JOURNAL OF FOREST SCIENCE, *62*, 2016 (9): 413–421

doi: 10.17221/51/2016-JFS

Generalized height-diameter models for *Fagus orientalis* Lipsky in Hyrcanian forest, Iran

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ABSTRACT: In this study, 39 generalized height-diameter prediction models were developed for Oriental beech (*Fagus orientalis* Lipsky) in the Hyrcanian forest in Iran. Data were collected from 75 permanent sample plots in uneven-aged stands of *F. orientalis*. A total of 1,067 individual tree height-diameter measurements were available for this study. For model testing a 10-fold cross validation method was used. The goodness of fit of the models was evaluated using six statistical measures including Akaike information criteria, Bayesian information criterion, root mean square error (RMSE), mean error, R^2 and R^2_{adj}. Results showed that the R^2 ranged from 0.62 to 0.78 and RMSE from 3.3 to 4.7 in the validation phase. Considering all the performance criteria, a model which uses DBH, dominant height, basal area per hectare and number of trees per hectare was found to be the best model to predict the height of Oriental beech from these data.

Keywords: Oriental beech; dominant height; forest models; evaluation; validation

Northern forests of Iran are important sources of genetic variation, biodiversity, commercial wood products, and various environmental services (Poorzady, Bakhtiari 2009). Covering about 1.85 million ha, these forests account for 15% of the total Iranian forests and 1.1% of the country's area. These forests are found from sea level to an elevation of 2,800 m a.s.l. and comprise various forest types because of their 80 woody species (trees and shrubs). One of the most abundant and economically important hardwood genera in the Hyrcanian forests is the genus *Fagus* Linnaeus. Beech forests account for approximately 17.6% of the total forest area, 30% of the standing volume and 23.6% of the stem number in the Hyrcanian forests in Iran (Ahmadi et al. 2013).

The relationship between tree height and DBH is an important element of forest structure that has long been used to characterize important aspects of tree growth, stand development and that helps define relationships among individuals. Height-diameter models are used to calculate measurable quantities needed to characterize forest growth and yield (Tewari et al. 2014) especially volume, weight and biomass. For inventory purposes, it is common practice to measure DBH on all trees in a plot while limiting the measurement of height to a subsample of trees because measurements of height can be more difficult and time consuming (Ahmadi et al. 2013). There are many height-diameter equations for different tree species in various regions that can be fitted to linear or non-linear functions. In most of these models DBH is a predictor variable for estimating total tree height. Additional stand variables are often included to improve prediction for a wide range of stand conditions and management practices. Therefore, a single height-diameter relationship may not be useful in all the possible situations that can be found in different stands (Lappi 1997; Eerikäinen 2003).

One alternative is to develop a height-diameter model separately for each stand. Although this approach may provide accurate estimates for an individual stand, it is time consuming (inefficient) and expensive. Another method that includes DBH while adding other individual tree and stand variables such as age, site index, stand basal area, stand density and dominant or mean height has been used (Newton, Amponsah 2007). This approach allows calibration of developed models for a wider range of stand conditions (Calama, Montero 2004; Sánchez-González et al. 2008).

In several studies the inclusion of additional stand-level variables improved tree height estimates (Bi et al. 2000; Staudhammer, LeMay 2000; López Sánchez et al. 2003; Sharma, Zhang 2004). For example, relative tree position variables were used by Temesgen and von Gadow (2004), and Newton and Amponsah (2007) utilized stand dominant height as a measure of stand-level competition. Temesgen et al. (2007) stated that stand-level basal area per hectare and crown competition were useful variables for predicting tree height. Krisnawati et al. (2010) found that using a generalized height-diameter model with stand-level variables of age and site index led to the best model based on fit and model validation. Temesgen et al. (2014) used nine models for estimating height as a function of individual tree diameter and site variables. A previous study in Tarbiat Modares University (TMU) forests showed that the Richards, Weibull, and Schnute functions provided satisfactory height predictions, but the Richards function was recommended for further analysis due to having better performance (Ahmadi et al. 2013).

This study aims at developing equations that can be used to predict the height-diameter relationship in *Fagus orientalis* Lipsky stands in Hyrcanian forests, Iran, by considering a number of stand variables which may influence the relationship and select an appropriate model for further validation and estimation of Oriental beech tree height.

MATERIAL AND METHODS

The study was carried out in Hyrcanian forests in Mazandaran Province, northeastern Iran. Data used in this study were collected on 75 circular sample plots of 0.1 ha in size established in TMU forests between 36°31'56''N and 36°32'11''N latitudes and 51°47'49''E and 51°47'56''E longitudes.

The study area consists of mixed and uneven-aged forests, dominated by *F. orientalis* associated with *Carpinus betulus* Linnaeus, *Acer velutinum* Boissier, *Parrotia persica* C.A. Meyer, *Sorbus torminalis* (Linnaeus) Crantz, *Quercus castaneifolia* C.A. Meyer, *Alnus subcordata* C.A. Meyer, *Acer laetum* C.A. Meyer, *Prunus avium* (Linnaeus) Linnaeus, *Ulmus glabra* Hudson and *Tilia begoniifolia* Steven species. The forests are managed following close-to-nature principles with single selection harvesting techniques. The bedrock is mainly limestone-dolomite with a silty-clay-loam soil texture (Ahmadi et al. 2013).

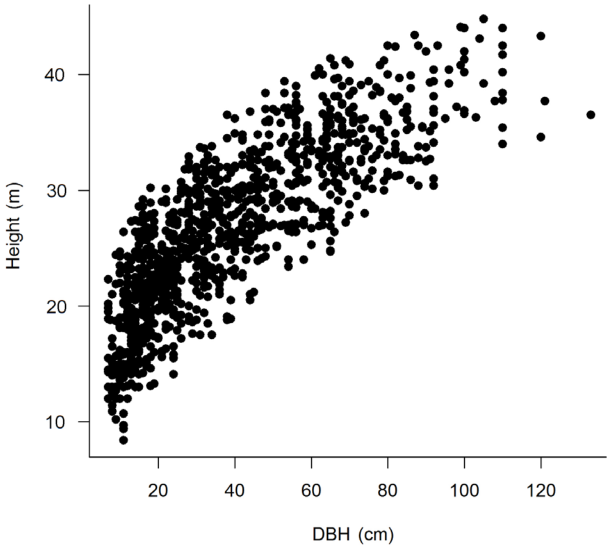

Fig. 1. Scatter plot of total height against DBH for 1,067 Oriental beech sample trees

For each of the sample plots, the minimum tree diameter recorded was 7 cm. At plot establishment, the following data were recorded for Oriental beech trees within each plot: compartment number, DBH at 1.30 m, and total height. Diameter was measured with a calliper and total height was measured using a Vertex IV instrument (Haglöf Sweden AB, Långsele, Sweden). A total of 1,067 individual tree height-diameter measurements were available for this study (Fig. 1).

The mean, minimum and maximum values and standard deviations of the stand variables are given in Table 1.

Stand variables such as quadratic mean diameter, a measure of central tendency, was measured us-

Table 1. Descriptive statistics for 1,067 Oriental beech sample trees in the Tarbiat Modares University forest in northeastern Iran

Variable	Mean	Min	Max	SD
DBH (cm)	39.60	8	133	23.63
Height (m)	25.05	8.2	46.10	7.60
BAL (m^2)	2.56	0	5.67	1.20
BA (m^2)	3.52	2.40	5.68	0.80
N	284	90	540	108
D_{max} (cm)	87.89	67	133	16.10
D_{dom} (cm)	69.94	52.2	91	10.75
H_{dom} (m)	31.49	25.12	39.59	3.31
DQ (cm)	44.80	26.22	68.17	10.06
H_{mean} (m)	25.10	20.32	32.66	3.08

BAL – basal area in larger trees, BA – stand basal area, *N* – number of trees per hectare, D_{max} – maximum diameter in each plot, D_{dom} – dominant diameter in each plot, H_{dom} – dominant height in each plot, DQ – quadratic diameter at breast height, H_{mean} – mean of height in each plot ing the Marshall and Curtis (2002) method. The average values of the five largest trees in each plot were considered as dominant diameter and height. The basal area in larger trees is another variable sometimes used as a competition measure in growth models. It is defined as total basal area per acre in trees that are larger than the subject tree (the tree for which a prediction is being made). This variable was calculated according to Wykoff et al. (1982).

A wide variety of generalized height-diameter models have been discussed in the literature for modelling the relationships between tree height and DBH by additional site variables. A complete list of selected generalized height-diameter models used in the present study is given in Table 2. In total, 39 candidate models were considered.

The nls (nonlinear least squares) function in free statistical R software package (Version 3.1.2, 2013) was used to fit the 39 candidate models. The assumption of homoscedasticity was investigated by plotting the fitted values versus the residuals (Ritz, Streibig 2008). For model validation, a *k*-fold cross validation method (R Development Core Team 2013) with k = 10 was used. Using the value of 10 has been the most common practice (Olson, Delen 2008; Diamantopoulou, Özçelik 2012). In this method the data are randomly partitioned into k equal subsamples with one subsample considered as the validation data, and the other subsamples (k – 1) used for parameter estimation.

Model performance criteria. The goodness of fit of the models was evaluated using six statisti-

Table 2. Generalized height-diameter functions and additional variables

Model	Additional variable	Expression	Reference
1	BAL-*N*-BA	$Height = 1.3 + e^{a1 + a2\,BAL + \frac{a3 + a4\,BAL + a5\,N + a6\,BA}{DBH + 1}}$	Temesgen and von Gadow (2004)
2	BAL-*N*-BA	$Height = 1.3 + e^{a1 + a2\,BAL + \frac{a3 + a4\,BAL + a5\,N + a6\,BA}{DBH + c}}$	Ratkowsky (1990)
3	DQ-BA	$Height = 1.3 + (a1 + a2\,BA)\,e^{b\left(1 - \frac{DBH}{DQ}\right) DBH^{c}}$	Hui and von Gadow (1993)
4	H_{dom}-DQ	$Height = a1\,Hdom\left(1 - a2\,e^{-\frac{b\,DBH}{DQ}}\right)^{a3}$	Schröder and Álavarez González (2001)
5		$Height = 1.3 + (a1 + a2\,Hdom - a3\,DQ)\,e^{-\frac{b}{DBH}}$	
6		$Height = 1.3 + (a1 + a2\,Hdom - a3\,DQ)\,e^{-\frac{b}{\sqrt{DBH}}}$	
7	H_{dom}-DQ-BA	$Height = 1.3 + (a1 + a2\,Hdom - a3\,DQ + a4\,BA)\,e^{-\frac{b}{\sqrt{DBH}}}$	Schröder and Álavarez González (2001)
8	H_{dom}-D_{dom}	$Height = 1.3 + \frac{1}{\left[a1\left(\frac{1}{DBH} + \frac{1}{Ddom}\right) + \left(\frac{1}{Hdom - 1.3}\right)^{1/3}\right]^{3}}$	Mønness (1982)
9	H_{dom}-D_{dom}	$Height = 1.3 + (Hdom - 1.3)\left(\frac{DBH}{Ddom}\right)^{a1}$	Cañadas et al. (1999)
10	D_{dom}-H_{dom}	$Height = 1.3 + \frac{DBH}{\frac{Ddom}{Hdom - 1.3} + a1\,(Ddom - DBH)}$	Cañadas et al. (1999)
11	H_{dom}-D_{dom}	$Height = 1.3 + \frac{(Hdom - 1.3)\left(1 - e^{a1\,DBH}\right)}{1 - e^{a1\,Ddom}}$	Cañadas et al. (1999)
12		$Height = 1.3 + \frac{1}{\left(a1\left(\frac{1}{DBH} - \frac{1}{Ddom}\right) + \sqrt{\frac{1}{Hdom - 1.3}}\right)^{2}}$	
13	H_{dom}-DQ	$Height = 1.3 + (Hdom - 1.3)\,e^{a1\left(1 - \frac{DQ}{DBH}\right) + a2\left(\frac{1}{DQ} - \frac{1}{DBH}\right)}$	Gaffrey (1988)
14	H_{mean}-DQ	$Height = 1.3 + (Hmean - 1.3)\,e^{a1\left(1 - \frac{DBH}{DQ}\right)}\,e^{a2\left(\frac{DBH}{DQ} - \frac{1}{DBH}\right)}$	Sloboda et al. (1993)
15	H_{dom}	$Height = 1.3 + a1\,Hdom^{a2}\,DBH^{a3\,Hdom^{a4}}$	Hui and von Gadow (1993)
16	H_{dom}-DQ	$Height = 1.3 + (a1 + a2\,Hdom - a3\,DQ)\,e^{-\frac{a4}{DBH}}$	López Sánchez et al. (2003)

Table 2. to be continued

Model	Additional variable	Expression	Reference
17	H_{dom}-DQ	$Height = 1.3 + (a1 + a2\,Hdom - a3\,DQ)\,e^{-\frac{a4}{\sqrt{DBH}}}$	Schröder and Álavarez González (2001)
18	H_{mean}-DQ	$Height = Hmean\left(a1 + a2\,Hmean + \frac{a3\,Hmean}{DQ} + a4\,DBH + \frac{a5\,N\,DBH}{DQ^2\,Hmean}\right)$	Cox (1994)
19		$Height = a1 + a2\,Hmean + a3\,DQ^{0.95} + a4\,e^{-0.08\,DBH} + a5\,Hmean^3\,e^{-0.08\,DBH} + a6\,DQ^3\,e^{-0.08\,DBH}$	
20	H_{dom}-D_{dom}	$Height = 1.3 + (Hdom - 1.3)\,e^{\frac{a1\,(1 - Ddom)}{DBH} + a2\left(\frac{1}{Ddom} - \frac{1}{DBH}\right)}$	Crecente-Campo et al. (2010)
21	BA-N	$Height = 1.3 + a1\,BA^{a2}\left(1 - e^{-a3\,N^{a4}\,DBH}\right)$	Sharma and Zhang (2004)
22		$Height = 1.3 + a1\,BA^{a2}\left(1 - e^{-a3\,N^{a4}\,DBH}\right)^{a5}$	
23	H_{dom}-D_{dom}	$Height = a1 + Hdom\left(1 - e^{-\frac{a2\,DBH}{\ln(Ddom)}}\right)^{a3}$	Sonmez (2009)
24	H_{dom}-D_{dom}	$Height = \frac{\left(1 - e^{a1\,DBH}\right)^{a2}}{\left(1 - e^{a1\,Ddom}\right)^{a2}}\,Hdom$	Cimini and Salvati (2011)
25		$Height = \frac{\left(1 - e^{a1\,DBH}\right)^{a2 + a3\,Hdom}}{\left(1 - e^{a1\,Ddom}\right)^{a2 + a3\,Hdom}}\,Hdom$	
26		$Height = \frac{\left(1 - e^{(a1 + a2\,Hdom)\,DBH}\right)^{a3 + a4\,Hdom}}{\left(1 - e^{(a1 + a2\,Hdom)\,Ddom}\right)^{a2 + a3\,Hdom}}\,Hdom$	
27	H_{dom}-D_{dom}	$Height = \left(1.3^{a1} + \frac{\left(Hdom^{a1} - 1.3^{a1}\right)\left(1 - e^{-a2\,DBH}\right)}{1 - e^{-a2\,Ddom}}\right)^{\frac{1}{a1}}$	Stankova and Diéguez-Aranda (2013)
28		$Height = 1.3 + \frac{1}{\left(a1\left(\frac{1}{DBH} - \frac{1}{Ddom}\right) + \sqrt{\frac{1}{Hdom - 1.3}}\right)^2}$	
29		$Height = 1.3 + \frac{(Hdom - 1.3)\left(1 - e^{a1\,DBH}\right)}{1 - e^{a1\,Ddom}}$	
30		$Height = 1.3 + (Hdom - 1.3)\,e^{a1\left(\frac{1}{Ddom} - \frac{1}{DBH}\right)}$	
31	H_{dom}-DQ	$Height = a1\,Hdom\left(1 - e^{-\frac{a2\,DBH}{DQ}}\right)^{a3}$	Stankova and Diéguez-Aranda (2013)
32	H_{dom}-D_{dom}	$Height = 1.3 + e^{a1 + a2\,Hdom + \frac{a3\,Ddom}{DBH + 1}}$	Stankova and Diéguez-Aranda (2013)
33	H_{dom}-D_{dom}	$Height = 1.3 + (Hdom - 1.3)\,e^{a1\left(1 - \frac{Ddom}{DBH}\right) + a2\left(\frac{1}{Ddom} - \frac{1}{DBH}\right)}$	Sánchez-González et al. (2008)
34		$Height = 1.3 + \frac{Hdom - 1.3}{1 - a1\left(1 - \left(\frac{Ddom}{DBH}\right)^{a2}\right)}$	
35		$Height = 1.3 + (Hdom - 1.3)\left(\frac{DBH}{Ddom}\right)^{a1}$	
36		$Height = 1.3 + \frac{(Hdom - 1.3)\left(1 - e^{-a1\,DBH}\right)}{1 - e^{-a1\,Ddom}}$	
37		$Height = 1.3 + (Hdom - 1.3)\,e^{a1\left(\frac{1}{DBH} - \frac{1}{Ddom}\right)}$	
38		$Height = 1.3 + \frac{Hdom - 1.3}{1 + a1\,(Hdom - 1.3)\left(\frac{1}{DBH} - \frac{1}{Ddom}\right)}$	
39	H_{dom}-BA-N	$Height = 1.3 + a1\,Hdom^{a2}\left(1 - e^{a3\left(\frac{N}{BA}\right)^{a4} DBH}\right)^{a5}$	Sharma and Parton (2007)

BAL – basal area in larger trees, N – number of trees per hectare, BA – stand basal area, DQ – quadratic diameter at breast height, H_{dom} – dominant height in each plot, D_{dom} – dominant diameter in each plot, H_{mean} – mean of height in each plot, a, b, c, e – model parameters cal measures including Akaike information criteria (AIC), Bayesian information criterion (BIC), root mean square error (RMSE), mean error (ME) defined as the difference between the observed and fitted tree heights and prediction error, coefficient of determination (R^2) and adjusted coefficient of determination (R^2_{adj}). These statistics may be expressed as Eqs 1–6:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)^2}{\sum_{i=1}^{n}(H_i - \bar{H})^2} \quad (1)$$

where:

n – number of observations,

H_i – observed value,

$\hat{H}_i$ – estimated value,

$\bar{H}$ – mean of the observed values.

$$R^2_{adj} = 1 - (1 - R^2) \times \frac{n-1}{n-p-1} \quad (2)$$

where:

n – number of observations,

p – number of parameters.

$$\mathrm{ME} = \frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)}{n} \quad (3)$$

where:

n – number of observations,

H_i – observed value,

$\hat{H}_i$ – estimated value.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)^2}{n}} \quad (4)$$

where:

n – number of observations,

H_i – observed value,

$\hat{H}_i$ – estimated value.

$$\mathrm{AIC} = n\ln(\mathrm{RMSE}) + 2p \quad (5)$$

where:

n – number of observations,

p – number of parameters.

$$\mathrm{BIC} = n\ln(\mathrm{RMSE}) + \ln(n) \quad (6)$$

where:

n – number of observations.

RESULTS

The measures of performance for 39 generalized height growth functions for the calibration dataset are summarized in Table 3. Considerable differences were observed between the predictive abilities of the generalized height-diameter models. Models with the lowest RMSE, BIC and AIC values and the R^2 and R^2_{adj} closest to unity have the best performance (Ahmadi et al. 2013). The highest R^2 is obtained by models 39, 23, 25 and 26. For the calibration dataset models 39 and 25 have the best fit and models 13 and 8 the poorest based on the RMSE criterion. If the information criteria are taken into account, models 13 and 8 perform poorly and models 39 and 25 have the lowest AIC and BIC.

The statistics RMSE, AIC, ME, BIC, R^2 and R^2_{adj} for the validation dataset vary significantly across models and are presented in Table 3. The results of performance criteria for validation data suggest that there is a small difference between the criteria values for calibration and validation data. For validation data, the highest and the lowest R^2 is achieved in models 39 (0.782) and 8 (0.618), respectively. Adjusted R^2 values are roughly the same as R^2 for each model, and model 39 has the highest value. Based on RMSE, model 39 generally performs better than the other models. The values of AIC range from 1,300 to 1,664, and BIC from 1,324 to 1,674 for validation data (Table 3). Among the 39 generalized height-diameter functions, model 39 had the lowest AIC and BIC. The result of 10-fold cross-validation also indicates model 39 scores better for performance measures.

Residual plots support the hypothesis of normality, homogeneity of variance and independence of residuals (López Sánchez et al. 2003; Sonmez 2009). Plotting model residuals versus predicted values showed that for all 39 models the residuals were randomly distributed and no trends were obvious. Fig. 2 shows plots of residuals versus predicted heights and observed against predicted heights in the fitting phase for model 25.

DISCUSSION

A wide variety of both local and generalized height-diameter models are available in the forestry literature (Ek 1974; Huang et al. 1992, 2000; Fang, Bailey 1998; Peng 1999; von Gadow et al. 2001; Soares, Tomé 2002; López Sánchez et al. 2003; Temesgen, von Gadow 2004). Since the variability of observed tree heights for the Oriental beech data available for this study were not adequately explained with DBH alone, additional stand-level independent variables were included. When generalized height-diameter functions were fitted, the predictive abilities of the models showed considerable differences.

For selecting the final generalized height-diameter equation, models were ranked with regard to their

Table 3. Performance criteria for 39 generalized height-diameter models for the fitting and validation data

Model	Fitting performance criterion						Validation performance criterion					
	R^2	RMSE	ME	AIC	R^2_{adj}	BIC	R^2	RMSE	ME	AIC	R^2_{adj}	BIC
1	0.6827	4.04	0.032	1,502	0.6809	1,532	0.6798	4.06	0.030	1,507	0.678	1,537
2	0.7169	3.814	–0.001	1,442	0.7150	1,477	0.7137	3.836	–0.003	1,448	0.7118	1,483
3	0.6538	4.218	0.0018	1,544	0.6525	1,564	0.652	4.229	0.0003	1,547	0.6507	1,566
4	0.7713	3.435	0.053	1,325	0.7704	1,345	0.7699	3.446	0.050	1,328	0.7691	1,348
5	0.7348	3.7	0.0587	1,404	0.7338	1,424	0.7327	3.714	0.059	1,408	0.7317	1,428
6	0.7716	3.427	0.023	1,322	0.7708	1,342	0.7696	3.442	0.022	1,327	0.7688	1,347
7	0.7733	3.414	0.022	1,320	0.7722	1,345	0.7712	3.43	0.021	1,325	0.7702	1,350
8	0.6195	4.647	0.823	1,651	0.6174	1,681	0.6184	4.651	0.821	1,642	0.618	1,647
9	0.7642	3.492	0.174	1,336	0.7639	1,341	0.7637	3.495	0.173	1,337	0.7635	1,342
10	0.7215	4.351	0.973	1,571	0.7213	1,576	0.7209	4.356	0.970	1,572	0.7206	1,577
11	0.7391	3.881	0.079	1,449	0.7389	1,454	0.7384	3.887	0.077	1,451	0.7382	1,456
12	0.7526	3.657	–0.121	1,385	0.7524	1,390	0.7521	3.661	–0.122	1,387	0.7518	1,392
13	0.7494	4.731	–2.481	1,662	0.7489	1,672	0.7484	4.741	–2.483	1,664	0.7479	1,674
14	0.6937	3.969	–0.0004	1,475	0.6931	1,485	0.6919	3.981	–0.004	1,478	0.6913	1,488
15	0.7633	3.488	–0.011	1,341	0.7624	1,361	0.7617	3.5	–0.014	1,345	0.7608	1,364
16	0.7348	3.7	0.058	1,404	0.7338	1,424	0.7327	3.714	0.059	1,408	0.7317	1,428
17	0.7716	3.427	0.023	1,322	0.7708	1,342	0.7696	3.442	0.022	1,327	0.7688	1,347
18	0.7429	3.635	–0.004	1,387	0.7417	1,412	0.7396	3.658	–0.011	1,394	0.7384	1,419
19	0.699	3.933	6.672	1,473	0.6973	1,503	0.6957	3.954	0.004	1,479	0.694	1,509
20	0.7303	3.75	0.252	1,414	0.7298	1,424	0.729	3.76	0.253	1,417	0.7285	1,427
21	0.6717	4.164	0.172	1,530	0.6705	1,550	0.6698	4.178	0.172	1,534	0.6686	1,554
22	0.7153	3.825	–0.0006	1,441	0.714	1,466	0.7113	3.852	–0.005	1,449	0.7099	1,474
23	0.7745	3.404	4.478	1,313	0.7739	1,328	0.7734	3.413	–0.0002	1,316	0.7727	1,331
24	0.7652	3.489	0.161	1,337	0.7647	1,347	0.7644	3.495	0.160	1,339	0.764	1,349
25	0.7753	3.399	0.064	1,312	0.7747	1,326	0.7743	3.407	0.062	1,314	0.7737	1,329
26	0.775	3.402	0.084	1,314	0.7741	1,334	0.7738	3.411	0.083	1,317	0.7729	1,337
27	0.7652	3.489	0.161	1,337	0.7648	1,347	0.7644	3.495	–0.004	1,339	0.764	1,349
28	0.7526	3.657	–0.121	1,385	0.7524	1,390	0.7521	3.661	–0.122	1,387	0.7518	1,392
29	0.7391	3.881	0.079	1,449	0.7389	1,454	0.7384	3.887	0.077	1,451	0.7382	1,456
30	0.7336	3.873	–0.290	1,447	0.7333	1,452	0.7329	3.879	–0.290	1,448	0.7327	1,453
31	0.7712	3.436	0.053	1,323	0.7705	1,338	0.7703	3.443	0.052	1,325	0.7697	1,340
32	0.7457	3.62	0.045	1,379	0.745	1,394	0.7441	3.632	0.045	1,382	0.7434	1,397
33	0.7435	3.771	–0.310	1,420	0.743	1,430	0.7422	3.782	–0.312	1,423	0.7417	1,433
34	0.7687	3.464	0.106	1,330	0.7683	1,339	0.7681	3.468	0.105	1,331	0.7677	1,341
35	0.7642	3.492	0.174	1,336	0.7639	1,341	0.7637	3.495	0.173	1,337	0.7635	1,342
36	0.7391	3.881	0.079	1,449	0.7389	1,454	0.7384	3.887	0.077	1,451	0.7382	1,456
37	0.7336	3.873	–0.290	1,447	0.7333	1,452	0.7329	3.879	–0.290	1,448	0.7327	1,453
38	0.7391	3.881	0.079	1,449	0.7389	1,454	0.7384	3.887	0.077	1,451	0.7382	1,456
39	0.7838	3.333	0.005	1,295	0.7828	1,319	0.7818	3.349	0.004	1,300	0.7808	1,324

R^2 – coefficient of determination, RMSE – root mean square error, ME – mean error, AIC – Akaike information criteria, R^2_{adj} – adjusted coefficient of determination, BIC – Bayesian information criterion

performances (R^2_{adj}, RMSE, AIC and BIC) from the calibration and 10-fold cross-validation datasets. Models that ranked within the first 3 are presented in Table 4. Model comparisons were done based on the rank and residual distribution. Results of 10-fold cross-validation and evaluation of models by the R^2_{adj}, R^2, RMSE, AIC, and BIC criteria showed that models 23, 25, 26 and 39 had better performance than the other models. Plotting the residuals also showed that for all these models the residuals were randomly distributed and had heterogeneous residuals. Models with four covariates (dominant height, dominant diameter, stand basal area and number of trees per hectare) in addition to DBH were found to be better than other combinations of covariates for predicting the total height of *F. orientalis* trees.

Table 4. Summary of ranks of model performance for Oriental beech

Model performance	Calibration data	Validation data
R^2_{adj}	39 (1), 25 (2), 23–26 (3)	39 (1), 25 (2), 23–26 (3)
RMSE	39 (1), 25 (2), 26 (3)	39 (1), 25 (2), 26 (3)
AIC	39 (1), 25 (2), 23 (3)	39 (1), 25 (2), 23 (3)
BIC	39 (1), 25 (2), 23 (3)	39 (1), 25 (2), 23 (3)
R^2	39 (1), 25 (2), 23 (3)	39 (1), 25 (2), 23 (3)

R^2_{adj} – adjusted coefficient of determination, RMSE – root mean square error, AIC – Akaike information criteria, BIC – Bayesian information criterion ME – mean error, R^2 – coefficient of determination

A number of researchers found that adding stand variables to the height-diameter equation and using the generalized height-diameter models increased the precision (López Sánchez et al. 2003; Sharma, Parton 2007; Krisnawati et al. 2010; Temesgen et al. 2014). Dominant height, stand basal area, dominant diameter, age, number of trees per hectare, basal area in larger trees, density stress, developmental status and the combination of density stress and developmental status are among the stand variables reported in the literature.

Ahmadi et al. (2013) recommended the Richards function for further analysis due to having better performance in TMU forests for Oriental beech. Fitting this function to the data collected in this study indicated that approximately 71.5% of the total variation in height could be explained by the Richards function. Like in previous studies, the inclusion of stand characteristics improved the prediction accuracy of tree height estimation for *F. orientalis* trees. Model 39 which uses DBH, dominant height, stand basal area and number of trees per hectare as independent variables had the best performance and showed consistency between the calibration and validation data, therefore it was found to be the best model for predicting height when stand basal area and number of trees per hectare are known. In some studies, stand basal area has been found to affect height growth patterns (Coomes, Allen 2007; Huang et al. 2013). When stand basal area and number of trees per hectare are not known, model 25 which employs dominant height in each plot and dominant diameter in each plot may be a suitable alternative. Soares and Tomé (2002), Eerikäinen (2003), López Sánchez et al. (2003), and Calama and Montero (2004) used stand dominant height as predictor variables in addition to DBH in developing height-diameter models and their accuracy has also been improved.

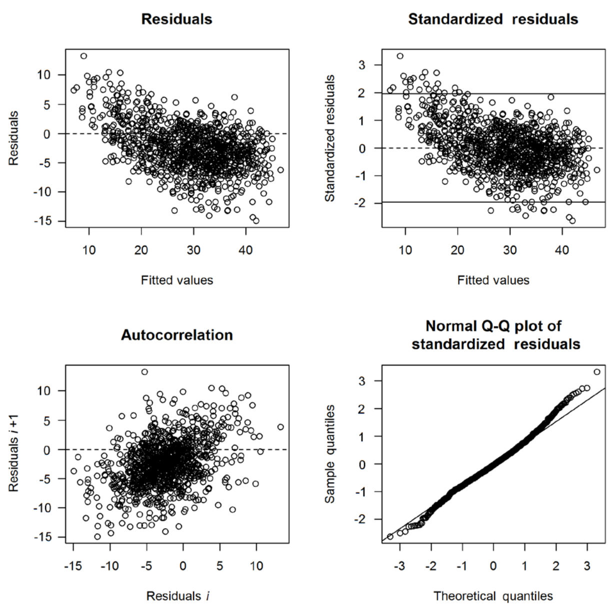

Fig. 2. Plot of residuals in the fitting phase for models 25

Developing simple and accurate models that allow forest managers to determine the tree heights in a stand from DBH data with reliability is of prime importance in forest management (Calama, Montero 2004). In this study, selected model 39 not only had the best performance, but also it is easy to apply. It is best used within the range of values used for model construction. Applications beyond this range should be used with caution and following additional testing.

Reliable estimates of tree height are needed for determining tree volume and stand yields. Measurement of tree height is less reliable and more expensive than measuring tree DBH. It is often limited to a sample of trees. By modelling the relationship between height and diameter, prediction of unmeasured heights can be done by suitable height-diameter functions. Including stand variables such as dominant height, basal area and number of trees in the height-diameter model allows generalizing the prediction for different stand and site conditions.

In this study, 39 height-diameter models were calibrated and cross validated for Oriental beech trees in the Hyrcanian forest, Iran. Model selection was based on performance measures. Results show that there existed little differences between models when DBH was the only independent variable. However, composite models that included stand variables improved model performance, as previous studies have also confirmed. The performance statistics showed that the 5-parameter model 39 which contains dominant height, stand basal area and number of trees per hectare as well as DBH is the most suitable and recommended for predicting the height-diameter relationships for Oriental beech trees in this study area.

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Received for publication May 3, 2016
Accepted after corrections Aug 3, 2016

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