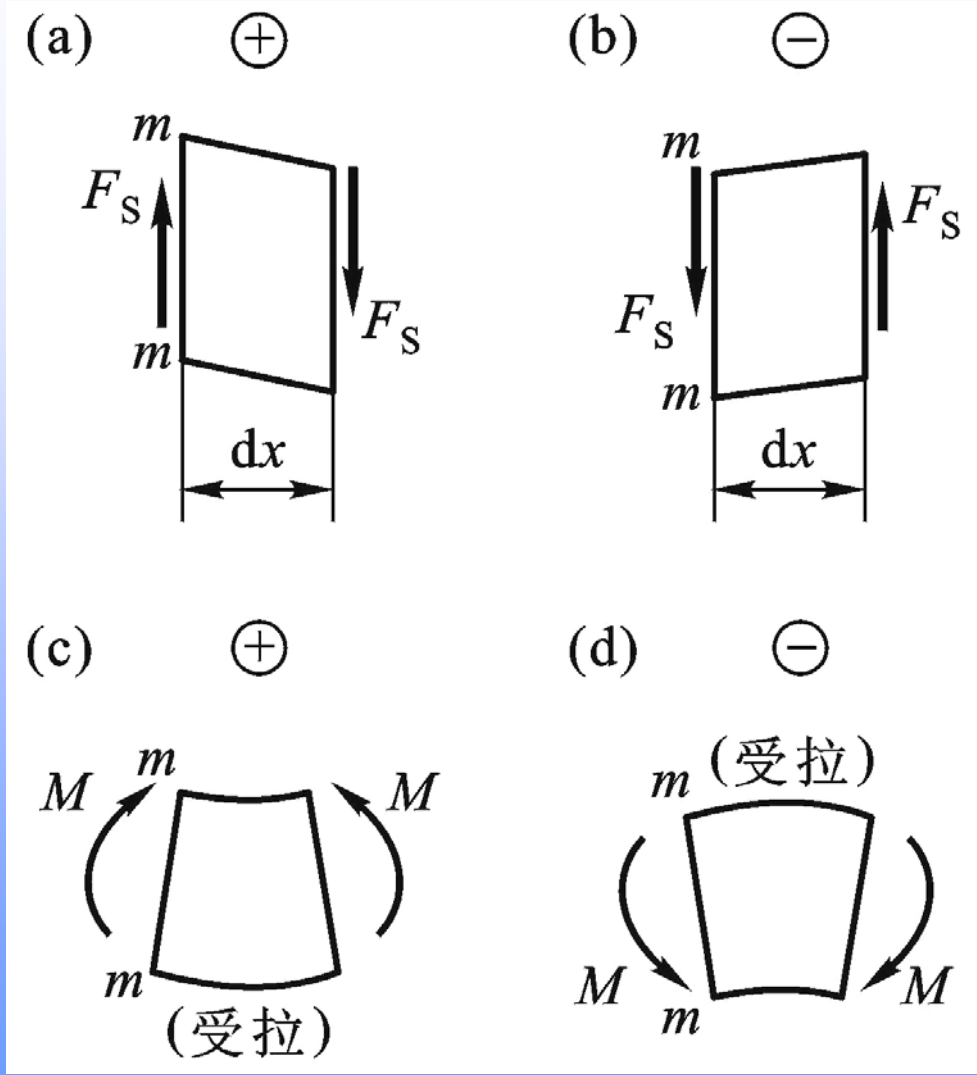
上次课回顾：

1、弯曲的概念

2、弯曲内力：剪力、弯矩

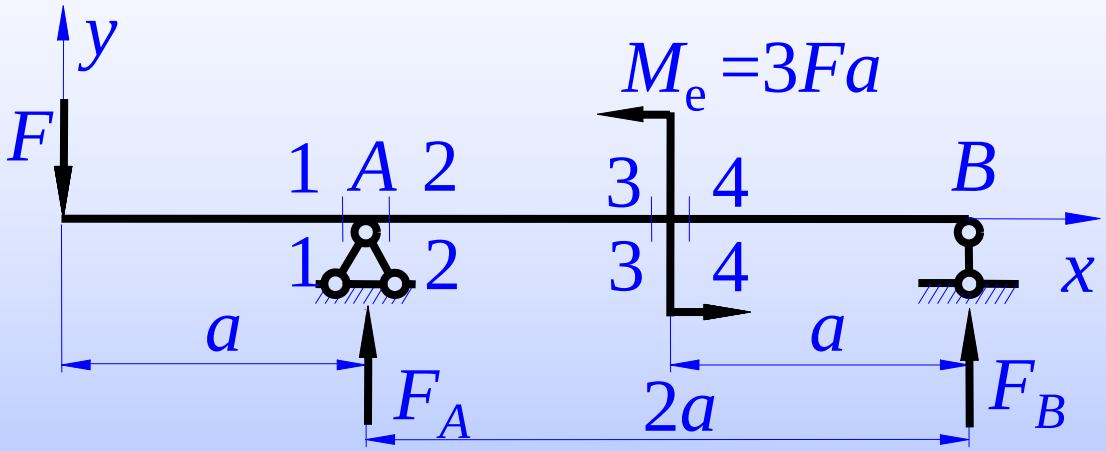
剪力和弯矩的符号规则：



左上右下，
剪力为正

左顺右逆，
弯矩为正

例4-2 求图示外伸梁在截面1—1、2—2、3—3和4—4横截面上的剪力和弯矩。

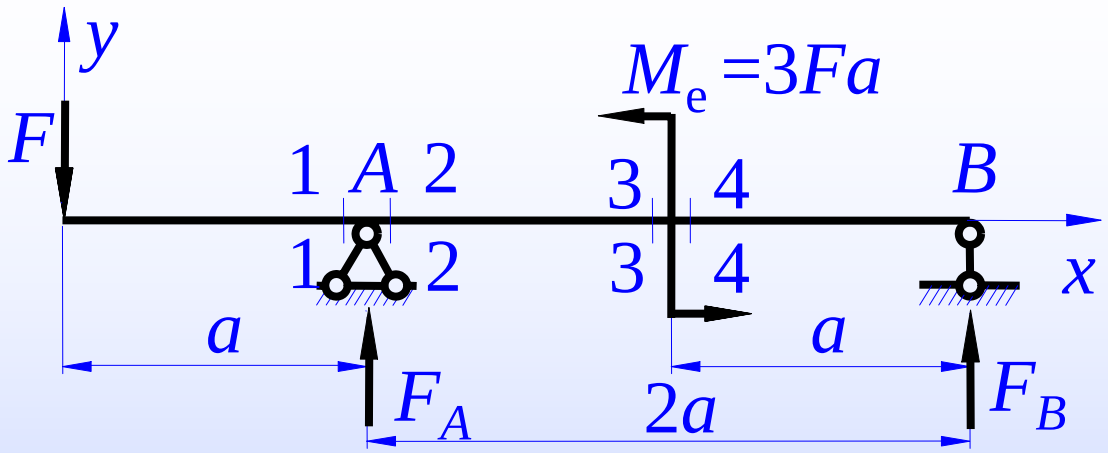


解：支反力为

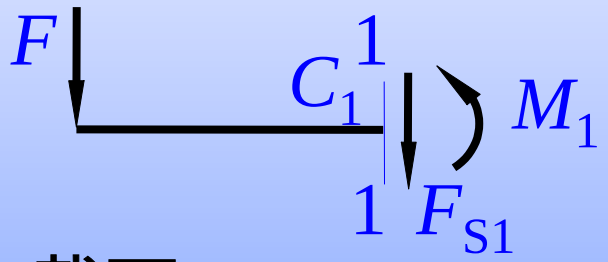
$$\sum M_A = 0 \qquad F_B \times 2a + 3Fa + F \times a = 0$$

$$F_B = -2F(\downarrow)$$

$$\sum F_y = 0 \qquad F_B + F_A = F \qquad F_A = 3F(\uparrow)$$



截面1—1

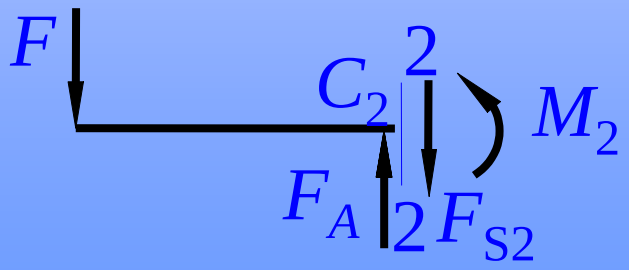


$$\sum F_y = 0 \qquad F_{S1} = -F$$

$$\sum M_{C1} = 0 \qquad M_1 + F \times a = 0$$

$$M_1 = -Fa\,(\text{顺})$$

截面2—2

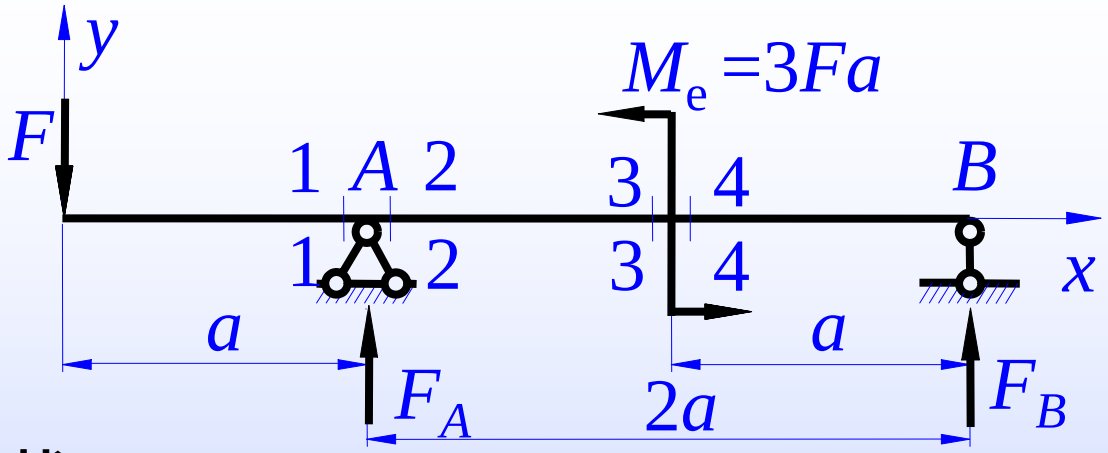


$$\sum F_y = 0 \qquad F_{S2} - F_A + F = 0$$

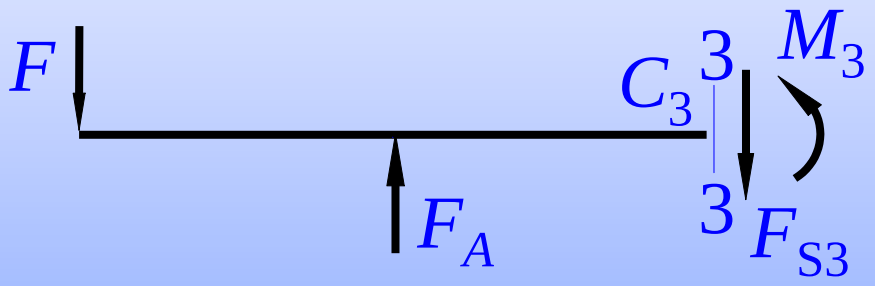
$$F_{S2} = F_A - F = 2F$$

$$\sum M_{C2} = 0 \qquad M_2 + F \times a = 0$$

$$M_2 = -Fa\,(\text{顺})$$



截面3—3



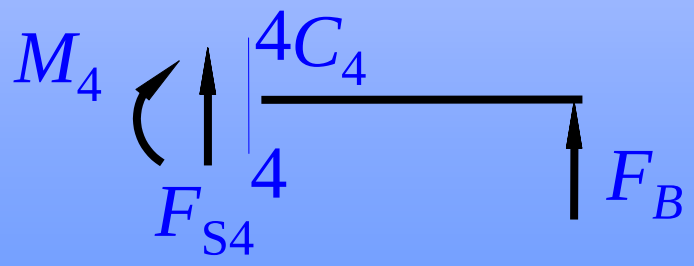
$$F_{S3}-F_A+F=0$$

$$F_{S3}=F_A-F=2F$$

$$M_3+F\times a-F_A\times a=0$$

$$M_3=Fa\,(\text{逆})$$

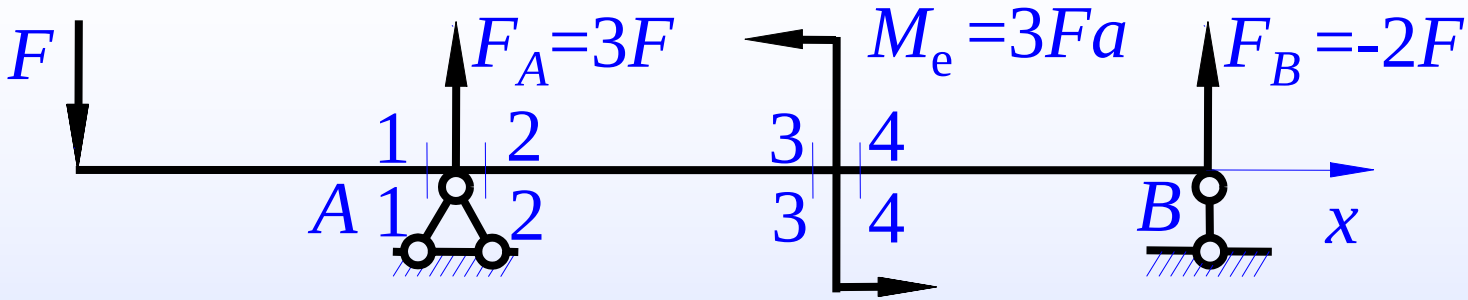
截面4—4



$$F_{S4}=-F_B=2F$$

$$M_4-F_B\times a=0$$

$$M_4=-2Fa\,(\text{顺})$$

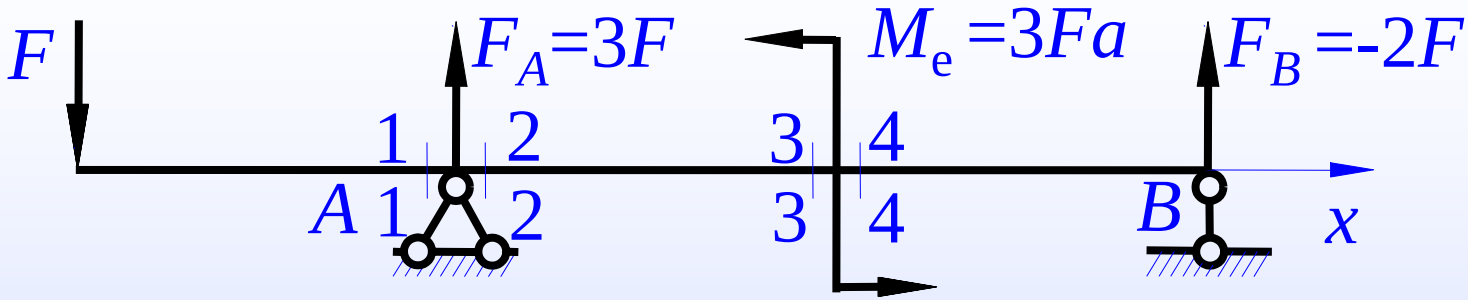


内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

1、横截面上的剪力和弯矩在数值上由截面左侧或右侧梁段分离体的静力平衡方程来确定。

剪力值= 截面左侧（或右侧）所有外力的代数和

弯矩值= 截面左侧（或右侧）所有外力对该截面形心的力矩代数和



内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

2、截面左侧梁段　向上的外力→正剪力→正弯矩

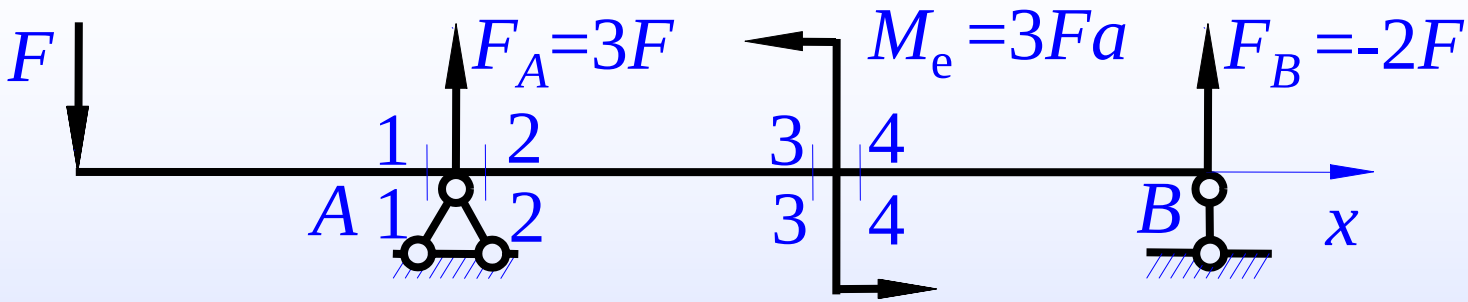
顺时针外力偶→正弯矩

截面右侧梁段　向上的外力→负剪力→正弯矩

顺时针外力偶→负弯矩

剪力：左上右下为正

弯矩：左顺右逆为正

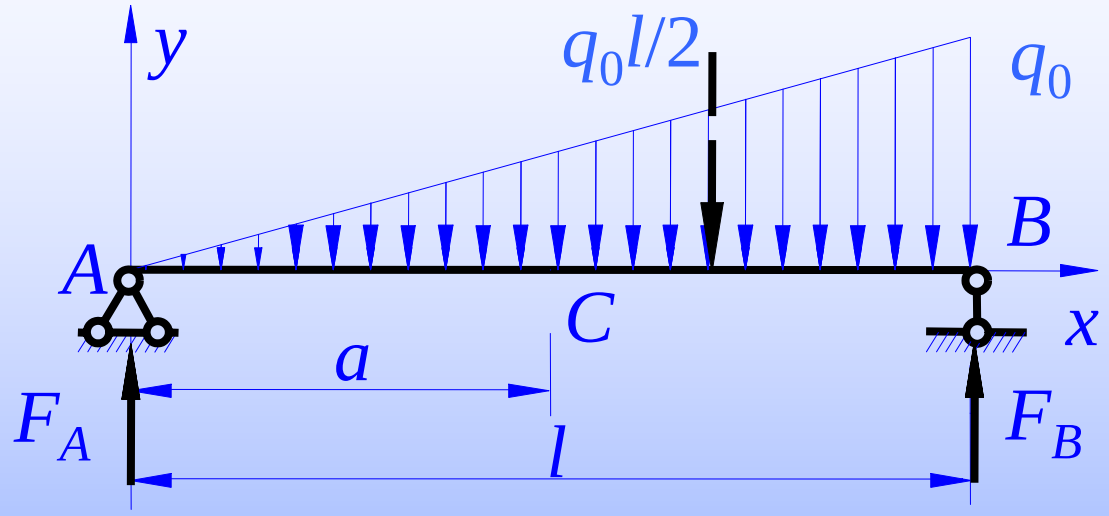


内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

3、在集中力作用处，剪力值发生突变，突变值=集中力大小；

在集中力偶作用处，弯矩值发生突变，突变值=集中力偶矩大小。

P_{96} 例4-3 图示简支梁受到三角形分布荷载的作用，最大荷载集度为q_0，试求截面C上的内力。

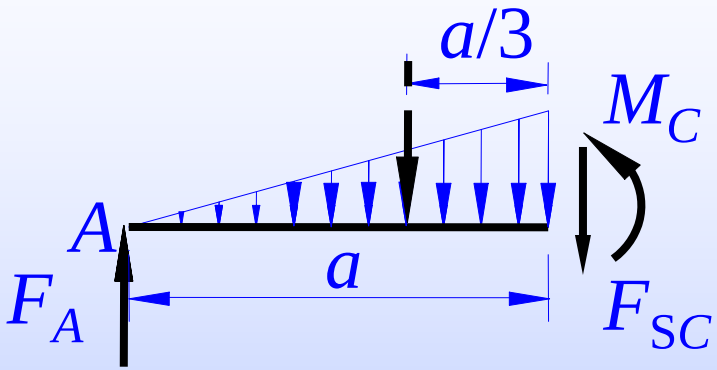


解：先求支反力

$$\sum M_A = 0 \qquad F_B l - \frac{q_0 l}{2} \times \frac{2l}{3} = 0 \qquad F_B = \frac{q_0 l}{3}$$

$$\sum M_B = 0 \qquad -F_A l + \frac{q_0 l}{2} \times \frac{l}{3} = 0 \qquad F_A = \frac{q_0 l}{6}$$

截面C的内力



$$q(x)=\frac{q_0 x}{l}$$

$$F_{SC}=F_A-\frac{a}{2}\times\frac{q_0 a}{l}=\frac{q_0 l}{6}-\frac{q_0 l^2}{2l}$$

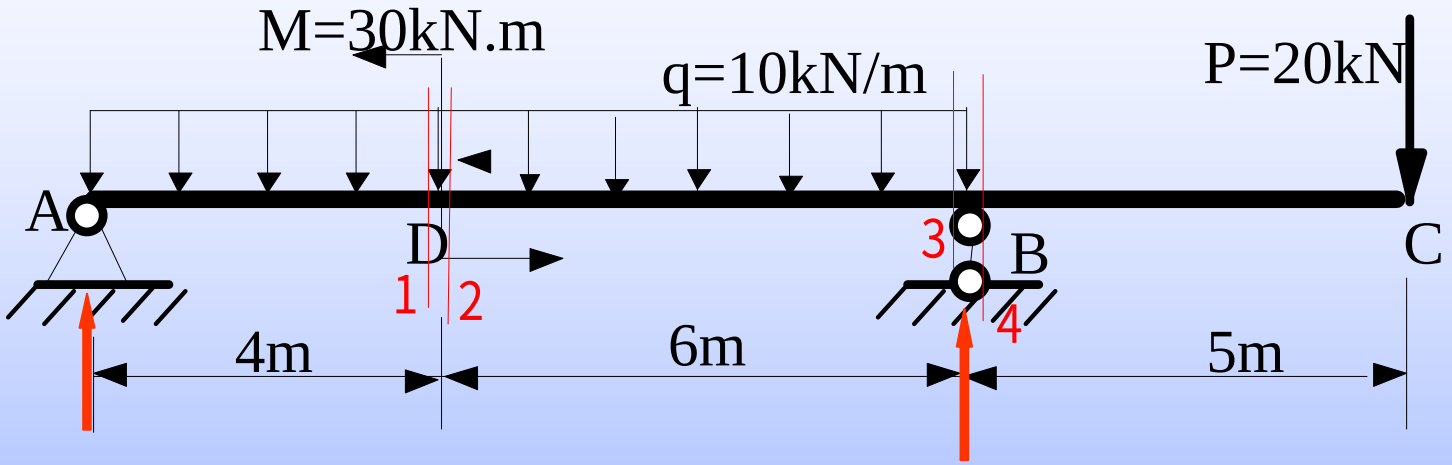
$$M_C=F_A\times a-\frac{a}{2}\times\frac{q_0 a}{l}\times\frac{a}{3}=\frac{q_0 l}{6}a-\frac{q_0 a^3}{6l}$$

思考：

是否可以将梁上的分布荷载全部用静力等效后的合力代替来求截面C的内力？

课堂练习

1．求图示外伸梁1，2，3，4截面的内力。



F_A=50-10+3=43kN

F_B=50+30-3=77kN

F_{s1}=43-10x4=3kN　　M_1=43x4-1/2x10x4^2=92kN.m

F_{s2}=3kN　　M_2=92-30=62kN.m

F_{s3}=20-77=57kN　　M_3=-20x5=-100kN.m

F_{s4}=20kN　　M4=-100kN.m

II、剪力方程和弯矩方程•剪力图和弯矩图

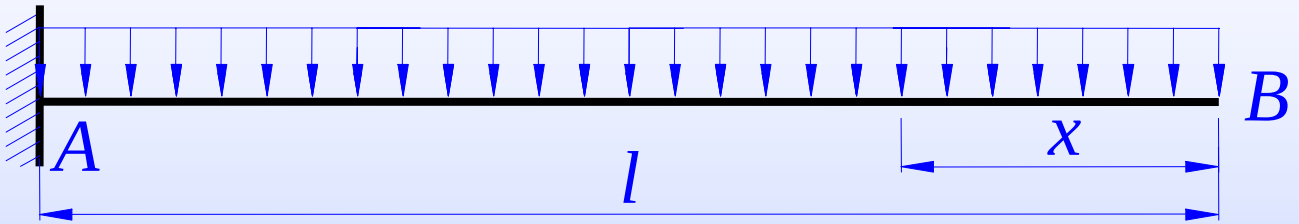
剪力方程　　$F_S = F_S(x)$

弯矩方程　　$M = M(x)$

反映梁的横截面上的剪力和弯矩随截面位置变化的函数式

显示剪力和弯矩随截面位移的变化规律的图形则分别称为剪力图和弯矩图。

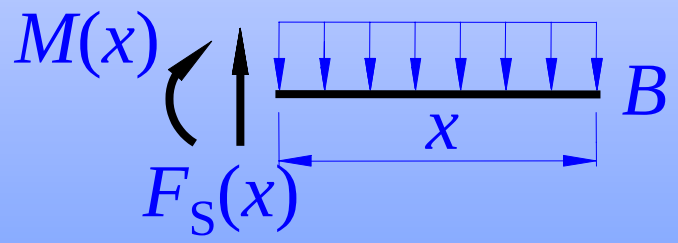
P_{98} 例4-4 图示悬臂梁受集度为q的满布均布荷载作用。试作梁的剪力图和弯矩图。



解：1、以自由端为坐标原点，则可不求反力

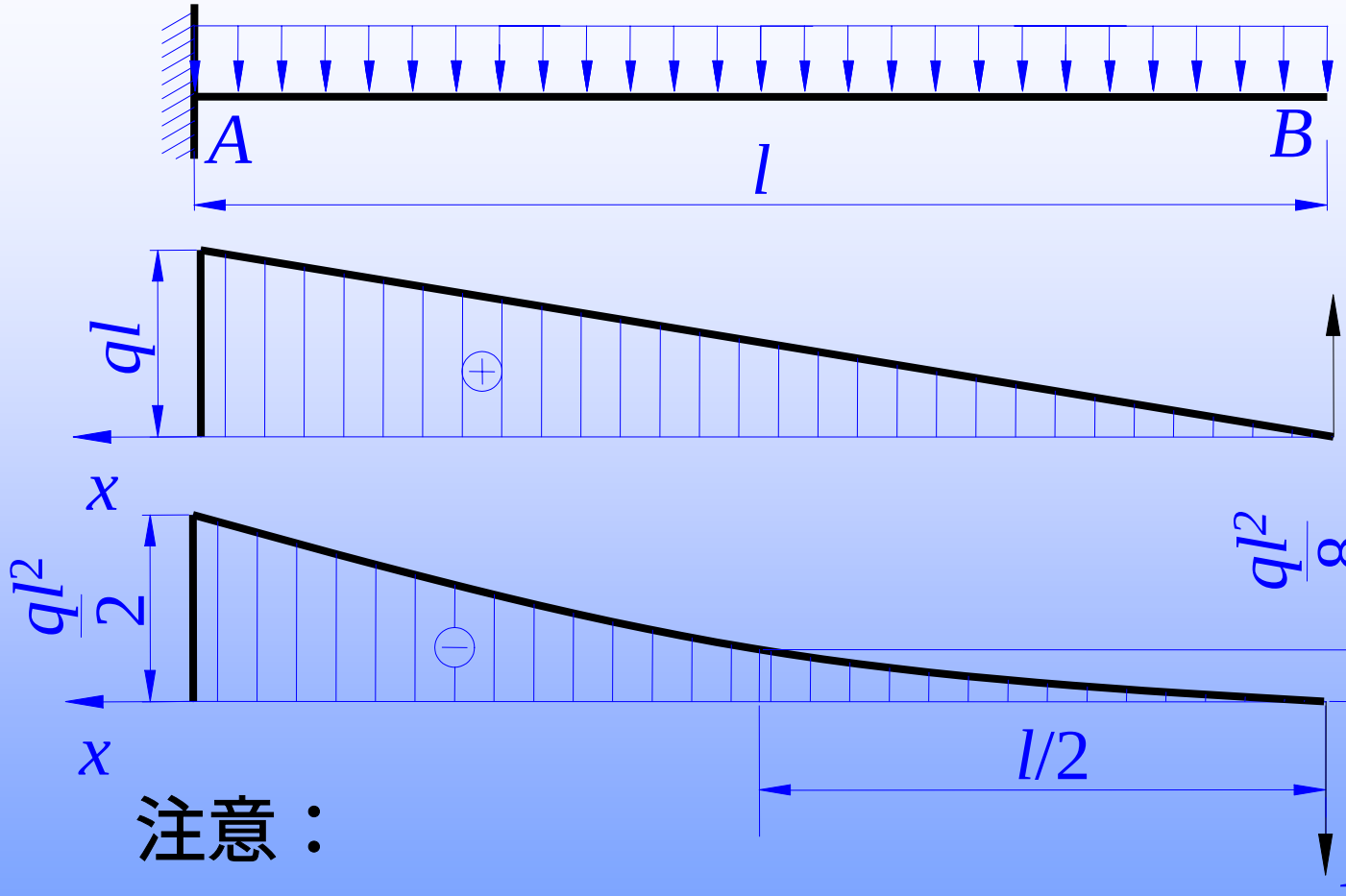
列剪力方程和弯矩方程：

$$F_S(x)=qx(0\le x<l)$$



$$M(x)=-qx\cdot\frac{x}{2}=-\frac{qx^2}{2}(0\le x<l)$$

2、 作剪力图和弯矩图



$$F_S(x) = qx$$

$$M(x) = -\frac{qx^2}{2}$$

$$F_{S,max} = ql$$

$$M_{max} = \frac{ql^2}{2}$$

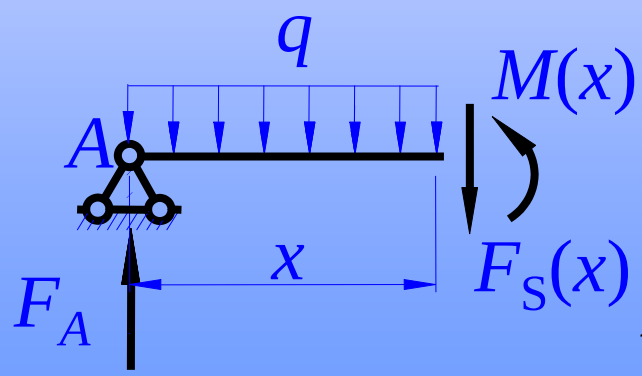
注意：

弯矩图中正的弯矩值绘在*x*轴的下方(即弯矩值绘在弯曲时梁的受拉侧)。

P_{99} 例4-5 图示简支梁受集度为q的满布荷载作用。试作梁的剪力图和弯矩图。

解：1、求支反力 $F_A = F_B = \dfrac{ql}{2}$

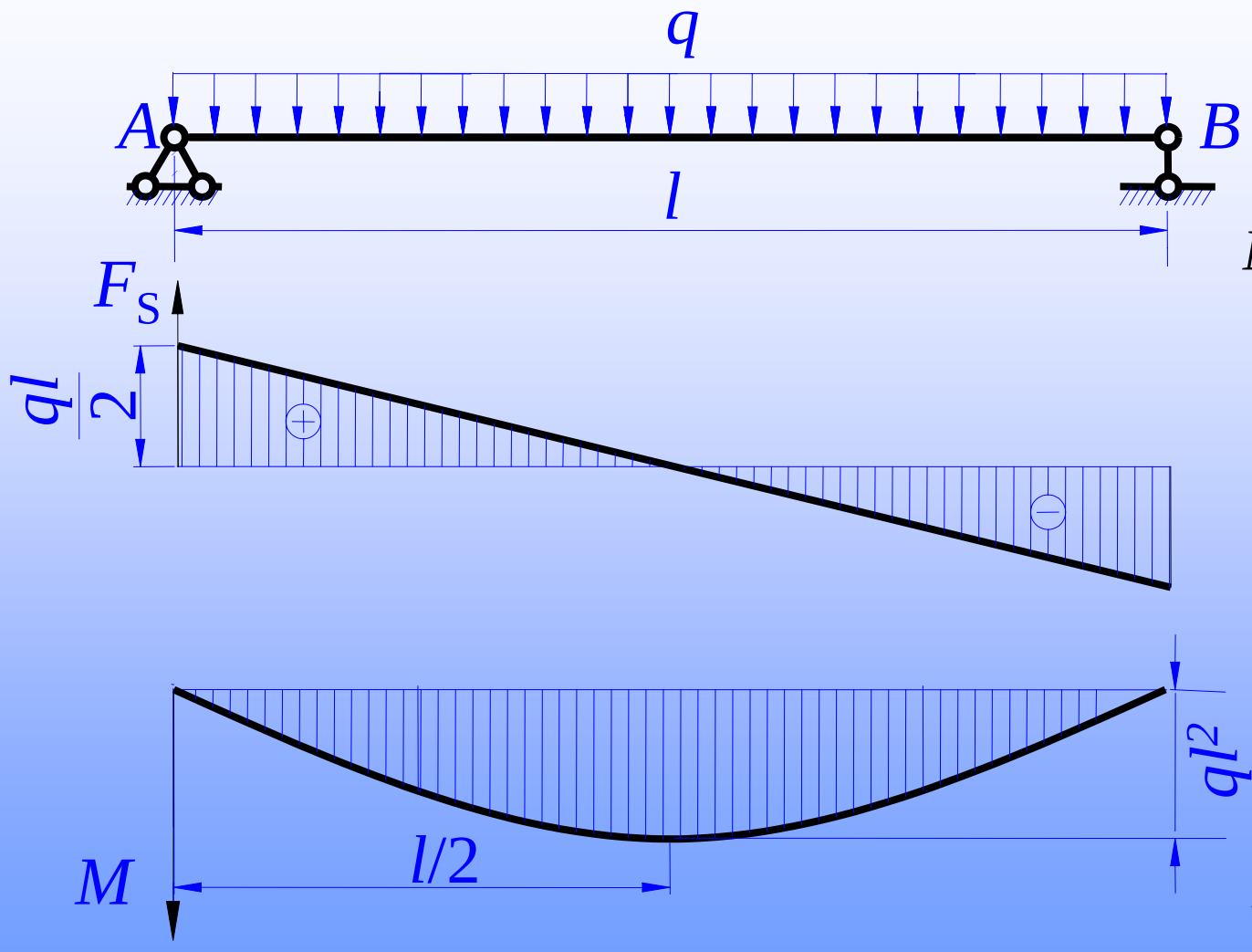
2、列剪力方程和弯矩方程



$$F_S(x) = F_A - qx = \frac{ql}{2} - qx$$

$$M(x) = F_A x - qx \times \frac{x}{2} = \frac{qlx}{2} - \frac{qx^2}{2}$$

3、作剪力图和弯矩图



$$F_S(x)=\frac{ql}{2}-qx$$

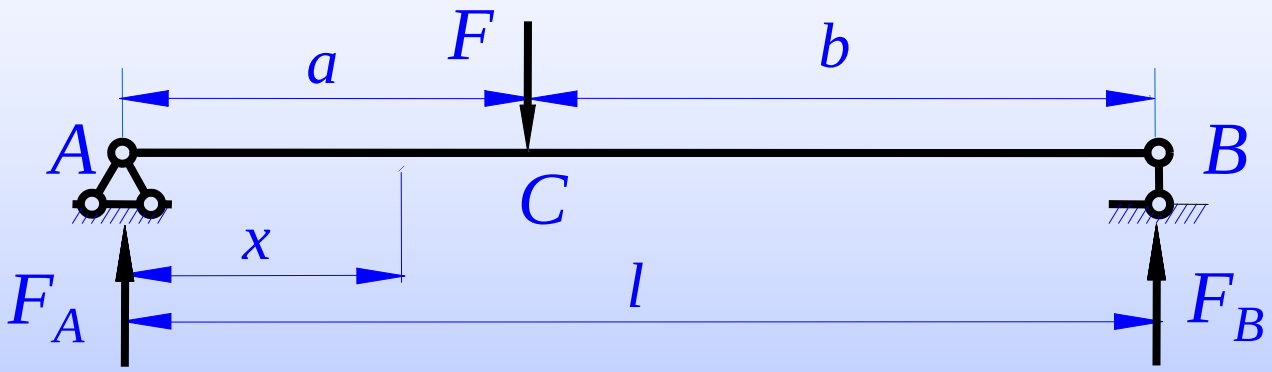
$$M(x)=\frac{qlx}{2}-\frac{qx^2}{2}$$

$$\left|F_{S,max}\right|=\frac{ql}{2}$$

$$M_{max}=\frac{ql^2}{8}$$

最大弯矩发生在剪力为零的截面。

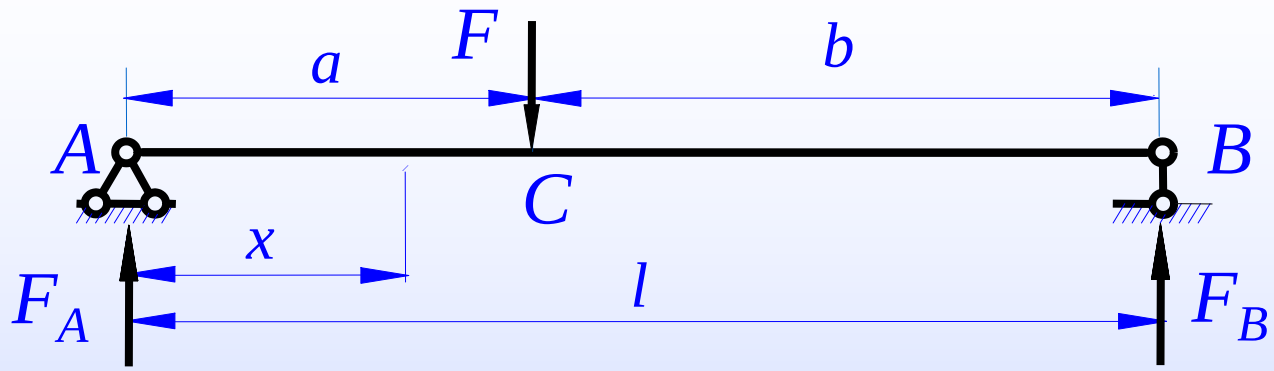
P$_{100}$ 例4-6 图示简支梁受集中荷载F作用。试作梁的剪力图和弯矩图。



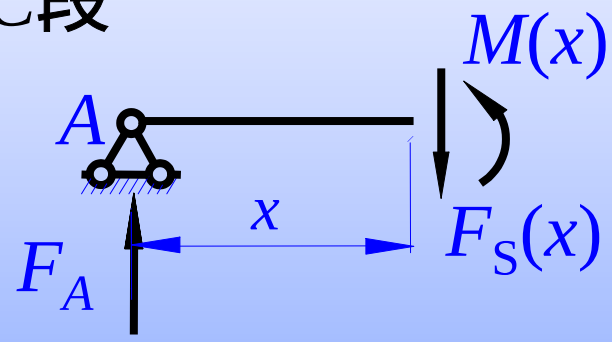
解：1、求支反力

$$F_A = \frac{Fb}{l} \qquad F_B = \frac{Fa}{l}$$

2、列剪力方程和弯矩方程 ——需分两段列出



AC段



$$F_S(x)=\frac{Fb}{l}(0<x<a)$$

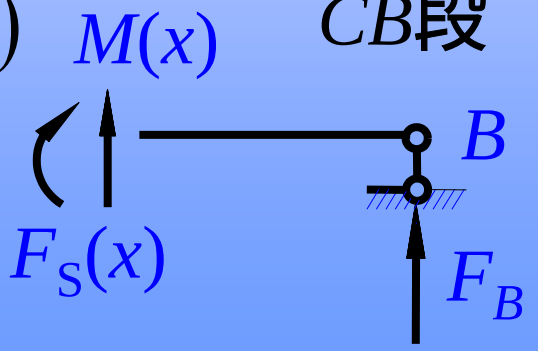
$$M(x)=\frac{Fb}{l}x(0\le x\le a)$$

$$F_S(x)=-F_B=-\frac{Fa}{l}(a<x<l)$$

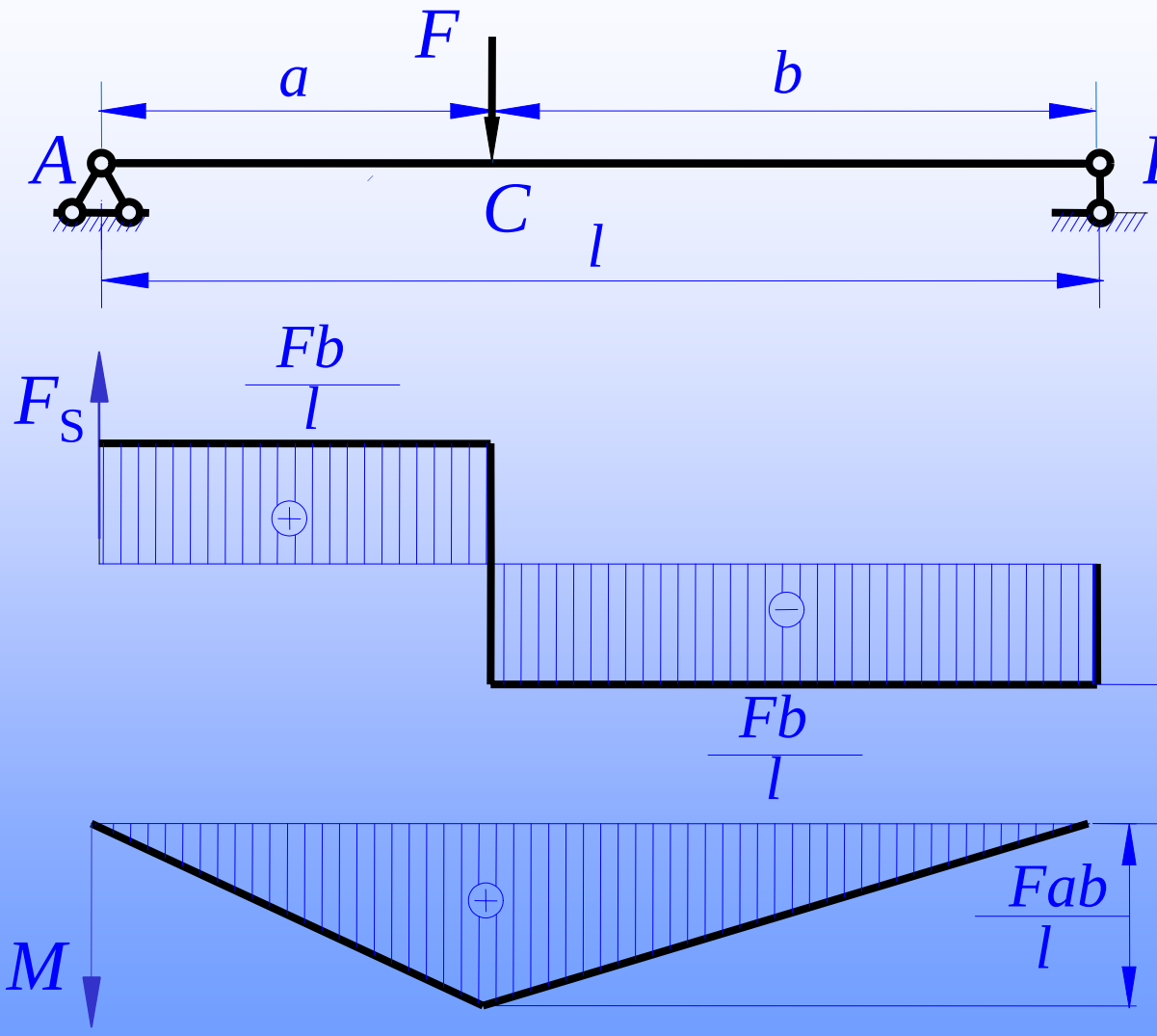
$$M(x)=F_B(l-x)=\frac{Fa}{l}(l-x)$$

$$(a\le x\le l)$$

CB段



3、作剪力图和弯矩图

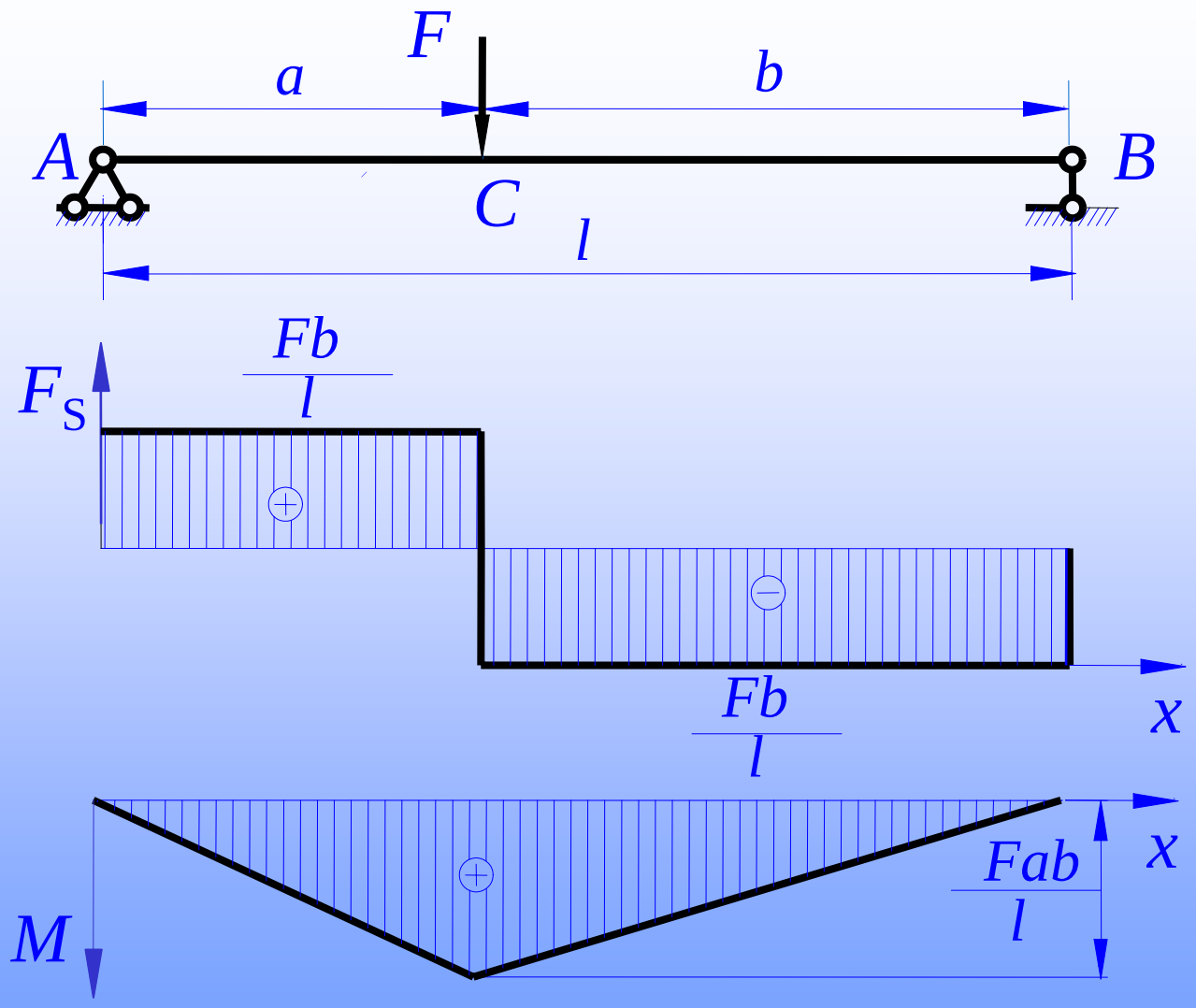


$$F_{S1}(x)=\frac{Fb}{l}$$

$$F_{S2}(x)=-\frac{Fa}{l}$$

$$M_1(x)=\frac{Fb}{l}x$$

$$M_2(x)=\frac{Fa}{l}(l-x)$$



$b>a$时

$$F_{S,max}=\frac{Fb}{l}$$

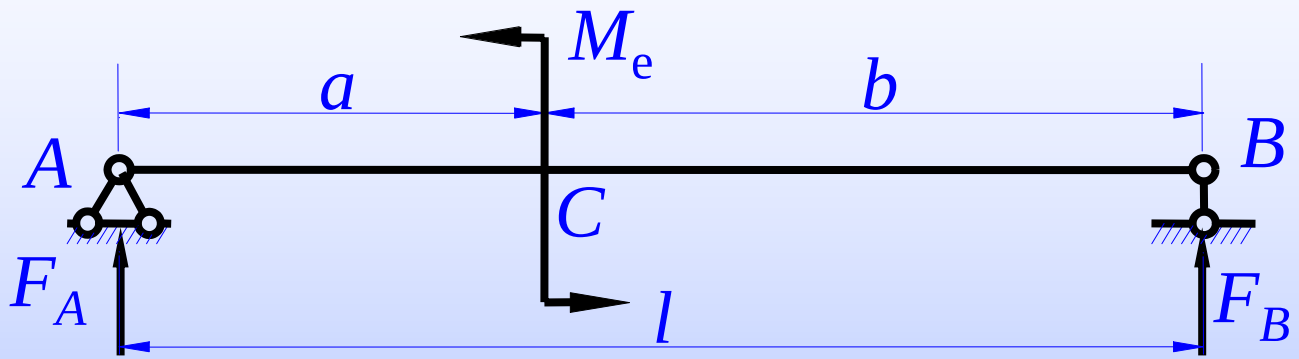
发生在AC段

$$M_{max}=\frac{Fab}{l}$$

发生在集中荷载作用处

$a=b=l/2$时，$M_{max}=\frac{Fl}{4}$为极大值。

例4-7　图示简支梁在C点受矩为M_e 的集中力偶作用。试作梁的剪力图和弯矩图。

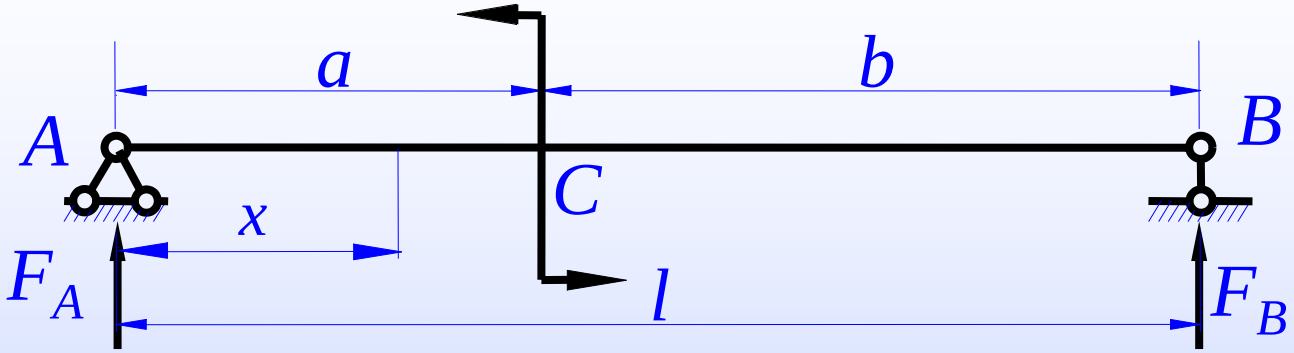


解：1、求支反力

$$\sum M_A = 0 \qquad M_e - F_A \times l = 0$$

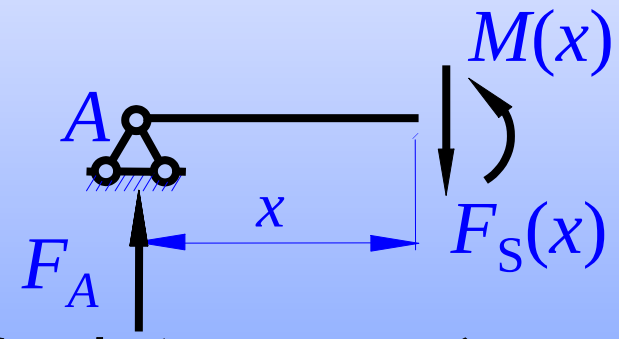
$$F_A = \frac{M_e}{l}(\uparrow) \qquad F_B = \frac{M_e}{l}(\downarrow)$$

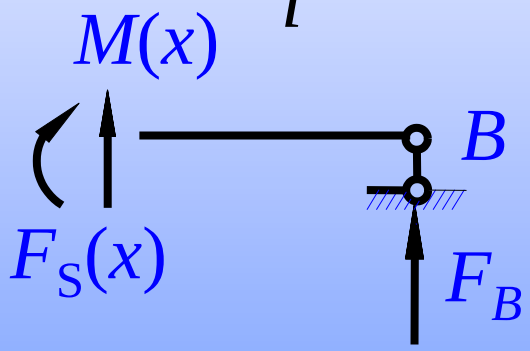
2、 列剪力方程和弯矩方程



剪力方程无需分段：

$$F_S(x)=F_A=\frac{M_e}{l}\quad(0<x<l)$$





弯矩方程——两段：

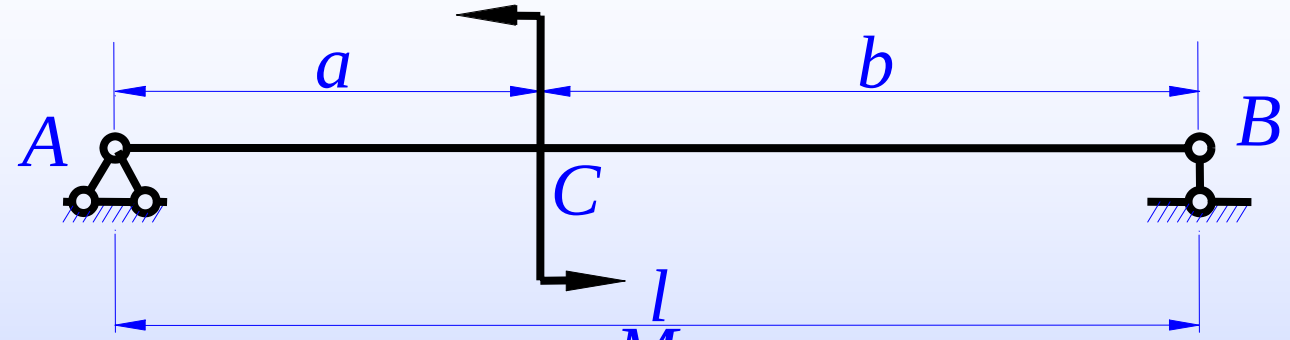
AC段：

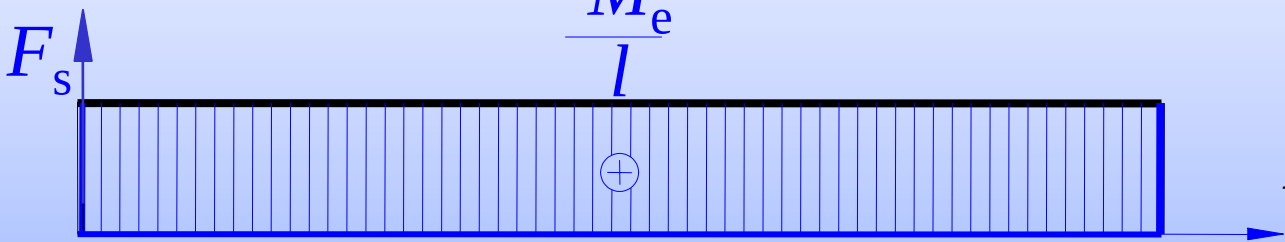
$$M(x)=F_Ax=\frac{M_e}{l}x\qquad(0\le x<a)$$

CB段：

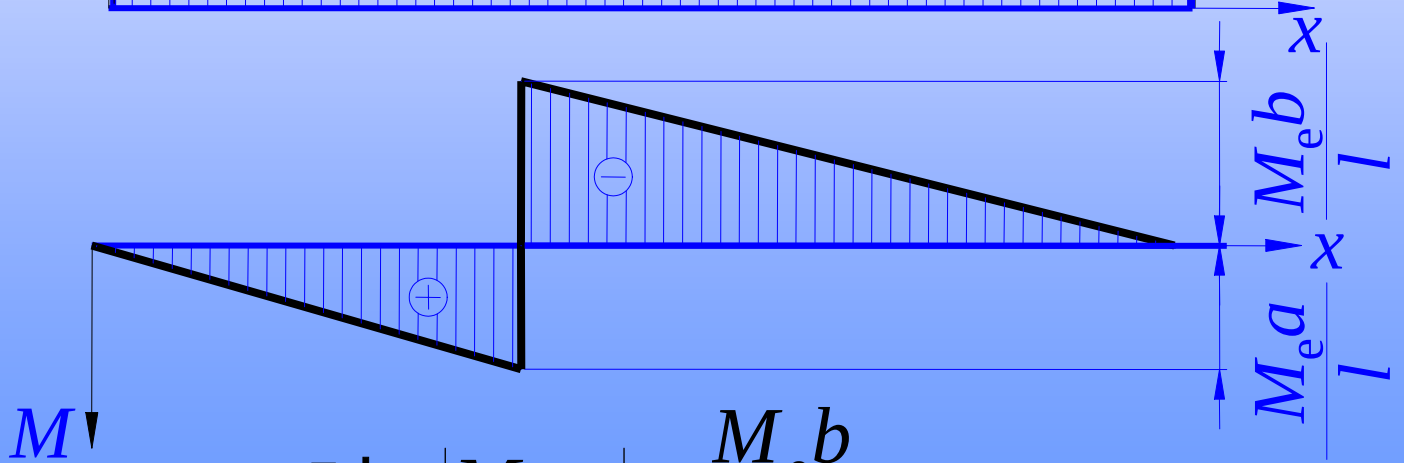
$$M(x)=F_Ax-M_e=-\frac{M_e}{l}(l-x)\quad(a<x\le l)$$

3、作剪力图和弯矩图





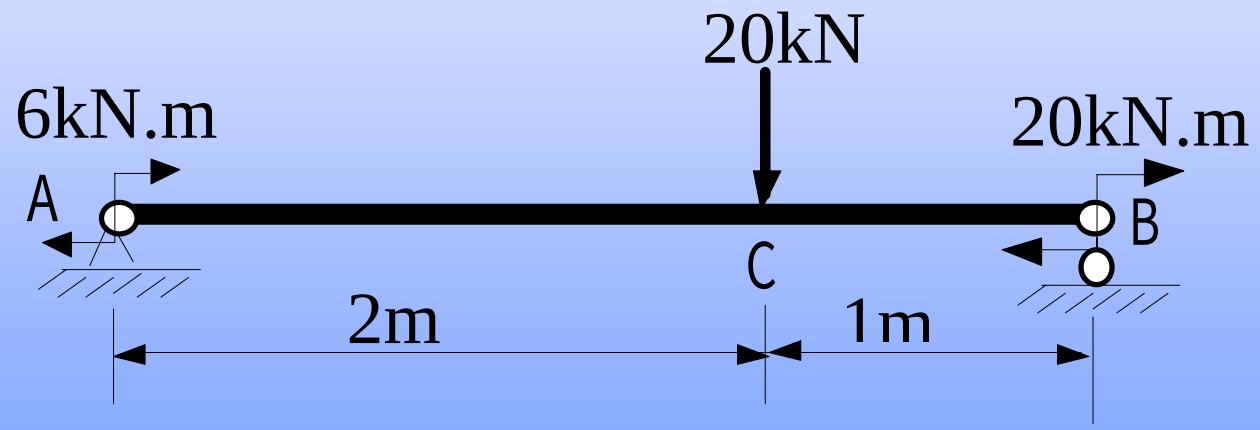
$$F_S(x)=\frac{M_e}{l}$$

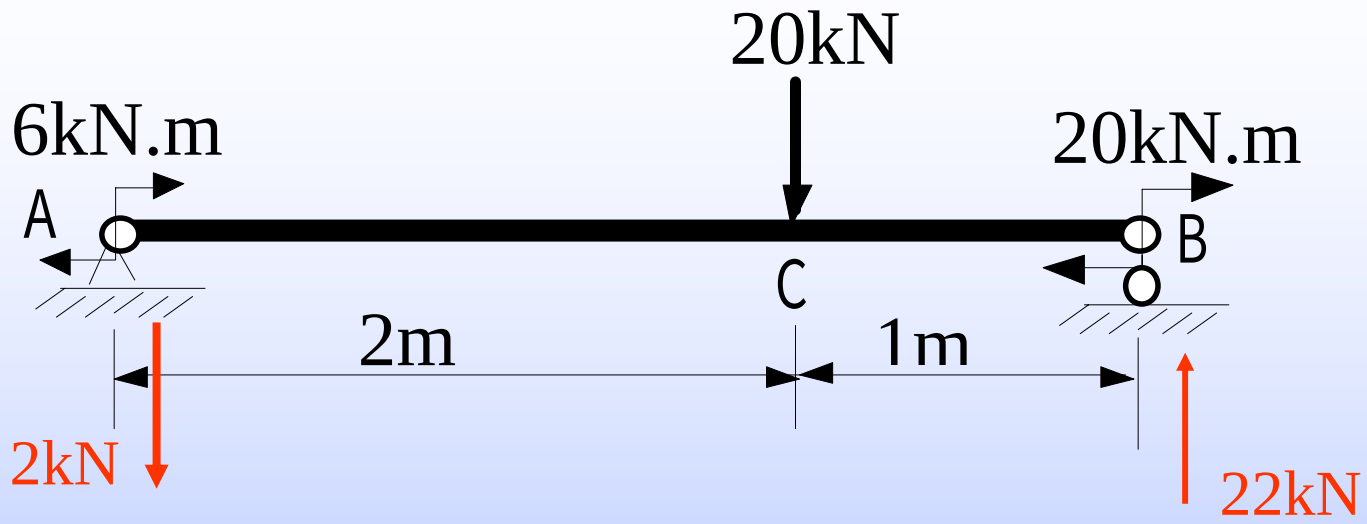


$b>a$时 $|M_{max}|=\frac{M_e b}{l}$ 发生在C截面右侧

课堂练习

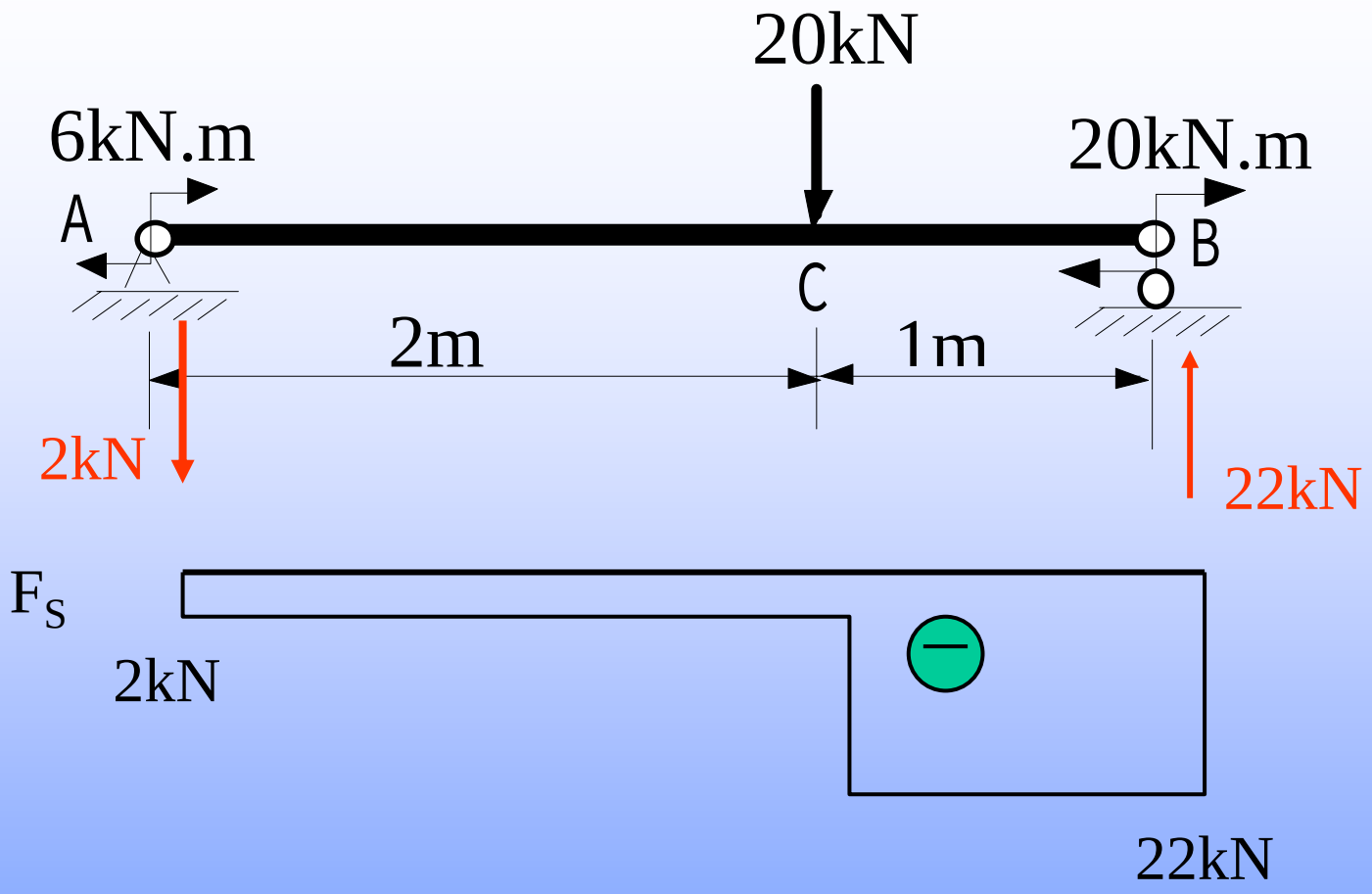
P142　4-2(e) 作图示梁的剪力图和弯矩图。





1)求约束力：

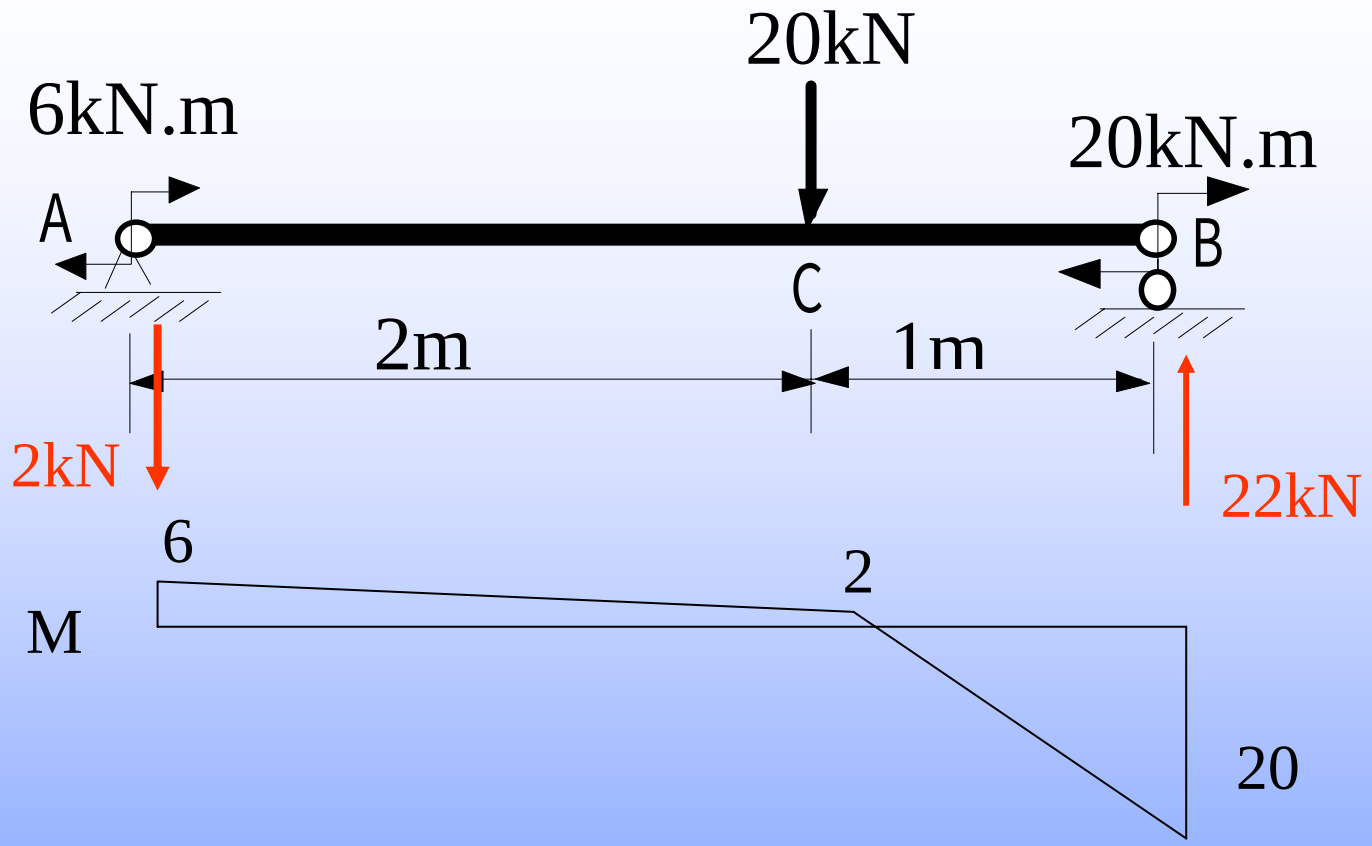
F_A=2kN （向下） F_B=22kN (向上）



2）剪力方程

AC段：$F_{s1} = -2kN$

CB段：$F_{s2} = -22kN$



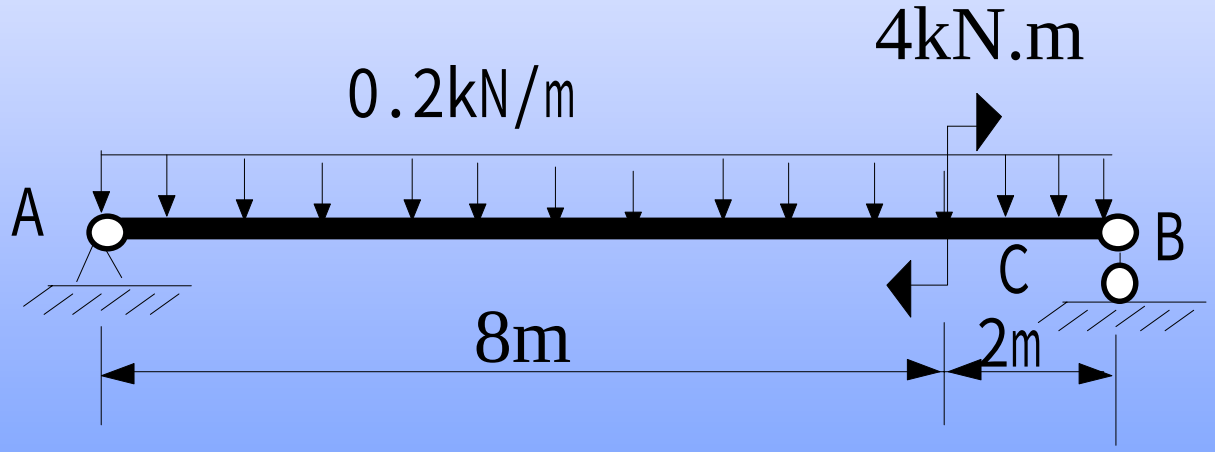
3）弯矩方程

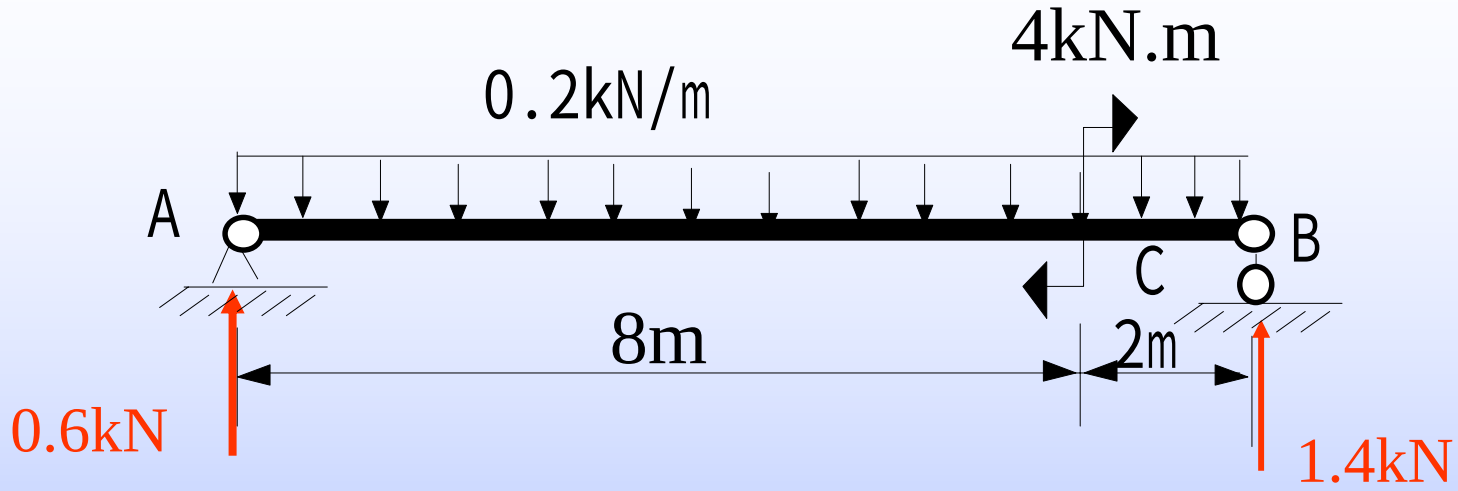
AC段：$M_1 = 6 - 2x \quad (0 < x \leq 2)$

CB段：$F_{s2} = -20 + 22x \quad (0 < x \leq 1)$

课堂练习

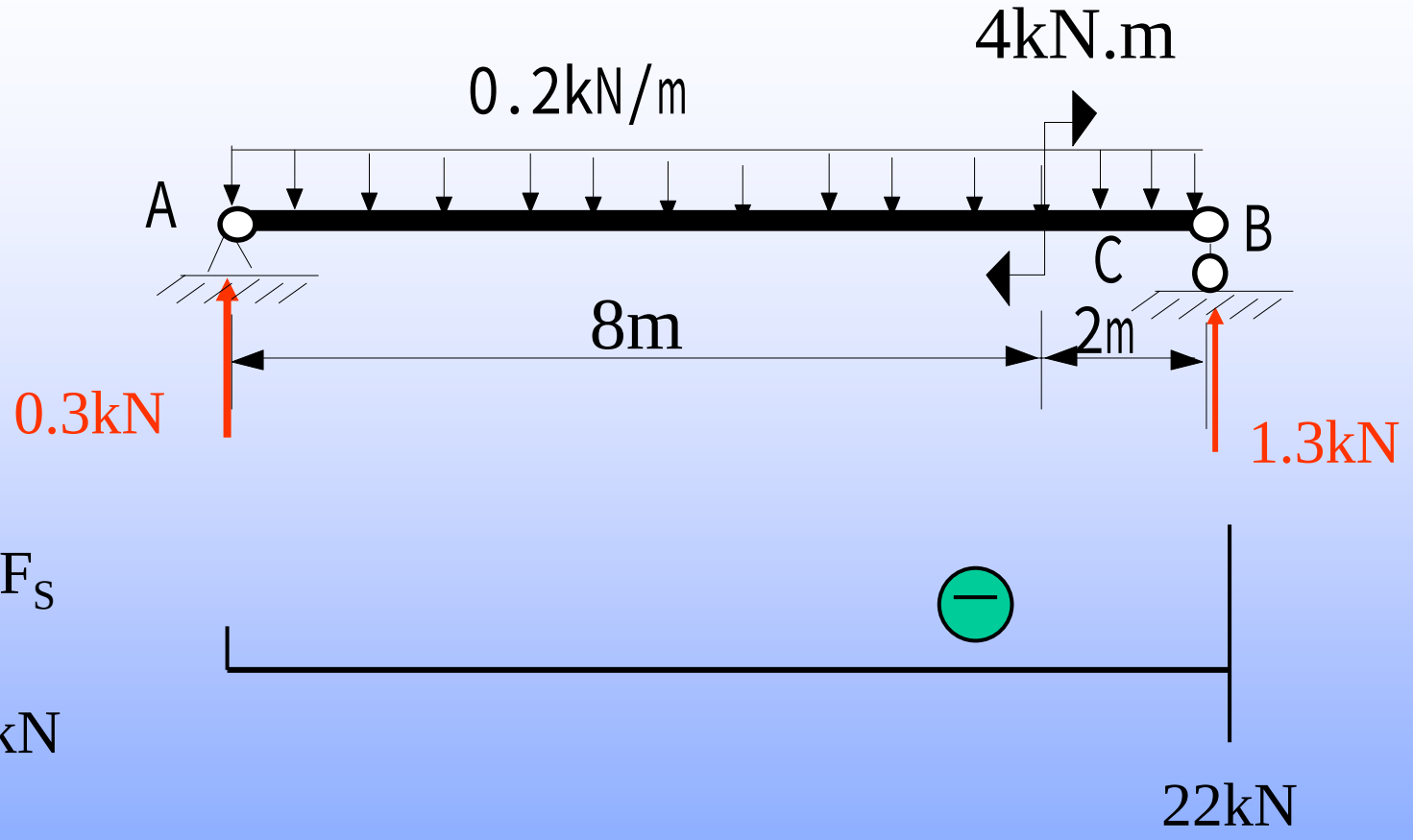
P142　4-2(d) 作图示梁的剪力图和弯矩图。





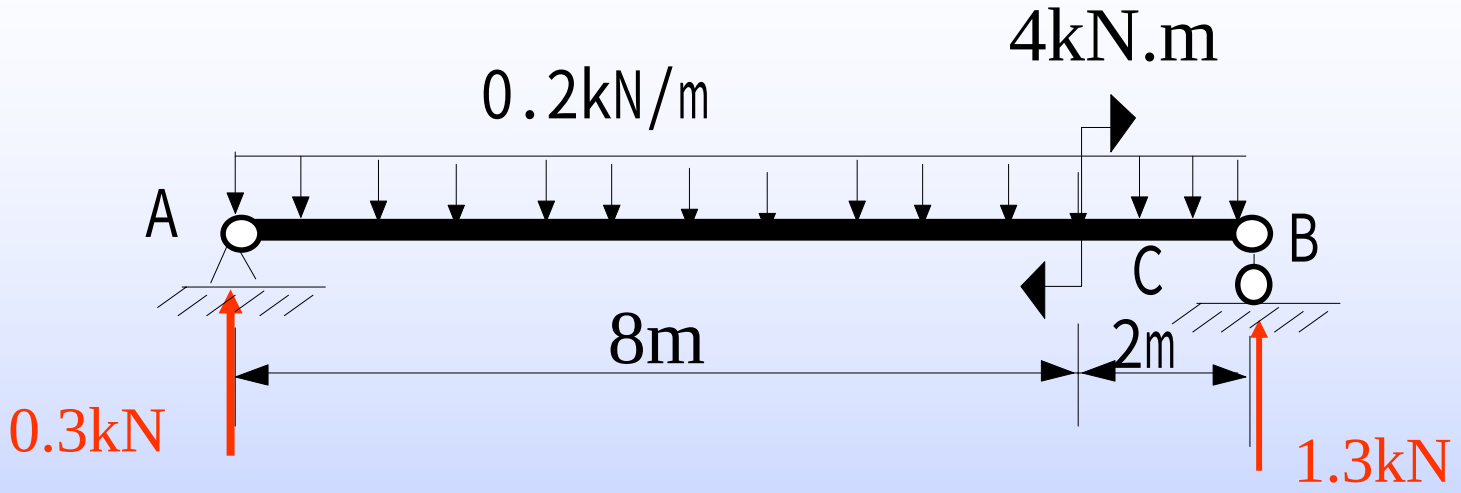
1)求约束力：

F_A=0.6kN （向下） F_B=1.4kN (向上）



2）剪力方程　AC段：$F_{s1}=0.3-0.2x \quad (0<x<8)$

CB段：$F_{s2}=-1.3-0.2x \quad (0<x<2)$

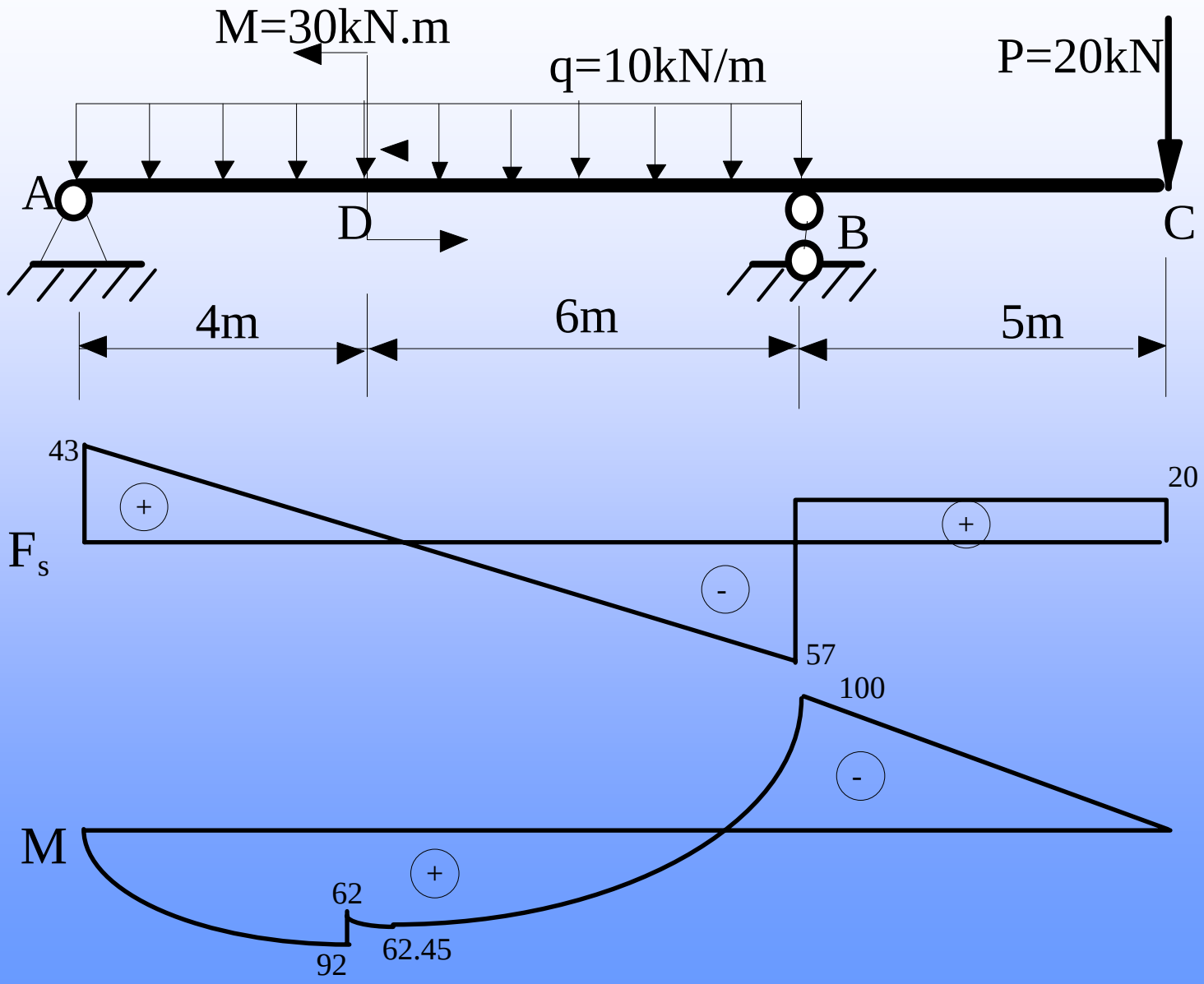


3）弯矩方程

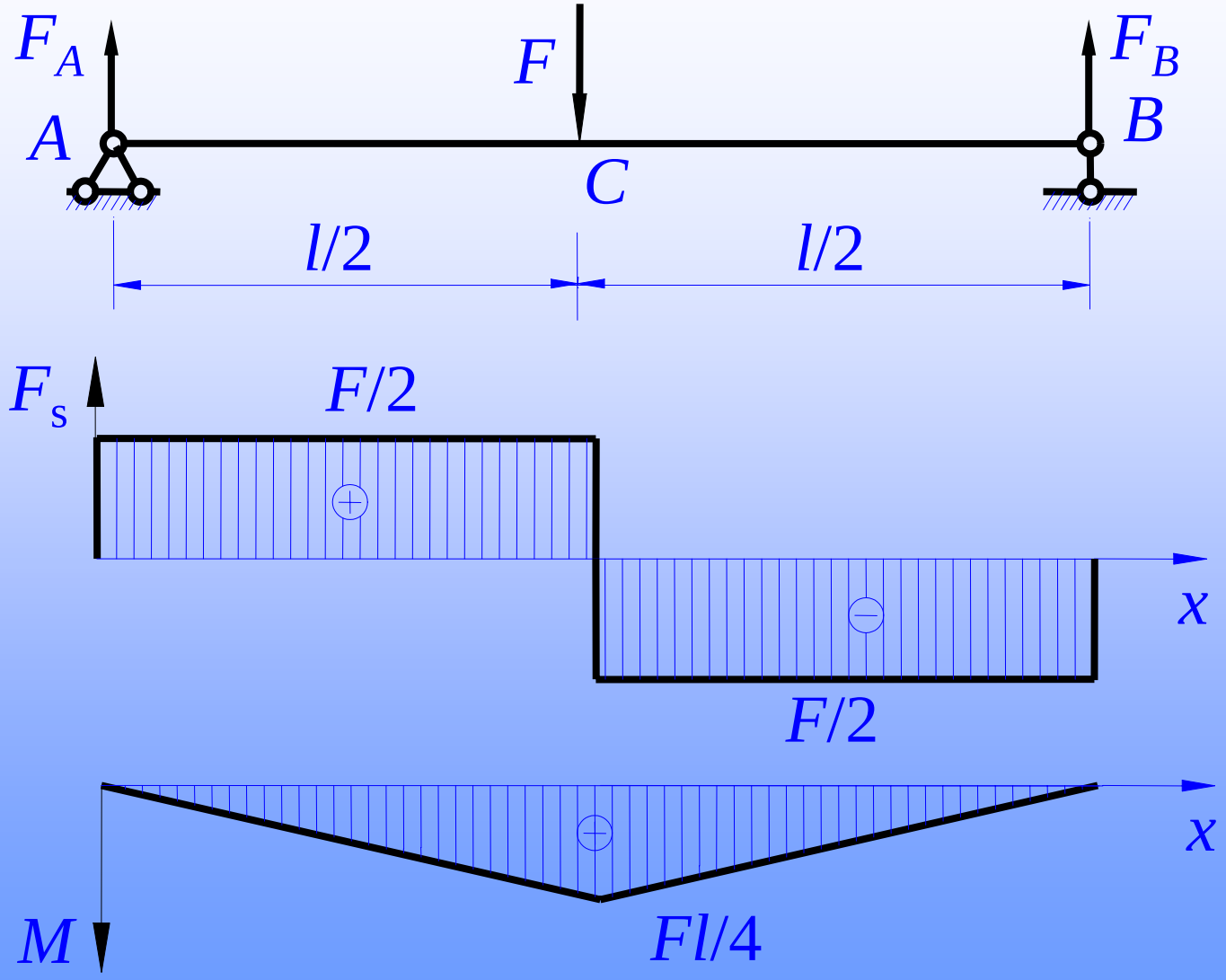
AC段：$M_1 = 6 - 2x \quad (0 < x \leq 2)$

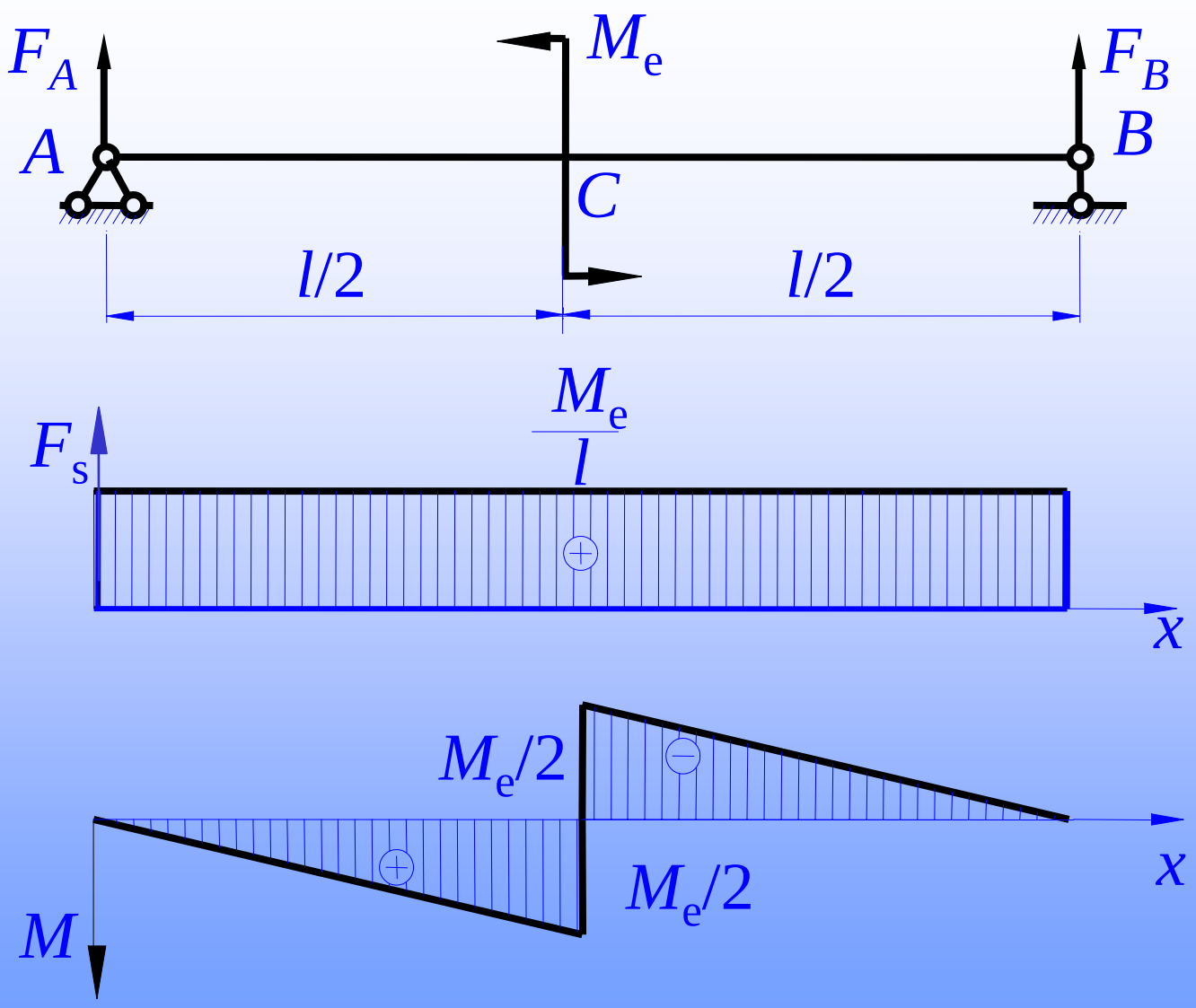
CB段：$F_{s2} = -20 + 22x \quad (0 < x \leq 1)$

例：作图示外伸梁的剪力图和弯矩图。



思考：对称性与反对称性




F_A
M_e
F_B
A
B
C
l/2
l/2
M_e
l
F_s
x
$M_e/2$
$M_e/2$
x
M

结论：

- 结构对称、外力对称时，弯矩图为正对称，剪力图为反对称
- 结构对称、外力反对称时，弯矩图为反对称，剪力图为正对称

作业：

4-3(b),(e),(h)

4-4(a),(c)