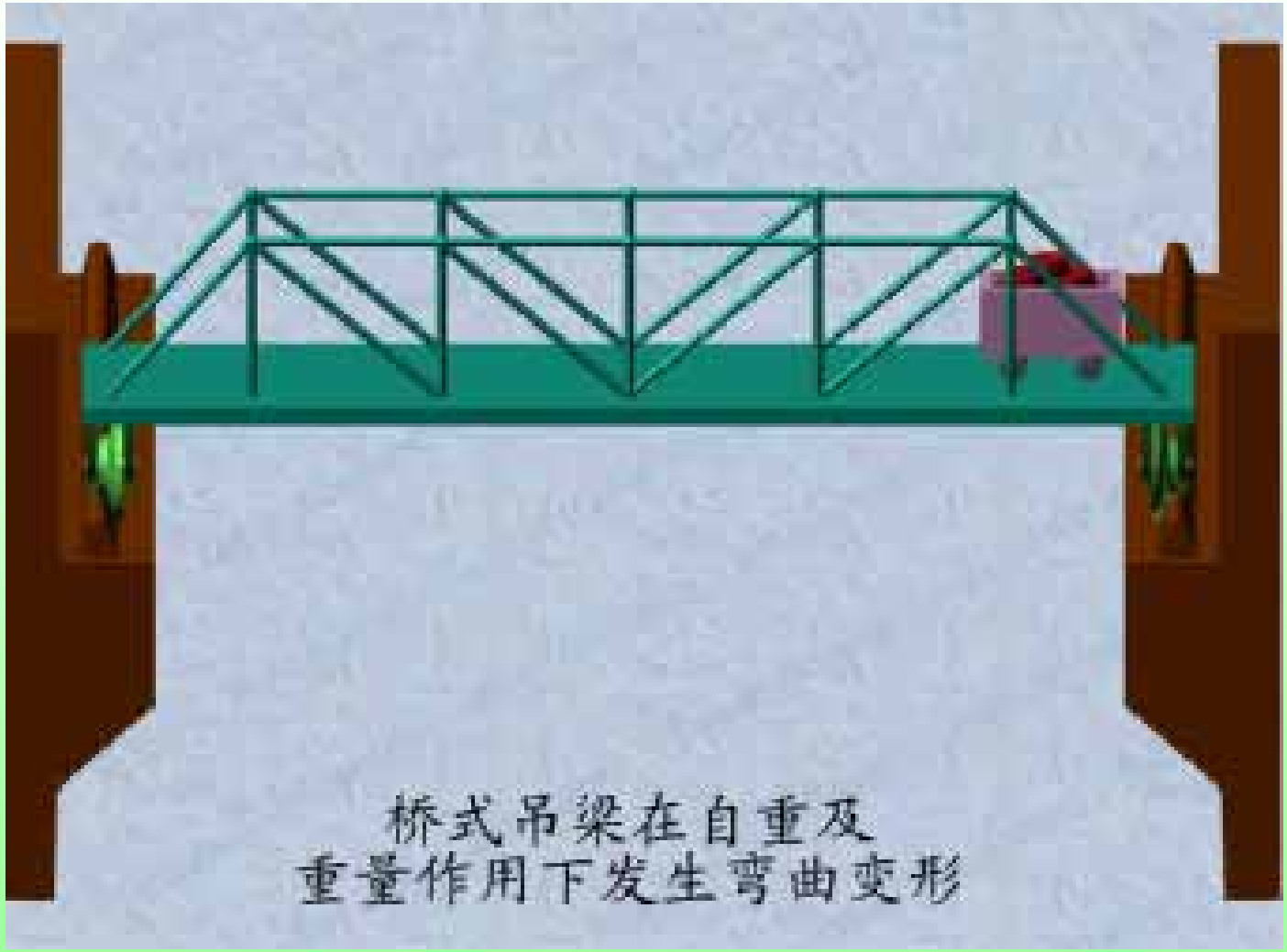
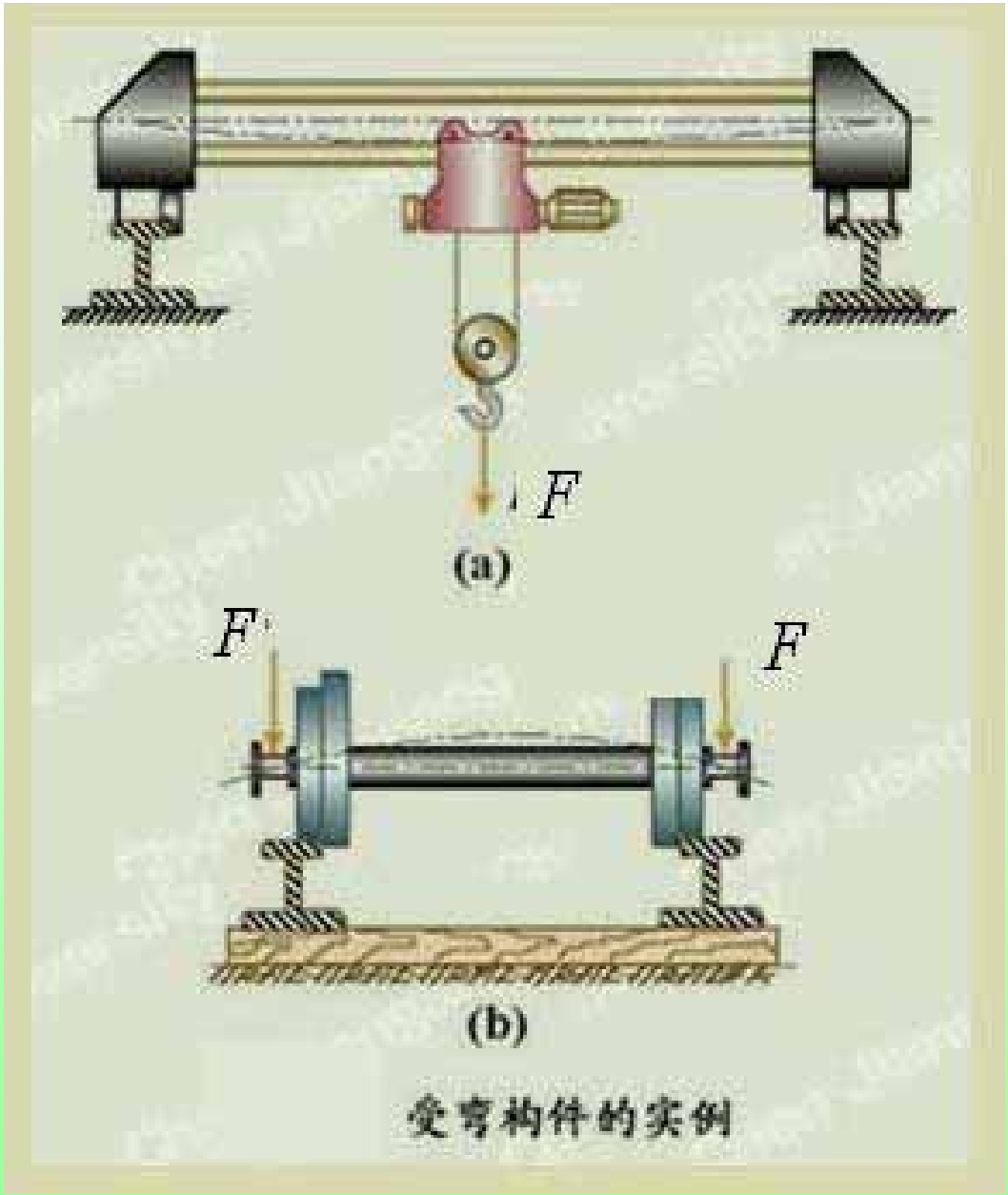
第四章　弯曲应力

§4-1 对称弯曲的概念及梁的计算简图



桥式吊梁在自重及
重量作用下发生弯曲变形

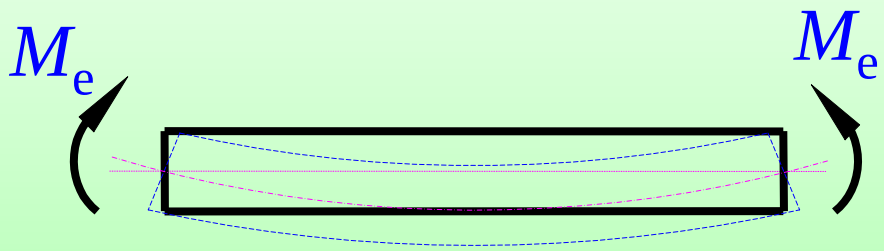


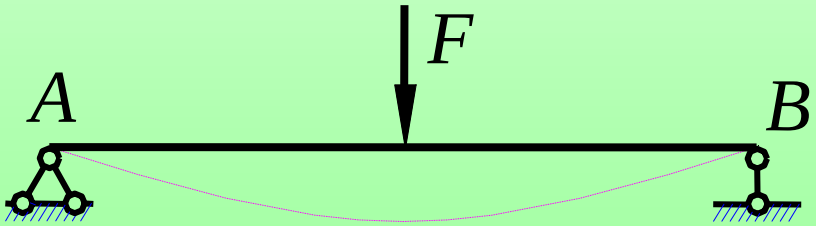
受弯构件的实例

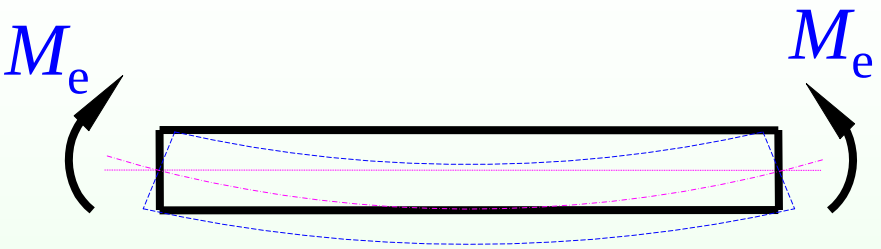
Ⅰ、弯曲的概念

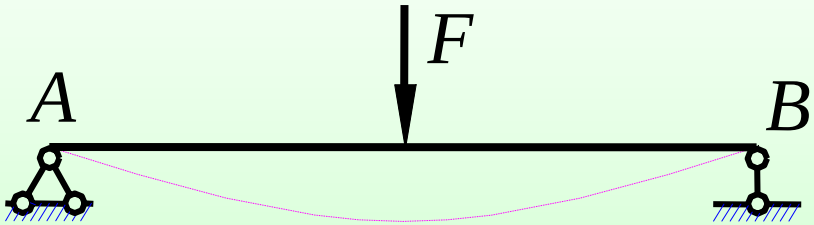
受力特点：

杆件受到垂直于杆轴线的外力（横向力）或外力偶（其矢量垂直于杆轴）作用。









变形特点：

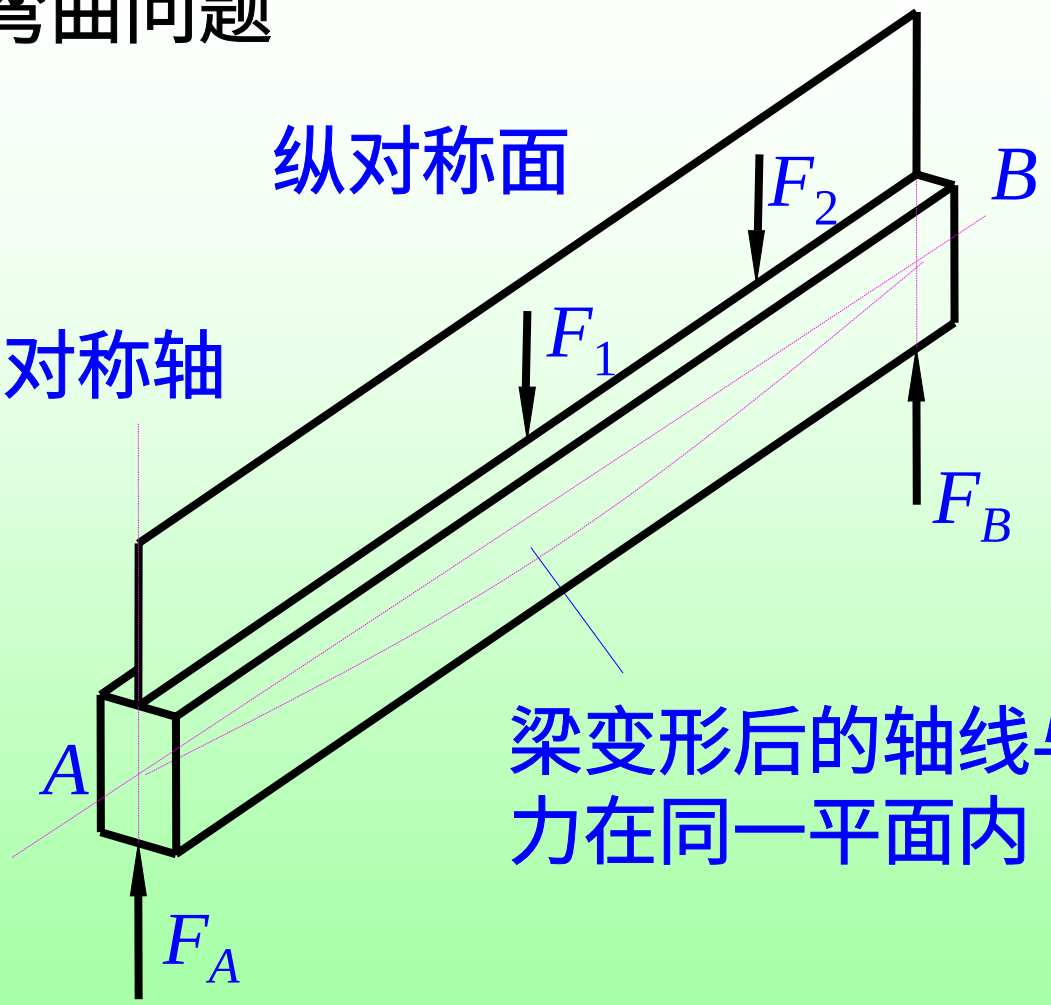
1、直杆的轴线在变形后变为曲线；

2、任意两横截面绕垂直于杆轴的轴作相对转动。

梁　——以弯曲为主要变形的杆件通称为梁。

最基本常见的弯曲问题

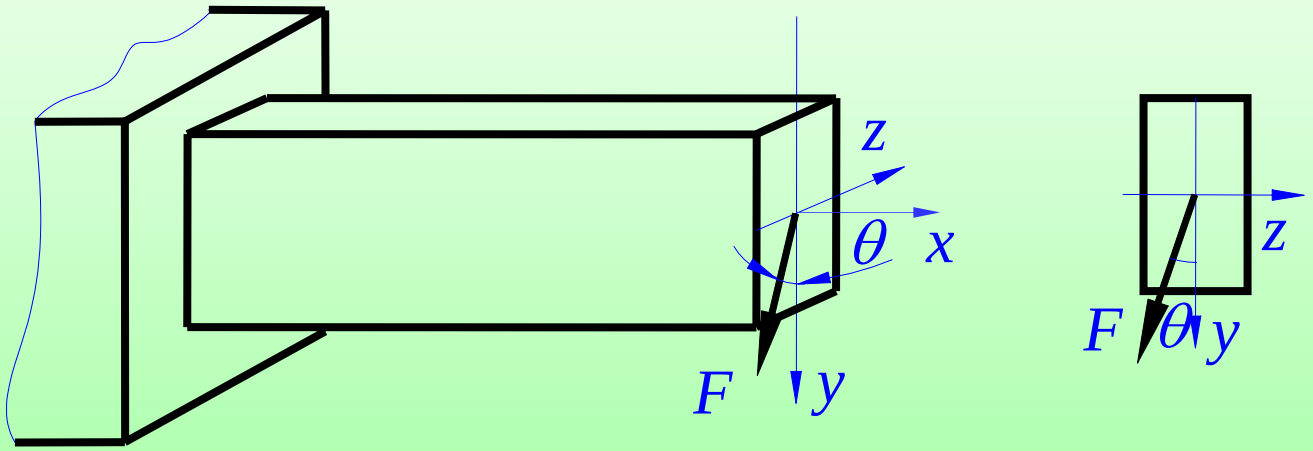
——对称弯曲



对称弯曲时梁变形后轴线所在平面与外力所在平面相重合，因而一定是平面弯曲。

非对称弯曲—— 1、梁不具有纵对称面；

2、梁有纵对称面，但外力没有作用在纵对称面内，从而变形后轴线所在平面与梁的纵对称面不一致。

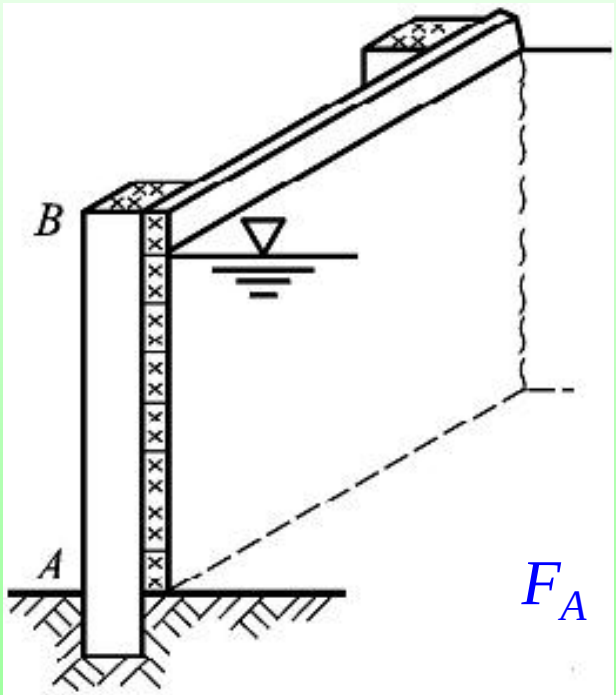


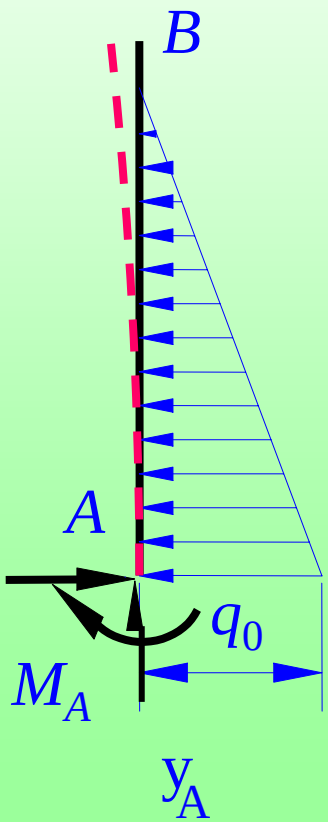
特定条件下，发生非对称弯曲的梁变形后其轴线所在平面也会跟外力所在平面相重合，因而也属于平面弯曲。

II、梁的计算简图

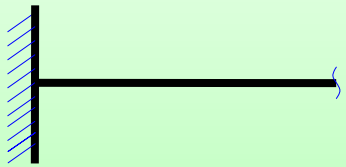
1、支座的基本形式

(1)固定端

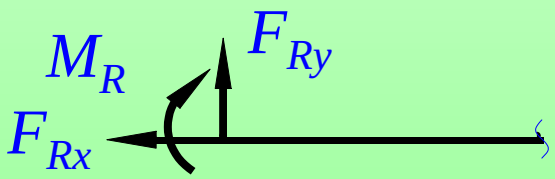




计算简图

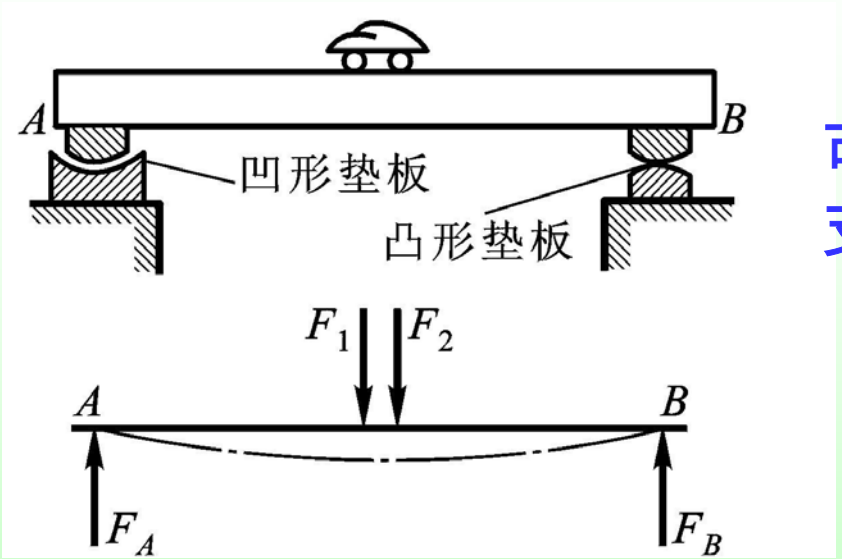


约束反力



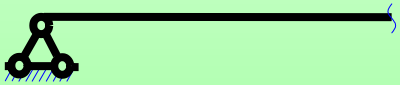
(2)固定铰支座和可动铰支座

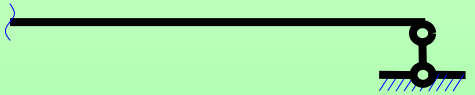
固定铰支座



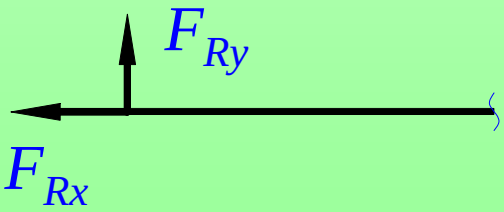
可动铰支座

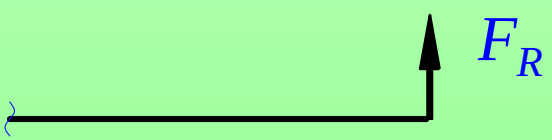
计算简图





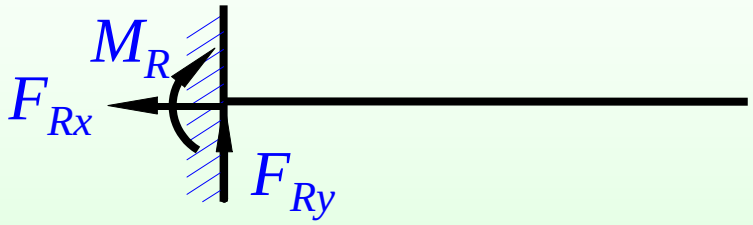
约束反力





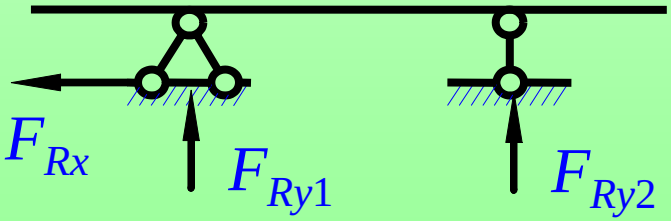
2、梁的基本形式

(1)悬臂梁



(2)简支梁

(3)外伸梁

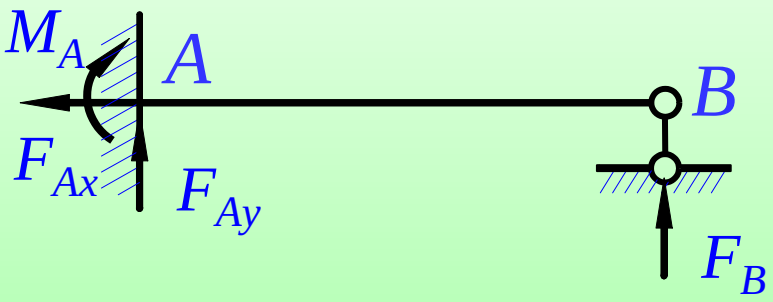


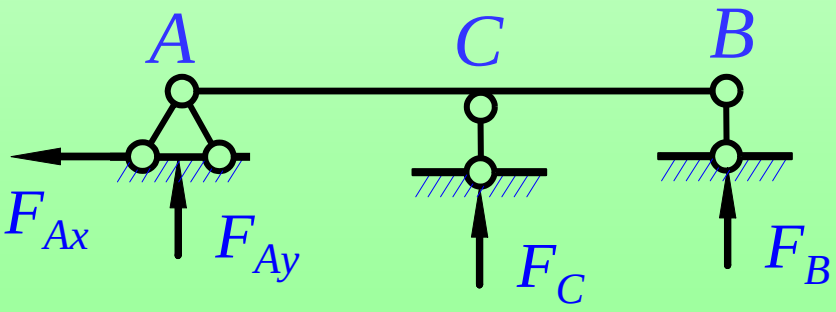
静定梁

3、静定梁和超静定梁

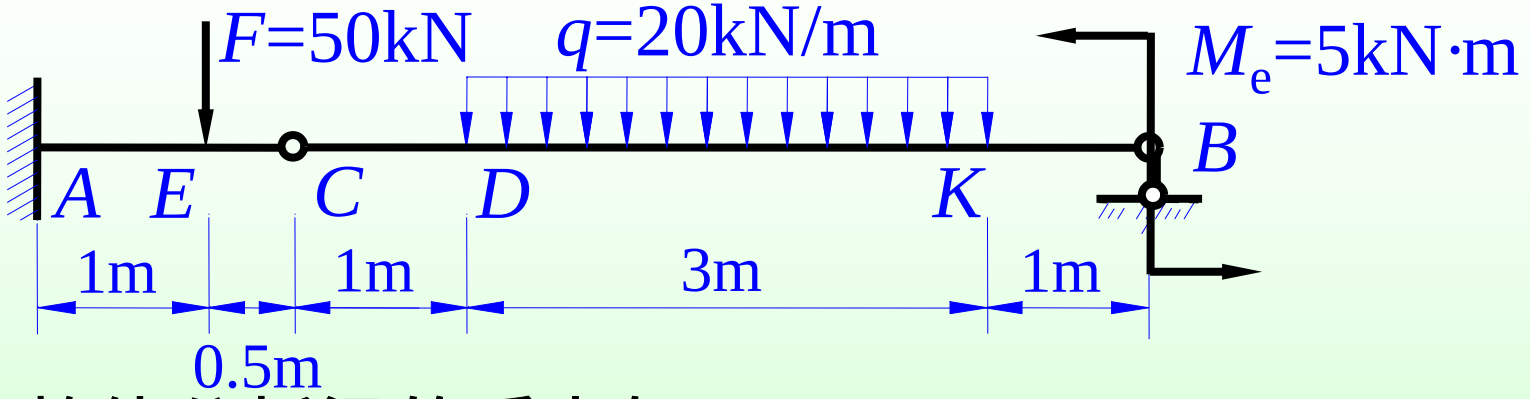
静定梁　梁的支反力均可由平面力系的三个独立的平衡方程求出。

超静定梁　梁的支反力单独利用平衡方程不能确定。

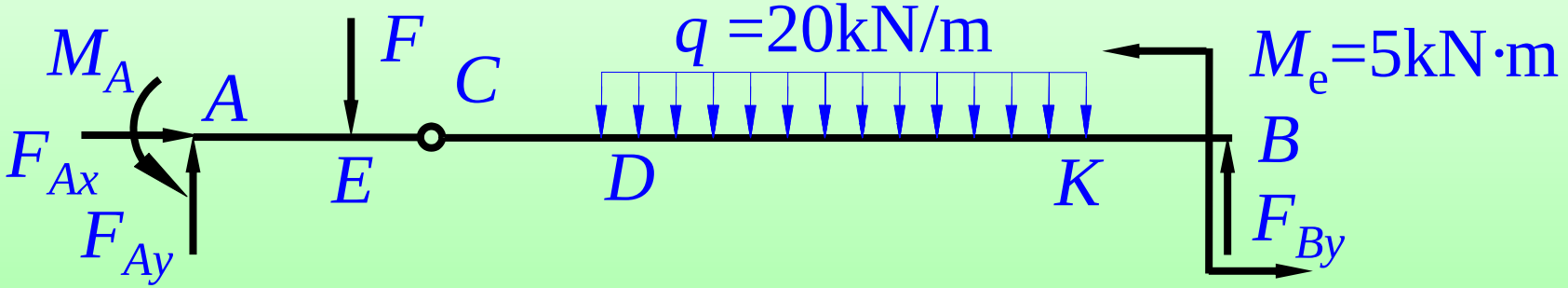




P_{93}例4-1 试求图示有中间铰C的梁AB的支反力。

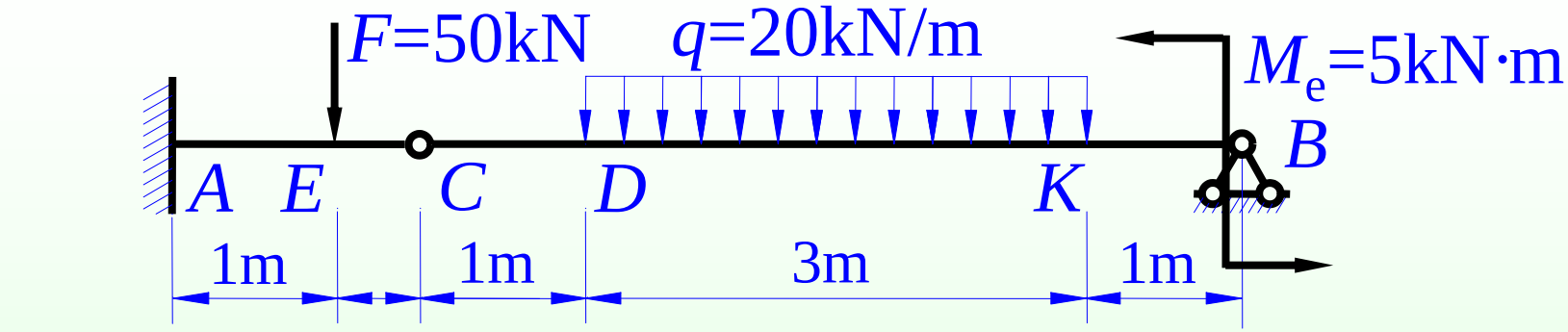


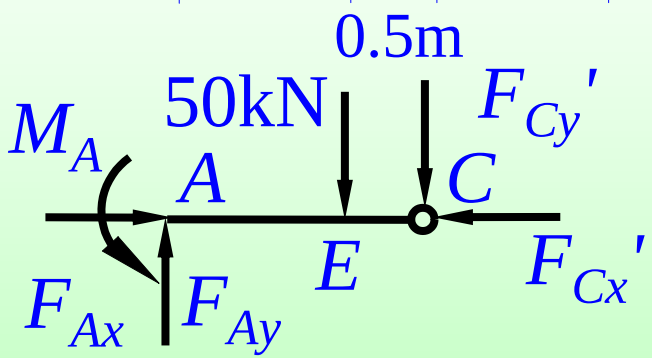
解： 整体分析梁的受力如图。



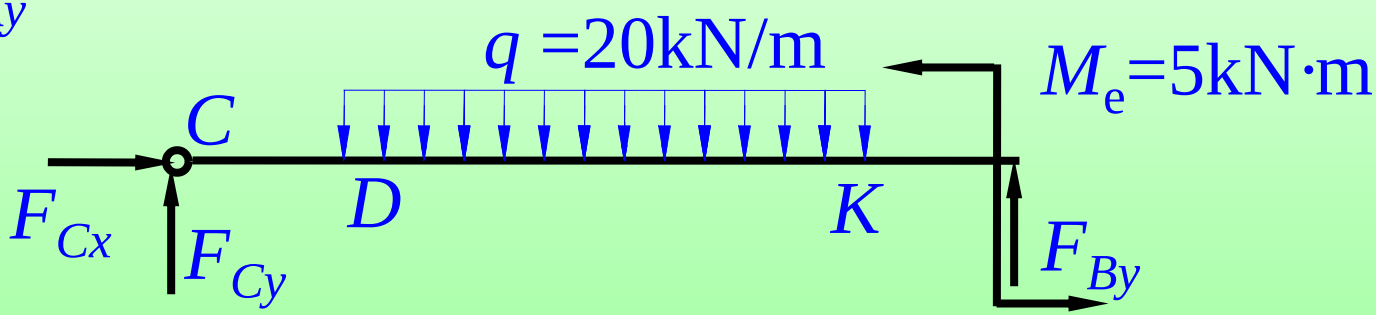
未知支反力： 4个　整体独立平衡方程：3个

关键在于中间铰不能传递力矩的特性，因而不论AC段或CB段均有 $\sum M_C = 0$



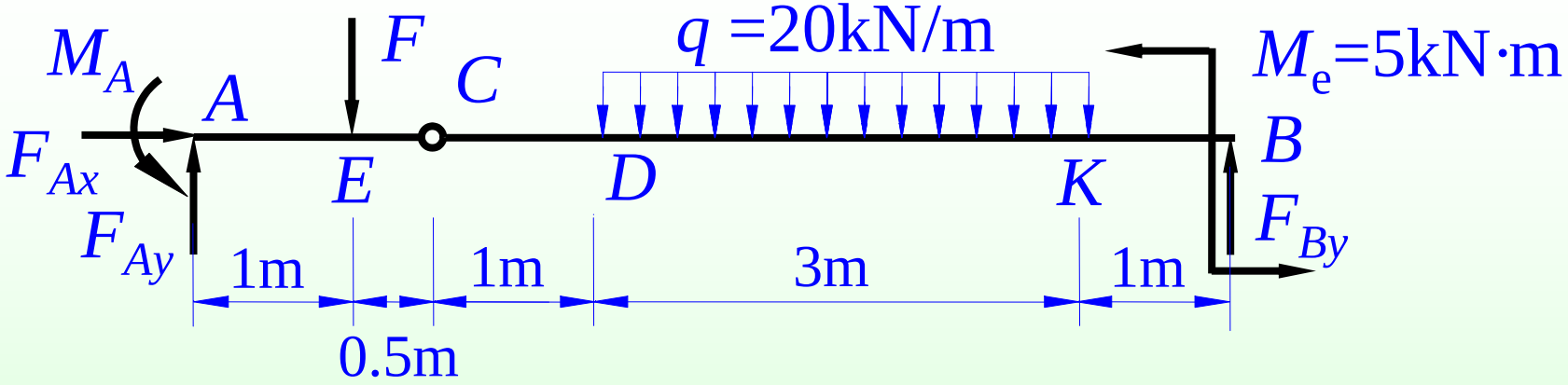


$$\sum M_C = 0$$



$$\sum M_C = 0 \quad -\left(20\times10^3\times3\times2.5\right)+5\times10^3+F_{By}\times5=0$$

$$F_{By}=29\text{kN}$$



$$\sum F_x = 0 \qquad F_{Ax} = 0$$

$$\sum F_y = 0 \qquad -F_{Ay} + 50 + (20 \times 3) - 29 = 0$$

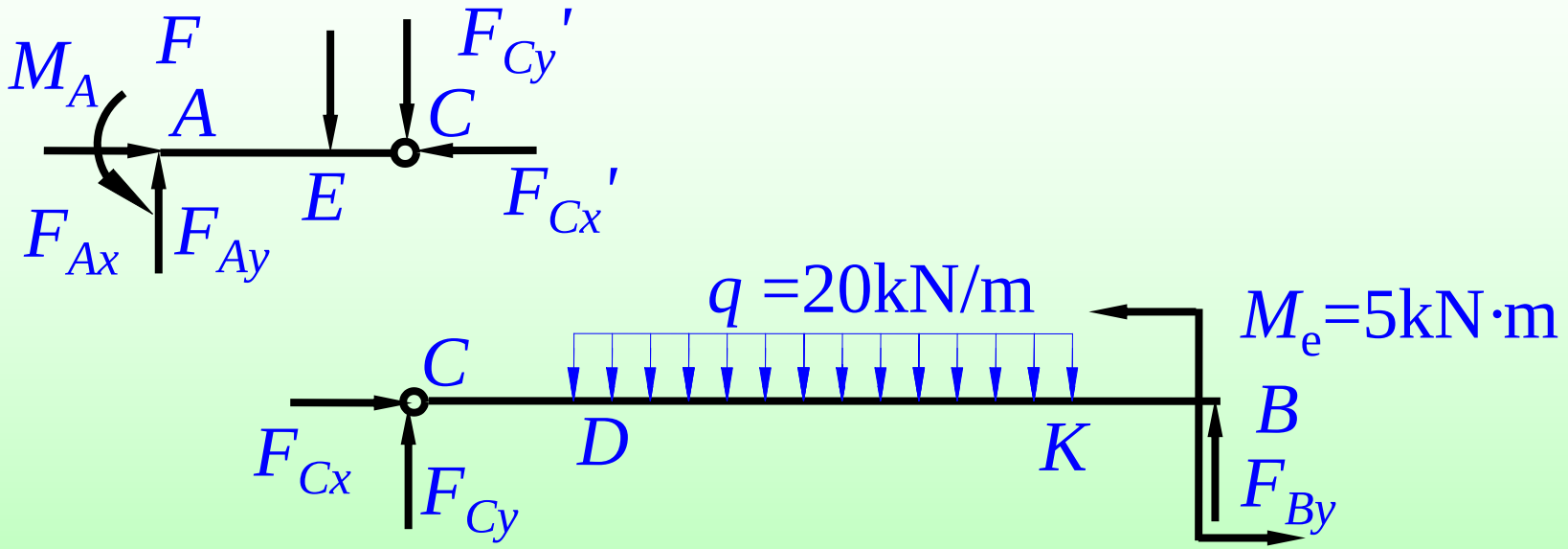
$$F_{Ay} = 81\text{kN}(\uparrow)$$

$$\sum M_A = 0$$

$$M_A - 50 \times 10^3 \times 1 + 5 \times 10^3 - 20 \times 10^3 \times 4 \times 3 = 0$$

$$M_A = 96.5\text{kN} \cdot \text{m}$$

带有中间铰的梁的受力特点：

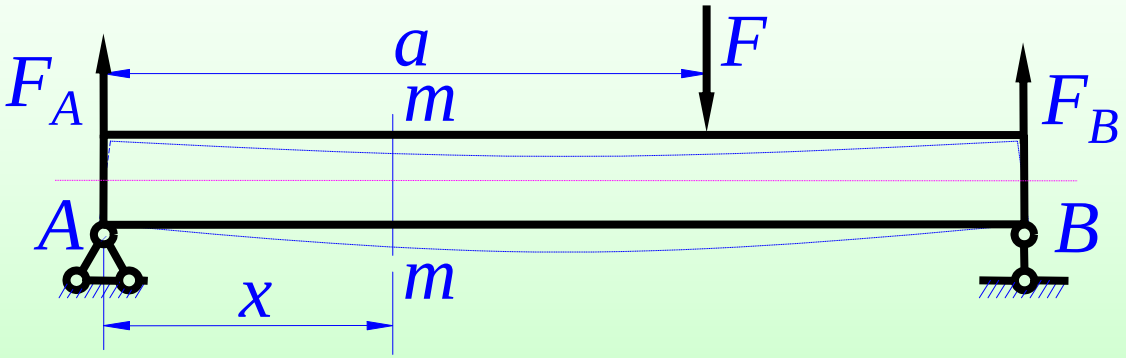


*CB*梁段上的荷载会传递到梁的*AC*段，称为副梁；

*AC*段上的荷载不会传递到梁的*CB*段，称为基本梁(或主梁)。

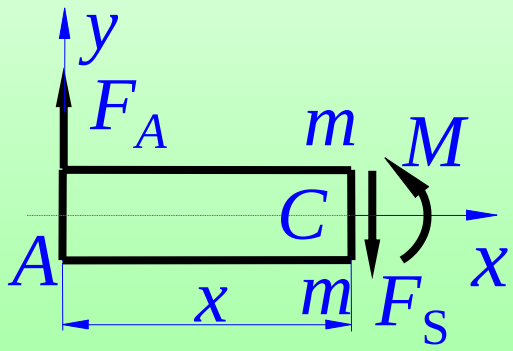
§4-2 梁的剪力和弯矩•剪力图和弯矩图

Ⅰ、梁的剪力和弯矩



$$F_A = \frac{F(l-a)}{l}$$

$$F_B = \frac{Fa}{l}$$



取左侧分离体分析任一横截面*m-m*上的内力

$$\sum F_y = 0 \qquad F_S = F_A = \frac{F(l-a)}{l}$$

$$\sum M_C = 0 \qquad M = F_A x = \frac{F(l-a)}{l} x$$

由其右边分离体的平衡条件同样可得

$$\sum F_y = 0$$

$$F_S - F + F_B = 0$$

$$F_S = F - F_B = \frac{F(l-a)}{l}$$

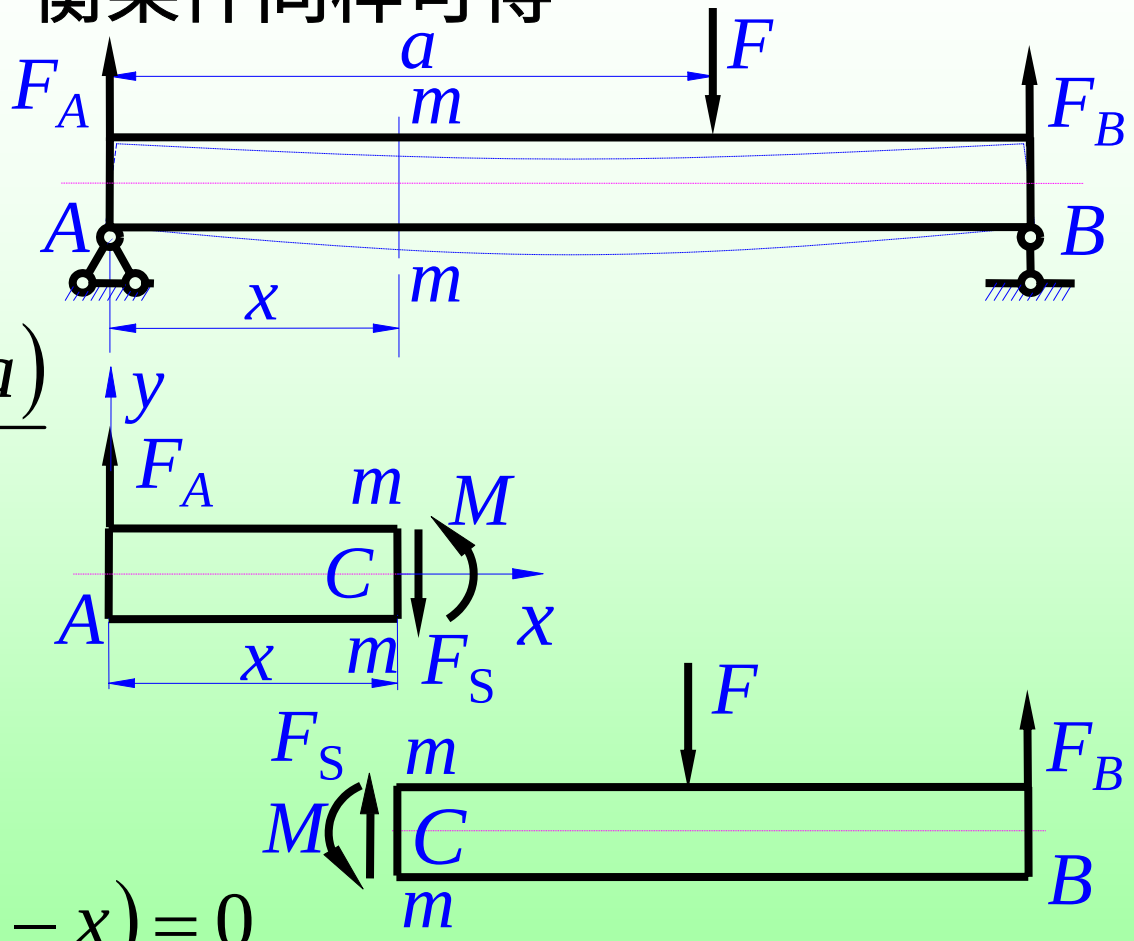
切向应力的合力，称为剪力

$$\sum M_C = 0$$

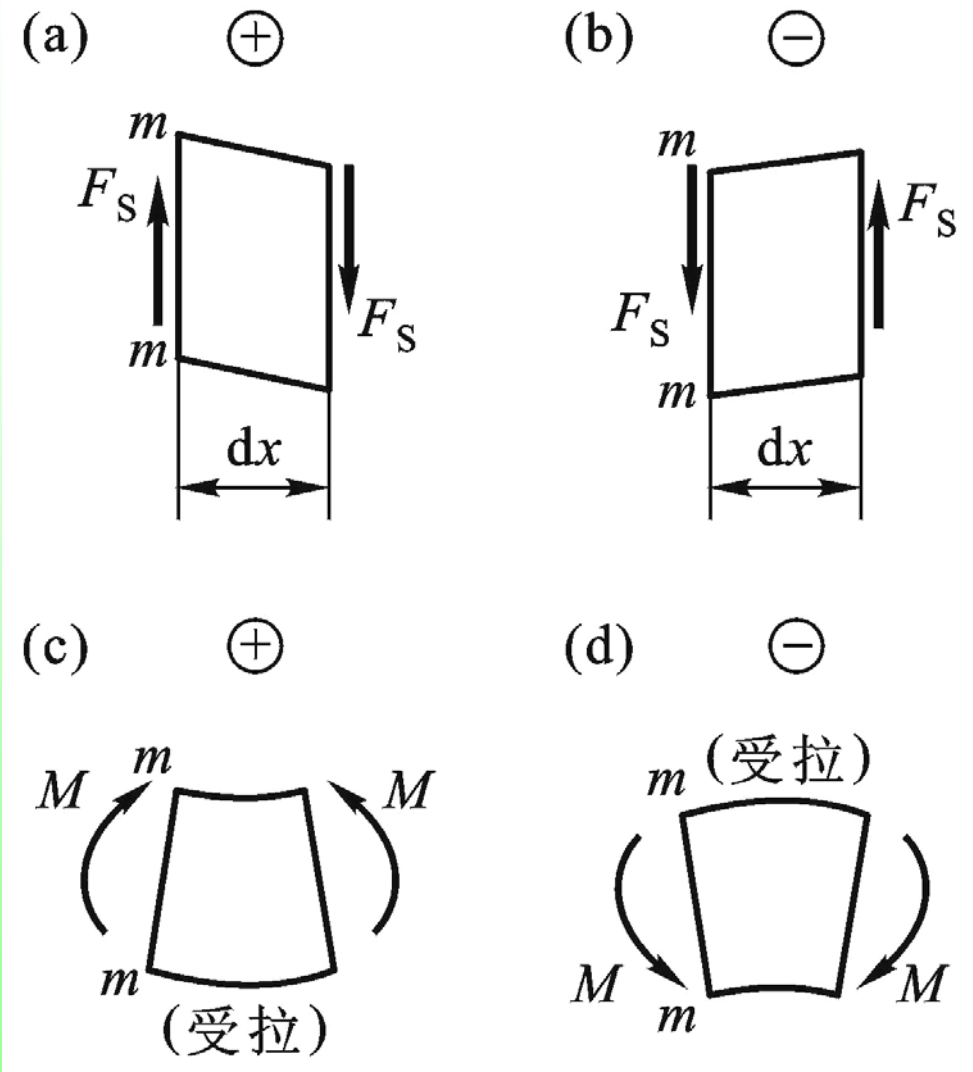
$$-M - F(a-x) + F_B(l-x) = 0$$

$$M = F_B(l-x) - F(a-x) = \frac{F(l-a)}{l}x$$

法向应力的合力，称为弯矩



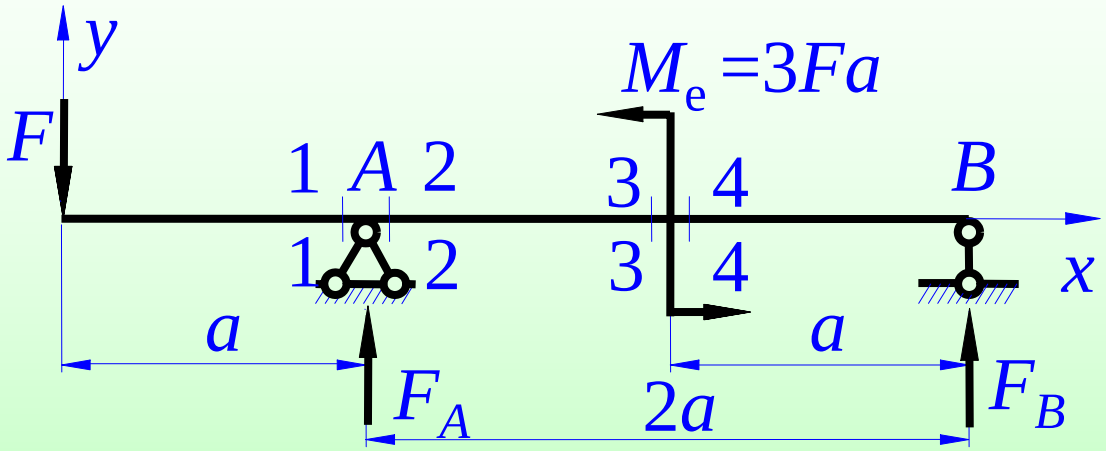
剪力和弯矩的符号规则：



左上右下，
剪力为正

左顺右逆，
弯矩为正

例4-2 求图示外伸梁在截面1—1、2—2、3—3和4—4横截面上的剪力和弯矩。

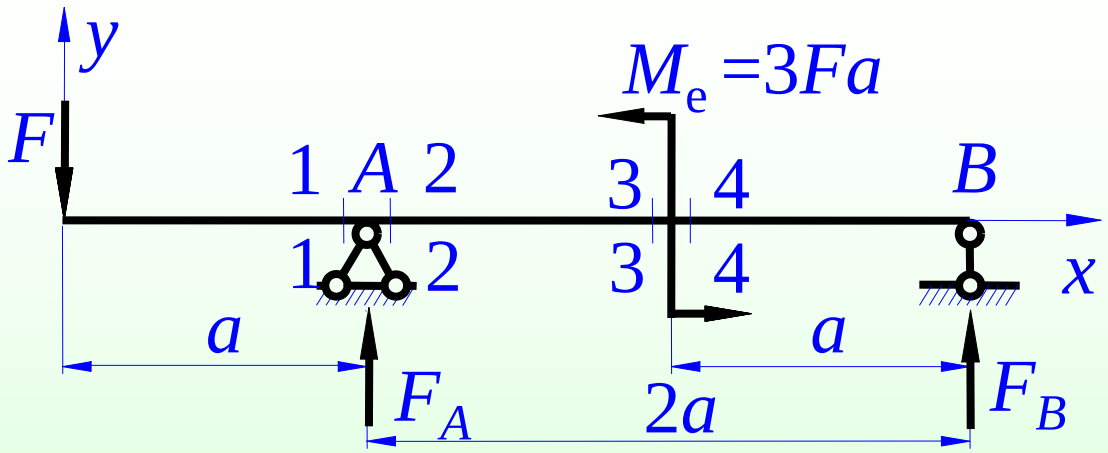


解：支反力为

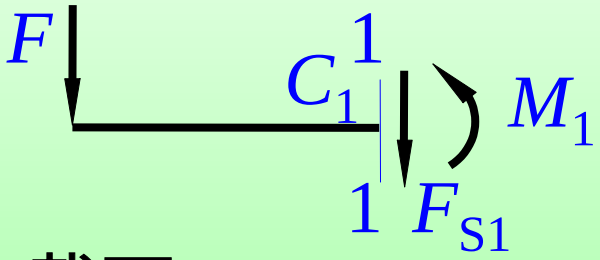
$$\sum M_A = 0 \qquad F_B \times 2a + 3Fa + F \times a = 0$$

$$F_B = -2F(\downarrow)$$

$$\sum F_y = 0 \qquad F_B + F_A = F \qquad F_A = 3F(\uparrow)$$



截面1—1

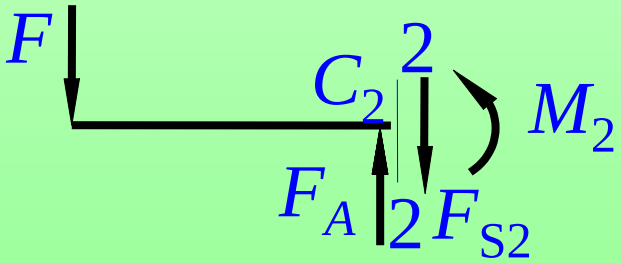


$$\sum F_y = 0 \qquad F_{S1} = -F$$

$$\sum M_{C1} = 0 \qquad M_1 + F \times a = 0$$

$$M_1 = -Fa\,(\text{顺})$$

截面2—2

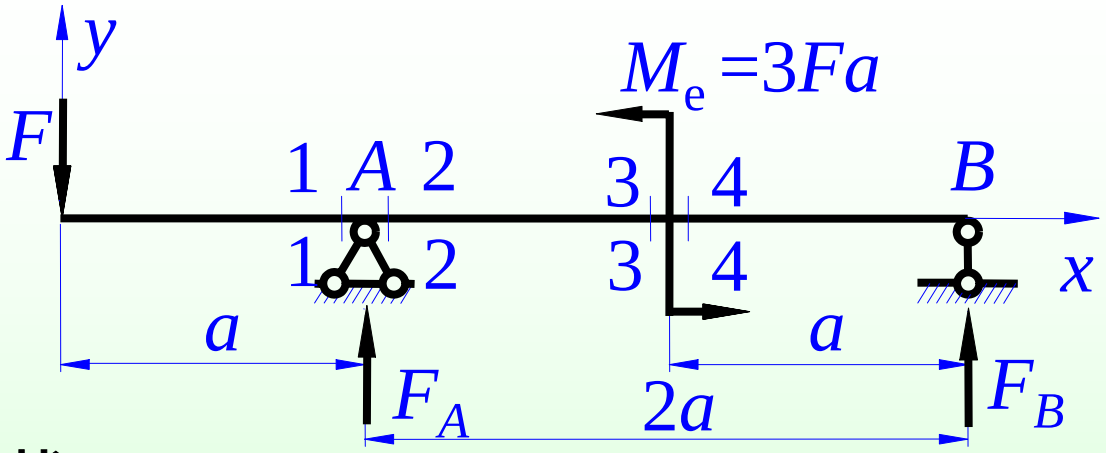


$$\sum F_y = 0 \qquad F_{S2} - F_A + F = 0$$

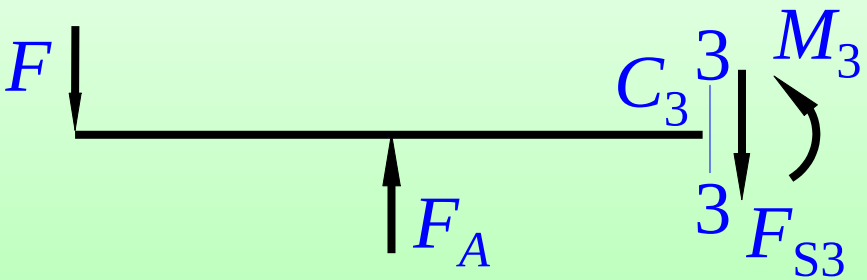
$$F_{S2} = F_A - F = 2F$$

$$\sum M_{C2} = 0 \qquad M_2 + F \times a = 0$$

$$M_2 = -Fa\,(\text{顺})$$



截面3—3



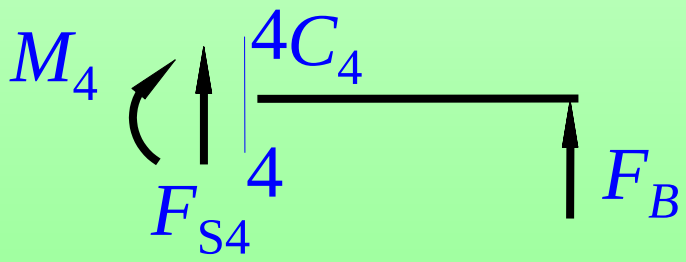
$$F_{S3}-F_A+F=0$$

$$F_{S3}=F_A-F=2F$$

$$M_3+F\times a-F_A\times a=0$$

$$M_3=Fa\,(\text{逆})$$

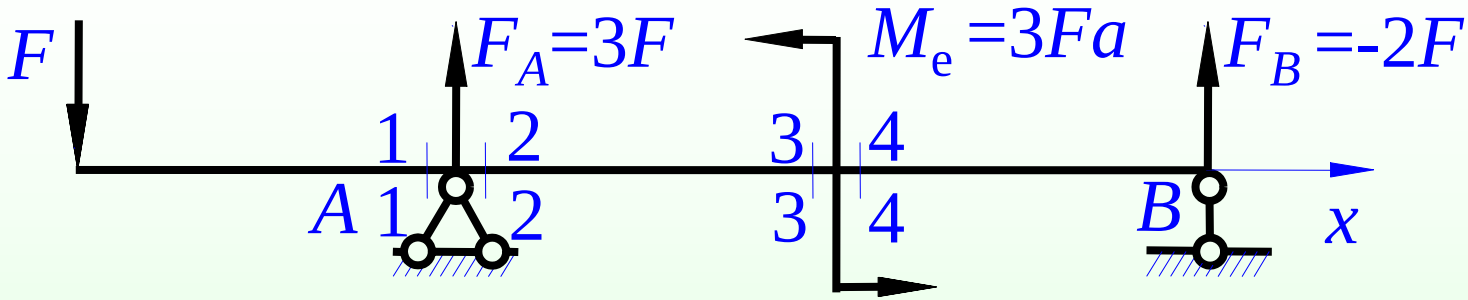
截面4—4



$$F_{S4}=-F_B=2F$$

$$M_4-F_B\times a=0$$

$$M_4=-2Fa\,(\text{顺})$$

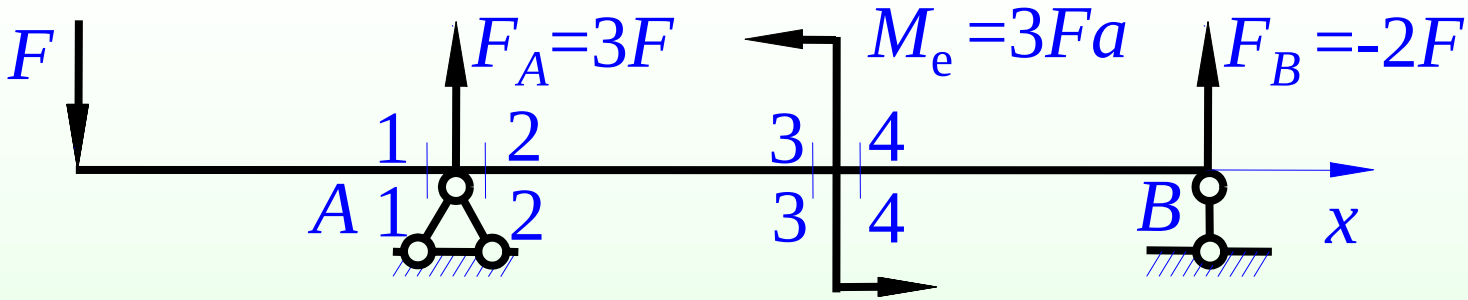


内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

1、横截面上的剪力和弯矩在数值上由截面左侧或右侧梁段分离体的静力平衡方程来确定。

剪力值= 截面左侧（或右侧）所有外力的代数和

弯矩值= 截面左侧（或右侧）所有外力对该截面形心的力矩代数和



内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

2、截面左侧梁段　向上的外力→正剪力→正弯矩

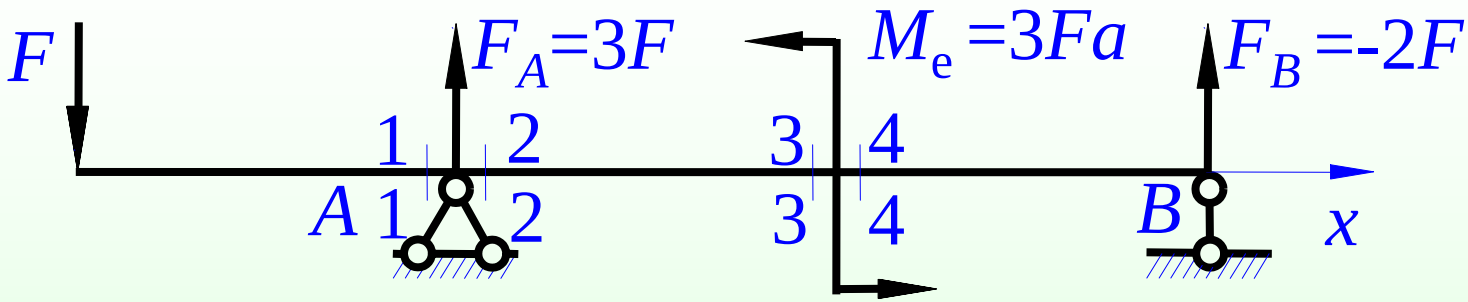
顺时针外力偶→正弯矩

截面右侧梁段　向上的外力→负剪力→正弯矩

顺时针外力偶→负弯矩

剪力：左上右下为正

弯矩：左顺右逆为正

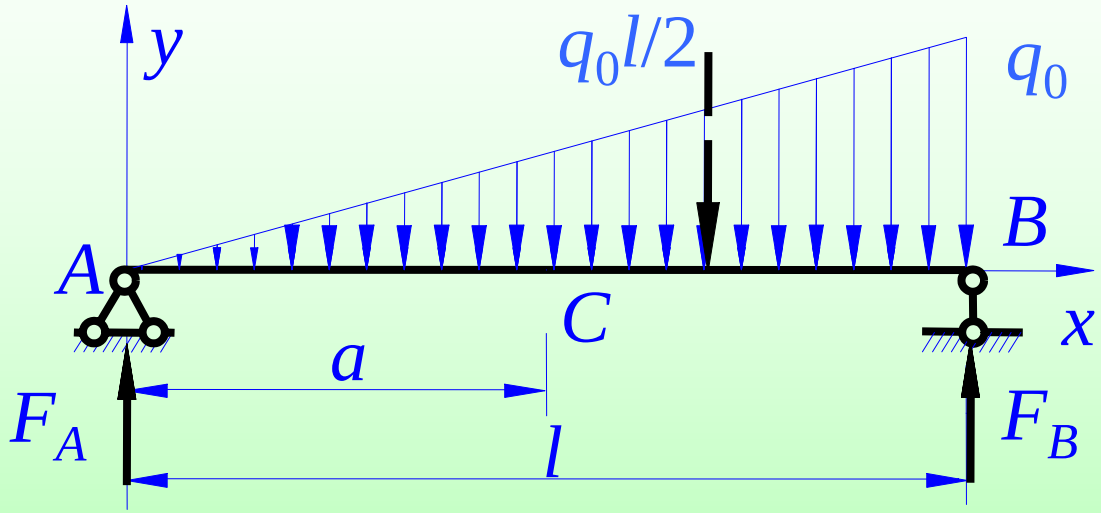


内力	1—1	2—2	3—3	4—4
F_S	-F	2F	2F	2F
M	-Fa	-Fa	Fa	-2Fa

3、在集中力作用处，剪力值发生突变，突变值=集中力大小；

在集中力偶作用处，弯矩值发生突变，突变值=集中力偶矩大小。

P_{96} 例4-3 图示简支梁受到三角形分布荷载的作用，最大荷载集度为q_0，试求截面C上的内力。

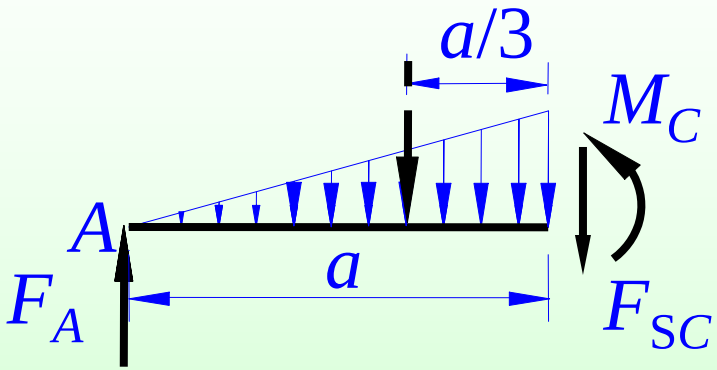


解：先求支反力

$$\sum M_A = 0 \qquad F_B l - \frac{q_0 l}{2} \times \frac{2l}{3} = 0 \qquad F_B = \frac{q_0 l}{3}$$

$$\sum M_B = 0 \qquad -F_A l + \frac{q_0 l}{2} \times \frac{l}{3} = 0 \qquad F_A = \frac{q_0 l}{6}$$

截面C的内力



$$q(x)=\frac{q_0 x}{l}$$

$$F_{SC}=F_A-\frac{a}{2}\times\frac{q_0 a}{l}=\frac{q_0 l}{6}-\frac{q_0 l^2}{2l}$$

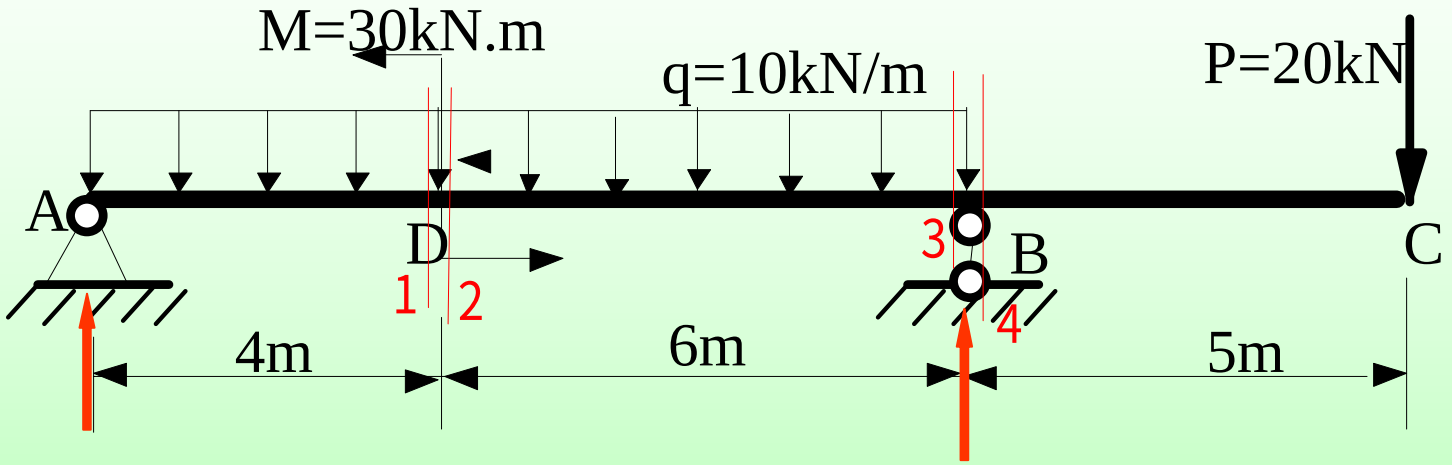
$$M_C=F_A\times a-\frac{a}{2}\times\frac{q_0 a}{l}\times\frac{a}{3}=\frac{q_0 l}{6}a-\frac{q_0 a^3}{6l}$$

思考：

是否可以将梁上的分布荷载全部用静力等效后的合力代替来求截面C的内力？

课堂练习

1．求图示外伸梁1，2，3，4截面的内力。



F_A=50-10+3=43kN

F_B=50+30-3=77kN

F_{s1}=43-10x4=3kN　　M_1=43x4-1/2x10x4^2=92kN.m

F_{s2}=3kN　　M_2=92-30=62kN.m

F_{s3}=20-77=57kN　　M_3=-20x5=-100kN.m

F_{s4}=20kN　　M4=-100kN.m

作业：P_{141} 4-1(b),(g);