# Playing Parties Against Eachother in Proportional Representation Systems*

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**Abstract**

When special interest groups can use threats of contributing to an opponent to gain support, they will never contribute to all parties in an election. In proportional representation systems where voters usually choose between many parties, contributions to multiple parties are possible. Contributions are made when a party is popular and not when a party is highly substitutable with other political parties.

*The views expressed in this paper are those of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management.

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# 1 Introduction

Campaign contributions have traditionally been thought of as a transaction involving only the contributor and the receiving candidate or political party. Such a perspective largely ignores how the possibility of contributing to an opponent could also affect the patterns of contributions and support. Chamon and Kaplan (2006) present a model of campaign contributions with multilateral contracting in a two-candidate race, where a special interest group's contribution is conditioned not only on the candidate's support for that special interest, but also implicitly on whether or not her opponent will support it. As a result, a candidate may support a special interest not only in order to receive a contribution but also in order to discourage that special interest from making a contribution to his or her opponent. The ability to threaten leverages the power of special interest groups, whose influence may be driven by implicit out-of-equilibrium contributions, generating a disconnect between their influence and the actual contributions we observe.

The multilateral contracting framework offers a number of advantages over previous approaches that have considered out-of-equilibrium contributions. In previous studies, such as Baron (1994), chapter 3 of Persson and Tabellini (2000) and chapter 10 of Grossman and Helpman (2001), out-of-equilibrium contributions after policy platforms have been announced lead to a collapse in equilibrium contributions. That is not the case in the multilateral contracting setting used in Chamon and Kaplan (2006), where equilibrium and out-of-equilibrium contributions coexist, with special interest groups mainly using out-of-equilibrium (equilibrium) contributions to induce support in close (lopsided) races. Thus, it is able to provide an explanation for the "missing money" puzzle, while still being able to explain when contributions actually take place.

This paper applies a modified version of the multilateral contracting framework used in Chamon and Kaplan (2006) to a race involving three or more candidates. The extension from a two-way race, which is more likely to be relevant in majoritarian systems, to a multi-way race, which is more likely to be relevant in proportional systems[1], allows us to analyze the implications of multilateral contracting for the latter. The main implication of multilateral contracting in a two-party race is that special interest groups should never give to directly opposing candidates in a same race.[2] This paper shows that contributions to one party will always be redundant. However, interest groups may contribute to more than one party in a multi-party race. This non-trivial extension shows that it is still possible for a special interest group to play one party against the other in a multi-party election, leveraging its influence well beyond the contributions actually made.

# 2 Model

We use a model of electoral competition building upon the frameworks of Grossman and Helpman (1996) and Baron (1994). Parties compete in a legislative election, modeled as a single race. Voters base their choice on the candidates' platforms and an "impression" component that is influenced by campaign expenditures. We consider special interest groups which do not inherently care about which party wins the election as long as their special interest policy is supported by the winner. Special interest groups (SIGs) are allowed to announce schedules of contributions which are contingent not only on the platform of the party receiving the offer (bilateral contacting) but also on the platform of the two other parties (multilateral contracting). Following Baron (1994), campaign contributions can "buy" some of the impressionable component of the vote, but catering to special interests will cost the politicians votes amongst the informed component of the vote.

Each voter makes her decision based not only upon what policies candidates will implement but also on her "impression" which is influenced by the amount of money spent on campaigns. Voters can choose from one of $|K|$ parties in a proportional legislative election, modeled as a single race.[3] In an abuse of notation we denote $|K|$

[1] Duverger's famous 1954 book shows that whereas majoritarian systems standardly have two parties with seats in the national legislature, proportional representation systems generally have more than two parties with seats.

[2] Chamon and Kaplan (2006) use contribution-level data form the U.S. Federal Election Comission to show that while many contributors give to both parties, it is very rare for them to give to directly opposing candidates in a same race.

[3] This would describe the election under a proportional system with closed party lists where the

$= K$. We furthermore assume that $K \geq 3$. Each party platform includes a "fixed" ideological policy that is exogenously given, and a "pliable" special interest policy whose position the party can chose so as to maximize its objective function. Voter $i$'s utility from voting for party $k$ is:

$$U_{ik} = W(\tau_k) + M_k + b_k + v_{ik}$$

where $W(\tau_k)$ is the utility from the SIG policy, $M_k$ are the campaign expenditures of party $k$ (the "impression" or uninformed component of the vote), $b_k$ is the average predilection for party $k$'s fixed ideological policy and $v_{ik}$ is the preference by voter $i$ for party $k$, which is given by the realization of a zero-mean stochastic shock that is i.i.d. type I extreme value (or Gumbel). Pliable special interest policies are assumed to be uniformly disliked by all voters ($W' < 0$).Voter $i$ prefers party $k$ over party $j$ if:

$$W(\tau_k) - W(\tau_j) + M_k - M_j + b_k - b_j > v_{ij} - v_{ik} \tag{1}$$

Since $v_{ik}$ is distributed i.i.d. type I extreme value, the difference $v_{ij} - v_{ik}$ is distributed logistic:

$$F\left(v_{ij} - v_{ik}\right) = F\left(v_{ijk}\right) = \frac{e^{v_{ijk}}}{1 + e^{v_{ijk}}}, \tag{2}$$

which together with (1) implies that the probability that person $i$ votes for party $k$ is[4]:

$$\frac{e^{W(\tau_k)+M_k+b_k}}{\sum_j e^{W(\tau_j)+M_j+b_j}} \tag{3}$$

where the subscript $j$ refers to all parties (including k). There is a continuum of voters who have identical preferences except for the realization of the i.i.d. shock $v$. Therefore, the probability that any given voter casts a ballot for party $k$, which is given by equation (3), is equal to the vote share $S_k$ for party $k$. The objective function of each party $k$ is to maximize its vote share $S_k$.[5] Modeling how the different party's platforms translate into the actual policies implemented would involve a complex

entire country was a single district.

[4]See Kenneth Train (2003) for a more detailed derivation of the logit probability formula.

[5]In a richer setting, coalitional bargaining considerations could feature in the party's objective function. For simplicity, we assume parties merely seek to maximize vote share.

excursion into coalitional bargaining and is thus beyond the scope of this paper. Following Persson and Tabellini (2000) chapter 8, we assume that the SIG policy implemented is the one from party $k$'s platform with probability $S_k$. One rationale for this assumption is that assuming no party gets a majority of the votes, each party will be picked with probability equal to their vote share to be formateur. If they are picked to be formateur, they implement their announced special interest policy. Diermeier, Eraslan and Merlo (2003) provide empirical evidence that in fact parties are picked to be formateur with probability roughly equal to their vote share. There is a single SIG, whose objective function is to maximize expected utility over the SIG policy and money:

$$U_{SIG} = \sum_k S_k W_{SIG}(\tau_k) + M_{SIG} - \sum_k M_k \tag{4}$$

where $W_{SIG}(\tau)$ is the utility it derives from the SIG policy $\tau$, $M_{SIG}$ is its starting level of monetary resources and $M_k$ is the level of contributions made to party $k$. We further assume that both money and policy are weakly positive: $M_k \in [0, M_{SIG}]$ and $\tau_k \in [0, \infty)$.

We now define a useful quantity for our analysis: the ratio of marginal utility that the SIG policy provides to the SIG relative to the marginal disutility it causes voters:

$$\theta(\tau) = -\frac{\frac{\partial W_{SIG}(\tau_k)}{\partial \tau_{SIG}}}{\frac{\partial W(\tau_k)}{\partial \tau_{SIG}}}$$

We assume that the ratio of SIG utility to voter disutility, $\theta(\tau)$, is non-increasing in $\tau$. In other words, at higher levels of the policy, the SIG does not care more on the margin about the policy relative to voters than at lower levels of the policy:

$$\frac{\partial \theta(\tau)}{\partial \tau} \leq 0$$

The SIG chooses multilateral contribution schedules to maximize utility given by equation (4) subject to parties maximizing vote share given by equation (3) and subject to the SIG's budget constraint and non-negativity constraints on its contributions. This is a difficult optimal control theory problem; however, as shown in Chamon and Kaplan (2006), its solution can be obtained from a much simpler associated principal-agent contract theory problem. Unlike in the standard bilateral contracting environment, a party needs not receive the same contribution in the other parties inside and outside options. Moreover, since the SIG can fully commit to a schedule in advance and because the amount of money it gives to a candidate in the other candidate's outside option does not actually get paid in equilibrium (as opposed to in the bilateral contracting problem), there is no cost to the SIG of threatening to optimally allocate all of its money to the other parties so as to maximize inflicted damage to party in question in determining that party's outside option. We denote party k's outside option by $\underline{S_k}(\tau_k)$.

The Lagrangian associated with the SIG's maximization problem is:

$$\max_{\vec{M}_k,\vec{\tau}_k} \sum_k S_k(\cdot)W_{SIG}(\tau_k) + M_{SIG} - \sum_k M_k + \sum_k \lambda_k M_k +$$
$$\lambda_{SIG}\left[M_{SIG} - \sum_k M_k\right] + \sum_k \mu_k\left[S_k(\cdot) - \underline{S_k}(\tau_k)\right]$$

where $\lambda_k$, $\lambda_{SIG}$ and $\mu_k$ are the Kuhn-Tucker Lagrange multipliers associated with the non-negativity constraint of contributions to party $k$, the SIG's budget constraint and the participation constraint for party $k$. The FOCs with respect to $M_k$ and $\tau_k$ are (respectively):

$$M_K : \sum_j \frac{\partial S_j}{\partial M_k} W_{SIG}(\tau_j) - 1 + \lambda_k - \lambda_{SIG} + \sum_j \mu_j \frac{\partial S_j}{\partial M_k} = 0 \tag{5}$$

$$\tau_k : \sum_j \frac{\partial S_j}{\partial \tau_k} W_{SIG}(\tau_j) + S_k(\cdot)\frac{\partial W_{SIG}}{\partial \tau_k} + \sum_j \mu_j \frac{\partial S_j}{\partial \tau_k} - \sum_{j\neq k} \mu_j \frac{\partial \underline{S_j}}{\partial W(\tau_k)} \frac{\partial W(\tau_k)}{\partial \tau_k} = 0 \tag{6}$$

Noting that $\partial S_j/\partial \tau_k = (\partial S_j/\partial M_k)(\partial W(\tau_k)/\partial \tau_k)$, the expressions above imply:

$$\lambda_k = 1 + \lambda_{SIG} - S_k(\cdot)\theta(\tau_k) - \sum_{j\neq k} \mu_j \frac{\partial \underline{S_j}}{\partial W(\tau_k)} \tag{7}$$

With simple but tedious algebraic manipulations, using the fact that $S_k = \frac{e^{W(\tau_k)+M_k b_k}}{\sum_j e^{W(\tau_j)+M_j+b_j}}$, we can show that:

$$\sum_k S_k(\cdot)\theta(\tau_k) = -\sum_k \mu_k \sum_{j\neq k} \frac{\partial \underline{S_j}}{\partial W(\tau_k)}, \tag{8}$$

which implies that summing (7) for all parties yields:

$$\sum_k \lambda_k = k + k\lambda_{SIG} > 0 \tag{9}$$

We can now show that whereas one-sidedness of donations does not generalize from the two party case, all-but-one-sidedness does:

**Proposition 1** *The SIG does not contribute to at least one of the parties.*

***Proof.*** *Since $k > 0$ and $\lambda_{SIG} \geq 0$, $\lambda_k$ must be greater than zero for some $k$, implying that the non-negativity constraint of contributions is binding for that party, and as a result that party receives zero contributions.* ■

Intuitively, if all parties are receiving money, then the SIG can lower donations to all parties by the amount given to the party receiving the least; all parties will be just as well off and so no IR constraints will be violated. Moreover, the party receiving the lowest amount will no longer be receiving contributions. It is not necessarily possible, however, to move more than one party to zero contributions because doing so may entail making negative donations to at least one party, which is not feasible.

# 3 Discussion

Equation (7) is able to provide a number of interesting insights about the behavior of SIG contributions. The Kuhn-Tucker Lagrange multiplier $\lambda_k$ on the non-negativity of contributions to party $k$ is affected positively by two terms which are the same for all parties (i.e. not specific to $k$): 1 and $\lambda_{SIG}$. The former is the marginal value of money, which is equal to one in the SIG's maximization problem. It also corresponds to the shadow value of negative contributions (i.e. the marginal benefit the SIG would obtain if it were able to extract money from a candidate). The term $\lambda_{SIG}$ is the Kuhn-Tucker Lagrange multiplier on the SIG's budget constraint. When that budget constraint is binding it will increase the shadow value of negative contributions, which makes it more likely that the non-negativity constraint on contributions binds. The remaining two terms in equation (7) are party-specific: $-S_k(\cdot)\theta(\tau_k)$and $-\sum_{j\neq k}\mu_j\frac{\partial S_j}{\partial W(\tau_k)}$. These terms have negative signs, which means that the higher their absolute value, the more likely that party $k$ receives equilibrium contributions.

**Proposition 2** *All else equal, party $k$ is more likely to receive contributions ($\lambda_k < 0$) when it wins a large proportion of the seats ($S_k$ is high).*

This result is very intuitive. The stronger the party, the more attractive it becomes to the SIG since it is able to more easily, or with greater likelihood, implement its SIG policy.

The remaining term in (7) is $-\sum_{j \neq k} \mu_j \frac{\partial S_j}{\partial W(\tau_k)}$. Note that $\frac{\partial S_j}{\partial W(\tau_k)} \leq 0$. That is, an increase in utility (or decrease in disutility) voters get from party $k$'s SIG platform has a negative effect on the vote share of other parties by increasing their outside options if their outside options are binding. As a result, $-\sum_{j \neq k} \mu_j \frac{\partial S_j}{\partial W(\tau_k)}$ is at least weakly positive. Moreover, the Kuhn-Tucker Lagrange multiplier $\mu_j$ is positive when the outside option for party $j$ is binding and zero otherwise. Thus, that term is large when the impact of party k's policy changes is high for parties held to their outside option. Since at least one party receives no money and is held to their outside option, $-\sum_{j \neq k} \mu_j \frac{\partial S_j}{\partial W(\tau_k)}$ is always positive. Our final proposition states:

**Proposition 3** *All else equal, party $k$ is more likely to receive contributions ($\lambda_k < 0$) when it is less substitutable with parties held to their outside options ( $-\sum_{j \neq k} \mu_j \frac{\partial S_j}{\partial W(\tau_k)}$ is high).*

We can illustrate this result with a simple example. Suppose there are three parties: $Left$, $Center$ and $Right$. The $Center$ party is the most popular, but is also the most substitutable with the other parties at their outside options. On the one hand, the strength of the $Center$ party would make it more likely to receive equilibrium contributions from the SIG. But on the other hand, its substitutability with respect to the other parties would make it more vulnerable to implicit out-of-equilibrium threats. Depending on parameter values and specific functional forms, the latter channel may dominate and the $Center$ party may end-up not receiving contributions despite its popularity. This substitutability effect can lead to a very interesting and somewhat counter-intuitive outcome, whereby the party that would benefit the most from contributions may end up receiving none. In fact, it could end up being the only party to receive none.

## 4 Conclusion

In this paper, we have extended a simplified version of the multilateral contracting model for interest group campaign contributions presented in Chamon and Kaplan (2006) to a proportional representation setting with more than two parties. The main result is that any given interest group will never give to all parties. This result stands in contrast with traditional bilateral contracting settings (e.g. Grossman and Helpman 1996) where usually all parties receive contributions.

While a full characterization of the pattern of contributions is beyond the scope of this paper[6], a number of interesting insights were obtained. First, popular parties tend to receive contributions. Secondly, and more interestingly, parties which are substitutable with others are less likely to receive contributions. This insight could only be obtained in a setting in which there are more than two choices. If moderate or centrist parties tend to be the more substitutable ones, then those parties will, ceterus paribus, be the ones most hurt by SIG activity. As a result, unless those parties are the most popular to begin with, SIG activity can contribute to polarization of vote shares. This finding also suggests an interesting application to the two-party setting, where absenteeism is also an option. All else equal, the party whose turn-out is easier to suppress should be the one most vulnerable to out-of-equilibrium threats.

Admittedly, while very tractable, our model of proportional representation is naive at best. Inroducing coalition formation and inter-party bargaining considerations could lead to additional insights on interest group behavior.

Our results imply a subtle distinction between two and multi-party political systems. They provide empirical predictions of political-threat based models of campaign contributions. Interest groups may give to more than one side when three or more parties are competing though they never give to all parties in a multi-party system. Moreover, a party is less likely to receive money when other parties are close substitutes. To empirically verify these predictions, data on itemized contributions are important; the lack of such data across countries currently constrains empirical work. Chamon and Kaplan (2006) document a one-sided pattern of contributions in the United States (where itemized data is available). Comparison with similar data from proportional systems could help further establish multilateral contracting as the main

[6] This is done in Chamon and Kaplan (2006), for the case of a two party system.

framework for understanding interest group behavior. This would be an exciting area for future research.

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