

An Experiment on Auctions with Endogenous Budget Constraints

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Abstract

We perform laboratory experiments comparing auctions with endogenous budget constraints. A principal imposes a budget limit on a bidder (an agent) in response to a principal-agent problem. In contrast to the existing literature where budget constraints are exogenous, this theory predicts that tighter constraints will be imposed in first-price auctions than in second-price auctions, offsetting any advantages attributable to the lower bidding strategy of the first-price auction. Our experimental findings support this theory: principals are found to set significantly lower budgets in first-price auctions. Our results help to explain the prevalence of ascending (i.e., second-price) formats in high-stakes auctions.

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1. Introduction

Beginning with important articles by Che and Gale (1996, 1998), an active literature has explored the implications of budget constraints in auctions. This literature models environments in which bidders have well-defined values for the items being auctioned, but may also face binding budget constraints. For example, a telecommunications entrant may value a spectrum license at \$800 million, but may be limited to a budget of \$500 million.¹ The existing literature identifies a number of interesting consequences. For example, a standard format such as the second-price auction may no longer be efficient in the sense of allocating items to the bidders who value them the most, as the bidder with the highest value may not have the highest budget. More surprisingly, budget constraints may cause first-price auctions to outperform second-price auctions with respect both to efficiency and revenues. Since bidders shade their bids in first-price auctions but bid full value in second-price auctions, bidders are less likely to find their budgets to be binding in first-price auctions. This upsets revenue equivalence and results in first-price auctions producing higher revenues. Moreover, since bids are relatively more likely to reflect bidders' values than their limited budgets, first-price auctions may also yield more efficient outcomes than second-price auctions.

However, most conclusions to date about auctions with budget-constrained bidders have depended crucially on a modeling assumption that their budgets are determined *exogenously*.² Recent work by Burkett (2011) demonstrates that conclusions change qualitatively if, instead, the choice of budgets is allowed to be *endogenous*. In Burkett's work, the budget constraint is a control mechanism that a principal (e.g., the corporate board) imposes on an agent (e.g., the manager delegated to bid for an asset) in order to curb managerial discretion such as empire building. Therefore, a principal seeking to constrain its agent ought to set a relatively more stringent budget when the agent bids in a first-price rather than in a second-price auction, as

¹ There is longstanding evidence that bidders in spectrum auctions face significant budget constraints. In describing the Nationwide Narrowband Auction (FCC Auction #1), Cramton (1995) wrote: "Budget constraints undoubtedly played a role in the bidding." More recently, Bulow, Levin and Milgrom (2009) emphasized two issues—exposure problems and budget constraints—arguing that the latter are "ubiquitous" in large spectrum auctions.

² Experimental results on auctions with budget constraints are limited and we are not aware of any other experimental paper with endogenous budget decisions in auctions (for example, Pitchic and Schotter (1998) studied sequential auctions where budget is common knowledge.)

identical budgets will leave the agent unconstrained in more states of the world in a first-price auction. Comparing revenues without allowing the principal's choice of budget to depend on the auction format may have no greater justification than comparing revenues without allowing the bidder's strategy to depend on the auction format.

In this paper, we attempt to test the above reasoning experimentally. The “bidder” (the agent) seeks to acquire an asset, but will derive a private benefit from acquiring the asset, above and beyond mere profit maximization. The “principal” can limit the bidder's discretion by imposing a budget constraint on bids. Each player observes a signal of the asset's value before moving: the principal chooses the budget and the agent chooses the bid based on their respective signals. In our laboratory experiments, the variable of greatest interest is the principal's choice of budget—we wish to see whether it is set independently of the auction format, or whether the principal sets a lower budget for a first-price auction than for a second-price auction. The bidder's choice of bid is only of secondary interest and, in some treatments, the role of the bidder will be replaced by a computer program rather than being a human subject.

[Figure 1]

Our experimental results can be seen most easily in Figure 1, which displays box plots of the budgets selected by the principal for each decile of signals from $[0,100]$ for both auction formats. Each box indicates the interquartile range (IQR) and the whiskers extend to the furthest data point within $1.5 \times \text{IQR}$. The grey (left) boxes display the budgets selected by the principal in first-price auctions and the black (right) boxes display the budgets selected in second-price auctions. It is apparent to the naked eye that budgets are set substantially lower in first-price than in second-price auctions for all signal deciles except $[0,10]$. The exogeneity of the budget choice is also rejected by statistical tests.

Figure 1 displays clear results with a pair of human subjects in each experiment—one taking the role of the principal and one taking the role of the bidder. The results are even sharper in treatments where the human bidders are replaced by computerized bidders, as displayed later in Figure 3. Since the computerized bidders consistently follow predetermined rules, we are able to elicit more information about the principals' behavior in these sessions. Using this additional information, we show that these data support the prediction that the principals constrain the same set of bidder types across auction formats, a key implication of the theoretical model.

Burkett (2011) demonstrated theoretically, in the model tested here, that the principal tightens the budget precisely so as to neutralize the change in auction format from second- to first-price. Consequently, the second-price auction with an endogenous budget constraint generates exactly the same theoretical allocation as the first-price auction with an endogenous budget constraint—a restoration of the revenue equivalence theorem. In particular, the outcome of the second-price auction is expected to attain the same degree of efficiency as the first-price auction, and both are expected to yield equal revenues. We test—and are unable to reject—the efficiency hypothesis in our experimental data. This finding is particularly relevant for environments (e.g., spectrum auctions) where budget constraints may be important and the seller may be motivated primarily by efficiency considerations. We also test—and do reject—the hypothesis of equal revenues. However, the latter experimental finding is unsurprising in light of the traditional experimental literature and is what we had expected to find. The experimental auctions literature (without budget constraints) has consistently found that bidders in the first-price auction bid higher than the risk-neutral Nash equilibrium, leading to higher revenues in the first-price auction.³ Given this prior evidence, it would have been surprising if adding a pre-auction budgeting decision by a principal had somehow eliminated the difference in revenues of the two auction formats that is generally observed in the laboratory.⁴

Our model and experimental findings help to explain why ascending auctions are commonly used in high-stakes settings such as the sale of telecommunications spectrum, where budget constraints are routinely imposed on managers. Note that most ascending auction formats used for spectrum effectively yield second pricing; therefore, the use of ascending auctions would be surprising within the context of the literature on exogenous budget constraints. However, with endogenous budget constraints, this empirical regularity becomes much easier to understand.

³ See Cox, Roberson and Smith (1982) and Cox, Smith and Walker (1988) as the seminal papers, and Kagel (1995) for a detailed survey. Risk aversion (Cox, Smith and Walker (1988)), anticipation of regret (Filiz-Ozbay and Ozbay, 2007), joy of winning (see, for example, Goeree, Holt and Palfrey, 2002), fear of losing (Delgado et al., 2008, Cramton et al., 2012a, 2012b), and level- k thinking (Crawford and Iriberry, 2007) have been offered as possible explanations of the overbidding phenomenon.

⁴ One interpretation of the results from Burkett (2011) is that the budgets in the model function like bids that are not always “active”. If the subjects recognize this, one might expect similarities between the budgeting decisions in this experiment and bidding decisions in the existing literature.

The rest of this paper is structured as follows. In Section 2, we specify the theoretical model and explore its properties. In Section 3, we describe the experimental design, and in Section 4, we give the experimental results. Section 5 concludes.

2. Model

The models tested in the experiment are standard first- and second-price sealed-bid, independent private values auction models with two bidders, extended to include a pre-auction budgeting stage. In the budgeting stage, each bidder receives a budget from a principal. Both the principal and the bidder receive a payoff in the event that the bidder wins the item at the auction; however, the principal's payoff is always lower than the bidder's. This is due to an additional private payoff that the bidder receives from the item that does not accrue to the principal. It is the presence of this private payoff that motivates the principal to restrain the bidder with a budget.

Formally, the game occurs in two stages. In the first stage, each principal receives a signal about the value of the item and decides on a budget for the bidder based on this information. Neither the principal's signal nor the budget choices are observed by the other principal-bidder pair. Having observed their budgets, each bidder in the second stage observes her valuation for the good and decides on a bid for the auction which may not exceed the budget set by her principal.⁵ The winner of the auction is the principal-bidder team with the highest bid. We consider first-price and second-price payment rules.

Payoffs

The payoffs to both the principal and the bidder are determined only by the information received by the bidder. Specifically, we assume that if bidder $i \in \{1,2\}$ observes a valuation of t_i , principal i has a valuation for the item given by δt_i , where $0 < \delta < 1$. If bidder i submits the winning bid in the auction and pays a price p , then bidder i receives a payoff proportional to $t_i - p$ and principal i receives a payoff proportional to $\delta t_i - p$.⁶ That is, the bidder and the

⁵ As will be clear from our equilibrium analysis, the principal's signal is irrelevant information for the bidder in this setup since the equilibrium unconstrained bid is a function of only the bidder's valuation.

⁶ The payoffs are proportional to those expressions to avoid double counting the total profits. For example, the bidder and the principal might be equity holders in a firm with shares σ_b and σ_p , respectively (where $\sigma_b + \sigma_p \leq 1$). The bidder is assumed to receive $\sigma_b(t_i - p)$ and the principal to

principal are both risk neutral and receive a payoff that is determined by the difference between their respective valuations and the price paid for the good.

Information

The signal received by principal i is denoted by s_i , assumed to be uniformly distributed on $[0,100]$. The signals of principals i and j are independent. The principal does not observe her valuation for the good, but knows that her valuation for the good, δt_i , is uniformly distributed on $[0, s_i]$. In other words, s_i determines the upper limit of the principal's valuation. Based on the realization of s_i , the principal decides on a budget for the bidder, given by w_i . Having observed her budget, w_i , bidder i observes her valuation for the object, t_i , which given the assumption on the principal's valuation is uniformly distributed on $[0, s_i/\delta]$. Although the theoretical results hold for general distributions, we chose these distributions for the experiment, because we wish to focus on the budgeting decision and hence would like the game to be as simple as possible from the principal's perspective. Note that as δ decreases (increases) the upper limit on the bidder's valuation increases (decreases) and the agency problem becomes more (less) severe.

The timing of the game is depicted in Figure 2. The dashed edges indicate the dependence relations between the signals, while the solid edges indicate the actions taken by the participants (budgets were always referred to as "caps" in the experiment).

[Figure 2]

Equilibrium

We consider a symmetric equilibrium of this model. The equilibrium will consist of the bidder's unconstrained choice of bid for a given valuation, $b(t)$, and the principal's choice of budget for each possible signal, $w(s)$. It can be shown that given these two choices the bid submitted to the auction is $\min\{b(t), w(s)\}$.

Assuming that the bidders employ bidding strategies that are strictly increasing and continuous functions of their valuations, each principal's choice of budget constraint is equivalent to choosing a cutoff type, \hat{t} , above which the bidder is constrained. In other words for

receive $\sigma_p(\delta t_i - p)$. This formulation identifies the term $(1 - \delta)\sigma_b t_i$ (the difference between the bidder's payoff and $\sigma_b(\delta t_i - p)$) as the bidder's private payoff from obtaining the good.

a choice of budget constraint, $w(s)$, we can define a cutoff type as the t that satisfies $w(s) = b(\hat{t}(s))$ where $b(t)$ is the bidder's unconstrained choice of bid corresponding to the value t .

The first consequence of this representation is that the bid submitted at the auction is now $b(\min\{t, \hat{t}(s)\})$, so that the winning bidder is the bidder with the higher value of $\min\{t, \hat{t}(s)\}$. We refer to this quantity as the bidder's effective type. The bids submitted at the auction are then equivalent to the bids submitted in a standard independent private values auction where valuations are distributed according to $\min\{t, \hat{t}(s)\}$.

As is shown in Burkett (2011), the equilibrium choice of $\hat{t}(s)$ is the same in the first- and second-price auctions when bidders' signals are independent and is the solution to the following equation:

$$E[\delta t | t \geq \hat{t}(s), s] = \hat{t}(s). \quad (1)$$

A detailed derivation of the equilibrium is in the Appendix.⁷ In our setup, the solution to Equation (1) is $\hat{t}(s) = s/(2 - \delta)$. This in turn implies that the distribution of effective types is given by the following:

$$G(x) = P(\min\{t, \hat{t}(s)\} \leq x) = \left(2 - \delta - \delta \ln\left(\frac{2 - \delta}{100} x\right)\right) \frac{x}{100}.$$

In the second-price auction the bidder still has a weakly dominant strategy to bid her own value. That is, in the second-price auction $b_{SP}(t) = t$. In the first-price auction, the equilibrium bid functions are determined according to the expected value of the opponent's effective type given a winning bid:⁸

$$b_{FP}(x) = E[\min\{t, \hat{t}(s)\} | \min\{t, \hat{t}(s)\} \leq x] = \frac{4 - 3\delta - 2\delta \ln\left(\frac{2 - \delta}{100} x\right)}{4 - 2\delta - 2\delta \ln\left(\frac{2 - \delta}{100} x\right)} \frac{x}{2}. \quad (2)$$

To summarize, in the second-price auction the equilibrium unconstrained bid function is given by $b_{SP}(t) = t$ and the budget function is given by $w_{SP}(s) = b_{SP}(\hat{t}_{SP}(s)) = \frac{s}{2 - \delta}$. In the

⁷ Proposition 1.A in the appendix states the uniqueness property of this equilibrium.

⁸ Although $b_{FP}(\cdot)$ in Equation (2) looks complicated it is approximately linear for the δ used in our experiments (see Figure 4).

first-price auction, the unconstrained bid function is given by $b_{FP}(t)$ in Equation (2), and the budget function is given by $w_{FP}(s) = b_{FP}(\hat{t}_{FP}(s)) = b_{FP}\left(\frac{s}{2-\delta}\right)$.

The notable results from this analysis are that the first- and the second-price auction raise the same expected revenue and have the same expected efficiency for any $0 < \delta < 1$.⁹ This is a direct consequence of the bids being determined by the distribution of the effective types, $\min\{t, \hat{t}(s)\}$, which as noted above is unchanged between the first- and second-price auctions. Moreover, a principal with signal s sets a lower budget in the first-price than in the second-price auction. This is because $w_{FP}(s) = b_{FP}\left(\frac{s}{2-\delta}\right) < \frac{s}{2-\delta} = w_{SP}(s)$. These results also extend to a model with more than two principal-bidder pairs and valuations with common-value components (Burkett (2011)).

3. Experimental Design

The experiments were run at the Experimental Economics Lab at the University of Maryland (EEL-UMD). All participants were undergraduate students at the University of Maryland.¹⁰ The main experiment involved five sessions of second-price sealed-bid auctions (SP) and five sessions of first-price sealed-bid auctions (FP). In our control treatments we had five sessions of FP and five sessions of SP where the bidders were computerized and principals were human subjects. In each session there were 16 subjects. No subject participated in more than one session and we did not have any pilot session. Therefore, we had 80 subjects per treatment with 320 subjects in total. The random draws were balanced in the sense that we used the same sequence of random number “seed” signals for each auction format, so the random value draws for SP matched the random draws for FP.¹¹ A new set of random draws was used for each session in each format, etc. Participants were seated in isolated booths. Each session lasted less than two

⁹ In fact, one can make the stronger assertion that the two auction formats agree in their allocations for every possible realization of the signals. This is a consequence of the winner being the one with the highest value of $\min\{t, \hat{t}(s)\}$ in both cases.

¹⁰ EEL-UMD is a relatively new lab and one or two auction experiments are conducted in a year. So we are confident that our very rich subject pool is not overly experienced in auction experiments.

¹¹ The random draws were balanced within the main and control treatments not in between. This is because in the main treatments, we had eight bidders and eight principals in a session and in the control treatments we had sixteen principals in the lab where the bidders were computerized players.

hours.¹² Bidder instructions are in the Appendix. To test the subjects' understanding of the instructions, they had to answer a sequence of multiple choice questions. The auctions did not begin until each subject answered all of the multiple choice questions correctly. The experiment is programmed in z-Tree (Fischbacher 2007).

We start with explaining the design for the main treatments where both principals and bidders were subjects. Later we will describe the control treatments with computerized bidders.

In each session, each subject participated in 30 auctions. The first 5 auctions were practice ones and they were only paid for the last 25 rounds. At the beginning of a session, each subject was assigned a role randomly: principal or bidder.¹³ The role of a subject was kept fixed throughout the session. There were eight principals and eight bidders in the lab in each session. At each round a principal was randomly matched with a bidder and formed a team of two subjects. Then two teams were randomly matched to participate in an auction. We made sure that not the same group of people played against each other in two consecutive rounds.¹⁴

In each auction, one fictitious item was offered to two randomly matched teams. All decisions were anonymous. At the conclusion of each auction, the players learned the outcome of the auction. In particular, each subject learned her actual value, her and opponent team's actual bids, whether her team had received the object, the price paid by the winning team, and her own payoff.¹⁵ The anonymity in conjunction with subjects only learning the outcome of their own game in each round was designed to generate a sequence of one-shot games. The screen shots of the experiment were in the instructions (see the Appendix.)

In the beginning of an auction, each principal received a private signal from the uniform distribution from $[0,100]$, independently. They did not know their value for the auctioned item at

¹² In a typical session, the instructions were described for 20-30 minutes while the actual play lasted for about an hour.

¹³ In the experiment, we referred to each principal as Participant A and each agent as Participant B, to avoid any name driven bias.

¹⁴ The anonymity, in conjunction with subjects learning the outcome of only their own match was designed to generate a sequence of one-shot games.

¹⁵ They learned the opponent's payoff when the opponent lost—it must have been zero—but we did not tell them the opponent's payoff when the opponent wins because, in that case, the subjects could determine the actual value of the opponent and his bidding strategy to some extent. Since we used random matching in each round to generate single-shot games, we aimed to minimize the learning about the strategy of the other subjects.

this time but they knew that the value was distributed uniformly on $[0,s]$ when the principal's signal is s . Then the principal was asked to set a budget for her bidder.

After each principal set a budget, each bidder observed her value and the budget set by her principal. The value of a bidder was 2.5 times more than the value of the corresponding principal. This sets δ of Section 2 equal to $2/5$.¹⁶ Therefore, the value of a bidder was from the uniform distribution on $[0, 2.5s]$ when the corresponding principal's signal is s . Then the bidder was asked to enter her bid, which was not allowed to exceed the budget.

After each bidder submitted a bid in behalf of her team, the team with the highest bid won the auction and paid its bid (in the first-price treatment) or the opposing team's bid (in the second-price treatment).

In the control treatments, where we aimed to better understand the principals' behavior, the bidders were computerized. Again we tested first- and second- price auctions. All the specifications such as the distribution of values and signals, number of bidders in an auction, and the auction rules were the same as in the main treatments. In each session, there were 16 principals in the experimental laboratory. The computerized bidders were programmed to play according to the equilibrium unconstrained bid functions as described in Section 2. We provided three tools to the human principals in order to explain to them the bidding strategy of computerized bidders: 1) The graph of the bidding function of the computerized bidder; 2) a table summarizing the bids corresponding to some actual values; and 3) an interactive tool in the software. The graph and the table were given as hard copies, and the interactive tool was a numbered line on each principal's computer screen. The signal received by the principal in a round was pointed to as the max value for the object on the numbered line. The principal could slide a black square between zero and the max value. The computer reported the corresponding unconstrained bid of the principal's computerized bidder every time the principal dropped the black square at a possible actual value on the line. We told the subjects that this tool was being provided to help them understand the bidding strategy of the computerized bidder when it was unconstrained by the principal's budget. An example of the computer screen of a principal with computerized bidder can be seen in the instructions provided in the Appendix.

¹⁶ We set $\delta = 2/5$ in the experiments because for this value of δ , the equilibrium strategies of first price auction are approximately linear.

The role of a subject in the control treatments was to decide on a budget after observing her signal for the round. Once each principal set the budget, the corresponding computerized bidder bid the minimum of the unconstrained bid corresponding to the actual value observed by the computerized bidder and the budget set by the principal.

All the amounts in the experiment were denominated in Experimental Currency Units (ECU). Subjects received \$8 as initial endowment to cover any possible losses in the experiment. The principals were more subject to potential losses since they did not know their values at the time of decision making. No subject lost all of her initial endowment.¹⁷ The final earnings of a subject was the sum of her payoffs in 25 rounds in addition to the initial endowment. The payoffs in the experiment were converted to US dollars at the conversion rate of 20 ECU = \$1 (for the principals) and 80 ECU = \$1 (for the bidders). Our calculations based on equilibrium predicted four times higher payoffs for the principals than the bidders in their variable payoffs. This was because of the difference between the valuations of principals and the bidders for the same auctioned item. Hence we set different conversion rates to make the earnings of subjects playing different roles comparable.¹⁸ By interpreting the sigma in footnote 5 as the conversion rate, one may note that the theory is independent of the conversion rates.¹⁹ Cash payments were made at the conclusion of the experiment in private. The average principal and bidder payments were \$23 and \$25 (including \$7 participation fee).

4. Experimental Results

The analysis presented in this section is based on 500 auctions we conducted per auction format with human bidders and 1000 auctions we conducted per auction format with

¹⁷ Bankruptcy is always a potential problem in auction experiments. We assured our subjects that they will earn positive amounts.

¹⁸ We are confident that using different exchange rates does not alter our findings since our findings in the main treatments and in the control treatments (where the agents are computerized and therefore there is only principals' exchange rate) are qualitatively the same.

¹⁹ An alternative method to balance the earnings of principals and bidders could be to provide them with different endowments. We did not use this method since we wanted to keep the relative weights of the variable and fixed portions of the bidders' expected payoff comparable for different roles.

computerized bidders. While testing differences between treatments, we report Mann-Whitney-Wilcoxon statistics for the session averages assuming that session averages are independent.²⁰

4.1. Budget and Bid Decisions

We start by examining the strategies of principals in treatments with and without computerized bidders. We compare the budgets selected in the first- and second-price auctions with the equilibrium budgets. The most basic prediction of the theory is that the principals choose lower budgets in the first-price auction.

Figure 1 shows the principals' choices of budgets across all sessions in first- and second-price auctions with human bidders. The figure shows box plots of the budgets²¹ for each of ten bins based on the signal observed by the principals. Figure 3 shows the same plot for the budgets submitted in treatments with computerized bidders. Both figures clearly show that, in the experiments, the principals set relatively lower budgets in first-price settings, with and without human bidders.²²

[Figure 3]

Moreover, in SP the mean of budgets is higher with computerized bidders than with human bidders ($p = 0.095$). However, the medians of budgets with and without computerized bidders in SP, either means or medians of budget decisions with and without computerized bidders in FP are not significantly different (all p -values are greater than 0.10).

Figures 1 and 3 show that in the aggregate level budget increases with signal and the relationship is approximately linear. Indeed, many of the principals' decisions in the data can be characterized by linear strategies, and as we discuss in the next paragraph, the equilibrium prescribes that the principals should be using linear strategies in the second-price auction and

²⁰ We also performed t -statistics by using each observation and the results were not qualitatively different in any of the comparisons except for the revenues in SP experiments and SP equilibrium prediction for computerized bidders in Table 7.

²¹ The box plots were created using standard techniques. The white lines represent the median; the box represents the interquartile range (IQR); the whiskers extend to the furthest data point within $1.5 \cdot \text{IQR}$; and the open circles are individual data points outside $1.5 \cdot \text{IQR}$. In Figure 1, 24 out of 28 of the outliers in the second-price auction represent decisions made by one subject.

²² Note that the data from the second price auctions with computerized bidders is noisier than its counterpart with human bidders. With human bidders only 28 of 1000 budget decisions were above 100 and with computerized bidders 179 of 2000 observations were above 100 in Figures 1 and 3, respectively.

approximately linear strategies in the first-price auction. To quantify the fit of linear strategies, we regressed the budget choices on the principal signals and the square of the principal signals for each individual principal. In the first-price auction, we reject the null hypothesis that the quadratic term is significant at the 1% level for 93% of the principals. In the second-price auction, we reject null for 83% of the principals. With computerized bidders in the first-price auction (second-price auction), we reject the null for 91% (84%) of the principals.²³

For the parameters used in the experiment, in SP the equilibrium budget strategy of a principal with signal s is:

$$w_{SP}(s) = b_{SP} \left(\frac{s}{2 - \delta} \right) = \frac{s}{2 - \delta} = \frac{5}{8} s = 0.625s .$$

In FP auction, the equilibrium bidding function specified in Equation (2) and hence the implied budget function of the principals is complicated. However, they are approximately linear for $\delta = 2/5$, the value used in the experiment, on the relevant domain. For linear approximation of the equilibrium budget function, if we regress equilibrium budget decision on signal for the signals used in the experiment, 0.276 is the estimated slope and $R^2 = 0.9999$:

$$w_{FP}(s) = b_{FP} \left(\frac{s}{2 - \delta} \right) \approx 0.276s .$$

We will use this linear approximation of equilibrium to compare it with our estimates for the parameters of the budget function in Table 1. Figure 4 compares the linear estimate (the dashed line) to the theoretical equilibrium budget function (solid line).

[Figure 4]

Table 1 reports regression results for budget decisions of the principals. A random effect model is used in the statistical analysis. Specifically, we assume that the budget set by principal i in round p is:

²³ In a separate analysis, we calculated the R^2 values from regressions of the budget on the principal signals for each individual principal. For principals in the first-price auction, 75% of the principals had R^2 values above 0.79, 50% were above 0.87 and 25% were above 0.93. The corresponding numbers in the second-price auction were 0.87, 0.94 and 0.97. With computerized bidders in the first-price auction (second-price auction), 75% of the principals had R^2 values above 0.72 (0.79), 50% were above 0.86 (0.92), and 25% were above 0.93 (0.96).

$$Budget_{ip} = \beta_0 + \beta_1 \times FP_i + \beta_2 \times Signal_{ip} + \beta_3 \times FP_i \times Signal_{ip} + \gamma_i + \varepsilon_{ip}.$$

In the specification above, γ_i is a random effect term, which is *iid* across i and distributed as $N(0, \sigma_\gamma^2)$, while ε_{ip} is an idiosyncratic error term, which is *iid* over i and p and distributed as $N(0, \sigma_\varepsilon^2)$. (We also assume that γ_i and ε_{jp} are independent for all i, j , and p .) We considered a version of this model that contained session fixed effects, but this did not have any effect on the standard errors.²⁴ FP_i is a dummy variable which is 1 if subject i participated in a first-price auction and zero otherwise. Our equilibrium analysis predicts that the budget set by a principal is: (i) linear in principal's signal for SP with intercept at zero and slope equal to 0.625; (ii) approximately linear for FP with slope equal to 0.276 and intercept at zero. In the model above β_0 and $\beta_0 + \beta_1$ are the constant terms for SP and FP, respectively; β_2 and $\beta_2 + \beta_3$ are the slopes for SP and FP, respectively. We find that in both auctions the constant terms are not significant and only the signals are significant as predicted by the theory. The estimated coefficient of signal is 0.749 in SP and 0.439 (= 0.749 – 0.310) in FP with human bidders. They are 0.932 in SP and 0.404 (= 0.932 – 0.528) in FP with computerized bidders. In all the treatments the regression coefficients suggest that the principals set higher budgets than the equilibrium predictions. We consider this result to be in line with the robust aggressive behavior in first and second-price auction experiments.

[Table 1]

A principal's linear strategy is completely characterized by the budget-to-signal ratio, since it passes through the origin. If we calculate a session average value of the budget-to-signal ratio and use a Mann-Whitney-Wilcoxon test to compare these ratios between the first- and the second-price auctions, we reject the hypothesis that the ratios in the first-price treatment are at least as high as those in the second-price treatment ($p = 0.004$ when we have human bidders and $p = 0.004$ when we have computerized bidders).²⁵ Figures 5a and 5b show the empirical density

²⁴ We also estimated these regressions controlling for the round number by including dummy variables indicating the first 10 rounds of the experiment in each treatment. However, these dummy variables were not significant at the 5% level and did not affect the estimates of interest when they were included, so they are excluded here. Alternatively, we tried OLS model with clustering and the results were qualitatively the same.

²⁵ Note that we continue to reject this hypothesis if we exclude the first session of the second-price treatment. The subject who set the outlier budget levels in Figure 1 participated in that session.

estimates of the budget ratios for human and computerized bidders, respectively. In addition to showing the raw data (dashed curve), we show the densities of the average budget ratios for each subject (solid curve). In both figures, we have excluded data where the average ratio exceeds 2 to emphasize the range where most of the data is concentrated. In Figure 5a, 0.1% of the raw first-price data (mean of 6.11) and 2.3% of the raw second-price data (mean of 46.22) is not shown. In Figure 5b, 0.1% of the first-price data (mean of 4.79) and 9.7% of the second-price data (mean of 4.77) is not shown. The larger number of outliers in this last case is evident in Figure 3 above.²⁶ The vertical lines mark the equilibrium predictions (recall that this is approximate in the first-price case and Figures 5a,b show the vertical line at 0.276 for the FP equilibrium).²⁷

[Figure 5]

The theory predicts that the principals set the same cutoff value for the bidders in both auctions. This means that a principal who observed signal s will set the budget so that the set of types of bidders for whom the constraint binds are the same. More precisely, we calculate that cutoff type as $\frac{s}{2-\delta} = 0.625s$. This result of course depends on the bidders using equilibrium strategies for their bid functions. In the computerized bidder treatment, we are able to invert the bid functions used by the bidders to infer the principals' choice of cutoff type. Inferring the cutoff type of the human bidders would involve making assumptions about the unconstrained behavior of the human bidders. Therefore, in the following analysis of the cutoff type, we only consider the computerized bidder treatment.

Given the inferred cutoff type choice in the computerized bidder treatment, we use the following model for the statistical analysis of the cutoff type:

$$\hat{t}_{ip} = \beta_0 + \beta_1 \times FP_i + \beta_2 \times Signal_{ip} + \beta_3 \times FP_i \times Signal_{ip} + \vartheta_i + u_{ip} .$$

²⁶ The larger fraction of outliers evident in the second-price treatments might be the result of the noisier feedback from the second-price design. The negative consequence of setting a high budget in either treatment is that one might have to (possibly) pay too high of a price for the item. In the first-price auction, the realization of this consequence requires that one's bidder also place a high bid, but in the second-price auction one's bidder must place a high bid and one's opponent must have a high budget and place a high bid which is rare.

²⁷ Instead of the vertical line at the mean value, if we insert the density of equilibrium budget/signal for FP in these Figures, we need to draw a density function with very small variance such that it concentrates around its mean (0.276) and its peak is too high to include in these Figures. That's why we present just the vertical line passing through the mean of equilibrium budget/signal realizations here.

ϑ_i is a random effect error term, which is *iid* across i and distributed as $N(0, \sigma_\vartheta^2)$, while u_{ip} is an idiosyncratic error term, which is *iid* (over i and p) and distributed as $N(0, \sigma_u^2)$. (We assume that ϑ_i and u_{jp} are independent for all i, j , and p .) We considered a version of the above model that included session fixed effects but again this did not affect the standard errors. The results from the regression are reported in Table 2.²⁸

[Table 2]

The theory predicts that in both first-price and second-price auctions, the principals set the same cutoff type which is proportional to their signals. This means that the theory predicts zero for β_0 , β_1 , and β_3 . Note that each of these coefficients (in the first, second and fourth rows) is insignificant in Table 2. The coefficient of the signal in both formats is predicted to be 0.625 by the theory but the estimated β_2 is 0.929. Therefore, the principals in FP and SP auctions do not constrain significantly different types of bidders when the bidders are computerized although they constrain a smaller set of bidder types than what the equilibrium predicts.

Because of the sequential nature of our experiments, we observe the bidders' unconstrained bidding strategy only when they submit a bid less than the budget. So we do not know what they would have bid if the budget allowed. Tables 3 and 4 report the number of observations where the submitted bid in the auction is constrained or unconstrained by the budget in the FP and SP auction experiments, respectively. This is done in comparison with the equilibrium prediction for those observed budgets. Note that if the equilibrium bid strategy was used, only 298 of the 1000 bids would be unconstrained in SP and that number is 383 out of 1000 in FP. More than 82% of the time the human bidders bid the budget when this is the behavior predicted by the equilibrium in SP and this happened in 71% of the cases in FP. In SP auctions, we observe the unconstrained bid of the human bidders in 86% of the cases where the equilibrium unconstrained bid was less than the budget and in 18% of the cases where the equilibrium bid was the budget. In FP auctions, the corresponding percentages were 83% and 29%.

²⁸ These regressions were also performed with controls for potential round effects. When a dummy for the first 10 rounds is included in the regressions, the coefficient on this dummy is positive and significant for the first-price treatment (but insignificant for the second-price treatment) suggesting that cutoff types were lower on average in the later rounds of the first-price treatment; however, including these controls affected neither the values of the β_2 or β_3 (in both cases the estimates changed by less than 0.001) nor the conclusions of the statistical tests reported so they are omitted from the discussion.

[Table 3]

[Table 4]

One way to evaluate the behavior of the bidders is to do the following test. First, the theory predicts that each bidder's value pins down their choice of unconstrained bid. So we can calculate each bidder's predicted choice conditional on the budget given to them in the experiment as the minimum of the predicted unconstrained bid and the observed budget. Figure 6 presents the distributions of these predicted bids (the light colored curves) and also the distributions of actually submitted bids (the dark colored curves) for each format. When we compare these distributions using a Kolmogorov-Smirnov test, the test does not reject the null hypothesis that the distributions are the same at the 5% level in either auction (p-values are 0.969 for both auctions).

[Figure 6]

Figure 6 suggests that in human bidder treatments, if we used the computerized bidders instead, the submitted bids would not change. Recall that budgets in FP with and without computerized bidders were not very different, either. In SP, principals set slightly higher budgets to computerized bidders than human bidders on average. Since Figure 6 indicates that the outcomes of the auctions with human bidders are the same as how they would be if computerized bidders were used instead, principals seem to have trusted their human bidders less in SP. We should keep in mind that the principals do not know the human bidders' unconstrained bids without experimenting with high budgets but they know the strategy of computerized bidders. As we will see in the revenue analysis later, higher budgets in SP auctions with computerized bidders do not lead to higher revenue than the corresponding human bidder treatment. This is because even though the budgets are high, the bidding behavior is limited by what the computer does as a bidder. Moreover, if there are only a few high-budget principals in a session, there is relatively little effect on revenues, as a high-budget person needs to be matched with another high-budget person in order for their budget to matter.

4.2. Efficiency and Revenue

In this section, we compare measures of efficiency and revenues arising in the experiments. As we have already seen, principals are observed to constrain approximately the same sets of

bidder types in each format in the experiments. Consequently, the level of efficiency of the first-price format is found to be insignificantly different from the level of efficiency of the second-price format. However, the phenomenon of overbidding (relative to risk-neutral equilibrium) that is typically observed in auction experiments persists in ours. As a result, the first-price auction is found to raise higher revenues than the second-price auction—although, as we shall see, for a different reason than is told in the literature on auctions with budget constraints.

Tables 5 and 6 summarize our efficiency findings for the human and computerized bidder cases, respectively, using two different measures of efficiency. The first rows report the percentages of auctions where the winning principal has the higher valuation. The second rows report the average surplus that is realized. This measure is defined as the winning principal's value divided by the highest value of the two principals, telling us the proportion of the available surplus that is realized in the auction experiments.

[Table 5]

Using the Mann-Whitney-Wilcoxon (MWW) test and a significance level of 5%, the average rate of efficient allocations is not significantly different between the first- and second-price ($p = 0.205$), or the first-price and equilibrium ($p = 0.396$), or the second-price and equilibrium ($p = 0.057$). Using MWW and a significance level of 5%, the average realized surplus is not significantly different between the first- and second-price ($p = 0.151$) or the first-price and equilibrium ($p = 0.222$), but it is significantly different between the second-price and equilibrium ($p = 0.008$).

The results on the efficiency of the allocations in the treatments with computerized bidders are presented in Table 6. There is no significant difference between the first-price and second-price with respect to either measure and none of them are significantly different from the equilibrium prediction (all the p -values are greater than 0.346). Moreover, all of the numbers in the last row (realized surplus) of Table 6 are strikingly close to one another.

[Table 6]

As for revenues, recall that the theory predicts that the principals choose to constrain the same sets of types in both auction formats. Revenue equivalence, however, is sensitive to the

particular sets of types that the principals constrain. In our treatments with computerized bidders, the principals constrained essentially the same types in first-price and second-price auctions, but constrained fewer types than the theory predicts in each (see Table 2). Moreover, as argued earlier, the principals' behavior in the experiment is close to linear. The proposition below shows that the first-price auction can be expected to raise higher revenues if the principals' deviation from the equilibrium has these properties while the bidders follow the equilibrium unconstrained bid function (which is how we programmed the computerized bidders).

Proposition 1. Suppose that the principals choose cutoff types according to linear strategies that constrain fewer types than the equilibrium (i.e. $\hat{t}(s) = \alpha s$ with $\alpha > 0.625$). Then the first-price auction raises more revenue than the second-price auction with computerized bidders which follow the equilibrium unconstrained bid function.

The seller revenues generated in four treatments as well as the equilibrium predictions based on the *ex post* draws are shown in Table 7. Aggregating average revenue to the session level, we performed Mann-Whitney-Wilcoxon tests of whether the session averages came from distributions with the same median. In line with Proposition 1, in the treatments with computerized bidders the test rejects the hypothesis between the first- and second-price auctions ($p = 0.032$) and between the first-price auction and equilibrium ($p = 0.008$). The test did not reject at the 5% level between the second-price auction and equilibrium ($p = 0.056$).

Table 7 reports the revenue results in treatments with human bidders as well. Although Proposition 1 addresses only the situation where the bidders follow the equilibrium strategies, we find similar results in the treatments with human bidders. In particular, we still find significantly different revenues in the first- and second-price auctions ($p = 0.008$). The revenue difference is significant between the first-price auction and equilibrium ($p = 0.008$) as well. The test did not reject at the 5% significance level that the session averages of the second-price auction came from a distribution with the same median as the equilibrium ($p = 0.095$).²⁹

[Table 7]

²⁹ Revenues were not significantly different between the treatments with human bidders and those with computerized bidders for both the first-price auction ($p = 0.690$) and the second-price auction ($p = 0.690$).

The tables above indicate that whether we have computerized bidders or human bidders does not alter the relative performance of the formats, qualitatively, in terms of efficiency and revenues.

5. Conclusion

Prior to Vickrey (1961), it might well have been commonplace for observers to wonder, “Why would a seller ever want to use a second-price, rather than a first-price auction? Given any set of bids, the seller would always do better to charge the winner the highest bid.” The obvious oversight is in not taking into account that bidding behavior is affected by a change in the rules.

Perhaps the literature on budget constraints makes a more subtle (but still analogous) oversight. When budget constraints are present, they can be presumed to exist for a reason; quite likely, they arise from a principal-agent problem. When a principal imposes a budget on an agent, the principal should determine a budget appropriate for the auction format. Just like the bidding behavior, the principal’s actions to restrain the bidding behavior should also be affected by a change in the rules.

The introduction of endogenous budgets may be particularly helpful in explaining two of the main predictive failures of the literature on auctions with budget constraints. First, as already noted in the Introduction, ascending (effectively second-price) formats are commonly used in high-stakes settings such as spectrum auctions. The use of ascending auctions would be difficult to reconcile with the prevalence of budget constraints, unless the budgets were determined endogenously, as is argued here. Second, recall that the literature on budget constraints not only predicts the second-price auction to be outperformed by the first-price auction, but it predicts the revenues of both formats to be exceeded by the all-pay auction (Che and Gale, 1996). Nevertheless, while the all-pay auction is a good model for phenomena such as political lobbying and patent races, the all-pay format is hardly ever used for conventional sales of valuable assets. Perhaps sellers recognize that principals would neutralize any advantages of the all-pay auction by imposing even more stringent budget constraints on their bidders.

In our experimental results, we found clear evidence that principals set demonstrably lower budgets for bidders when the format will be a first-price, rather than a second-price auction. This

result holds true robustly, whether the principal seeks to constrain human bidders or computerized bidders.

In the theoretical model we considered, the principals choose to constrain precisely the same sets of bidder types in each auction format, implying that the same allocation will arise in equilibrium under each auction format. As such, the budget constraint is no more likely to be binding in the second-price auction than in the first-price auction. Thus, the endogenous choice of budget “neutralizes” two of the key predictions of the existing literature on auctions with budget constraints. First, realized efficiency is no longer expected to be greater in the first-price than in the second-price auction. Second, revenues are no longer expected to be higher in the first-price than in the second-price auction.

The efficiency prediction was broadly confirmed in our experiments. The experimental principals chose to constrain approximately the same sets of bidder types in each auction format and, as a result, we were unable to reject that the measures of realized efficiency were equal. This finding is especially important in that, in many auction environments where budget constraints are likely to be most important (e.g., in spectrum auctions), the seller’s stated objective is efficiency, not revenue maximization.

Meanwhile, the prediction of revenue equivalence was rejected in our experiments: the first-price auction generated significantly higher revenues than the second-price auction. However, this came about for a different reason than in the literature with exogenous budget constraints. In the existing literature, the higher revenues resulted from budgets that were the same, for different auction formats. In our experiments with endogenous budget constraints, the higher revenues resulted from the principals setting higher budgets than in the equilibrium solutions, both with second-price and first-price formats. Bids higher than the risk-neutral Nash equilibrium are typically observed in laboratory experiments—see footnote 2. The selection of a budget limit is effectively the principal’s submission of a bid in the auction. Thus, the setting of budget limits higher than the equilibrium solutions is, in effect, another version of the overbidding phenomenon that is pervasive throughout experimental auctions.

Our experimental design yielded sharp conclusions as to the endogeneity of budget constraints in auctions: if budget limits are allowed to be chosen, then higher limits will be chosen in second-price than in first-price formats. Our experimental results also generally

supported the notion that budget constraints need not alter the efficiency rankings of different auction formats. However, our experimental design did not explore the sources of overbidding and it is not well suited for identifying what motivated the setting of higher-than-equilibrium budgets. This is an interesting question for future research.

Appendix

DERIVATION OF EQUILIBRIUM

Let t represent the valuation of the bidder, δt represent the valuation of the principal, and s represent the signal received by the principal. The distributions of these random variables in the experiment are:

$$s \sim U[0,100];$$

$$t \sim U\left[0, \frac{s}{\delta}\right]; \text{ and}$$

$$\delta t \sim U[0, s].$$

The relevant density functions are:

$$f(s) = \frac{1}{100}, \quad 0 \leq s \leq 100; \text{ and}$$

$$f(t|s) = \frac{\delta}{s}, \quad 0 \leq t \leq \frac{s}{\delta}.$$

Second-Price Auction

Let $b_{SP}(t)$ be the bidder's unconstrained choice of bid in the second-price auction. It is a weakly dominant strategy in this environment for the bidder to set $b_{SP}(t) = t$.

Let $w_{SP}(t)$ be the budget set by the principal. Since the bidder's unconstrained choice is the identity functions, the budget set by the principal is equivalently represented in terms of t as $\hat{t}_{SP}(s) \equiv w_{SP}(s)$. Also let the density of the opposing bids be given by $g_{SP}(x)$. The principal's objective is now:

$$\int_0^{\hat{t}_{SP}} \int_0^t (\delta t - x) g_{SP}(x) f(t|s) dx dt + \int_{\hat{t}_{SP}}^{\frac{s}{\delta}} \int_0^{\hat{t}_{SP}} (\delta t - x) g_{SP}(x) f(t|s) dx dt.$$

The first term represents the payoff in the event that the budget constraint does not bind, and the second is the payoff when the constraint does bind. The first order condition is:

$$g_{SP}(\hat{t}_{SP}) \int_{\hat{t}_{SP}}^{s/\delta} (\delta t - \hat{t}_{SP}) f(t | s) dt = 0$$

$$\int_{\hat{t}_{SP}}^{s/\delta} (\delta t - \hat{t}_{SP}) \frac{\delta}{s} dt = 0$$

$$\frac{s}{s - \delta \hat{t}_{SP}} \int_{\hat{t}_{SP}}^{s/\delta} (\delta t - \hat{t}_{SP}) \frac{\delta}{s} dt = 0$$

$$E(\delta t - \hat{t}_{SP} | t > \hat{t}_{SP}, s) = 0$$

$$\frac{s + \delta \hat{t}_{SP}}{2} - \hat{t}_{SP} = 0$$

$$\hat{t}_{SP} = \frac{s}{2 - \delta}.$$

For the second line, note that $g_{SP}(\hat{t}_{SP})$ is a constant. Note that this implies that the principal's optimal choice of budget does not depend on the distribution of opposing bids. In the third line, we are dividing by $P(t > \hat{t}_{SP} | s)$ to make the left side a conditional expectation. The final line is the principal's equilibrium choice of \hat{t}_{SP} , which in the second-price auction is also the equilibrium choice of budget constraint. To verify that this choice of \hat{t}_{SP} maximizes the principal's objective, notice that the sign of the principal's first order condition is negative for $\hat{t} > \hat{t}_{SP}$ and positive for $\hat{t} < \hat{t}_{SP}$. The constrained bid submitted by the bidder can be written as:

$$B(s, t) = \min \left\{ t, \frac{s}{2 - \delta} \right\},$$

which depends on both the bidder's and the principal's information.

First-Price Auction

Suppose that a type t bidder in a first-price auction wins with probability $G_{FP}(t)$ (assume for the moment that $G_{FP}(t)$ is differentiable and increasing and let $g_{FP}(t)$ be the corresponding density function), then a standard analysis concludes that the optimal choice of bid is given by

$$b_{FP}(t) = \frac{1}{G_{FP}(t)} \int_0^t x g_{FP}(x) dx.$$

This is the bidder's unconstrained choice of bid. To analyze the constrained bidder's choice, observe that $b_{FP}(t)$ is nondecreasing and continuous, so a budget constraint $w_{FP}(s)$ can be equivalently represented as a cutoff type, $\hat{t}_{FP}(s)$, defined by $w_{FP}(s) = b_{FP}(\hat{t}_{FP}(s))$. Write the bidder's objective in terms of a choice of type t' as

$$\pi_B(t, t') = G_{FP}(t')(t - b_{FP}(t')) = G_{FP}(t')(t - t') + \int_0^{t'} G_{FP}(x) dx,$$

where the equality follows after integrating by parts. The derivative of this expression with respect to t' is negative for $t' > t$ and positive for $t' < t$. It follows that a bidder who is restricted to setting $t' \leq \hat{t}_{FP}$ optimally sets $t' = t$ if $t \leq \hat{t}_{FP}$ and $t' = \hat{t}_{FP}(s)$, otherwise.

The bidder's behavior allows us to write the principal's objective in terms of the choice of \hat{t}_{FP} as

$$\int_0^{\hat{t}_{FP}} G_{FP}(t)(\delta t - b_{FP}(t))f(t | s)dt + \int_{\hat{t}_{FP}}^{s/\delta} G_{FP}(\hat{t}_{FP})(\delta t - b_{FP}(\hat{t}_{FP})) f(t | s) dt .$$

Plugging in for $b_{FP}(t)$ this becomes:

$$\begin{aligned} & \int_0^{\hat{t}_{FP}} \left(G_{FP}(t)\delta t - \int_0^t x g_{FP}(x) dx \right) f(t | s) dt \\ & + \int_{\hat{t}_{FP}}^{s/\delta} \left(G_{FP}(\hat{t}_{FP})\delta t - \int_0^{\hat{t}_{FP}} x g_{FP}(x) dx \right) f(t | s) dt . \end{aligned}$$

As with the second price auction, the solution for \hat{t}_{FP} does not depend on $G_{FP}(t)$ and is actually the same as \hat{t}_{SP} :

$$\hat{t}_{FP} = \frac{s}{2 - \delta} .$$

Summarizing, if the probability of a type t bidder winning is $G_{FP}(t)$, then the bidder optimally bids $b_{FP}\left(\min\left\{t, \frac{s}{2-\delta}\right\}\right)$. In a symmetric equilibrium, the bid functions are the same for each

bidder and the winner is the bidder with the higher value of $\min\left\{t, \frac{s}{2-\delta}\right\}$. The distribution of $\min\left\{t, \frac{s}{2-\delta}\right\}$ determines $G_{FP}(t)$. We find

$$G_{FP}(t) = \left[2 - \delta - \delta \ln\left(\left(\frac{2-\delta}{100}\right)t\right)\right] \frac{t}{100}.$$

This function is differentiable and increasing as was originally assumed. This makes the unconstrained bid function

$$b_{FP}(t) = \left[\frac{4 - 3\delta - 2\delta \ln\left(\frac{2-\delta}{100}t\right)}{4 - 2\delta - 2\delta \ln\left(\frac{2-\delta}{100}t\right)}\right] \frac{t}{2}.$$

Proposition 1.A: The equilibrium in the first-price auction with budget constraints is the unique equilibrium with increasing and differentiable budget functions.

PROOF OF PROPOSITION 1.A

Suppose that there exists another (possibly asymmetric) equilibrium in this model with the property that the budget functions ($w_i(s_i)$) are increasing and differentiable in the principals' signals. Each pair of budget functions leads to a unique equilibrium being played between the bidders. This follows from two results in Maskin and Riley (2003). First, their Lemma 2 shows that bidders best response functions are nondecreasing in the range of potentially winning bids, which implies that budget functions can be equivalently thought of in terms of cutoff types ($\hat{t}_i(s_i)$). We can then think of the auction as occurring between two bidders with types $\min\{t_i, \hat{t}_i(s_i)\}$, $i = 1, 2$. Proposition 1 in Maskin and Riley (2003) then implies that the equilibrium of this auction is unique. So if the principals both used the budgets prescribed in the equilibrium derived in this paper there is only one bidding equilibrium in the auction game.

Therefore, if there exists another equilibrium with increasing, differentiable budget functions, both budget functions must differ from the one derived above. Suppose that in such an equilibrium a type t bidder's probability of winning is $G_{FP}^*(t)$. The argument in the previous section implies that in this case the principal's optimal choice of cutoff type is $\frac{s}{2-\delta}$, which is a

contradiction. The critical observation is that the principal's optimal choice of cutoff type does not depend on the equilibrium being played in the subsequent auction.

PROOF OF PROPOSITION 1

Suppose the principals choose a cutoff type strategy according to $\hat{t}_\alpha(s) = \alpha s$, where in equilibrium $\alpha = \alpha^* = 5/8$ in both auction formats, and let the distribution of $\tilde{t}_\alpha \equiv \min\{t, \alpha s\}$ be given by $G_\alpha(x)$ with $G_\alpha^{(i)}(x)$ being the distribution of the i^{th} order statistic. Finally, denote the equilibrium unconstrained bid function derived above by $b_{\alpha^*}(x) = \frac{1}{G_{\alpha^*}(x)} \int_0^x y dG_{\alpha^*}(y)$. Then the experimental expected revenue in the first-price auction ($E[R_\alpha^{FP}]$) and the second-price auction ($E[R_\alpha^{SP}]$) with principals following a strategy $\hat{t}_\alpha(s)$ and the computerized bidders following the corresponding equilibrium unconstrained bid functions can be written as:

$$E[R_\alpha^{FP}] = \int_0^{100\alpha} b_{\alpha^*}(x) dG_\alpha^{(1)}(x) \quad \text{and} \quad E[R_\alpha^{SP}] = \int_0^{100\alpha} x dG_\alpha^{(2)}(x) .$$

In the formulas above, note that the integral limits are determined by the strategy of the principal, i.e. $\hat{t}_\alpha(s) = \alpha s$ and the integrands are determined by the bid functions of the computerized bidders who follow the equilibrium unconstrained bid functions, i.e. $b_{\alpha^*}(x)$ in FP and value bidding in SP. As the theory shows, with $\alpha = \alpha^*$ the two expressions are equal. Also, using a standard revenue equivalence argument we have:

$$E[R_\alpha^{SP}] = \int_0^{100\alpha} b_\alpha(x) dG_\alpha^{(1)}(x) ,$$

where $b_\alpha(x)$ is defined analogously to $b_{\alpha^*}(x)$. So the first-price auction raises more revenue when $\alpha > \alpha^*$ if and only if:

$$\begin{aligned} \int_0^{100\alpha} b_{\alpha^*}(x) dG_\alpha^{(1)}(x) &> \int_0^{100\alpha} b_\alpha(x) dG_\alpha^{(1)}(x) \\ \Leftrightarrow \int_0^{100\alpha} (b_{\alpha^*}(x) - b_\alpha(x)) dG_\alpha^{(1)}(x) &> 0 . \end{aligned}$$

In fact, for $\alpha > \alpha^*$ and all $x > 0$, $b_{\alpha^*}(x) > b_\alpha(x)$, so that the above expression holds. We calculate:

$$b_\alpha(x) = \left(1 - \frac{2\alpha - 1}{2 - 2(2\alpha - 1) \ln\left(\frac{x}{100\alpha}\right)}\right) \frac{x}{2}.$$

So:

$$b_{\alpha^*}(x) = \left(1 - \frac{1}{8 - 2 \ln\left(\frac{2x}{125}\right)}\right) \frac{x}{2}.$$

Therefore:

$$\begin{aligned} b_{\alpha^*}(x) - b_\alpha(x) &> 0 \\ \Leftrightarrow \frac{2\alpha - 1}{2 - 2(2\alpha - 1) \ln\left(\frac{x}{100\alpha}\right)} &> \frac{1}{8 - 2 \ln\left(\frac{2x}{125}\right)} \\ \Leftrightarrow 16\alpha - 10 &> 2(2\alpha - 1) \ln\left(\frac{8\alpha}{5}\right), \end{aligned}$$

which holds for $\alpha \in \left(\frac{5}{8}, \frac{5}{2}\right]$. Note that for $\alpha > \frac{5}{2}$ the bidder is never constrained.

References

- Bulow, Jeremy, Jonathan Levin, and Paul Milgrom (2009), “Winning Play in Spectrum Auctions,” NBER Working Paper No. 14765.
- Burkett, Justin (2011), “Endogenous Budget Constraints in Auctions,” working paper, University of Maryland.
- Che, Yeon-Koo and Ian Gale (1996), “Expected Revenue of All-Pay Auctions and First-Price Sealed-Bid Auctions with Budget Constraints,” *Economics Letters*, 50(3), pp. 373-379.
- Che, Yeon-Koo and Ian Gale (1998), “Standard Auctions with Financially Constrained Bidders,” *Review of Economic Studies*, 65(1), pp. 1-21.
- Cox, James C., Bruce Roberson, and Vernon L. Smith (1982), “Theory and Behavior of Single Object Auctions,” in Vernon L. Smith, ed., *Research in Experimental Economics*, Vol. 2. Greenwich, CT: JAI Press, pp. 1–43.
- Cox, James C., Vernon L. Smith, and James M. Walker (1988), “Theory and Individual Behavior of First-Price Auctions,” *Journal of Risk and Uncertainty*, 1(1), pp. 61–99.
- Cramton, Peter (1995), “Money Out of Thin Air: The Nationwide Narrowband PCS Auction,” *Journal of Economics and Management Strategy*, 4(2), pp. 267–343.
- Cramton, Peter, Emel Filiz-Ozbay, Erkut Y. Ozbay, and Pacharasut Sujarittanonta (2012a), “Discrete Clock Auctions: an Experimental Study,” *Experimental Economics*, 15(2), pp. 309-322.
- Cramton, Peter, Emel Filiz-Ozbay, Erkut Y. Ozbay, and Pacharasut Sujarittanonta (2012b), “Fear of Losing in a Clock Auction,” *Review of Economic Design*, 16(2), pp. 119-134.
- Crawford, Vincent and Nagore Iriberry (2007), “Level-k Auctions: Can a Nonequilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private Value Auctions?” *Econometrica*, 75(6), pp. 1721–1770.
- Delgado, Mauricio R., Andrew Schotter, Erkut Y. Ozbay and Elizabeth A. Phelps (2008), “Understanding Overbidding: Using the Neural Circuitry of Reward to Design Economic Auctions,” *Science*, 321, pp. 1849–1852.
- Filiz-Ozbay, Emel and Erkut Y. Ozbay (2007), “Auctions with Anticipated Regret: Theory and Experiment,” *American Economic Review*, 97(4), pp. 1407–1418.
- Fischbacher, Urs (2007), “z-Tree: Zurich Toolbox for Ready-made Economic Experiments,” *Experimental Economics* 10(2), pp. 171–178.

Goeree, Jacob K., Charles A. Holt, and Thomas R. Palfrey (2002), “Quantal Response Equilibrium and Overbidding in Private Value Auctions,” *Journal of Economic Theory*, 104, pp. 247–272.

Kagel, John H. (1995), “Auctions: A Survey of Experimental Research,” in John H. Kagel and Alvin E. Roth, eds., *The Handbook of Experimental Economics*. Princeton: Princeton University Press, pp. 501-585.

Maskin, Eric and John Riley (2003), “Uniqueness of Equilibrium in Sealed High-Bid Auctions,” *Games and Economic Behavior*, 45(2), pp. 395-409.

Pitchic, Carolyn and Andrew Schotter (1998), “Perfect Equilibria in Budget Constrained Sequential Auctions: An Experimental Study,” *RAND Journal of Economics*, 19(3), pp. 363-388.

Vickrey, William (1961), “Counterspeculation, Auctions, and Competitive Sealed Tenders,” *Journal of Finance*, 16(1), pp. 8–37.

TABLES AND FIGURES

Table 1. Random Effects Estimates of the Budget
Standard errors are in parenthesis

Variable	With Human Bidders	With Computerized Bidders
Constant	3.215 (3.888)	0.329 (2.135)
FP	-3.673 (5.499)	0.187 (3.019)
Signal	0.749*** (0.014)	0.932*** (0.013)
FP*Signal	-0.310*** (0.019)	-0.528*** (0.018)
N	2,000	4,000

* Statistically significant at the 5% level. ** Statistically significant at 1% level. *** Statistically significant at the 0.1% level.

Table 2. Random Effects Estimates of the Cutoff Function
Standard errors in parentheses

Variable	With Computerized Bidders
Constant	0.400 (2.427)
FP	-0.269 (3.433)
Signal	0.929*** (0.157)
FP*Signal	0.017 (0.022)
N	4,000

* Statistically significant at the 5% level. ** Statistically significant at 1% level. *** Statistically significant at the 0.1% level.

Table 3. Bids in SP auctions with human bidders

	Actual Bid < Actual Budget	Actual Bid = Actual Budget	# of Observations
Eqm. Unconstrained Bid<Actual Budget	257 (86%)	41 (14%)	298
Eqm. Unconstrained Bid≥Actual Budget	125 (18%)	577 (82%)	702

Table 4. Bids in FP auctions with human bidders

	Actual Bid < Actual Budget	Actual Bid = Actual Budget	# of Observations
Eqm. Unconstrained Bid<Actual Budget	317 (83%)	66 (17%)	383
Eqm. Unconstrained Bid≥Actual Budget	180 (29%)	437 (71%)	617

Table 5. Efficiency in the treatments with human bidders

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Rate of efficient allocations	0.850 (0.021)	0.874 (0.010)	0.804 (0.023)	0.874 (0.010)
Realized surplus	0.946 (0.008)	0.961 (0.003)	0.923 (0.012)	0.961 (0.003)

Standard errors of session means are in parentheses.

Table 6. Efficiency in the control treatments

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Rate of efficient allocations	0.863 (0.017)	0.852 (0.007)	0.855 (0.009)	0.852 (0.007)
Realized surplus	0.952 (0.008)	0.955 (0.002)	0.950 (0.003)	0.955 (0.002)

Standard errors of session means are in parentheses.

Table 7. Seller Revenue

	First-Price	First-Price Equilibrium	Second-Price	Second-Price Equilibrium
Average Revenue with Human Bidders	24.332 (0.658)	17.043 (0.173)	18.061 (0.402)	16.956 (0.363)
Average Revenue with Computerized Bidders	23.15 (1.064)	16.653 (0.194)	18.81 (0.875)	16.648 (0.192)

Standard errors of session means are in parentheses

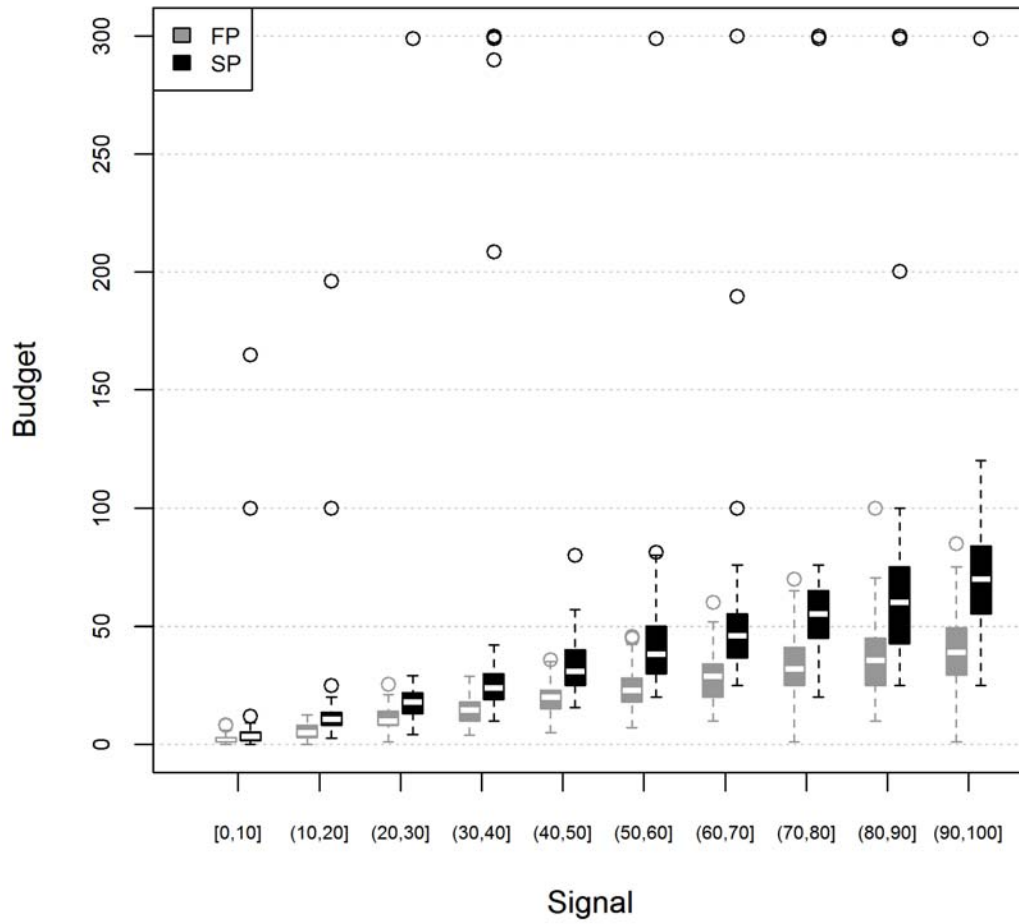


Figure 1. Budgets in First- and Second-Price Auctions with Human Bidders.

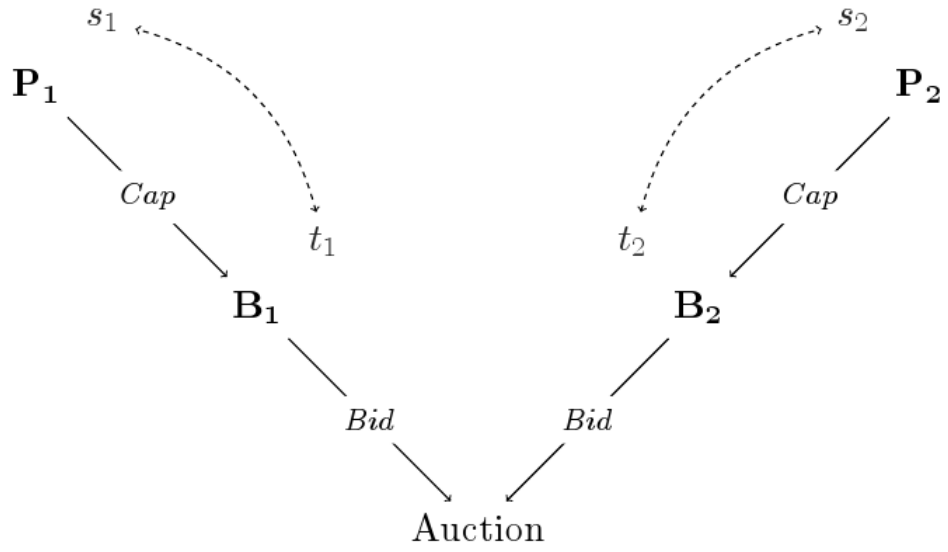


Figure 2. Timing of the Principal-Bidder Game

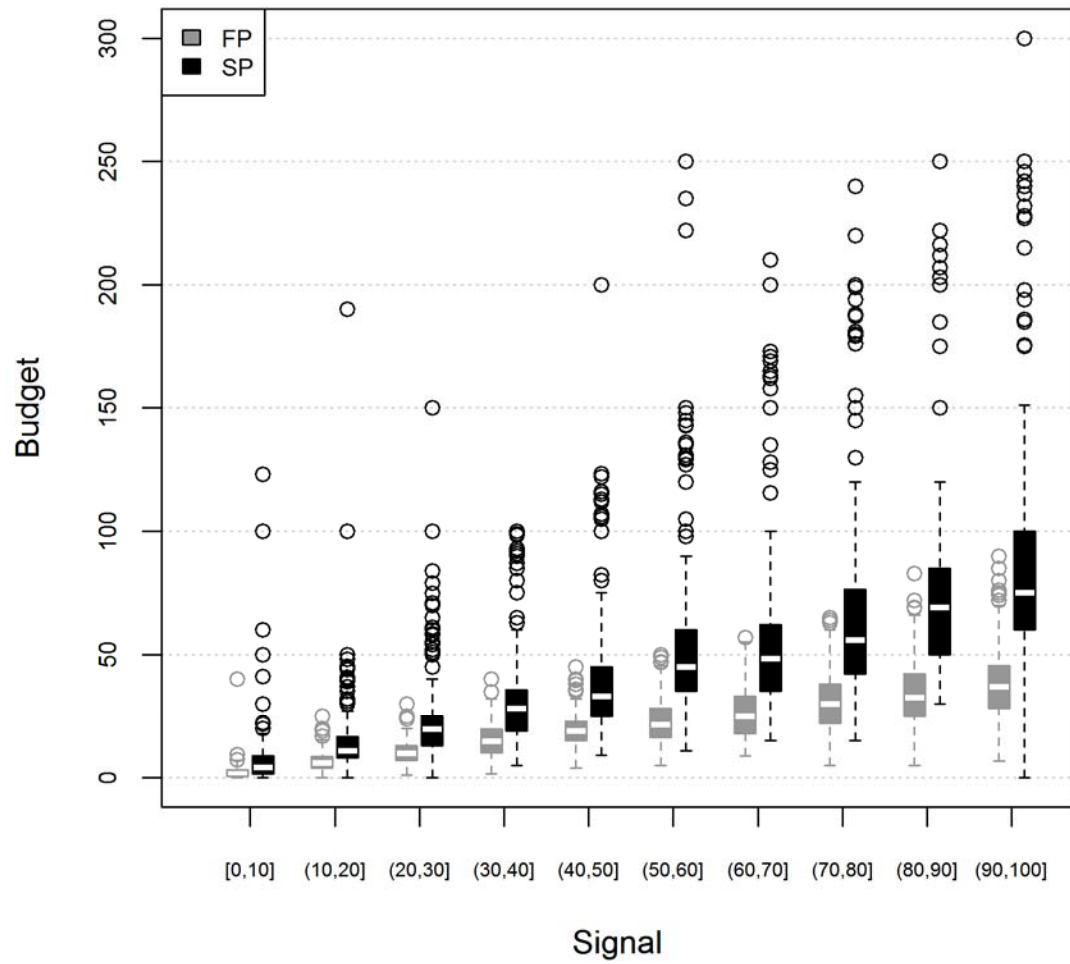


Figure 3. Budgets in First- and Second-Price Auctions with Computerized Bidders.

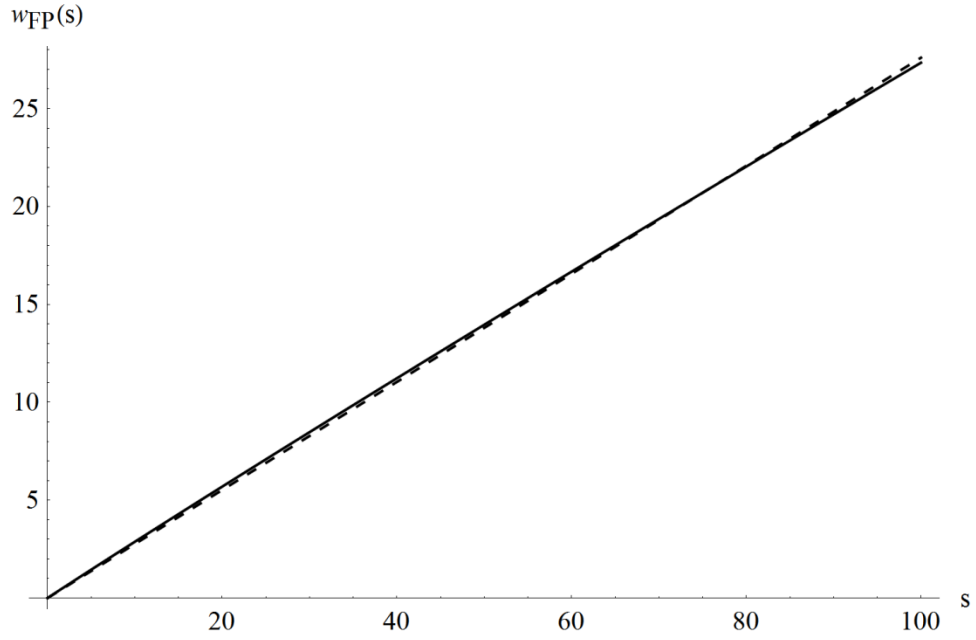


Figure 4: First-Price Budget Function and Linear Approximation ($\delta = 2/5$)

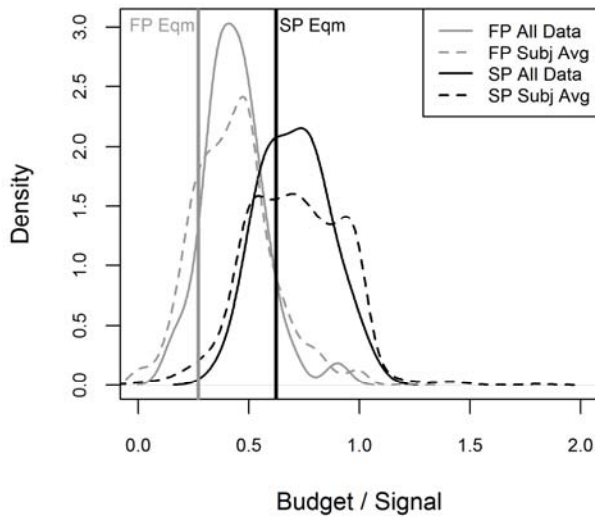


Figure 5a. Density of (budget/signal) in experiments with Human Bidders

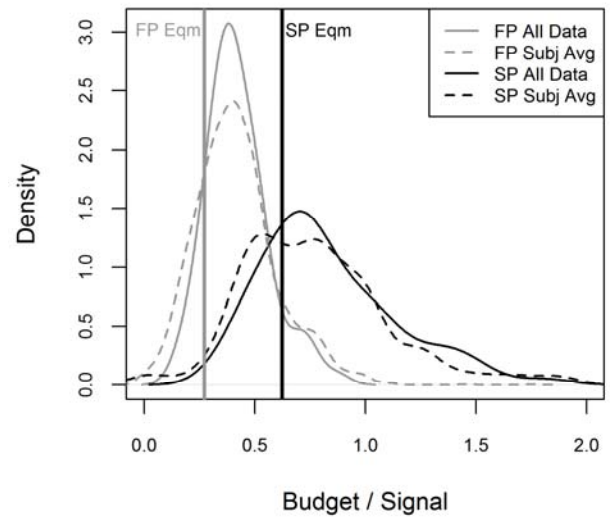


Figure 5b. Density of (budget/signal) in experiments with Computerized Bidders

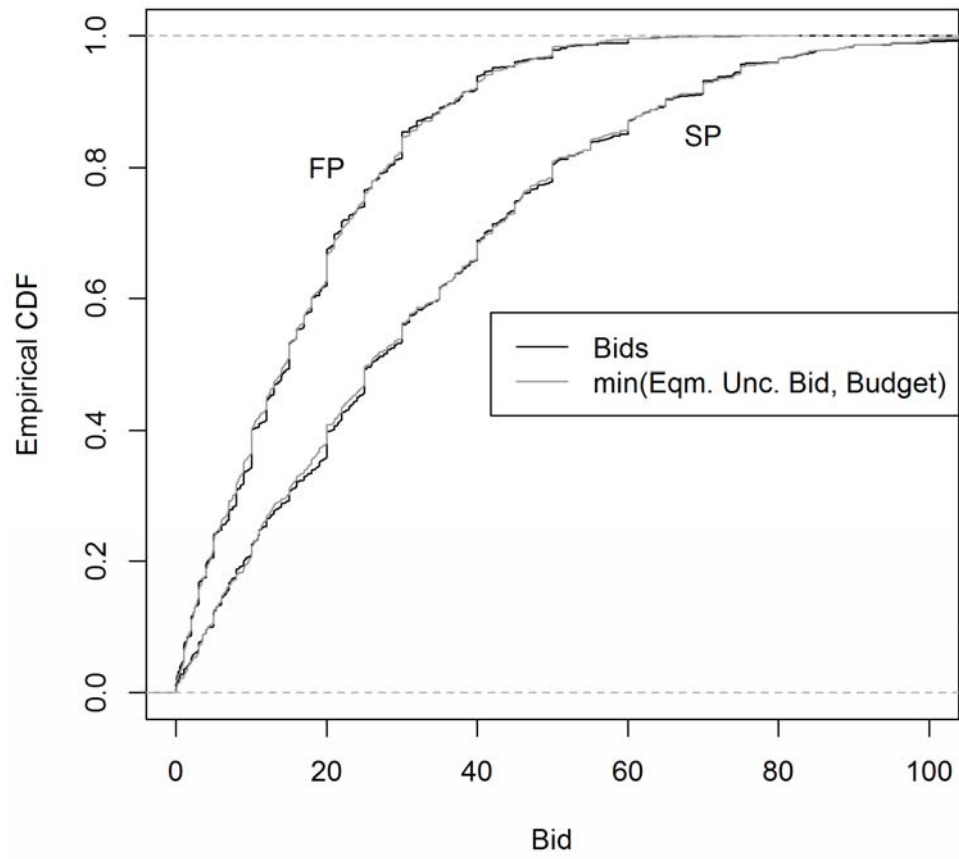


Figure 6. Empirical CDFs of Bids

INSTRUCTIONS FOR FIRST-PRICE SEALED-BID AUCTIONS

(Online appendix: not intended for publication)

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US\$.

- **In this experiment, you will participate in a sequence of auctions. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.**
- **At the beginning of the session, you will be assigned to one of the two types: A and B.**
- **Your type is fixed throughout the experiment.**
- **A single good will be auctioned off in each period.**

Matching in Each Period

In each period, each type A subject will be randomly matched with a type B subject and form a team of two. Then each team will be randomly matched with another team of two subjects and then two teams will participate in an auction. You will never know whom you are matched. You will not be matched with the same group of subjects in any two consecutive periods.

- **Each team consists of a type A and a type B subjects.**
- **Each team participates in an auction to obtain an auctioned good.**
- **Two teams participate in an auction.**

Values

For Type A subjects:

At the beginning of each period, each type A subject *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. The MAX VALUES of the two

type A subjects in two teams that are participating in the same auction are independently determined and most likely different.

Type A subject does not know her exact VALUE for the good at the time of decision making. All she knows is that her VALUE is a number contained in the interval [0, MAX VALUE]. Therefore, her VALUE is at minimum zero and cannot exceed her MAX VALUE. Again, any number between 0 and her MAX VALUE is equally likely. For example, let's say a type A subject receives a MAX VALUE of 45.32. Then her VALUE is uniformly distributed on interval [0, 45.32] and it can be any number less than or equal to 45.32. Let's say her VALUE is 21.00. This means that if her team obtains the good at the end of the period, she will receive 21.00 ECU from us.

For Type B Subjects:

Each Type B subject knows the true value of her Type A teammate. Each Type B subject's value for obtaining the good is 2.5 times her Type A teammate's value.

$\text{Type B's Value} = 2.5 \times \text{Type A's Value}$
--

Auction

Each auction occurs in two stages. In the first stage only the Type A subjects will be active. In the second stage, only the Type B subjects are active. Specifically,

Stage 1:

- Each Type A only observes her MAX VALUE. Her true value is something less than this MAX VALUE.
- Each Type A subject decides on a CAP which is the maximum amount she allows her type B teammate to bid.

Stage 2:

- Each Type B observes the exact value of the good for herself.
- Each Type B also observes the CAP decided on by her Type A teammate.
- Each Type B subject decides on how much she wants to bid in behalf of her team in the auction. Type B subjects are not allowed to bid above the CAP. The bid decided on is simply labeled BID.

- After both teams participating in the auction submit their BIDs, the one who has the highest BID obtains the good and pays her BID. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

- The following table summarizes the progression of stages

	Team 1		Team 2	
	Type A	Type B	Type A	Type B
Stage 1	Sees: MAX VALUE Chooses: CAP		Sees: MAX VALUE Chooses: CAP	
Stage 2		Sees: CAP,VALUE Chooses: BID (\leq CAP)		Sees: CAP,VALUE Chooses: BID (\leq CAP)

Earnings in a Period

When your team obtains the good at the end of a period (if your BID is the highest), then you will receive your VALUE for the good and will pay the team's **BID**. If your team does not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

<p>Earnings = Your VALUE – Your BID (If you obtained the auctioned good);</p> <p>Earnings = 0 (If you did not obtain the auctioned good).</p>

Recall that a Type B subject's value is 2.5 times more than her Type A teammate. Moreover, at the time of decision making, each Type B subject knows her value. However, Type A subjects only know their maximum possible value but not their actual value.

Sequence of Auctions:

When the current period is over, the next period will start. Each period, you will be randomly matched with a new teammate and participate in a new auction with a different opponent team. Your VALUE of the good in each period is independent of your VALUE in the previous periods.

Screens

Below, there is an example of how Type A's and Type B's screens may look.

If you are a Type A subject, you see your MAX VALUE. Remember that this is the highest amount your actual value can be. You DON'T know your actual value before the auction is over. You need to enter a CAP for your Type B teammate in the text box on your screen and click on SUBMIT.

Period

Trial 1 of 5

Remaining time [sec]: 0

MAX VALUE 72.95

CAP

Submit

If you are a Type B subject, you see your value for the good and the CAP that your Type A teammate decided on. After observing your value, you will enter your BID in the text box on your screen and click on SUBMIT. Your bid has to be less than or equal to your CAP.

Period

Trial 1 of 5

Remaining time [sec]: 0

Please reach a decision.

CAP 50.00
Your VALUE 149.45
BID

Submit

The screen below shows the information that a Type A person will see at the conclusion of a period. It displays Type A's MAX VALUE, VALUE and CAP. Then it shows the BID that the Type B teammate decided on. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Note that much of the information about your opponent is hidden.

Period	Trial 1 of 5	Remaining time [sec]: 10
	You	Opponent
MAX VALUE	72.95	--
Your VALUE	59.78	--
CAP	50.00	--
BID	50.00	2.00
Received Item	Yes	No
Price	50.00	N/A
Payoff	9.78	--

OK

The next screen shows the information that a Type B person will see at the conclusion of a period. It displays Type B's VALUE, CAP and BID. It will also give you the BID of your opponent team and calculates the winner, the price of the good and your payoff for the round. Again, much of the information about your opponent is hidden.

	You	Opponent
Your VALUE	149.45	--
CAP	50.00	--
BID	50.00	2.00
Received Item	Yes	No
Price	50.00	N/A
Payoff	99.45	--

Example

The tables below indicate all the MAX VALUEs, VALUEs and BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment Type A subjects will observe only their own MAX VALUEs and Type B subjects will observe their own VALUEs. No subject will know the values received by the opponent team.

1.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	84.62	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	5.00	20.50
Type B's VALUE	141.00	20.20
Type B's BID	5.00	9.03
Received Item	No	Yes
Price	N/A	9.03
Type A's Payoff	0	-0.95
Type B's Payoff	0	11.17

2.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	84.62	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	36.00	20.50
Type B's VALUE	141.00	20.20
Type B's BID	36.00	9.03
Received Item	Yes	No

Price	36.00	N/A
Type A's Payoff	20.40	0
Type B's Payoff	105.00	0

3.

	Team 1 (YOU)	Team 2
Type A's MAX VALUE (observed by Type A)	84.62	37.40
Type A's VALUE	56.40	8.08
Type A's CAP	70.00	20.50
Type B's VALUE	141.00	20.20
Type B's BID	59.44	9.03
Received Item	Yes	No
Price	59.44	N/A
Type A's Payoff	-2.96	0
Type B's Payoff	81.56	0

In all of the examples above, Type A of Team 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 20.50. Team 2's BID is 9.03.

Type A of Team 1 observes a MAX VALUE of 84.62. Her true value (which she does not know at the time of decision making), is 56.40. Therefore, the value of the Type B subject of Team 1 is 141 ($2.5 \times 56.40 = 141$).

Each of the three tables corresponds to different choices of CAP and BID for Team 1. In the first table, Type A chose a cap of 5.00 and Type B chose a BID of 5.00.

In the first example the BIDs of two teams are 5.00 and 9.03. Since the highest bid (9.03) is submitted by Team 2, Team 2 obtains the good and pays its BID (9.03). In this period, the subjects in Team 1 earn zero, Type A of Team 2 earns $8.08 - 9.03 = -0.95$ ECU, and Type B of Team 2 earns $20.20 - 9.03 = 11.17$. Note that Type A of Team 2 loses money in this period because her Type B teammate is allowed to submit a bid that is higher than the Type A's true value.

In the second example, everything is the same except Type A of Team 1 chooses a higher CAP (36.00) and Type B chooses a higher BID (36.00). Now Team 1 receives the item. The price is equal to the BID (36.00), so Type A of Team 1's payoff is $56.40 - 36.00 = 20.40$ ECU and Type B of Team 1's payoff is $141.00 - 36.00 = 105.00$ ECU.

In the third example, Type A of Team 1's CAP is now 70.00, and Type B's BID is now 59.44. The BIDs are 59.03 and 9.03. Team 1 receives the item for a price of 59.03, Type A's payoff is $56.40 - 59.03 = -2.96$ ECU, and Type B's payoff is $141.00 - 59.03 = 81.97$ ECU.

Total Payoffs

At the beginning of today's session both Type A and Type B subjects will receive an endowment of \$8. The endowment is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings —your earnings from 25 auctions plus your initial endowment. The conversion rate is \$1 = 20 ECU for Type A and \$1 = 80 ECU for Type B. In addition to this sum, you will be paid a \$7 participation fee.

Are there any questions?

Practice Questions

Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose a Type A subject observes a MAX VALUE of 76.00ECU. What is her possible VALUE?

- a) any value from 0 to 100.00
- b) any value from 0 to 76.00
- c) any value from 0 to 76.00 but not 76.00
- d) any value from 0 to 100.00 but not 76.00

2. Suppose a Type A subject entered 21.00 as the CAP. What are the possible BIDs that the Type B subject in her team can select?

- a) Any BID is possible.
- b) Any BID that is between 0 and Type B's VALUE is possible.
- c) Any BID that is between 0 and Type A's VALUE is possible.
- d) Any BID that is between 0 and the CAP is possible.

3. Fill the table below

	Team 1	Team 2
MAX VALUE (observed by Type A)	43.00	37.40
Type A's VALUE	4.00	10
Type A's CAP	38.00	21.00
Type B's VALUE		
Type B's BID	6.00	8.00
Received Item		
Price		
Type A's Payoff		
Type B's Payoff		

INSTRUCTIONS FOR FIRST-PRICE SEALED BID AUCTIONS (computerized bidders)

(Online appendix: not intended for publication)

Thank you for being part of our research. Various research foundations have provided funds for this research. This is an experiment in the economics of market decision making. The instructions are simple and, if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

All values in the experiment will be in terms of Experimental Currency Unit (ECU). At the end of the experiment, we will convert your total earnings in the session to US\$.

- **In this experiment, we will run a sequence of auctions in which you will act as the buyer of a fictitious good. There will be 5 practice periods and 25 real periods. You will be paid only for the real periods.**
- **A single good will be auctioned off in each period.**
- **In each period, a computerized bidder will bid on behalf of you in the auction. We will tell you the bidding rule of the computerized bidder later in these instructions**
- **If you do not limit your computerized bidder, it will place an UNCONSTRAINED BID which may be higher than the amount you would like it to bid.**
- **Your task will be to determine a CAP, which is the maximum amount that you allow your computerized bidder to bid.**
- **The computerized bidder's ACTUAL BID is the lesser of its UNCONSTRAINED BID and the CAP.**

Matching in Each Period

In each period, you will be randomly matched with another person in this room. You and that person will participate in the auction. You will never know who the other person is in your auction. You will not be matched with the same person in any two consecutive periods.

Values

At the beginning of each period, each person participating in the auction *privately* observes her maximum possible value (MAX VALUE) for the auctioned good. Your MAX VALUE is a number randomly selected from the interval [0,100] and rounded to the nearest cent. Each number is equally likely. Your

MAX VALUE and the MAX VALUE of the other person who participates in the same auction with you are independently determined.

Your VALUE for the good is the amount of ECU the experimenter will give you if you receive the item at the end of the period. You do not know your exact VALUE for the good at the time of decision making. All you know is that your VALUE is a number contained in the interval $[0, \text{MAX VALUE}]$. Therefore, your VALUE is at minimum zero and cannot exceed your MAX VALUE. Again, any number between 0 and your MAX VALUE is equally likely. For example, let's say you receive a MAX VALUE of 45.32. Then your VALUE is uniformly distributed on interval $[0, 45.32]$ and it can be any number less than or equal to 45.32. Let's say your VALUE is 21.00. This means that if you get the good at the end of the period, you will receive 21.00 ECU from us.

To reiterate, MAX VALUE is never higher than 100 and a VALUE is never higher than the corresponding MAX VALUE. Each person in an auction receives independent MAX VALUEs and independent VALUEs. Hence, your VALUE and MAX VALUE are most likely different from your opponent's VALUE and MAX VALUE.

Auction

- **Two persons participate in each auction. Each person is represented by a computerized bidder.**
- **You observe your MAX VALUE for the current period. Your computerized bidder observes your true VALUE for the good.**
- **After observing your MAX VALUE for the period, you need to decide the maximum amount that you will allow the computerized bidder to bid on your behalf. We call this amount your "CAP".**
- **You have been given two sheets explaining the bidding rule of your computerized bidder for each possible VALUE (unless you restrict it by a CAP). These two sheets provide you with the same information in two formats: one is a table, and one is a graph. The bids on the sheets are referred to as the computerized bidder's UNCONSTRAINED BID. Please take a look at these sheets and confirm that when your VALUE is, for example, 22.00 ECU, the UNCONSTRAINED BID will be 44.00.**

- The computerized bidder's **ACTUAL BID** is the lesser of its **UNCONSTRAINED BID** and the **CAP**:

$$\text{ACTUAL BID} = \text{minimum} \{ \text{UNCONSTRAINED BID} , \text{CAP} \}$$

- After both your and the other player's **ACTUAL BIDS** are submitted, the one who has the highest **ACTUAL BID** obtains the good and pays her **ACTUAL BID**. In case of a tie, each of the two participants in the auction will receive the good with equal probabilities.
- This is the end of the period.

Earnings in a Period

If you obtain the good at the end of the period (if your **ACTUAL BID** is the highest), then you will receive your **VALUE** for the good and you will pay your **ACTUAL BID**. If you did not obtain the good, you do not receive or pay any amount. In other words, your earnings in the current period are:

Earnings = Your VALUE – Your ACTUAL BID (If you obtained the auctioned good);

Earnings = 0 (If you did not obtain the auctioned good).

When the current period is over, the next period will start. Each period, you will be randomly matched with a new player and receive a new **MAX VALUE**. Therefore, your **VALUE** of the good in each period is independent of your **VALUE** in the previous periods.

Screens

Below, there is an example of how your screen may look. On top of your screen there is an interactive tool. The tool shows your MAX VALUE for the current period. Remember that your VALUE can be any number between zero and your MAX VALUE, but you do not know what it is. However, your computerized bidder knows your VALUE and bases its UNCONSTRAINED BID on your VALUE. By sliding the little black square between zero and your MAX VALUE, you may see the UNCONSTRAINED BID of your computerized bidder for the corresponding VALUE. This tool provides you with the exact same information as you may learn from the UNCONSTRAINED BID table or graph that we have provided to you. Please use whichever tool that you prefer in order to understand how the computerized bidder bids unless it is restricted by a CAP.

You need to enter your bidder's CAP for this period in the text box on your screen and click on SUBMIT. Remember that your ACTUAL BID will be what your computerized bidder's UNCONSTRAINED BID is **unless** you restrict it by a CAP, in which case it will be the lesser of the two numbers.

Period

Trial 1 of 5

Remaining time [sec]: 254

Drag the black square to see what bid your bidder will make for different possible VALUES, unless the bidder is constrained by the CAP.

Your MAX VALUE is 60.86

Your MAX VALUE

0 10 20 30 40 50 60 70 80 90 100

Possible VALUES

VALUE: 17.37

UNCONSTRAINED BID: 19.23

Your Bidder's CAP

Submit

The screen below is an example of the results screen after the conclusion of one auction. It displays your true VALUE, the UNCONSTRAINED BID that corresponds to that value, and the ACTUAL BID. It will also give you the ACTUAL BID of your opponent and calculate the winner, the price of the item and your payoff for the round. Note that much of the information about your opponent is hidden.

Period		Trial 1 of 5		Remaining time [sec]: 9	
	You	Opponent			
MAX VALUE	60.86	--			
VALUE	13.11	--			
UNCONSTRAINED BID	14.62	--			
CAP	25.00	--			
ACTUAL BID	14.62	12.00			
Received Item	Yes	No			
Price	14.62	N/A			
Payoff	-1.51	--			

Example

The tables below indicate all the MAX VALUEs, VALUEs and UNCONSTRAINED BIDs in an auction, and show the results for three different choices of a CAP. Recall that in the experiment you will only observe your own MAX VALUE and will *only* know that your VALUE is not higher than your MAX VALUE. You will *not* know the MAX VALUE or VALUE of the other player.

	Player 1 (YOU)	Player 2
MAX VALUE	84.62	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	59.44	9.03
CAP	5.00	20.50
ACTUAL BID	5.00	9.03
Received Item	No	Yes
Price	N/A	9.03
Payoff	0	-0.95

	Player 1 (YOU)	Player 2
MAX VALUE	84.62	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	59.44	9.03
CAP	36.00	20.50
ACTUAL BID	36.00	9.03
Received Item	Yes	No
Price	36.00	N/A
Payoff	20.40	0

	Player 1 (YOU)	Player 2
MAX VALUE	84.62	37.40
VALUE	56.40	8.08
UNCONSTRAINED BID	59.44	9.03
CAP	70.00	20.50
ACTUAL BID	59.44	9.03
Received Item	Yes	No
Price	59.44	N/A
Payoff	-2.96	0

In all of the examples above, Player 2 (your opponent) observes a MAX VALUE of 37.40 and she decides on a CAP of 20.50. Player 2's VALUE is 8.08 and therefore, her computerized bidder's UNCONSTRAINED BID is 9.03. However, since $20.50 > 9.03$, the ACTUAL BID of Player 2 is her UNCONSTRAINED BID.

Player 1 observes a MAX VALUE of 84.62. Each of the three tables corresponds to a different choice of CAP that Player 1 might have chosen. Player 1's private VALUE is 56.40 (she does not observe this at the time of deciding on the CAP but her computerized bidder knows the VALUE). In the first table, Player 1 chose a cap of 5.00. If you check the provided bidding sheet, you will see that Player 1's computerized bidder's UNCONSTRAINED BID is 59.44. Since $5.00 < 59.44$, the ACTUAL BID is 5.00.

In the first example the ACTUAL BIDS are 5.00 and 9.03. Since the highest bid (9.03) is submitted by Player 2, Player 2 obtains the good and pays her ACTUAL BID (9.03). In this period, Player 1 earns zero and Player 2 earns $8.08 - 9.03 = -0.95$ ECU. Note that Player 2 loses money in this period because the bidder is unconstrained and allowed to submit a bid that is higher than the true value.

In the second example, everything is the same except Player 1 chose a higher CAP (36.00). Now the ACTUAL BIDS are 36.00 and 9.03 and Player 1 receives the item. The price is equal to her ACTUAL BID (36.00), so Player 1's payoff is $56.40 - 36.00 = 20.40$ ECU.

In the third example, Player 1's CAP is now 70.00. The ACTUAL BIDS are now 59.03 and 9.03. Player 1 receives the item for a price of 59.03, so her payoff is $56.40 - 59.03 = -2.96$ ECU.

Total Payoffs

At the beginning of today's session you will receive an endowment of 160 ECU which is provided in order to cover any losses that you may make. Every period your earnings from that period are added to your initial endowment. At the end of today's session you will receive your cumulative earnings — your earnings from 25 auctions plus your initial endowment. The conversion rate from ECU to dollars is $\$1 = 20$ ECU. In addition to this sum, you will be paid a \$7 participation fee.

Are there any questions?

Practice Questions

Please answer the questions below. The experiment will start after everybody answers all the questions correctly. Please feel free to ask questions.

1. Suppose your MAX VALUE is 76.00ECU. What is your possible VALUE?
 - a) any value from 0 to 100.00
 - b) any value from 0 to 76.00
 - c) any value from 0 to 76.00 but not 76.00
 - d) any value from 0 to 100.00 but not 76.00

2. Suppose your MAX VALUE is 76.00 and you entered 21.00 as your CAP. Your computerized bidder observed your private VALUE of 69.00. What will your ACTUAL BID be?
 - a) 69.00
 - b) 21.00
 - c) 54.00
 - d) 76.00

3. Suppose your MAX VALUE is 43.00 and you entered 38.00 as your CAP. Your computerized bidder observed your VALUE of 4.00. What will your ACTUAL BID be?
 - a) 4.00
 - b) 4.57
 - c) 38.00
 - d) 43.00

4. Suppose Player 1's ACTUAL BID is 26.15 and Player 2's ACTUAL BID is 63.00. Who will obtain the good and what price the winner will pay?
 - a) Player 1 wins and pays 26.15
 - b) Player 1 wins and pays 63.00
 - c) Player 2 wins and pays 63.00
 - d) Player 2 wins and pays 26.15

5. Suppose Player 1's ACTUAL BID is 31.00 and Player 2's ACTUAL BID is 24.00. Player 1's VALUE for the good was 38, and Player 2's VALUE was 46. What will be the earnings of each player from this period.
 - a) Player 1 earns 7, Player 2 earns zero.
 - b) Player 1 earns 7, Player 2 earns 22.
 - c) Player 1 earns 14, Player 2 earns zero.
 - d) Player 1 earns zero, Player 2 earns 22.