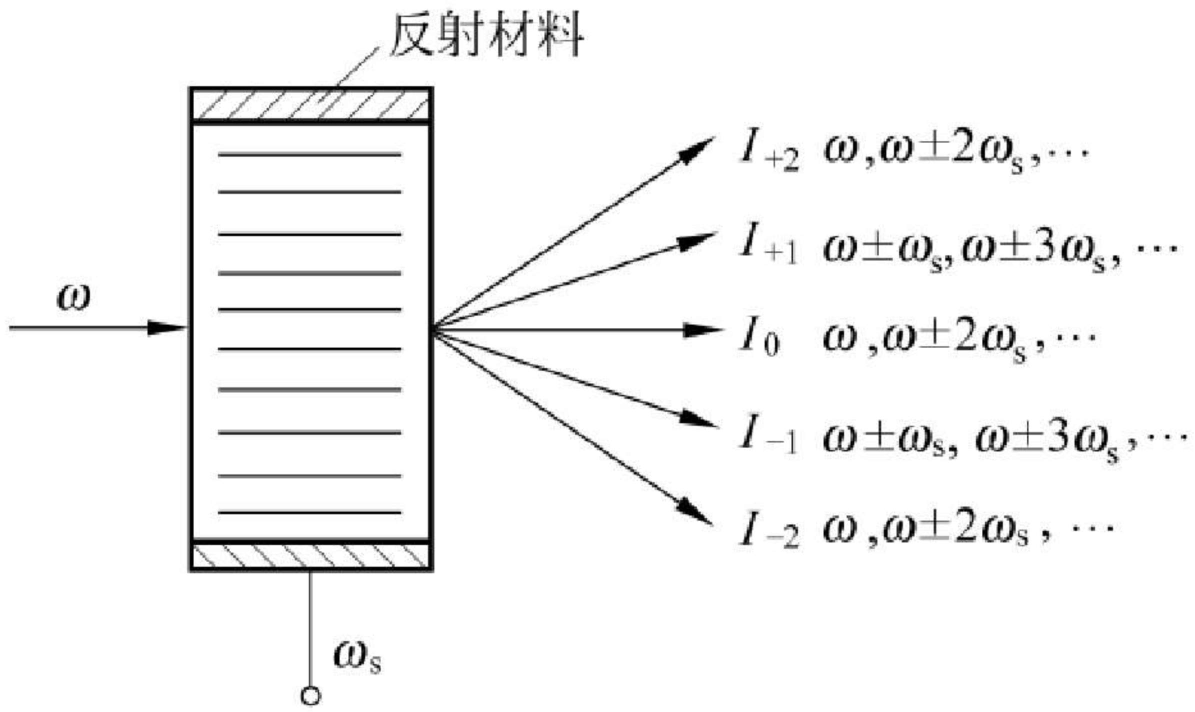


第m级衍射的极值光强为

$$I_m = I_i J_m^2(\delta) \quad \textbf{(7-199)}$$

I_i 为入射光强，δ 为附加光程差

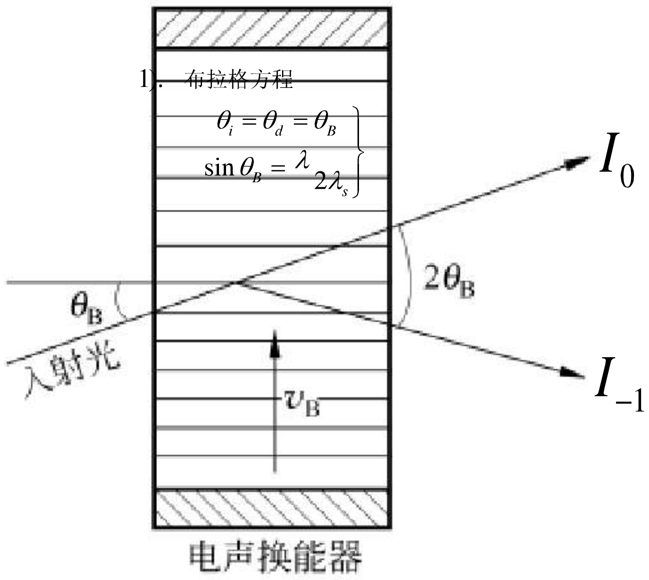
薄光栅


反射材料
ω
ω_s
I_{+2} $\omega,\omega\pm2\omega_s,\cdots$
I_{+1} $\omega\pm\omega_s,\omega\pm3\omega_s,\cdots$
I_0 $\omega,\omega\pm2\omega_s,\cdots$
I_{-1} $\omega\pm\omega_s,\omega\pm3\omega_s,\cdots$
I_{-2} $\omega,\omega\pm2\omega_s,\cdots$

7.10.3 布拉格衍射-厚光栅

1. 布拉格方程

$$\left.\begin{aligned}\theta_i = \theta_d = \theta_B \\ \sin\theta_B = \lambda/(2\lambda_s)\end{aligned}\right\}$$ **(7-203)**



产生衍射极大的条件

θ_B布拉格角

$$2\lambda_s \sin\theta = m\lambda\ ,\quad m = 0, \pm 1$$ **(7-202)**

2. 布拉格衍射光强

零级衍射光强 $I_0 = I_i \cos^2(\delta/2)$ **(7-205)**

一级衍射光强 $I_{\pm 1} = I_i \sin^2(\delta/2)$ **(7-206)**

I_i 为入射光强度， δ 为折射率变化引起的附加相移

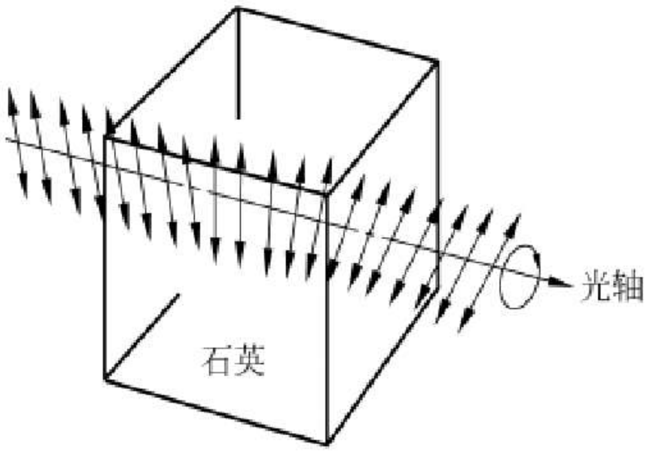
如 $\delta = \pi$， 则 $I_0 = 0$， $I_{\pm 1} = I_i$

$\Longrightarrow$ 转换效率很高的声光器件

7.11 旋光现象

7.11.1 旋光现象的观察和规律

1. 旋光现象



$$\theta = \alpha L \qquad \textbf{(7-209)}$$

$$\theta = \alpha c L \quad (溶液) \qquad \textbf{(7-210)}$$

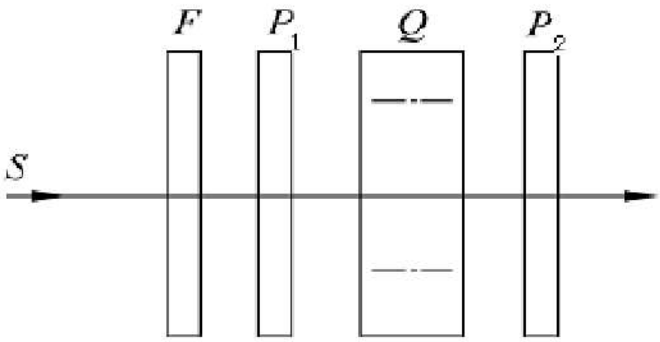
α —旋光率，

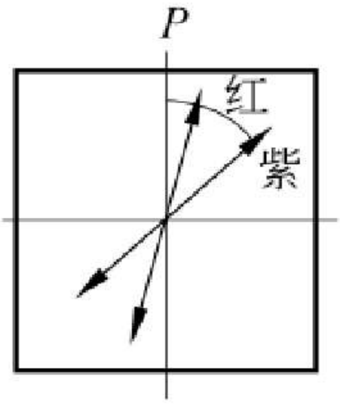
c —浓度

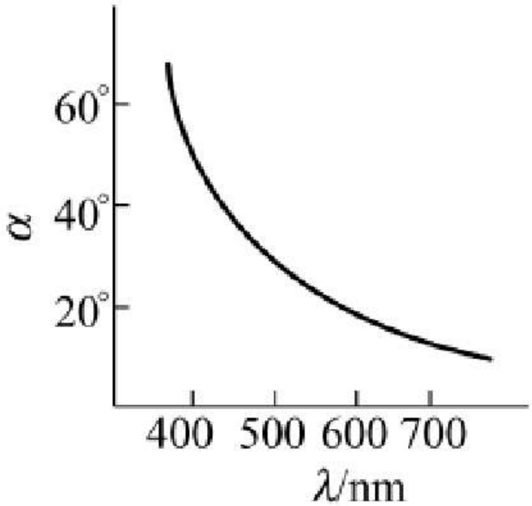
2. 晶体旋光现象的观察

F 滤光片　　P_1 偏振片

Q 石英晶片　P_2 检偏器

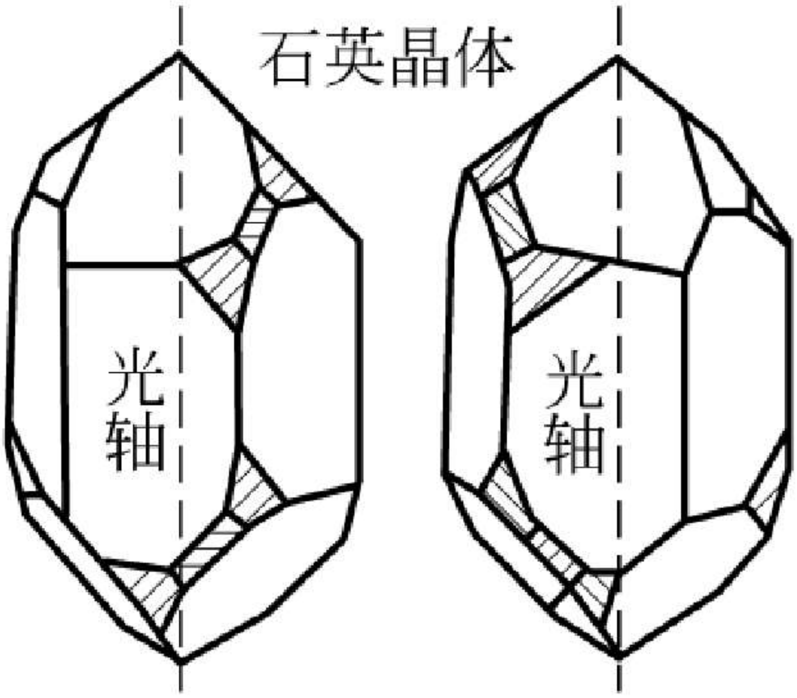






3. 旋光色散

4. 旋光现象和物质的结构有关

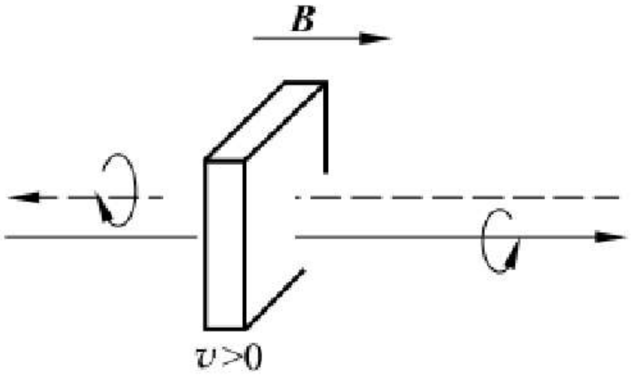


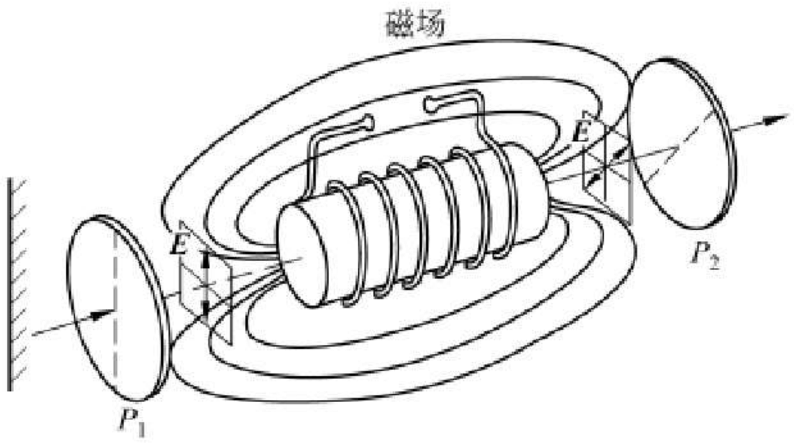
5. 有的液体或溶液也有旋光现象

7.12 磁致旋光效应 Faraday effect

7.12.1 法拉第磁致旋光效应

1. 观察与实验





2. 描述公式

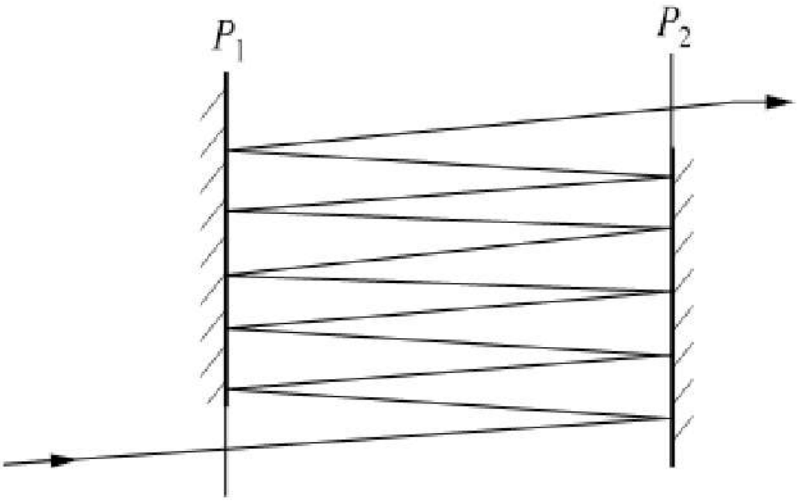
$$\theta = VBd \qquad (7\text{-}218)$$

B 磁感应强度

V Verdet常数，与物质有关

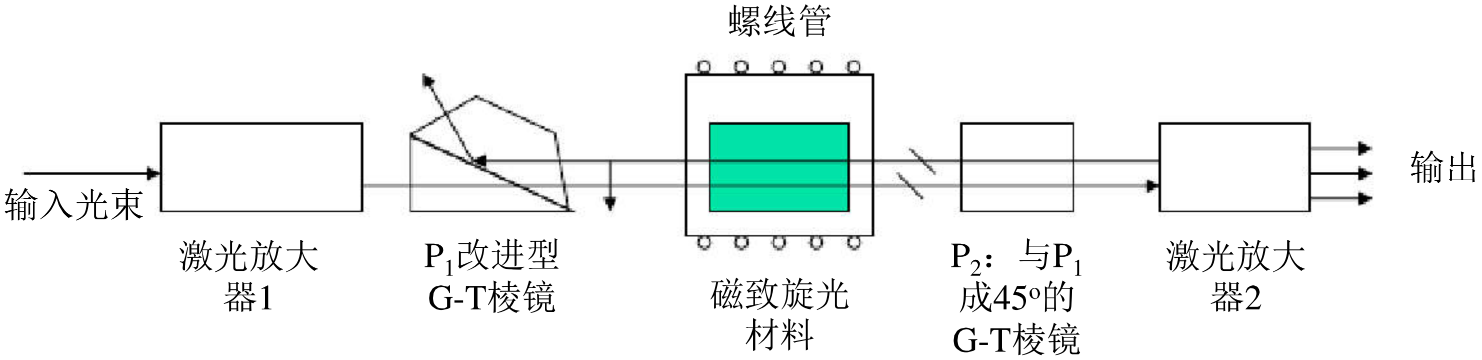
3. 特点

实际空间旋向决定于外加磁场，与光的传播方向无关。



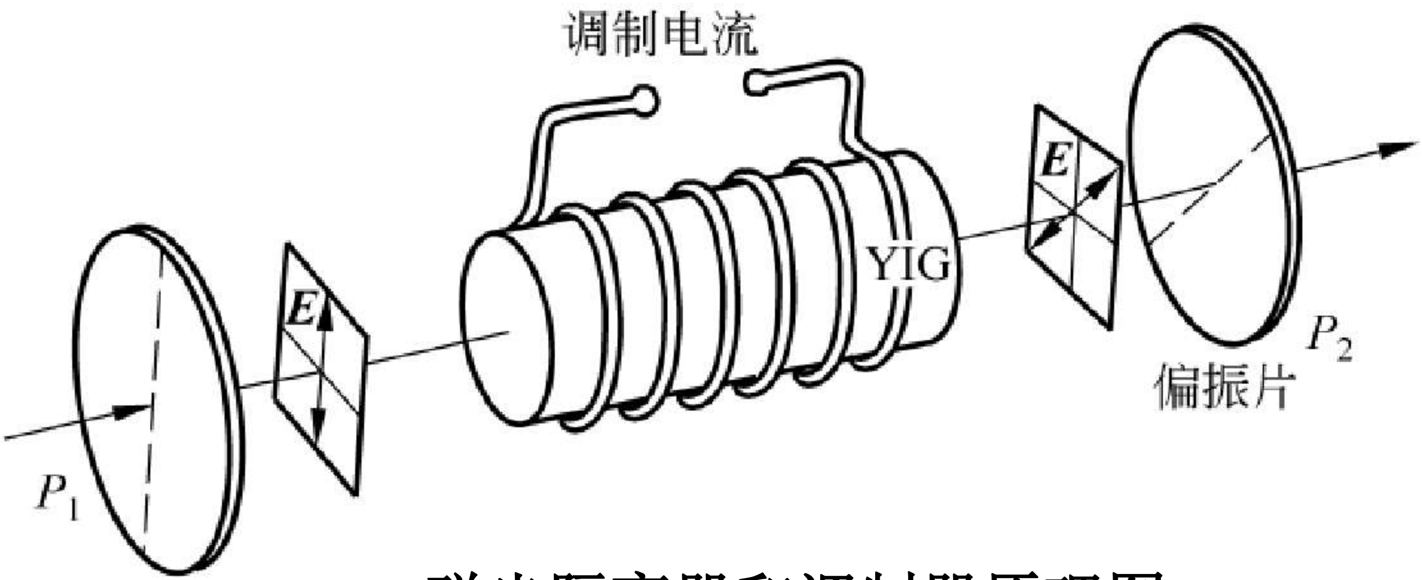
利用多次反射加强法拉第效应

Faraday 光隔离器应用：




See our NEW website at www.ofr.com
ISOLATORS
Size, Power and Wavelength-OFR has it covered.
OFR
INPUT
Features:
• > 64 Models
• Wavelengths: (350nm-10.6μm)
• Tunable, Fixed & Broadband
• mW-GW power handling
• Several polarizer options
• Small packages
• Large apertures
• OEM options

光传感器（磁场，电流……）



磁光隔离器和调制器原理图