Optimal Design and Quantum Benchmarks for Coherent State Amplifiers

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We establish the ultimate quantum limits to the amplification of an unknown coherent state, both in the deterministic and probabilistic case, investigating the realistic scenario where the expected photon number is finite. In addition, we provide the benchmark that experimental realizations have to surpass in order to beat all classical amplification strategies and to demonstrate genuine quantum amplification. Our result guarantees that a successful demonstration is in principle possible for every finite value of the expected photon number.

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Continuous-variable quantum systems, such as coherent light pulses, are promising information carriers for the new quantum technology [1,2]. One of the cornerstones of continuous-variable quantum information is the amplification of signals encoded into quantum states of the radiation field [3,4]. Unlike classical amplifiers, quantum amplifiers are subject to fundamental limits, typically expressed as a reduction of the signal-to-noise ratio (SNR) as a function of the amplification parameter [5–7]. Despite these limits, quantum amplifiers are an essential piece of technology [8], for they enable the detection of ultraweak signals—such as gravitational waves—that would not trigger the detectors otherwise.

Determining the ultimate quantum limits to amplification is both a topic of immediate technological import and a fundamental chapter of quantum theory, deeply connected with the no-cloning theorem, the uncertainty principle, and the quantum-classical transition in the limit of large amplification. Up to now, however, the performances of quantum amplifiers have been discussed mostly in classical terms (SNR), which are well suited for tasks such as signal detection, but less suited for applications in quantum information processing. For example, the role of the amplifier could be to coherently copy quantum data [9,10] and to broadcast them to the users of a quantum internet [11]. For quantum tasks, the most natural figure of merit is the fidelity between the desired output states and the states effectively produced by the amplifier, which can be interpreted operationally as the probability that the output state passes a test set up by a verifier who knows the input state.

In the fidelity setting, the works on optimal cloning of coherent states [12–15] give a first insight in the problem of optimal amplification, suggesting that two-mode squeezing should be the best deterministic process allowed by quantum mechanics. If confirmed in a realistic scenario, this conclusion would be of high practical importance, as it would allow one to construct the best possible amplifiers using an optical element that is already in the toolbox of most laboratories. However, the optimality of two-mode squeezing, long conjectured, has never been proved without invoking strong simplifying assumptions, either on the nature of the amplifier—typically assumed to be Gaussian—or on the probability distribution of the states to be amplified—typically assumed to be uniform over all coherent states. Both assumptions are far from trivial: On the one hand, it is well known that non-Gaussian operations often outperform Gaussian ones, even for the manipulation of coherent states [16]. Hence, there is no *a priori* reason to expect that the best amplifier of coherent states is Gaussian. From a fundamental point of view, any restriction on the allowed operations can hardly be satisfactory: if one wants to discover the ultimate quantum limits, one should not restrict the search to a subset, such as the subset of Gaussian operations, which has measure zero in the set of all possible operations. On the other hand, assuming a uniform distribution over coherent states means assuming that the expected photon number is infinite, or equivalently, that there is no bound on the energy of the source producing the coherent pulses—a quite unphysical assumption. In a realistic setting one can only have a large photon number, and in order to know how large this number should be to be effectively treated as infinite, one needs to gain first a full grasp of the finite photon number scenario.

Further motivation to go beyond the assumption of uniform distribution comes from the recent proposals of noiseless probabilistic amplifiers [17–22], whose performances are almost ideal for low photon numbers but decay exponentially as the photon number increases. In this case, it is most natural to test the performances of the amplifier on input states with low photon number, because these are the states where the amplifier is expected to work. Furthermore, in order to claim the demonstration of a genuine quantum amplifier, a real experiment should surpass the classical fidelity threshold (CFT) [23–26], i.e., the maximum fidelity achieved by "classical" amplifiers that produce an estimate of the input state and, conditional to the estimate, reprepare amplified states. In the case of probabilistic amplifiers, where the photon number is necessarily finite, it would be unfair to compare the experimental fidelity with a lower CFT computed for the uniform distribution. However, despite the urge to have suitable criteria to assess the new experimental breakthroughs on probabilistic amplification [19–22], the correct value of the CFT for probabilistic quantum amplifiers has never been derived up to now.

In this Letter we establish the ultimate limits on the fidelity of quantum and classical amplifiers, treating both the deterministic and probabilistic case without making any assumption on the type of amplifying process, and without making the assumption of infinite expected photon number. We focus on the realistic scenario where the coherent states are distributed according to a Gaussian prior, which is the most studied case for applications in coherent-state quantum cryptography [27–31], cloning [15], and teleportation or storage [23]. In the deterministic case, we show that the maximum quantum fidelity can be achieved through a two-mode squeezing process with the amount of squeezing depending critically on the variance of the prior. In the probabilistic case, the critical behavior persists, with a dramatic effect: for variances below the critical value, the optimal amplifier becomes non-Gaussian and its fidelity can be arbitrarily close to 1. We then provide the value of the classical fidelity threshold (CFT) that must be experimentally surpassed in order to demonstrate the implementation of a genuine quantum amplifier. The value of the CFT is the same for both deterministic and probabilistic protocols and, luckily, it guarantees that a successful demonstration is possible for every finite value of the expected photon number. For example, for a gain $g = 2$ and variance $1/3$, the value of the CFT is 50%, while the fidelity achieved by the optimal deterministic amplifier is 85%. The general techniques developed in this work are not limited to quantum amplification, but apply more broadly to the optimization of quantum devices for any desired quantum task, including, e.g., cloning, time reversal, and purification. At this level, they establish a tight relation between the demonstration of genuine quantum processing and the advantage of entanglement in the maximization of a suitable Bell-type correlation.

Let us start the derivation of our results. We begin from a general problem: finding the best physical process that approximates a desired transformation $\rho_x \mapsto \psi_x$, where $\{\rho_x\}_{x\in\mathsf{X}}$ is a set of (possibly mixed) input states, given with prior probabilities $\{p_x\}_{x\in\mathsf{X}}$, and $\{\psi_x = |\psi_x\rangle\langle\psi_x|\}_{x\in\mathsf{X}}$ is a set of pure target states. Finding the best coherent-state amplifiers is a special case of this problem, corresponding to the input $\rho_\alpha = |\alpha\rangle\langle\alpha|$ and the output $|\psi_\alpha\rangle = |g\alpha\rangle$, where $g > 1$ is the desired gain. To approximate the transformation $\rho_x \mapsto \psi_x$, we will consider the most general deterministic process, described by a quantum channel (completely positive trace-preserving map) $\mathcal{C}$. The performances of the channel will be ranked by the average fidelity $F = \sum_{x\in\mathsf{X}} p_x\langle\psi_x|\mathcal{C}(\rho_x)|\psi_x\rangle$. In addition to the deterministic processes we will also consider probabilistic ones, described by quantum operations (completely positive trace nonincreasing maps). The average fidelity of a quantum operation $\mathcal{Q}$, conditional on its occurrence, is given by $F' = \sum_{x\in\mathsf{X}} p_x\langle\psi_x|\mathcal{Q}(\rho_x)|\psi_x\rangle/(\sum_{x'\in\mathsf{X}} p_{x'}\mathrm{Tr}[\mathcal{Q}(\rho_{x'})])$. The optimal fidelity, defined as the supremum of the fidelity over all possible deterministic (probabilistic) processes, will be denoted by $F^{\det}$ (F^{prob}).

Theorem 1 ([32–34]) For deterministic processes, the optimal fidelity for the transformation $\rho_x \mapsto \psi_x$ is given by

$$F^{\det} = \inf_{\sigma>0,\mathrm{Tr}[\sigma]=1} \|A_\sigma\|_\infty$$
$$A_\sigma := \sum_{x\in\mathsf{X}} p_x|\psi_x\rangle\langle\psi_x| \otimes (\sigma^{-1/2}\rho_x\sigma^{-1/2})^T, \qquad (1)$$

where $\|A_\sigma\|_\infty$ denotes the operator norm $\|A_\sigma\|_\infty := \sup_{\|\Psi\|=1}\langle\Psi|A_\sigma|\Psi\rangle$, and T denotes the transpose.

For probabilistic processes, the optimal fidelity is given by

$$F^{\mathrm{prob}} = \|A_\tau\|_\infty \qquad \tau := \sum_{x\in\mathsf{X}} p_x\rho_x. \qquad (2)$$

Theorem 1 is a powerful tool for the optimization of quantum devices: since every quantum state $\sigma > 0$ gives an upper bound on the fidelity, finding a channel that achieves any of these bounds means finding an optimal channel.

In addition to the performances of the best quantum processes, it is important to know the CFT for the transformation $\rho_x \to \psi_x$. The CFT is the maximum fidelity that can be achieved with a classical, measure-and-prepare protocol, where the input state is measured with a positive operator-valued measure $\{P_y\}_{y\in\mathsf{Y}}$ and, conditionally on outcome y, a state ρ'_y is prepared. In the deterministic case, the fidelity of the protocol is the fidelity of the measure-and-prepare channel $\tilde{\mathcal{C}}(\rho) = \sum_{y\in\mathsf{Y}}\mathrm{Tr}[P_y\rho]\rho'_y$. In the probabilistic case, the positive operator-valued measure $\{P_y\}_{y\in\mathsf{Y}}$ includes an outcome $y = ?$, conditionally to which no output state is produced. The fidelity is then the fidelity of the measure-and-prepare quantum operation $\tilde{\mathcal{Q}}(\rho) = \sum_{y\in\mathsf{Y},y\neq ?}\mathrm{Tr}[P_y\rho]\rho'_y$. In the following, the CFT will be denoted by $\tilde{F}^{\det}$ ($\tilde{F}^{\mathrm{prob}}$) in the deterministic (probabilistic) case.

Theorem 2 [33] For deterministic protocols, the CFT for the transformation $\rho_x \to \psi_x$ is given by

$$\tilde{F}^{\det} = \inf_{\sigma>0,\mathrm{Tr}[\sigma]=1} \|A_\sigma\|_\times, \qquad (3)$$

where $\|A_\sigma\|_\times$ denotes the injective cross norm [35] $\|A_\sigma\|_\times := \sup_{\|\varphi\|=\|\psi\|=1}\langle\varphi|\langle\psi|A_\sigma|\varphi\rangle|\psi\rangle$.

For probabilistic protocols, the CFT is given by

$$\tilde{F}^{\mathrm{prob}} = \|A_\tau\|_\times. \qquad (4)$$

Remark: quantum-classical gap and Bell-type correlations—Note that the trace of the separable operator

A_σ with a quantum state is a Bell-type correlation. Remarkably, Eqs. (2) and (4) state that for probabilistic processes the gap between the quantum fidelity and the CFT is equal to the gap between the maximum Bell correlation achievable with entangled states and the maximum Bell correlation achievable with separable states. This relation establishes a tight connection between the demonstration of genuine quantum processing and the violation of suitable Bell-type inequalities.

We are now ready to tackle the optimal design of quantum amplifiers and to find the corresponding CFT. To account for the prior information about the input, we introduce a probability distribution $p(\alpha)$, normalized as $\int(d^2\alpha/\pi)p(\alpha) = 1$. The most popular choice for $p(\alpha)$, typically considered in the literature [23,27–31], is a Gaussian distribution with mean α_0 and variance $V=1/\lambda$. The idealized "uniform prior" can be retrieved here in the limit $\lambda \to 0$. Note that it is not restrictive to consider probability distributions centred around $\alpha_0 = 0$: indeed, both in the deterministic and probabilistic case, the fidelity does not change if one (i) replaces the prior $p(\alpha)$ by $p(\alpha - \alpha_0)$, (ii) displaces the input state by $-\alpha_0$, and (iii) displaces the output of the amplifier by $g\alpha_0$. For $\alpha_0 = 0$, the Gaussian $p_\lambda(\alpha) = \lambda e^{-\lambda|\alpha|^2}$ represents the distribution of coherent states generated by a classical oscillator obeying the Boltzmann distribution and $\langle n\rangle = 1/\lambda$ is the expected photon number. A controlled way to generate Gaussian-distributed coherent states is to prepare a two-mode squeezed state and perform a heterodyne measurement on one mode.

To determine the optimal deterministic amplifiers, it is useful to assess first the performances that can be achieved using two-mode squeezing, i.e., using quantum channels of the form

$$\mathcal{C}_r(\rho) = \mathrm{Tr}_B[e^{r(a^\dagger b^\dagger - ab)}(\rho \otimes |0\rangle\langle 0|)e^{-r(a^\dagger b^\dagger - ab)}], \quad (5)$$

where r is the squeezing parameter, a and b are the annihilation operators of the input mode and of an ancillary mode, respectively, and Tr_B denotes the partial trace over the ancillary Hilbert space. Optimizing the value of the squeezing parameter, one obtains the fidelity [33]

$$F^{\text{squeez}}_{g,\lambda} = \begin{cases} \frac{\lambda+1}{g^2}, & \lambda \le g-1 \\ \frac{\lambda}{\lambda+(g-1)^2}, & \lambda > g-1. \end{cases} \quad (6)$$

Note the discontinuity of the first derivative of the fidelity at the critical value $\lambda_c^{\text{det}} = g - 1$. This value separates two different domains: for $\lambda \le \lambda_c^{\text{det}}$ the optimal amount of squeezing in Eq. (5) is $r = \cosh^{-1}(g/(\lambda + 1))$, while for all values $\lambda > \lambda_c^{\text{det}}$ the optimal value is $r = 0$, corresponding to no squeezing at all. In other words, when the prior information about the input state is large (i.e., when the variance is small), the best amplifying strategy consists in leaving the state unamplified. In the case of 1-to-2 cloning, this fact was noted by Cochrane *et al.* [15], who assumed from the start cloning processes based on two-mode squeezing. Armed with Theorem 1, we are now in position to prove that no deterministic process can beat two-mode squeezing:

Theorem 3 (*optimal design of deterministic amplifiers* [33]) Two-mode squeezing is the best deterministic process for the amplification of Gaussian-distributed coherent states.

For probabilistic amplifiers, however, the situation is very different. Evaluating Eq. (2) we get [33]

$$F^{\text{prob}}_{g,\lambda} = \begin{cases} \frac{\lambda+1}{g^2}, & \lambda \le g^2-1 \\ 1 & \lambda > g^2-1. \end{cases} \quad (7)$$

The difference with the deterministic case is dramatic: above the critical value $\lambda_c^{\text{prob}} = g^2 - 1$, probabilistic processes allow for noiseless amplification. Fidelity arbitrarily close to $F^{\text{prob}}_{g,\lambda}$ can be reached as follows.

Theorem 4 (*optimal design of probabilistic amplifiers* [33]) The best probabilistic amplifier for Gaussian-distributed coherent states is (i) for $\lambda \le \lambda_c^{\text{det}}$, the two-mode squeezer (5) with squeezing parameter $r = \cosh^{-1}[g/(\lambda+1)]$ (ii) for $\lambda_c^{\text{det}} < \lambda \le \lambda_c^{\text{prob}}$, a quantum operation $\mathcal{Q}_N(\rho) = Q_N\rho Q_N$ with $Q_N \propto \sum_{n=0}^N[(\lambda+1)/g]^n|n\rangle\langle n|$, achieving fidelity $F^{\text{prob}}_{g,\lambda} = (1+\lambda)/g^2$ exponentially fast in the limit $N \to \infty$ (iii) for $\lambda > \lambda_c^{\text{prob}}$, a quantum operation $\mathcal{Q}_N(\rho) = Q_N\rho Q_N$ with $Q_N \propto \sum_{n=0}^N g^n|n\rangle\langle n|$, achieving the fidelity $F^{\text{prob}}_{g,\lambda} = 1$ exponentially fast in the limit $N \to \infty$. Note that for $\lambda > g - 1$ the optimal quantum operations are non-Gaussian, whereas for $\lambda = 0$ ("uniform prior") the optimal deterministic and probabilistic amplifiers coincide and are Gaussian. Noiseless amplification is only possible when the expected photon number is finite.

Suppose now that an experiment aims at demonstrating quantum amplification—or equivalently, cloning—of a coherent state. Thanks to Theorem 2, we can easily find the analytical expression of the CFT, also specifying the best measure-and-prepare channel. The result applies to both deterministic and probabilistic protocols, and, as an extra bonus, provides a concise derivation of the quantum benchmark for teleportation and storage of coherent states found by Hammerer *et al.* [23], which is retrieved here in the special case of no amplification ($g = 1$).

Theorem 5 (*benchmark for quantum amplifiers* [33]) The CFT for the amplification of Gaussian-distributed coherent states is given by

$$\tilde{F}_{g,\lambda} = \frac{1+\lambda}{1+\lambda+g^2} \quad (8)$$

both for deterministic and probabilistic protocols. The above value is achieved by a heterodyne measurement $P(\hat{\alpha})d^2\hat{\alpha}/\pi = |\hat{\alpha}\rangle\langle\hat{\alpha}|d^2\hat{\alpha}/\pi$ followed by the conditional preparation of the coherent state $|\frac{g\hat{\alpha}}{1+\lambda}\rangle$.

Equations (6)–(8) represent good news for experimental demonstrations. They prove that genuine quantum amplification can be demonstrated for every finite value of the expected photon number. As an illustration, consider the demonstration of probabilistic amplification provided by Zavatta *et al.* in Ref. [22]. In this case, the amplifier is designed to achieve gain $g = 2$. By Eq. (7), noiseless amplification requires at least $\lambda \geq 3$, which is actually a reasonable value in the experiment (choosing $\lambda = 3$ puts the maximum amplitude tested in the experiment, $|\alpha_{\max}|^2 \approx 1.0$, at three standard deviations from the mean photon number $\langle n \rangle = 1/3$, effectively cutting off the values $|\alpha| > 1$). For $\lambda = 3$, Eqs. (6) and (7) give $F^{\text{squeez}}_{g=2,\lambda=3} = 85\%$ and $\tilde{F}_{g=2,\lambda=3} = 50\%$ for the fidelity of the best deterministic amplifier and for the CFT, respectively [36]. The average of the experimental fidelities $F_{\exp} \approx 0.99/0.91/0.67$, corresponding to the amplitudes $|\alpha| \approx 0.4/0.7/1.0$, gives a value that is well above the benchmark for genuine quantum processing, but also very close to the value that can be achieved by deterministic amplifiers. One should observe, however, that the small number of values of $|\alpha|$ probed in the experiment precludes an accurate data analysis, as the average over few values of α is very sensitive to statistical fluctuations. Our analysis suggest that, although the available data show a neat quantum advantage over measure-and-prepare strategies, further experimental investigations would be desirable to enable a statistically significant analysis of the advantage of probabilistic amplifiers. To guarantee a fair sampling, the ideal setup would be to test the amplifier on Gaussian-distributed coherent states generated randomly by a heterodyne measurement on one side of a two-mode squeezed state.

The classical limit of quantum amplifiers.—For $\lambda \leq g - 1$, the gap between the quantum fidelity and the CFT is equal to the gap between entangled and separable states in the Bell correlation $\langle A_\tau \rangle$. The gap vanishes in the limit $g \to \infty$, and the fundamental reason is that an amplifier with infinite gain is classical, like a cloning device producing infinite clones [37–39]. This point is made very clear by our results: denoting by $\mathcal{C}_{g,\lambda}$ and by $\tilde{\mathcal{C}}_{g,\lambda}$ the optimal quantum amplifier and the optimal measure-and-prepare amplifier, for $\lambda \leq g - 1$ we have the remarkable relation [33] $\tilde{\mathcal{C}}_{g,\lambda} = \mathcal{A}_{g/\sqrt{g^2+(\lambda+1)^2}} \mathcal{C}_{\sqrt{g^2+(\lambda+1)^2},\lambda}$, where $\mathcal{A}_\eta$ is the attenuation channel transforming the coherent state $|\alpha\rangle$ into $|\eta\alpha\rangle$, $\eta \leq 1$. In words, the best measure-and-prepare strategy with gain g is equivalent to the best quantum strategy with gain $g' = \sqrt{g^2 + (\lambda + 1)^2}$, followed by an attenuation of $\eta = g/\sqrt{g^2 + (\lambda + 1)^2}$ that reduces the gain from g' to g. When the desired gain is large compared to the prior information available ($g \gg \lambda$) we have $g' \approx g$ and $\eta \approx 1$, which imply $\tilde{\mathcal{C}}_{g,\lambda} \approx \mathcal{C}_{g,\lambda}$.

In conclusion, we established the ultimate quantum limits to the deterministic and probabilistic amplification of Gaussian-distributed coherent states, without making any assumption on the nature of the amplifier and without making the unrealistic assumption of uniform distribution over coherent states. For probabilistic amplifiers, we discovered the presence of a critical value of the expected photon number, below which noiseless amplification becomes possible. Furthermore, we provided the quantum benchmark that has to be surpassed in order to establish the successful experimental demonstration of a genuine quantum amplifier. Our results show an intriguing link between genuine quantum amplification and the maximization of a suitable Bell-type correlation, and, in addition, they guarantee that a successful demonstration is possible for any finite value of the expected photon number.

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