Estimation of spatial panel data models with time varying spatial weights matrices

Wei Wang
Antai College of Economics and Management
Shanghai Jiao Tong University

Jihai Yu*
Guanghua School of Management
Peking University

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Abstract

This paper investigates the quasi-maximum likelihood (QML) estimation of spatial panel data models where spatial weights matrices can be time varying. We show that QML estimate is consistent and asymptotically normal. We also derive the asymptotic distribution of average impact coefficients (direct, indirect, total). Monte Carlo results are reported to investigate the finite sample properties of QML estimates and impact coefficients.

JEL classification: C13; C23; R15
Keywords: Spatial autoregression, Panel data, Time varying spatial weights matrices, Fixed effects, Maximum likelihood, Impact analysis

*Corresponding author. Tel: 8610-62760702; fax: 8610-62753624. Address: Room 349, Guanghua School of Management Building #2, Peking University, Beijing, China 100871.

1. Introduction

For spatial panel data models, the spatial weights matrix can be based on contiguity or distances among regions, which is usually time invariant. However, the spatial weights matrix can also be constructed from economic/socioeconomic distances or demographic characteristics, which might be changing over time. One may wonder whether we can easily handle the models with time varying spatial weights, and whether ignoring time variation in spatial weights matrices would have substantial consequences on estimates. These motivate our investigation on the spatial panel data model with time varying spatial weights matrices. Lee and Yu (2012) investigate the time varying weights matrices in a dynamic spatial panel model setting, where the number of time periods T is assumed to be large. In the current paper, we consider the static spatial panel model with both individual and time fixed effects, where T could be finite or large, and investigate the quasi-maximum likelihood (QML) estimation.

Compared to the development in the estimation and testing in spatial panel data models, the estimation and statistical inference of impact effects are rarely carried out in empirical applications. LeSage and Pace (2009) provide a computationally efficient simulation approach to produce empirical estimates of dispersion for scalar summary measures of impacts. Debarsy et al. (2011) extend the preceding approach to the dynamic spatial panel data models with a time invariant spatial weights matrix. Elhorst (2012) provides Matlab routines for the bias-corrected estimates in Lee and Yu (2010) and relevant impact analysis. The current paper will provide the estimation and inference for those impacts based on the QML estimates.

The rest of the paper is organized as follows. Section 2 introduces the model and establishes asymptotic properties of QML estimators. Section 3 investigates the impact estimates and their asymptotic inference. Section 4 provides Monte Carlo results. Section 5 concludes the paper. Due to space limit, lemmas and proofs are collected in a supplement file available upon request.

2. The Model and Asymptotic Properties of the QML Estimate

The model considered is

$$Y_{nt} = \lambda_0 W_{nt} Y_{nt} + X_{nt}\beta_0 + \mathbf{c}_{n0} + \alpha_{t0} l_n + V_{nt}, \quad t = 1, 2, ..., T, \tag{1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, ..., y_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, ..., v_{nt})'$ are $n \times 1$ column vectors, and v_{it}'s are *i.i.d.* across i and t with zero mean and variance σ_0^2. The X_{nt} is an $n \times K$ matrix of individually and time varying nonstochastic regressors, $\mathbf{c}_{n0}$ is an $n \times 1$ column vector of individual effects, and α_{t0} is the tth element of the $T \times 1$ fixed time effect vector $\boldsymbol{\alpha}_{T0}$ with l_n being $n \times 1$ vector of ones. The spatial weights matrix W_{nt} is nonstochastic and it could be time varying. We assume that W_{nt} is row-normalized as in common practice.

Similar to Lee and Yu (2010), we can use the eigenvector matrix of $J_n = I_n - \frac{1}{n} l_n l_n'$ to eliminate the time effects. However, we will directly estimate the individual effects.[1] By denoting $S_{nt}(\lambda) = I_n - \lambda W_{nt}$ for an arbitrary λ and $\tilde{X}_{nt} = X_{nt} - \frac{1}{T}\sum_{t=1}^{T} X_{nt}$, the concentrated log likelihood (with individual effects and time effects concentrated out) is

$$\ln L_{n,T}(\theta) = -\frac{(n-1)T}{2}\ln 2\pi - \frac{(n-1)T}{2}\ln\sigma^2 - T\ln(1-\lambda) + \textstyle\sum_{t=1}^{T}\ln|S_{nt}(\lambda)| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\tilde{V}_{nt}'(\theta)J_n\tilde{V}_{nt}(\theta), \tag{2}$$

where $\tilde{V}_{nt}(\theta) = \widetilde{S_{nt}(\lambda)Y_{nt}} - \tilde{X}_{nt}\beta$ with $\widetilde{S_{nt}(\lambda)Y_{nt}} = S_{nt}(\lambda)Y_{nt} - \frac{1}{T}\sum_{t=1}^{T} S_{nt}(\lambda)Y_{nt}$ and $J_n\tilde{V}_{nt}(\theta) = J_n[\widetilde{S_{nt}(\lambda)Y_{nt}} - \tilde{X}_{nt}\beta - \tilde{\alpha}_t l_n]$ because $J_n l_n = \mathbf{0}$.

We consider the properties of QMLE when n is large while T can be finite or large. For asymptotic analysis of the QML estimators, we assume the following regularity conditions.

Assumprion 1. W_{nt}'s are row-normalized nonstochastic spatial weights matrices with zero diagonals.

Assumprion 2. The disturbances $\{v_{it}\}$, $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$, are *i.i.d.* across i and t with zero mean, variance σ_0^2 and $\mathrm{E}|v_{it}|^{4+\eta} < \infty$ for some $\eta > 0$.

Assumprion 3. The elements of X_{nt}, $\mathbf{c}_{n0}$ and $\boldsymbol{\alpha}_{T0}$ are nonstochastic and bounded, uniformly in n and t. Also, $\lim_{n\to\infty}\frac{1}{nT}\sum_{t=1}^{T}\tilde{X}_{nt}'J_n\tilde{X}_{nt}$ exists and is nonsingular.

Assumprion 4. $S_{nt}(\lambda)$ is invertible for all t and for all $\lambda \in \Lambda$, where the parameter space Λ is compact and λ_0 is in the interior of Λ.

Assumprion 5. W_{nt}'s and $S_{nt}^{-1}(\lambda)$'s are uniformly bounded (uniformly in t for W_{nt}'s, and uniformly in $\lambda \in \Lambda$ and t for $S_{nt}^{-1}(\lambda)$'s) in both row and column sums in absolute value.

Assumprion 6. n is large, where T can be finite or large.

In Lee and Yu (2010) with time invariant weights matrix, the direct approach (estimating the individual effects directly) will yield bias for the variance parameter. Denote $\theta_T = \theta_0 - (\mathbf{0}_{1\times(K+1)}, \frac{1}{T}\sigma_0^2)'$. The asymptotic analysis for the direct approaches is based on θ_T. For the time varying spatial weights matrices case in the current paper, because we transform the data to eliminate the time effects but directly estimate the individual effects, we expect that the bias for the variance parameter remains. Thus, we will similarly base our asymptotic analysis on θ_T, and make bias correction for the variance parameter.

Denoting $G_{nt} = W_{nt}S_{nt}^{-1}$, $\widetilde{G_{nt}X_{nt}} = G_{nt}X_{nt} - \frac{1}{T}\sum_{t=1}^{T} G_{nt}X_{nt}$ and $\tilde{G}_{nt} = G_{nt} - \frac{1}{T}\sum_{t=1}^{T} G_{nt}$. The information matrix $\Sigma_{\theta_T,nT} = -\mathrm{E}\left(\frac{1}{(n-1)T}\frac{\partial^2 \ln L_{n,T}(\theta_T)}{\partial\theta\partial\theta'}\right)$ is equal to

$$\Sigma_{\theta_T,nT} = \frac{1}{\sigma_T^2}\begin{pmatrix} \mathrm{E}\mathcal{H}_{nT}^c & * \\ \mathbf{0}_{1\times(K+1)} & 0 \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{K\times K} & * & * \\ \mathbf{0}_{1\times K} & \frac{1}{(n-1)T}\sum_{t=1}^{T}\left[\mathrm{tr}(G_{nt}'J_nG_{nt}) + \mathrm{tr}((J_nG_{nt})^2)\right] & * \\ \mathbf{0}_{1\times K} & \frac{1}{\sigma_T^2(n-1)T}\sum_{t=1}^{T}\mathrm{tr}(J_nG_{nt}) & \frac{1}{2\sigma_T^4} \end{pmatrix}, \tag{3}$$

[1] We can eliminate the individual effects by eigenvector matrix of $J_T = I_T - \frac{1}{T}l_Tl_T'$. But, due to the time varying feature of spatial weights matrices, the transformed equation is no longer an SAR process and the QML approach cannot be applied directly. Thus, we will adopt a direct approach where we eliminate the time effects but estimate the individual effects directly.

where $\mathcal{H}^c_{nT} = \frac{1}{(n-1)T}\sum_{t=1}^T(\tilde{X}_{nt}, (\widetilde{G_{nt}X_{nt}}\beta_0 + \tilde{G}_{nt}\mathbf{c}_{n0}))' J_n(\tilde{X}_{nt}, (\widetilde{G_{nt}X_{nt}}\beta_0 + \tilde{G}_{nt}\mathbf{c}_{n0}))$. The limit of $\Sigma_{\theta_T,nT}$ is nonsingular if $\lim_{n\to\infty} \mathrm{E}\mathcal{H}^c_{nT}$ is nonsingular or

$$\lim_{n\to\infty}(\frac{1}{(n-1)T}\sum_{t=1}^T\left[\mathrm{tr}(G'_{nt}J_nG_{nt}) + \mathrm{tr}((J_nG_{nt})^2)\right] - 2[\frac{1}{T}\sum_{t=1}^T\frac{\mathrm{tr}(J_nG_{nt})}{n-1}]^2) \neq 0. \tag{4}$$

For asymptotic distribution, denote

$$\begin{aligned}\Omega_{\theta_T,nT} &= \frac{T-1}{T}\frac{\mu_3}{\sigma_0^4}\begin{pmatrix} \mathbf{0}_{K\times K} & * & * \\ \frac{1}{(n-1)T}\sum_{t=1}^T\sum_{i=1}^n (J_nG_{nt})_{ii}(J_n\tilde{X}_{nt})_i & \frac{2}{(n-1)T}\sum_{t=1}^T\sum_{i=1}^n (J_nG_{nt})_{ii}(J_n\widetilde{G_{nt}X_{nt}}\beta_0 + J_n\tilde{G}_{nt}\mathbf{c}_{n0})_i & * \\ \frac{1}{2\sigma_T^2(n-1)T}\sum_{t=1}^T\sum_{i=1}^n \mathrm{E}(J_n\tilde{X}_{nt})_i & \frac{1}{2\sigma_T^2(n-1)T}\sum_{t=1}^T\sum_{i=1}^n \mathrm{E}(J_n\widetilde{G_{nt}X_{nt}}\beta_0 + J_n\tilde{G}_{nt}\mathbf{c}_{n0})_i & 0 \end{pmatrix} \\ &+\frac{T-1}{T}\frac{\mu_4 - 3\sigma_0^4}{\sigma_0^4}\begin{pmatrix} \mathbf{0}_{K\times K} & * & * \\ \mathbf{0}_{1\times K} & \frac{1}{(n-1)T}\sum_{t=1}^T\sum_{i=1}^n (J_nG_{nt})_{ii}^2 & * \\ \mathbf{0}_{1\times K} & \frac{1}{2\sigma_T^2(n-1)T}\sum_{t=1}^T \mathrm{tr}(J_nG_{nt}) & \frac{1}{4\sigma_T^4}\end{pmatrix}.\end{aligned}$$

Assumprion 7. Either $\lim_{n\to\infty}\mathrm{E}\mathcal{H}^c_{nT}$ is nonsingular or (4) holds.

Assumption 7 assures that the information matrix is nonsingular.

Theorem 1 *Under Assumptions 1-7, the QMLE $\hat{\theta}_{nT}$ that maximizes (2) has*

$$\sqrt{(n-1)T}(\hat{\theta}_{nT} - \theta_T) \xrightarrow{d} N(0, \lim_{n\to\infty}\frac{T}{T-1}(\Sigma_{\theta_T,nT})^{-1}(\Sigma_{\theta_T,nT} + \Omega_{\theta_T,nT})(\Sigma_{\theta_T,nT})^{-1}). \tag{5}$$

Therefore, the variance estimate $\hat{\sigma}^2_{nT}$ does not converge to σ_0^2 when T is a fixed finite value as n tends to infinity. It will be consistent only when T is large. The $(\hat{\beta}'_{nT}, \hat{\lambda}_{nT})'$ of $\hat{\theta}_{nT}$ will be consistent even when T is small. From Theorem 1, it is straightforward to construct the bias corrected estimates $\hat{\theta}^1_{nT} = (\hat{\beta}'_{nT}, \hat{\lambda}_{nT}, \frac{T}{T-1}\hat{\sigma}^2_{nT})' = A_{nT}\hat{\theta}_{nT}$ where $A_{nT} = diag(\mathbf{1}_{K+1}, \frac{T}{T-1})$. Correspondingly,

$$\sqrt{(n-1)T}(\hat{\theta}^1_{nT} - \theta_0) \xrightarrow{d} N(0, \lim_{n\to\infty}\Xi_{\theta_0,nT}), \tag{6}$$

where $\Xi_{\theta_0,nT} = \frac{T}{T-1}A_{nT}(\Sigma_{\theta_T,nT})^{-1}(\Sigma_{\theta_T,nT} + \Omega_{\theta_T,nT})(\Sigma_{\theta_T,nT})^{-1}A_{nT}$.

3. Impact Analysis

We take the partial derivatives $\frac{\partial Y_{nt}}{\partial X'_{nt,k}} = (I_n - \lambda_0 W_{nt})^{-1}\beta_{k0}$ and define

$$R_{nk} = (I_n - \lambda_0 W_{nt})^{-1}\beta_{k0} \tag{7}$$

as the *impact matrix* associated to the k^{th} explanatory variable (for notational simplicity, we omit the t-subscript here). In contrast to the classical linear model, diagonal elements of this matrix are different from one another, and off-diagonal elements are non null and the matrix is not symmetric.

The diagonal elements of this matrix, $diag\{R_{nk}\}$, represent the *direct impacts* including feedback effects, where individual i affects individual j and individual j also affects individual i as well as longer paths which might go from individual i to j to k and back to i. Those feedback effects corresponding to $diag\{R_{nk}\}-\beta_{k0}I_n$ are inherently heterogenous, due to differentiated interaction terms in the W_{nt} matrix. The off-diagonal elements of the impact matrix $R_{nk}-diag\{R_{nk}\}$ represent *indirect impacts*, which can be seen as the difference of the *total impacts*, R_{nk}, and the direct impacts $diag\{R_{nk}\}$.

Given the matrix presentation of these impacts, it is useful to use some summary scalar measures. The average direct impact, average total impact, and average indirect impact are respectively defined as

$$\begin{aligned} f_{k,direct}(\theta) &\equiv n^{-1}trR_{nk}(\theta), \qquad (8)\\ f_{k,total}(\theta) &\equiv n^{-1}l_n'R_{nk}(\theta)l_n, \\ f_{k,indirect}(\theta) &\equiv n^{-1}l_n'R_{nk}(\theta)l_n - n^{-1}trR_{nk}(\theta). \end{aligned}$$

As we have $\hat{\theta}^1_{nT}$ from (6), the distributions of $f_{k,direct}(\hat{\theta}^1_{nT})$, $f_{k,total}(\hat{\theta}^1_{nT})$ and $f_{k,indirect}(\hat{\theta}^1_{nT})$ can be obtained to make statistical inferences of these impact estimates. Thus,

$$\begin{aligned} &\sqrt{(n-1)T}(f_{k,direct}(\hat{\theta}^1_n) - f_{k,direct}(\theta_0)) \xrightarrow{d} N(0, \lim_{n\to\infty}(\frac{\partial f_{k,direct}(\theta_0)}{\partial\theta'}\Xi_{\theta_0,nT}\frac{\partial f_{k,direct}(\theta_0)}{\partial\theta})), \\ &\sqrt{(n-1)T}(f_{k,total}(\hat{\theta}^1_n) - f_{k,total}(\theta_0)) \xrightarrow{d} N(0, \lim_{n\to\infty}(\frac{\partial f_{k,total}(\theta_0)}{\partial\theta'}\Xi_{\theta_0,nT}\frac{\partial f_{k,total}(\theta_0)}{\partial\theta})), \qquad (9)\\ &\sqrt{(n-1)T}(f_{k,indirect}(\hat{\theta}^1_n) - f_{k,indirect}(\theta_0)) \xrightarrow{d} N(0, \lim_{n\to\infty}(\frac{\partial f_{k,indirect}(\theta_0)}{\partial\theta'}\Xi_{\theta_0,nT}\frac{\partial f_{k,indirect}(\theta_0)}{\partial\theta})), \end{aligned}$$

where $\frac{\partial f_{k,direct}(\theta)}{\partial\theta}$, $\frac{\partial f_{k,total}(\theta)}{\partial\theta}$ and $\frac{\partial f_{k,indirect}(\theta)}{\partial\theta}$ can be obtained from (7) and (8).

Theorem 2 *Under Assumptions 1-7, the estimates for the impacts in (8) are consistent and asymptotically normally distributed as specified in (9).*

4. Monte Carlo

We conduct a Monte Carlo experiment to evaluate the performance of QMLEs and impact estimators. The DGP is from equation (1) using $\theta_0 = (\beta_0', \lambda_0, \sigma_0^2)' = (1, 0.5, 1)'$, and X_{nt}, $\mathbf{c}_{n0}$, α_{t0} and V_{nt} are generated from independent standard normal distributions. We use $T = 10$, 50, and $n = 49$, 196. For time varying spatial weights matrices, we choose an alternating pattern. When t is odd, W_{nt} is a square tessellation where each unit only interact with its left and right neighbors (for the left and right edge units, they have then only one neighbor). We call this a left-right matrix. When t is even, W_{nt} is a queen matrix, which presents a square tessellation with a connectivity of eight for the inner fields on the chessboard and three and five for the corner and border fields, respectively. All these weights matrices are row-normalized. For each set of generated sample observations, we obtain the bias $\hat{\theta}^1_{nT} - \theta_0$ and do this 1000 times. We also report the empirical standard deviation (SD), the empirical root mean square error (RMSE), and coverage probability (CP) of these 1000 estimates. Results are summarized in the columns under "W_{nt}" in Table 1. We see that the estimators have small biases and their CPs are close to the specified 95% confidence level. For different cases of n and T, when T is larger or n is larger, biases and SDs are smaller. This is consistent with our theoretical results, as the variances of the estimators are of the order $O(\frac{1}{nT})$.

We report some results using misspecified time invariant weights matrices, while the DGP still has the above alternating pattern. The columns under "$W_{average}$" are the results where the time average of left-right and queen matrices is used in the estimation of the model. Columns under "W_{under}" in are the results where only the left-right matrix is used, and columns under "W_{over}" are the results where only queen matrix is used. We can see the performance of estimates under misspecification is not as good as that of the correct specification. Also, overspecification has a better result than the underspecification in the estimation and inference of λ.

We also investigate the estimation of impact analysis under the correct specification of time varying weights matrices and misspecification of time invariant weights matrix. We see that the estimates of the impact coefficients are satisfactory with small biases (columns under W_{nt}). Under misspecifications of time invariant weights matrix, we have biases in the impact coefficients. Similar to the biases for parameter estimates, the misspecification of weights matrix by the time average of the left-right and queen matrices has smaller biases than the over- and under- specifications; and the over-specification of weights matrix yields smaller biases than the under- specification. However, we see that estimates of direct impact coefficients are less influenced by the misspecification of weights matrix. Take DGP (1) for an example, the magnitude of bias for λ is 27% for the under-specification and 24% for the over-specification. For the impact coefficients, the direct impact estimate has about 7% ($\frac{1.1802-1.1091}{1.1802}$) bias for the under-specification and 6% ($\frac{1.1009-1.0492}{1.0492}$) bias for the over-specification. These biases are much smaller than the total impact and indirect impact. LeSage and Pace (2010) state that even though we might have opposite biases in the estimation of λ and β, the estimations of the impact coefficients are less influenced by the misspecification of spatial weights matrices. The current finding for the average direct impact confirms LeSage and Pace (2010), while the average indirect and total impacts still have large biases.

5. Conclusion

In this paper, we investigate the QML estimation of the SAR panel data model with both individual and time fixed effects and time varying spatial weights matrices. We prove that the QML estimate is consistent and asymptotically normal under this setting. Impact analysis and its statistical inference is also investigated.

Through Monte Carlo experiments, we see that when the true process has substantially time varying spatial weights matrices, a model misspecification with a time invariant spatial weights matrix will cause biases in estimates; however, the average direct impact estimates are less influenced.

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Table 1: Performance of Parameter Estimators and Impact Coefficients

Parameter Estimators			Correct specification W_{nt}			Misspecification $W_{average}$			Misspecification W_{under}			Misspecification W_{over}		
DGP	T,n		β	λ	σ^2	β	λ	σ^2	β	λ	σ^2	β	λ	σ^2
(1)	10,49	Bias	-0.0018	-0.0016	-0.0076	0.0264	0.0305	0.1879	0.0223	-0.1386	0.1635	0.0711	-0.1182	0.4582
		SD	0.0488	0.0330	0.0713	0.0528	0.0468	0.0918	0.0522	0.0359	0.0863	0.0611	0.0835	0.1377
		RMSE	0.0657	0.0445	0.0947	0.0752	0.0687	0.2088	0.0735	0.1425	0.1871	0.1026	0.1450	0.4694
		CP	0.9530	0.9540	0.9300	0.9150	0.8760	0.3770	0.9240	0.0090	0.4850	0.7720	0.5890	0.0070
(2)	10,196	Bias	-0.0009	-0.0003	-0.0013	0.0264	0.0219	0.1742	0.0247	-0.1309	0.1553	0.0697	-0.1106	0.4216
		SD	0.0239	0.0172	0.0353	0.0262	0.0229	0.0454	0.0260	0.0188	0.0442	0.0300	0.0394	0.0665
		RMSE	0.0326	0.0229	0.0472	0.0424	0.0366	0.1791	0.0415	0.1319	0.1607	0.0766	0.1156	0.4244
		CP	0.9560	0.9450	0.9440	0.8230	0.8380	0.0090	0.8390	0.0000	0.0240	0.3300	0.0900	0.0000
(3)	50,49	Bias	-0.0001	0.0006	-0.0005	0.0294	0.0318	0.1943	0.0254	-0.1380	0.1704	0.0749	-0.1147	0.4684
		SD	0.0209	0.0138	0.0299	0.0229	0.0206	0.0404	0.0226	0.0161	0.0377	0.0263	0.0359	0.0634
		RMSE	0.0282	0.0189	0.0404	0.0405	0.0399	0.1976	0.0379	0.1386	0.1740	0.0795	0.1186	0.4704
		CP	0.9460	0.9530	0.9530	0.7490	0.6280	0.0000	0.8020	0.0000	0.0000	0.1660	0.0420	0.0000
(4)	50,196	Bias	0.0002	0.0002	-0.0006	0.0278	0.0225	0.1715	0.0260	-0.1308	0.1536	0.0714	-0.1082	0.4182
		SD	0.0104	0.0072	0.0146	0.0114	0.0097	0.0194	0.0114	0.0081	0.0179	0.0129	0.0171	0.0304
		RMSE	0.0141	0.0098	0.0199	0.0304	0.0249	0.1723	0.0288	0.1310	0.1545	0.0725	0.1090	0.4187
		CP	0.9520	0.9410	0.9530	0.3050	0.3610	0.0000	0.3530	0.0000	0.0000	0.0000	0.0000	0.0000

Impact Coefficients

Correct specification			true value					W_{nt}				
DGP	T,n		$f_{d,low}$	$f_{d,high}$	f_{total}	$f_{id,low}$	$f_{id,high}$	$f_{d,low}$	$f_{d,high}$	f_{total}	$f_{id,low}$	$f_{id,high}$
(1)	10,49	Mean	1.1802	1.0492	2	0.8198	0.9508	1.1785	1.0475	1.9980	0.8195	0.9506
		SD						0.0627	0.0512	0.1566	0.1055	0.1276
(2)	10,196	Mean	1.1675	1.0458	2	0.8325	0.9542	1.1667	1.0449	1.9994	0.8327	0.9545
		SD						0.0298	0.0248	0.0790	0.0556	0.0662
(3)	50,49	Mean	1.1802	1.0492	2	0.8198	0.9508	1.1809	1.0493	2.0035	0.8225	0.9542
		SD						0.0259	0.0217	0.0638	0.0434	0.0525
(4)	50,196	Mean	1.1675	1.0458	2	0.8325	0.9542	1.1679	1.0460	2.0014	0.8335	0.9554
		SD						0.0130	0.0108	0.0337	0.0235	0.0280

Misspecification			$W_{average}$			W_{under}			W_{over}		
DGP	T,n		f_d	f_{total}	f_{id}	f_d	f_{total}	f_{id}	f_d	f_{total}	f_{id}
(1)	10,49	Mean	1.1198	2.2075	1.0877	1.1091	1.6056	0.4965	1.1009	1.7627	0.6618
		SD	0.0602	0.2445	0.2067	0.0589	0.1189	0.0744	0.0621	0.2475	0.2258
(2)	10,196	Mean	1.1086	2.1516	1.0430	1.1089	1.6255	0.5166	1.0971	1.7586	0.6615
		SD	0.0288	0.1128	0.0961	0.0287	0.0599	0.0395	0.0303	0.1145	0.1046
(3)	50,49	Mean	1.1222	2.2026	1.0804	1.1117	1.6083	0.4965	1.1036	1.7540	0.6505
		SD	0.0257	0.1047	0.0893	0.0250	0.0507	0.0325	0.0265	0.1041	0.0952
(4)	50,196	Mean	1.1101	2.1533	1.0432	1.1101	1.6269	0.5167	1.0988	1.7630	0.6641
		SD	0.0126	0.0480	0.0408	0.0126	0.0260	0.0170	0.0129	0.0499	0.0459

Note: θ_0= $(1, 0.5, 1)$. The $f_{d,low}$ is the direct impact under W_{under}, and $f_{d,high}$ is the direct impact under W_{over}. The $f_{d,low}$ and $f_{d,high}$ are corresponding indirect impacts.