

Mortality Decline, Human Capital Investment, and Economic Growth ^{*}

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Abstract

We examine the role of increased life expectancy in raising human capital investment during the process of economic growth. We develop a continuous time, overlapping generations model in which individuals make optimal schooling investment choices in the face of a constant probability of death. We present analytic results, followed by results from a calibrated version of the model using realistic estimates of the return to schooling. Mortality decline produces economically significant increases in schooling and consumption. Allowing schooling to vary endogenously produces a much larger response of consumption and capital to mortality decline than is observed when schooling is held fixed.

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1 Introduction

“.....decline in the death is an indispensable prerequisite for modern economic growth. ”

Simon Kuznets (1973).

Two of the most salient aspects of the process of economic growth are the decline in mortality and the growth of investment in human capital. These phenomena are visible in both long-term historical data for the countries that developed early as well as broadly, and in more accelerated form, in the post World War II period. Over the nineteenth century, for example, life expectancy at birth in England rose from 37.3 years to 48.2 years, and by 1930 it had reached 60.8. The average number of years of schooling rose from 2.3 for the cohort born 1801-5 to 9.1 for the cohort born 1897-1906.¹ Averaging across lower income countries, life expectancy at birth rose from 42.2 in 1950 to 63.3 in 1990. Gross secondary school enrollment increased from 17.1% in 1960 to 46.9% in 1990.²

Both the decline in mortality and the increase in schooling are intimately related to several aspects of the growth process. Mortality fell both directly because of higher incomes (which led to better nutrition) and because of advances in health technology. Mortality decline, in turn, triggered the process of demographic transition, in which, with a varying lag, fertility rates fell.³ The increase in human capital accumulation has been attributed to a increase in the return to schooling (Foster and Rosenzweig, 1996; Mincer, 1996). Higher investment in human capital has in turn been linked to changes in fertility behavior, via a quality-quantity tradeoff (Becker and Lewis, 1973); to an increase in the growth rate of technology (Lucas, 1988); and directly to a higher level of output (Mankiw, Romer, and Weil, 1992).

In this paper we look at a direct relationship between mortality and human capital accumulation. Specifically, we study the effect of mortality decline in raising human capital accumulation by increasing the horizon over which investments in schooling will be paid off. Although much of the decline in mortality has been in infancy, before any investment in schooling will have taken place, there have also been significant declines in mortality later in life. For example in Sweden, male life expectancy at age five rose from 48.3 to 57.7 between 1800 and 1900. Over the same period life expectancy at birth rose from 35.9 to 54.3. In

¹Livi-Bacci (1997), table 4.3. Matthews, Feinstein, and Odling-Smee (1982), table E.1.

²Life expectancy is from Schultz (1997). School enrollment is from World Bank (1998). The samples used in the two sources differ slightly.

³See Easterlin (1996), ch. 6, for a discussion of these issues.

India, male life expectancy at age 10 rose from 39.0 to 48.8 over the period 1951-71. Over the same period, male life expectancy at birth rose from 32.4 to 46.4.⁴

The effect of mortality on education has been investigated both empirically and theoretically. Ram and Schultz (1979) argued that improvements in mortality were an important incentive to increase investment in education, and that the post-war experience of India was consistent with this incentive effect being significant. Preston (1980) calculated the degree to which reductions in mortality raised the internal rate of return to investments in education, but found that increases in the return to schooling were not sufficient to explain large increases in enrollment. Meltzer (1992) extended Preston's work, arguing that the elasticity of enrollment with respect to rate of return was sufficiently large that mortality-induced increases in returns could indeed explain large movements in enrollment. Meltzer also argued that reductions in morbidity which have accompanied declining mortality have further raised the return to investments in schooling.

Two papers have incorporated this effect of life expectancy into general equilibrium models. Ehrlich and Lui (1991) embedded the effect of changing mortality in an overlapping generations model in which children provide old-age support for their parents. In the model, improvements in longevity lower fertility, raise educational investment, and raise the long run growth rate of output. Ehrlich and Lui also presented evidence on the effects of mortality on growth and fertility that is consistent with their model. Meltzer (1992) extended Becker, Murphy, and Tamura's (1990) model of human capital, fertility, and growth by introducing a relationship between mortality and human capital investment. He showed that mortality decline may promote economic growth through an increase in the educational investment. In both the Ehrlich-Lui and Meltzer models, time is discrete, and individuals live a maximum of three periods.

In this paper we examine the relation between mortality and human capital investment using a variant of the continuous time, overlapping generations model of Blanchard (1985). Individuals face a constant probability of death, and choose an optimal level of schooling to maximize the discounted value of expected consumption. By using a simple demographic structure we are able to aggregate across generations and solve for general equilibrium. We can thus sort out the direct effects of mortality decline on capital accumulation, interest rates, and labor supply from the effects of increased schooling that is induced by mortality decline.

⁴Keyfitz and Flieger (1968), Ram and Schultz (1979).

Further, by using a model with realistic time periods, we are able to use estimates of the return to schooling in a calibrated version of the model.

The rest of this paper is structured as follows. In Section 2, we introduce our model of schooling and growth. We first analyze the model in partial equilibrium, with constant wage and interest rates, and then turn to general equilibrium analysis, concentrating on the steady state. In Section 3 we conduct comparative static analyses of the effect of mortality change on the steady state, comparing the case where schooling responds to changes in mortality to the case where schooling is fixed. We present analytic results, followed by results from a calibrated version of the model using realistic estimates of the return to schooling. Section 4 concludes.

2 A Model of Schooling and Growth with Finite Horizons

2.1 The Structure of the Model

We consider a continuous time, overlapping generations model. Individuals face a probability of dying per unit time, ρ , which is constant throughout life. Similarly, an individual's life expectancy, $1/\rho$, is constant.

There are two important aspects to our approach to modeling mortality. The first is that the probability of death is constant at all ages, which is obviously unrealistic, since it fails to capture some life-cycle aspects of human experience, but is made for analytic convenience. The second is that reductions in mortality take the form of a lowering of death probabilities at all ages (not just in old age, for example). This aspect of our approach is more consistent with the data.⁵

At every instant of time a new cohort is born. Even though there is individual uncertainty about the time of death, it is assumed that population is large enough that the size of a cohort can be viewed as declining deterministically through time. To normalize the size of population, we assume the size of a new cohort is also ρ . A cohort born at time b has a

⁵In Sweden, for example, the probability of death at age 25 fell from .044 in 1800 to .023 in 1900 (a reduction of 48%) and to .005 in 1985 (a further reduction of 78%). The probability of death at age 45 fell from .093 in 1800 to .026 in 1900 (72%) and to .018 in 1985 (31%). At age 65 the probability of death fell from .236 to .140 (41%) to .127 (9%). In Chile, the probability of death at age 25 fell from .099 in 1909 to .011 in 1980 (a reduction of 89%). The probability of death at age 45 in the same years fell from .120 to .038 (68%) and the probability of death at age 65 fell from .244 to .174 (29%) (Keyfitz, Preston and Schoen (1972) and Keyfitz and Flieger (1968, 1990)).

size as of time t equal to $\rho e^{(-\rho(t-b))}$, and the size of the total population at any time t is $\int_{-\infty}^t \rho e^{(-\rho(t-b))} db = 1$.⁶

Individuals are born with no wealth. They are endowed with one unit of time per period, and receive utility only from consumption. They invest in education at the beginning of their lives, then work until they die.⁷ Their wages depend on their human capital, which is a function of schooling. The only cost of schooling is the foregone earnings.⁸

The earnings of an individual who is no longer in school are given by

$$E = wh(s), \tag{1}$$

where w is the wage per unit of human capital and $h()$ gives the quantity of human capital as a function of schooling. The standard analysis in the labor literature is to estimate a “structural earnings function” of the form

$$\ln(E) = \text{constant} + f(s). \tag{2}$$

These two equations imply that human capital will be given by $h = e^{f(s)}$, while the constant term in equation (2) will correspond to $\ln(w)$. The standard assumptions about the $f()$ function (Willis, 1986) are that

$$f_s > 0, \quad f_{ss} < 0. \tag{3}$$

As in the models of Yaari (1965) and Blanchard (1985), uncertainty about the date of an individual’s death creates the need for an annuity contract. Older individuals who have accumulated wealth face the possibility of dying before they can spend it. Insurance companies will allow individuals to trade claims on their wealth in the event that they die for payments if they live. Under the assumption that the annuity business is perfectly competitive, and that the market interest rate is r , the interest rate paid on these annuities will be $r + \rho$.

⁶Introducing exogenous population growth in the form of larger new cohorts over time would be straightforward.

⁷In the model there is no constraint that prevents individuals from going to school part time and working part time, but such behavior can be shown not to be optimal.

⁸In a more complex model, education choices would be made by parents who maximize an intergenerational utility function, and choices over education would be integrated with the fertility decision. The key effect on which we focus – that increasing life expectancy would raise the period over which investments in schooling are paid off, and thus raise the optimal quantity of schooling – would still be present in such a model.

In addition to these standard annuity contracts, however, there is a second sort of annuity in this model. Individuals must borrow in the early part of their lives to pay for consumption while they are getting an education, and there is a risk that they will die before they are able to repay their loans. Thus there will be a market for annuitized loans, in which borrowers will pay an interest rate of $r + \rho$, but in which the loan will be forgiven if the borrower dies.⁹

2.2 Individual's Maximization Problem

Denote consumption, assets and human capital of an individual who born at time b as of time t as $c(b, t)$, $k(b, t)$ and $h(b, t)$ respectively. Individuals born at time b belong to generation b . Until aggregation over individuals in different generations, we can drop b index and focus on individual consumption, assets, and human capital.

Individuals maximize expected utility from consumption.

$$\max \int_b^{\infty} \left[\ln(c(z)) e^{-(\theta+\rho)(z-b)} \right] dz, \quad (4)$$

where θ is the pure rate of time discount and $e^{-\rho(z-b)}$ is probability of being alive at time z . We assume log utility for convenience.¹⁰

The accumulation of assets is described by the equations:

$$\dot{k}(z) = (r + \rho)k(z) - c(z) \quad z \in [b, b + s]. \quad (5)$$

$$\dot{k}(z) = (r + \rho)k(z) + hw - c(z) \quad z \in [b + s, \infty]. \quad (6)$$

$$k(b) = 0. \quad (7)$$

⁹Obviously annuity contracts of this form are a modeling device which is not meant to be taken literally. These annuity contracts can be seen as a proxy for a number of arrangements which will yield similar results. The same result could be obtained in a model in which education levels are chosen by an infinitely-lived dynasty, composed of finitely-lived members, as in Barro and Sala-i-Martin (1995), chapter 9. The problem is to choose s to maximize

$$\int_s^{\infty} e^{-(r+\rho)t} w e^{f(s)} dt = \frac{w e^{f(s) - (r+\rho)s}}{r + \rho}.$$

This yields (13) as a first order condition.

¹⁰Results using *CRRA* utility were qualitatively similar.

Individuals maximizes (4) subject to (5),(6), and (7), taking w and r as given. The optimal path of consumption is given by

$$\dot{c}(z) = [(r + \rho) - (\theta + \rho)]c(z) = [r - \theta]c(z), \quad (8)$$

which can be re-written as

$$c(z) = c(b)e^{(r-\theta)(z-b)}. \quad (9)$$

Using equations (5)-(7) and (9), we can solve for the differential equations characterizing the path of assets during the two phases of an individual's life. During an individual's schooling (ages 0 through s), the evolution of assets is given by

$$k(z) = \frac{c(b)}{(\theta + \rho)} \left[e^{(r-\theta)(z-b)} - e^{(r+\rho)(z-b)} \right]. \quad (10)$$

For a working individual, the path of assets is given by

$$k(z) = \frac{-wh}{(r + \rho)} + \frac{c(b)}{(\theta + \rho)} \left[e^{(r-\theta)(z-b)} \right] + \frac{wh}{(r + \rho)} \left[e^{(r+\rho)(s-z)} \right] - \frac{c(b)}{(\theta + \rho)} \left[e^{(r+\rho)(z-b)} \right]. \quad (11)$$

By imposing transversality, we can solve for the initial value of consumption, $c(b)$:

$$c(b) = \frac{(\theta + \rho)}{(r + \rho)} \left[we^{f(s)} e^{-(r+\rho)s} \right]. \quad (12)$$

By maximizing the initial level of consumption with respect to s , individual chooses the optimal level of schooling. The first order condition is

$$f_s = r + \rho, \quad (13)$$

which says that an individual goes to school until his marginal rate of return from schooling is equal to the effective interest rate. This is the condition from Rosen's model of optimal schooling (See Willis, 1986). Substituting (12) into (9), (10), and (11), an individual who born at time b will have consumption and assets at time t given by

$$c(b, t) = \frac{(\theta + \rho)}{(r + \rho)} \left[we^{f(s)} e^{-(r+\rho)s} \right] \left[e^{(r-\theta)(t-b)} \right] \quad t \in [b, \infty), \quad (14)$$

$$k^{sc}(b, t) = \frac{(we^{f(s)} e^{-(r+\rho)s})}{(r + \rho)} \left[e^{(r-\theta)(t-b)} - e^{(r+\rho)(t-b)} \right] \quad t \in [b, b + s], \quad (15)$$

$$k^{wk}(b, t) = \frac{we^{f(s)}}{(r + \rho)} \left[e^{-(r+\rho)s} e^{(r-\theta)(t-b)} - 1 \right] \quad t \in [b + s, \infty), \quad (16)$$

where k^{sc} is the wealth of a person who is still in school and k^{wk} is the wealth of a person who is working. Assuming that $r > \theta$, (which we show must hold true in general equilibrium), consumption will be rising over the course of an individual's life.

Figure 1 shows an individual's wealth as a function of age.¹¹ Wealth is zero at birth, and it declines while he is in school, reaching a minimum at the point in time where he starts working. During his early working years, the individual pays down his debt, and eventually begins to hold positive wealth. Although wages are constant during working life, consumption is increasing. This consumption path in turn requires that the level of wealth also increase over the course of working life. Unlike a standard life-cycle model in which wage income ceases at some point during life and in which there is a deterministic date of death, individuals in this model will continue to work and to accumulate assets as long as they live. They hold wealth in order to take advantage of the difference between the interest rate and their rate of time discount, even though this means that they will die without having consumed all of their wealth.

2.3 General Equilibrium and Aggregation

The production function for the total output is

$$Y(t) = AK(t)^\alpha H(t)^{1-\alpha}, \quad (17)$$

where $K(t)$ denotes total physical capital and $H(t)$ denotes the total human capital of workers in the economy at time t . From the maximization problem of the perfectly competitive firms,

¹¹The figure is drawn using values of the parameters and endogenous variables in Table 1.

the wage rate per unit of individual human capital, w , is the marginal product of human capital and the interest rate is the marginal product of physical capital:¹²

$$w(t) = A(1 - \alpha) \left(\frac{K(t)}{H(t)} \right)^\alpha. \quad (18)$$

$$r(t) = A\alpha \left(\frac{K(t)}{H(t)} \right)^{\alpha-1}. \quad (19)$$

To derive the aggregate variables we sum over generations. Aggregate consumption at time t will be;

$$C(t) = \int_{-\infty}^t c(b, t) \rho e^{-\rho(t-b)} db. \quad (20)$$

Recall that the size of the generation born at b as of t is $\rho e^{-\rho(t-b)}$. Integrating this expression yields

$$C(t) = \frac{(\theta + \rho)\rho}{(r + \rho)(\rho - r + \theta)} \left[w e^{f(s)} e^{-(r+\rho)s} \right]. \quad (21)$$

In the appendix we show that in equilibrium, $\theta < r < \rho + \theta$. The intuition for this result is as follows. If $r \leq \theta$, individuals will want consumption to decline over the course of their lives. This would imply that the optimal policy would be to always be in debt, and so there will be no capital accumulation.¹³ If $\rho < r - \theta$, by contrast, then the growth rate of consumption among the individuals within a cohort who do not die will be higher than the rate at which members of the cohort are dying, in which case total consumption of the cohort will be rising over time, and aggregate consumption will be infinite.

Aggregate human capital at time t will be

¹²We assume zero depreciation for convenience.

¹³This is a result of finite horizons (i.e an overlapping generations structure). To see the intuition, ignore the schooling aspect of our model. If individuals had discount rates equal to the interest rate, then they would want flat consumption. Given that the wage is constant in steady state, this would imply simply consuming wages in every period and never accumulating capital. The situation is different in an infinite horizon setting (i.e a Ramsey model). In this case, constant consumption in the steady state indeed implies that $\theta = r$. See Blanchard and Fisher (1989), p. 124.

$$H(t) = \int_{-\infty}^{t-s} h(b, t) \rho e^{-\rho(t-b)} db. \quad (22)$$

Notice that the upper limit of the integral is $t - s$ instead of t , since people born in the last s periods are still in school and are thus not supplying their human capital to the labor market. This integral implies (remembering that h is only a function of schooling):

$$H(t) = e^{f(s) - \rho s}. \quad (23)$$

Lastly, the aggregate capital stock as of time t will be;

$$K(t) = \int_{-\infty}^{t-s} k^{wk}(b, t) \rho e^{-\rho(t-b)} db + \int_{t-s}^t k^{sc}(b, t) \rho e^{-\rho(t-b)} db. \quad (24)$$

Integrating (24) yields;

$$K(t) = \frac{w e^{f(s)}}{(r + \rho)} \left[\left(\frac{\rho}{\rho - r + \theta} + \frac{\rho}{r} \right) e^{-(r+\rho)s} - \left(1 + \frac{\rho}{r} \right) e^{-\rho s} \right]. \quad (25)$$

2.4 Existence and Uniqueness of the Steady State

In steady state, the aggregate quantities of consumption, C , capital, K , and human capital, H , the wage, w , interest rate r , and the age at which individuals leave school, s , are all constant. Equations (13), (18), (19), (21), (23), and (25) implicitly determine the steady state values of these endogenous variables.

To solve the model explicitly, we need to specify a functional form for $f(s)$, the return on schooling, which is defined in equations (2) and (3). To derive our analytic results, we use a logarithmic form: $f(s) = \ln(s)$. In our calibration we use a more realistic form of the $f(s)$ function. The logarithmic functional form implies that the optimal quantity of schooling is simply

$$s = \frac{1}{r + \rho}. \quad (26)$$

We use an asterisk to designate steady-state values. Dividing equation (25) by (23) gives physical to human capital ratio in steady state:

$$\frac{K^*}{H^*} = \frac{w^*}{(r^* + \rho)} \left[\left(\frac{\rho}{\rho - r^* + \theta} + \frac{\rho}{r^*} \right) e^{-r^* s^*} - 1 - \frac{\rho}{r^*} \right]. \quad (27)$$

Substituting (18), (19) and (26) into (27) gives the following equality which should hold at the steady state:

$$\frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r^*)(\rho + \theta - r^*)} = e^{\frac{r^*}{\rho + r^*}}. \quad (28)$$

In the appendix we show that there exists a unique value of r^* that satisfies this equation. This also implies unique steady state values for K^*, H^*, C^*, w^*, s^* .

An important implication of equation (28) is that the interest rate is not dependent on A , the parameter which measures productivity. Further, this implies that the level of schooling, which depends on the interest rate and the level of mortality, will also not be affected by changes in productivity.

The intuition for this result is as follows. First, note that changes in productivity have offsetting direct effects on the optimal quantity of education. Higher productivity raises the costs and benefits of education by the same factor. Further, increases in productivity do not affect the quantity of schooling via the interest rate: a standard property of neoclassical growth models is that the interest rate is constant along balanced growth paths with changing technology.

A further implication of the result that schooling does not depend on the level of productivity is that the model which we have examined in this paper can easily be combined with a model of growth due to changing technology. For example, in our calibrations in section 3.2, we show that an increase in life expectancy from age 33 to age 83 can raise consumption via the channels we study by a factor of 2.6. Suppose that one were examining a country in which life expectancy had risen by this much over a period of 100 years, but in which consumption had risen by a factor of 8. Of the annual growth rate of consumption of 2.10%, the change in life expectancy could account for only 0.96% per year. The remaining growth could then be accounted for by the growth of A . Assuming a value of $\alpha = 0.3$, this would imply that A grew at a rate of .80% per year.

3 Comparative Static Results

There are three exercises that we want to conduct with our model. First, we want to examine the effect of changing mortality on the equilibrium values of all of the endogenous variables. Second, we want to compare the general equilibrium case to the case where schooling is held constant. This will allow us to assess the importance of the effect of mortality on schooling on which we have focused. Finally, we want to compare the effect of mortality on schooling in general equilibrium to the partial equilibrium case in which the wage and interest rates are held constant. This will highlight the importance of examining the determination of schooling in a general equilibrium framework, and will also allow for a comparison of the effects of mortality on schooling and consumption in a closed versus an open economy.

The first half of this section examines these issues analytically. The second half uses a realistically specified model for the return to schooling to produce quantitative results.

3.1 Analytical Results

All of the results in this section refer to steady states, so for convenience we suppress the asterisk indicating steady state. In the appendix we show that r varies positively with ρ , as would be expected from the simple intuition that shorter lives lead to lower wealth accumulation, and thus to a higher marginal product of capital.

$$\frac{dr}{d\rho} > 0. \quad (29)$$

Once this result is established, results for the other endogenous variables follow quickly. From equation (26), it is straightforward to show that;

$$\frac{ds}{d\rho} = -\frac{dr/d\rho + 1}{(r + \rho)^2} < 0. \quad (30)$$

From equations (23) and (26), it follows that;

$$\frac{dH}{d\rho} = -\frac{e^{-\frac{\rho}{r+\rho}}}{(r + \rho)^2} \left(\frac{r(dr/d\rho) + 1}{r + \rho} + 1 \right) < 0. \quad (31)$$

Equation (19) implies the physical to human capital ratio is, $K/H = (r/A\alpha)^{1/\alpha-1}$. Thus it is easy to show;

$$\frac{d(K/H)}{d\rho} = -\frac{1}{A\alpha(1-\alpha)} \left(\frac{r}{A\alpha}\right)^{\frac{2-\alpha}{\alpha-1}} \frac{dr}{d\rho} < 0. \quad (32)$$

Equation (18) together with the above expression for $d(K/H)/d\rho$ implies;

$$\frac{dw}{d\rho} = -\left(\frac{r}{A\alpha}\right)^{\frac{1}{\alpha-1}} \frac{dr}{d\rho} < 0. \quad (33)$$

Since $K = (K/H)H$, $dK/d\rho$ will be;

$$\frac{dK}{d\rho} = H \frac{d(K/H)}{d\rho} + (K/H) \frac{dH}{d\rho},$$

$$\frac{dK}{d\rho} = -\frac{e^{-\frac{\rho}{r+\rho}}}{r+\rho} \left[\left(\frac{1}{A\alpha(1-\alpha)} \left(\frac{r}{A\alpha}\right)^{\frac{2-\alpha}{\alpha-1}} \frac{dr}{d\rho} \right) + \left(\frac{r}{A\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{r}{(r+\rho)^2} \left(\frac{dr}{d\rho} + 1\right) + \frac{1}{r+\rho} \right) \right] < 0. \quad (34)$$

Lastly, the general equilibrium response of aggregate consumption to a change in ρ , follows from the fact that aggregate consumption, C , is equal to aggregate output, Y , at the steady state (since there is no depreciation, population growth, or technical change). Thus:

$$\frac{dC}{d\rho} = \frac{dY}{d\rho} = A\alpha \left(\frac{K}{H}\right)^{\alpha-1} \frac{dK}{d\rho} + A(1-\alpha) \left(\frac{K}{H}\right)^{\alpha} \frac{dH}{d\rho} < 0, \quad (35)$$

which follows directly from equations (31) and (34).

3.1.1 The Effect of Endogenous Schooling

In the model of Blanchard, in which there is no schooling at all, a reduction in mortality raises the steady state levels of capital and consumption, just as it does in our model. Thus, to show the importance of endogenous schooling we need to show that the general equilibrium effects of mortality change are larger when schooling adjusts than when it stays constant. This is the issue we examine in this section. We show that the effect of a change in mortality on human capital, physical capital, and consumption is smaller in absolute value in the case where schooling is held constant than in the case where schooling varies endogenously.

We begin by calculating $dr/d\rho$ holding s constant at its general equilibrium level. Performing calculations similar to those in the appendix, one can show that

$$\left. \frac{dr}{d\rho} \right|_{\bar{s}=s^*} = \frac{r(r-\theta)(2\rho+\theta)}{(3r-\rho-2\theta)\rho(\rho+\theta)} > 0. \quad (36)$$

With constant s , $dH/d\rho$ becomes:

$$\left. \frac{dH}{d\rho} \right|_{\bar{s}=s^*} = -\frac{e^{-\frac{\rho}{r+\rho}}}{(r+\rho)^2} < 0, \quad (37)$$

which is clearly smaller in absolute value than the result with endogenous s in equation (31). The response of capital stock becomes:

$$\left. \frac{dK}{d\rho} \right|_{\bar{s}=s^*} = -\frac{e^{-\frac{\rho}{r+\rho}}}{r+\rho} \left[\left(\frac{1}{A\alpha(1-\alpha)} \left(\frac{r}{A\alpha} \right)^{\frac{2-\alpha}{\alpha-1}} \frac{dr}{d\rho} \right) + \left(\frac{r}{A\alpha} \right)^{\frac{1}{\alpha-1}} \left(\frac{1}{r+\rho} \right) \right] < 0, \quad (38)$$

which is less than the value in equation (34).

Finally, since both physical and human capital respond less to a change in mortality in the case of fixed schooling than in the case where schooling is endogenous, the same must be true of output and thus consumption.

3.1.2 Partial vs General Equilibrium

Our last goal is to compare the general equilibrium response of schooling and consumption to a change in mortality to the response of these variables in case where wages and interest rates are held fixed – conditions which would hold, for example, in a small, open economy subject to factor price equalization.

From (26), holding r constant, we can calculate the partial equilibrium response of schooling to a change in the death rate:

$$\left. \frac{ds}{d\rho} \right|_{\bar{r}} = -\frac{1}{(r+\rho)^2} < 0, \quad (39)$$

which is clearly smaller in absolute value than the general equilibrium result in equation (30). By the same token, the partial equilibrium response of aggregate human capital stock to a change in ρ is:

$$\left. \frac{dH}{d\rho} \right|_{\bar{r}} = -\frac{e^{-\frac{\rho}{r+\rho}}}{(r+\rho)^2} \left(\frac{r}{r+\rho} + 1 \right) < 0, \quad (40)$$

which is smaller in absolute value than the derivative in equation (31).

To show how the response to a change in mortality compares in the partial and general equilibrium in the case of consumption, first notice that we can write equation (21) as follows.

$$C = \frac{w(\rho + \theta)\rho}{(r + \rho^2)(\rho + \theta - r)e}. \quad (41)$$

It is straightforward to show that the partial equilibrium derivative is negative,

$$\left. \frac{dC}{d\rho} \right|_{\bar{r}, \bar{w}} = -\frac{w(\rho + \theta)\rho}{(r + \rho^2)(\rho + \theta - r)e} \left[\frac{2\theta + \rho - r}{(\rho + \theta)(\rho + r)} + \frac{r - \theta}{\rho(\rho + \theta - r)} \right] < 0. \quad (42)$$

Second, by using the two equations above one can show that,

$$\frac{dC}{d\rho} = \left. \frac{dC}{d\rho} \right|_{\bar{r}, \bar{w}} + \frac{dr}{d\rho} \left[\frac{1}{\rho + \theta - r} - \frac{2}{r + \rho} - \frac{\alpha}{(1 - \alpha)r} \right]. \quad (43)$$

In the appendix we show the existence of some tighter bounds for r so that the term in the square brackets can be shown to be positive.¹⁴ The above equation, together with (29) and (35), imply that the partial equilibrium response of consumption to a change in mortality is greater in absolute value than the general equilibrium response.

3.2 Calibration

Our analytical results were derived by assuming a specific form for the function $f(s)$, the derivative of which is the return to schooling. We assumed $f(s) = \ln(s)$. This function satisfies the properties assumed in the schooling models of Mincer and Rosen (Willis, 1986), specifically $f_s < 0$, $f_{ss} > 0$. It is not particularly realistic, however. In this section we examine our results using a more realistic earnings function. The price of this realism is that we are not able to produce analytic solutions.

Bils and Klenow (1996) posit the following form for the $f(s)$ function:

¹⁴The algebra for this result is available from the authors upon request.

$$f(s) = \frac{\Theta}{1 - \Psi} s^{1-\Psi}. \quad (44)$$

The Mincerian return to schooling is thus $f'(s) = \Theta/s^\Psi$. Using data from Psacharopoulos (1994) on a sample of 56 countries, Bils and Klenow regressed estimates of Mincerian returns on country schooling levels to estimate Ψ and Θ .¹⁵ Their estimates are $\Psi = .58$ and $\Theta = .32$.

Using these estimates for the return to schooling, we examine the steady state of our model for two different values of ρ , corresponding to two different values of life expectancy. In table 1, we use $\rho = .03$, corresponding to a life expectancy of 33 years. In table 2, we use $\rho = .012$, corresponding to a life expectancy of 83 years. These values bracket the observed experience of life expectancy in recent history. Since our results on the effects of mortality reduction on schooling do not differ greatly between the two tables, we are confident that the choice of ρ is not affecting our conclusions.

The second column of each table shows the steady state values of the endogenous variables s , H , r , w , and K , as well as two measures of consumption: the average level of consumption, C , and the level of consumption in the first instant of life, $c(b)$. The third column shows the elasticities of each of these variables with respect to ρ , the mortality probability. All of the elasticities in tables 1 and 2 have the same sign as in our analytic exercise.¹⁶ Note that an increase in ρ corresponds to a *decline* in life expectancy. In our discussion of tables 1 and 2 we find it more natural to talk about the effects of increased life expectancy, and so all of the signs of the elasticities in the discussion below will be the opposite of those in the tables.

The tables show that the effects of reduced mortality are economically significant. At low life expectancy (table 1), a one percent reduction in mortality will lead to a one percent increase in the length of schooling. For an economy with high life expectancy this effect is diminished only slightly: a one percent increase in life expectancy leads to an increase in schooling of seven-tenths of one percent. Overall, an increase in life expectancy from 33 years to 83 years leads to the length of schooling more than doubling. Thus, although the increase in longevity could not account for all of the increase in schooling that has been observed, it

¹⁵Specifically, they estimate

$$\ln(\hat{\lambda}) = \ln(\Theta) - \Psi \ln(s) + \epsilon,$$

where $\hat{\lambda}$ is the estimated return to schooling and ϵ is an error term.

¹⁶We performed a numerical analysis using the log specification, which was used to derive analytical results in section 3.1. The results were quite similar to those presented here.

can explain a significant fraction.¹⁷

At both high and low life expectancies, there is a relatively large response of both total human capital and total physical capital to improvements in life expectancy, and there are correspondingly small effects of life expectancy on the interest rate and on the wage per unit of human capital.¹⁸ At both high and low life expectancies, the elasticity of average consumption with respect to mortality decline is roughly equal to one. Some of this higher average consumption is due directly to lower mortality: since consumption is rising over the course of life, higher life expectancy will, *ceteris paribus*, lead to higher average consumption. But most of the increase in average consumption, the table shows, is due to higher $c(b)$, that is, higher consumption at the beginning of life.

The fourth columns of tables 1 and 2 show the elasticities of the endogenous variables with respect to mortality under the assumption that the length of schooling is held constant. The variable s is set at the value that holds in general equilibrium. By comparing the second and third columns we are thus able to look directly at the role being played by the endogenous variation in schooling. As can be seen from the table, even when schooling is held constant, there is an increase in H , the total quantity of human capital supplied, in response to a reduction in mortality. This is because when mortality is low, holding the length of schooling constant, a larger fraction of the population will be composed of people who have completed their schooling. But the change in H when s is held constant is only one-third as large as when s is allowed to vary. Similarly, the size of the capital stock increases directly in response to a decline in mortality, but the size of this increase is much larger in the case where schooling varies endogenously.

The increase in average consumption in response to a reduction in mortality is roughly twice as large in the case where schooling varies endogenously as in the case where schooling is held fixed. This comparison shows the important role that is played by the endogenous adjustment of schooling. It is interesting to note, however, that the increase in *initial* consumption is larger in the case where schooling is held fixed than in the case where schooling

¹⁷Since future improvements in mortality in the most developed countries are unlikely to match those that took place over the last 150 years, our model implies that the growth in education that has been witnessed over this period will not be repeated. Jones (1998) has similarly argued that the growth of human capital accumulation over this period represents a one-time adjustment.

¹⁸One of the most important effects that we would expect from longer life expectancy would be an increase in saving, as people expect to live into old age and thus save for life-cycle reasons. In this paper such an effect is absent because people work until they die. In a companion paper Kalemli-Ozcan and Weil (1998) explore this effect in a model where individuals can choose to retire if they live long enough.

is allowed to vary. The reason for this is that when schooling does not adjust in response to a mortality decline, the interest rate falls by more than it does in the case where schooling is endogenous. This lower interest rate leads to a flatter path of lifetime consumption.

The fifth columns of tables 1 and 2 examine the response of schooling and consumption to a change in mortality, under the assumption that neither wages nor interest rates change. An important phenomenon to note here is that the change in schooling in response to mortality decline is significantly larger in the general equilibrium case than in the partial equilibrium case (63 percent at low life expectancy and 73 percent at high life expectancy). This is the effect of changes in the interest rate: when mortality falls, the interest rate does so as well. As equation (13) showed, a lower interest rate will raise the optimal quantity of schooling.¹⁹ Thus partial equilibrium analyses of the effect of mortality on schooling will miss an important channel of causation.

One possible worry about the results presented in these tables is that some of the increase in life expectancy that results from a decline in mortality will take place at ages where the individual has left the labor force. Obviously these extra years of life will not affect the incentive to acquire human capital. To check the robustness of our results to this effect, we carried out the following experiment. We simulated a partial equilibrium version of the model, holding the interest rate constant at its steady state value, as is done in the fifth columns of tables 1 and 2. We then compared the elasticity of schooling with respect to mortality in the case where there is no retirement (which exactly matches the analytic exercise carried out in the tables) with the case where individuals stop working at age 65. In the case of low life expectancy, the elasticity of schooling with respect to ρ moves from -.64 to -.70. In the case of high life expectancy, the elasticity moves from -.40 to -.35. Extending these results to a general equilibrium model would be analytically intractable, but these partial equilibrium results give us some confidence that allowing for retirement in our model would not greatly alter the outcome.

¹⁹Another difference between partial and general equilibrium is that the former holds constant the wage. But changes in the wage do not have any effect on the optimal quantity of schooling, since both the opportunity cost of and the benefits from schooling are proportional to the wage.

4 Conclusion

Reduced mortality and increased investment in education are two of the most significant aspects of the process of economic growth. In this paper we have examined the effects of declining mortality on the incentive to invest in education. Higher life expectancy raises the optimal quantity of schooling because investments in education will earn a return over a longer period of time. The general equilibrium effect of reduced interest rates enhances this effect.

Using empirical estimates of the return to schooling, we showed that the magnitude of the effect of lower mortality on the length of schooling is economically significant. We also showed that the effect of lower mortality on consumption is significantly larger in the case where schooling is endogenous than in the case where schooling is taken to be fixed. This is another way of showing that the mortality-induced increase in education is an important consideration.

Our analysis leaves many questions unanswered, however. First, we have modeled the change in mortality as being completely exogenous. Obviously, this leaves out an important link running from higher income to lower mortality. In the context of our model, such a link would produce a multiplier effect from exogenous shocks to either income or mortality. For example, an exogenous shock to income would lower mortality and increase the optimal quantity of schooling. Higher schooling would further raise income, producing a feedback effect. While such a model could be important for analyzing the historical experience of the richest countries, however, it must be noted that the large changes in mortality in the post-World War II developing world were *not* been the product of higher income (see Preston, 1978a).

A second limitation of our analysis is that we have restricted education's role to be only as a factor of production, thus not allowing for any interaction between education and technology. Indeed, our model is one in which technology is stagnant. However, recent literature has drawn a link from increased education to a higher rate of technological progress. Thus a more extensive model could draw a link all the way from lower mortality to more rapid technological growth.

Finally, and possibly most significantly, our model of education and mortality has ignored any interaction with the fertility decision. Lower mortality and higher education have both been important correlates of reduced fertility. More educated parents face a higher op-

portunity cost of child-rearing, and in a low-mortality environment, parents are more likely to substitute child quality for child quantity. Thus lowered mortality will have mutually-reinforcing effects that lower fertility and raise education.

Appendix

Existence and Uniqueness of the Steady State:

Recall the reduced form steady state equation²⁰ (28):

$$\frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r)(\rho + \theta - r)} = e^{\frac{r}{\rho+r}}.$$

Consider values of r between 0 and θ . In this range *LHS* is strictly smaller than the *RHS*:

$$\frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r)(\rho + \theta - r)} = \frac{\rho(\rho + \theta)(1 - \alpha)}{\rho(\rho + \theta) + r(\theta - r)} \leq 1 - \alpha < 1 < e^{\frac{r}{\rho+r}}.$$

In fact this is also true for the range $0 \leq r \leq \rho/2 + \theta$:

$$\frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r)(\rho + \theta - r)} < \frac{\rho(\rho + \theta)}{(\rho + r)(\rho + \theta - r)} \leq \frac{\rho + 2r}{\rho + r} = 1 + \frac{r}{\rho + r} < e^{\frac{r}{\rho+r}}.$$

The last part follows from the fact that $1 + \frac{r}{\rho+r}$ is the first term of the Taylor series expansion for $e^{\frac{r}{\rho+r}}$. The weak inequality part is true since $\frac{\rho(\rho+\theta)}{\rho+\theta-r} \leq \rho + 2r$ when $r \leq \rho/2 + \theta$.

If we consider the case when $r \rightarrow \rho + \theta$, *LHS* is strictly bigger than the *RHS*:

$$\frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r)(\rho + \theta - r)} \rightarrow \infty > e^{\frac{\rho+\theta}{2\rho+\theta}}.$$

Therefore the range for r is: $\rho/2 + \theta < r < \rho + \theta$. In this range *LHS* begins below *RHS* and ends above it, so there exists a fixed point in this range. But the question remains whether this steady state solution is unique. To determine this, we examine the derivatives of both *LHS* and *RHS* with respect to r . It is easy to show that:

$$\frac{dLHS}{dr} = \frac{\rho(\rho + \theta)(2r - \theta)(1 - \alpha)}{(\rho + r)^2(\rho + \theta - r)^2} > 0.$$

The above derivative is positive given $\theta < r < \rho + \theta$. One can also show that:

$$\frac{d^2LHS}{dr^2} = \frac{\rho(\rho + \theta)(1 - \alpha)}{(\rho + r)^3(\rho + \theta - r)^3} [2(\rho + r)(\rho + \theta - r) + 2(2r - \theta)^2] > 0,$$

²⁰For simplicity we drop * which indicates the values for steady state.

which is also positive for the given range of r . So LHS is an increasing convex function. Examining the RHS :

$$\frac{dRHS}{dr} = \frac{\rho e^{\frac{r}{\rho+r}}}{(\rho+r)^2} > 0.$$

So RHS is also increasing in r . We can also evaluate the second derivative:

$$\frac{d^2RHS}{dr^2} = -\frac{\rho(\rho+2r)e^{\frac{r}{\rho+r}}}{(\rho+r)^4} < 0.$$

Thus RHS is an increasing concave function. Since LHS begins below RHS and ends above RHS for the range $\theta < r < \rho + \theta$ and since both sides are monotonic increasing functions of r , there exists a *unique* steady state solution.

The Tighter Bounds for r :

One can prove the existence of tighter bounds for r than the ones given above. We can analyze the reduced form steady state equation more as we did above and can show that,

$$\gamma\rho + \theta < r < \beta\rho + \theta,$$

where $\gamma = 1 - (1 - \alpha)\frac{2\rho+2\theta}{3\rho+2\theta}e^{-\frac{\rho+2\theta}{3\rho+2\theta}}$ and $\beta = 1 - (1 - \alpha)\frac{\rho+\theta}{2\rho+\theta}e^{-\frac{\rho+\theta}{2\rho+\theta}}$.

These bounds follows from the fact that if $r \geq \theta + \beta\rho$, $LHS > RHS$, i.e.,

$$\frac{\rho(\rho+\theta)(1-\alpha)}{(\rho+r)(\rho+\theta-r)} > \frac{\rho(\rho+\theta)(1-\alpha)}{(2\rho+\theta)(1-\beta)\rho} = e^{\frac{\rho+\theta}{2\rho+\theta}} > e^{\frac{r}{\rho+r}},$$

and also if $r \leq \theta + \gamma\rho$, $LHS < RHS$, i.e.,

$$\frac{\rho(\rho+\theta)(1-\alpha)}{(\rho+r)(\rho+\theta-r)} < \frac{\rho(\rho+\theta)(1-\alpha)}{(3/2\rho+\theta)(1-\gamma)\rho} = e^{\frac{\rho+2\theta}{3\rho+2\theta}} < e^{\frac{r}{\rho+r}}.$$

These results establish the tighter bounds.

The Effect of the Death Rate on the Interest Rate:

Examining the reduced form steady state equation, (28) again:

$$\frac{\rho(\rho+\theta)(1-\alpha)}{(\rho+r)(\rho+\theta-r)} = e^{\frac{r}{\rho+r}}.$$

We also know that $\rho/2 + \theta < r < \rho + \theta$, should hold to have a unique steady state solution. Taking the logs of both sides of the above equation and re-writing it as an implicit function will give:

$$G(r; \rho) = \frac{r}{\rho + r} + \ln(\rho + r) + \ln(\rho + \theta - r) - \ln(\rho) - \ln(\rho + \theta) - \ln(1 - \alpha) = 0.$$

The implicit function theorem implies:

$$\frac{dr}{d\rho} = -\frac{G_\rho}{G_r},$$

where subscripts denote the partial derivatives. We can evaluate this piece by piece:

$$\begin{aligned} G_\rho &= \left(\frac{\rho}{(\rho + r)^2} - \frac{\rho}{\rho^2} \right) + \left(\frac{1}{\rho + \theta - r} - \frac{1}{\rho + \theta} \right) = r \left(\frac{-(2\rho + r)}{\rho(\rho + r)^2} + \frac{1}{(\rho + \theta)(\rho + \theta - r)} \right), \\ &= \left(\frac{r}{\rho(\rho + r)^2(\rho + \theta)(\rho + \theta - r)} \right) \left[\rho(\rho + r)(r - \theta) + r(\rho + \theta)(r - \theta) + \rho(\rho + \theta)(2r - \rho - 2\theta) \right], \end{aligned}$$

which is positive in the given range of r .

$$G_r = \frac{\rho}{(\rho + r)^2} + \frac{1}{\rho + r} - \frac{1}{\rho + \theta - r} = \frac{(\rho + r)(\theta - 2r) + \rho(\rho + \theta - r)}{(\rho + r)^2(\rho + \theta - r)} = -\frac{\rho(3r - 2\theta - \rho) + r(2r - \theta)}{(\rho + r)^2(\rho + \theta - r)},$$

which is negative for the given range of r . These imply $dr/d\rho > 0$:

$$\frac{dr}{d\rho} = \frac{r[\rho(\rho + r)(r - \theta) + r(\rho + \theta)(r - \theta) + \rho(\rho + \theta)(2r - \rho - 2\theta)]}{\rho(\rho + \theta)[\rho(3r - 2\theta - \rho) + r(2r - \theta)]} > 0.$$

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Table 1: Calibration Results with Low Life Expectancy

Variables	Steady State Value	Elasticity with respect to ρ		
		General Equilibrium	General Equilibrium with fixed Schooling	Partial Equilibrium
s	10.69	-1.04		-0.64
H	5.70	-0.89	-0.32	-0.67
r	0.05	0.36	0.49	
w	1.50	-0.16	-0.21	
K	71.70	-1.41	-1.02	
$c(b)$	3.66	-0.78	-0.98	-0.19
C	12.18	-1.04	-0.53	-2.54

Notes: $\rho = 0.03$, $\alpha = 0.3$, $\theta = 0.03$, $A = 1$. The fourth column holds s fixed at its steady state value. The fifth column holds r and w fixed at their steady state values.

Table 2: Calibration Results with High Life Expectancy

Variables	Steady State Value	Elasticity with respect to ρ		
		General Equilibrium	General Equilibrium with fixed Schooling	Partial Equilibrium
s	23.51	-0.69		-0.40
H	13.30	-0.92	-0.28	-0.66
r	0.04	0.21	0.26	
w	1.67	-0.09	-0.11	
K	242.92	-1.23	-0.66	
$c(b)$	7.24	-0.69	-0.79	-0.23
C	31.80	-1.01	-0.40	-3.76

Notes: $\rho = 0.012$, $\alpha = 0.3$, $\theta = 0.03$, $A = 1$. The fourth column holds s fixed at its steady state value. The fifth column holds r and w fixed at their steady state values.

Figure 1: The Evolution of the Individual Capital Stock