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A Note on the Extinction of Renewable Resources[1]

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This paper presents two sets of conditions under which a sole owner of a renewable resource stock who maximizes a nonlinear benefit function would find it more profitable to harvest the stock to extinction than follow a continuous harvesting strategy. When the minimum viable resource stock is positive, extinction is optimal as long as the initial resource stock is sufficiently small, regardless of the discount rate. When the minimum viable resource stock is zero and the discount rate exceeds the growth potential of the species extinction is optimal for sufficiently small initial stocks. © 1988 Academic Press, Inc.

1. INTRODUCTION

Much of the literature on the management of a renewable resource by a single owner is concerned with the effect of a high discount rate on the profitability of extinction. When the discount rate exceeds the maximum growth rate of the species the resource is said to be submarginal and extinction is usually more profitable than continuous harvesting [1, 2]. By contrast, a discount rate below the growth potential of the species usually implies that the resource is safe from extinction.[2] In a 1977 paper Tracy Lewis and Richard Schmalensee [3, Proposition 9] show that extinction is optimal for certain recruitment functions even if the discount rate is below the growth potential of the species. Specifically, if there is a critical population below which net births are negative, extinction will be optimal for small initial stocks. Their proof of this result, however, is incorrect.

This paper provides a correct proof of Lewis and Schmalensee's extinction theorem and shows that the result also holds when the recruitment function exhibits critical depensation; i.e., when the function has an initial convex segment usually associated with a minimum viable population. Following this we prove a result for the case in which the minimum viable population is zero. In this case the extinction theorem no longer holds, and the resource is not in danger of extinction at low interest rates. Extinction is optimal, however, for some initial stocks if the discount rate exceeds the growth potential of the species.

These results are proved using the model of Lewis and Schmalensee, which is presented, for convenience, in Section 2. Section 3 contains the proof of the Lewis–Schmalensee extinction theorem. The effect of the discount rate on extinction is examined in Section 4.

[1] In writing this paper I have benefitted from discussions with Tracy Lewis and William R. Porter.

[2] The *growth potential* of the species is defined to be the rate of change in net births as the resource stock approaches the minimum viable population.

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2. THE MODEL

The problem facing the manger of a fishery is to choose a time path of harvesting, $H(t)$, and a terminal date, T, to maximize the present discounted value of net benefits from fishing

$$V = \int_0^T [B(H(t),S(t)) - F] e^{-\rho t} dt. \quad (1)$$

Equation (1) is maximized subject to a constraint on the rate of growth of the stock, S,

$$\dot{S} = g(S(t)) - H(t), \quad (2)$$

and to the constraints

$$H(t) \geqslant 0, \quad S(T) \geqslant 0, \quad S(0) = S_0. \quad (3)$$

In (1) F represents a fixed cost associated with fishing, such as the cost of locating fish, and is incurred only if $H(t) > 0$.

Instantaneous benefits from fishing are assumed to be a strictly concave function of the harvest rate and the stock and are assumed bounded from above. In addition, the benefit function has the following properties:

$$B(0, S) = 0, \quad S \geqslant 0; \quad B_S(0, S) = 0, \quad S \geqslant 0;$$
$$B_S(H, S) > 0, \quad B_{HS}(H, S) > 0, \quad H, S > 0;$$
$$\exists H_0 \text{ such that } B(H_0, S_0) > F.$$

As Lewis and Schmalensee show, the preceding assumptions imply that marginal benefits from harvesting are positive when the rate of harvest is zero,

$$B_H(0, S) > 0, \quad S \geqslant 0.$$

We shall also assume that marginal benefits are finite; i.e., the resource is not essential for survival.

The natural rate of growth of the fish is given by $g(S)$, which is zero at stocks $\underline{S}$ and $\bar{S}$,

$$g(\underline{S}) = g(\bar{S}) = 0, \quad \bar{S} > \underline{S} \geqslant 0.$$

We assume that $g(S)$ is strictly concave in the interval $(\underline{S}, \bar{S})$ and that it is nonpositive for stocks below the minimum viable population, $\underline{S}$. A final assumption is that the discount rate is smaller than the slope of the recruitment function at $\underline{S}$,

$$\rho < g_S(\underline{S}).$$

The necessary conditions for (1) to have a maximum subject to (2) and (3) yield the differential equations [3, p. 538]

$$\dot{\lambda} = \lambda[\rho - g_S(S)] - B_S(H(\lambda, S), S) \equiv G^\lambda(\lambda, S) \tag{4}$$

$$\dot{S} = g(S) - H(\lambda, S) \equiv G^S(\lambda, S), \tag{5}$$

which have multiple equilibria in the interval $[\underline{S}, \bar{S}]$.

Following Lewis and Schmalensee, paths that approach equilibria of the system will be called continuous harvesting paths. Because these paths are confined to the interval $[\underline{S}, \bar{S}]$ the conditions of [4, Theorem 1] are satisfied. Any continuous harvesting path satisfying the necessary conditions is therefore optimal among the class of infinite horizon paths.

Consider now the class of paths for which T is finite. These will be called abandonment paths, and the subset of these for which $S(T) < \underline{S}$ will be termed extinction paths. Since harvesting ceases once total or marginal benefits become negative, the terminal stock along an abandonment path, $\hat{S}$, is the largest S for which

$$B_H(H, S) = 0 \qquad \text{and} \qquad B(H, S) \leqslant F.$$

3. EXTINCTION WHEN $\underline{S} > 0$

It is now possible to prove Proposition 9 of Lewis and Schmalensee. Suppose that the terminal stock, $\hat{S}$, is below the minimum viable population, $\underline{S}$. Then beginning at $S_0 > \underline{S}$ one can always find an extinction path—for example, the path $B_H(H,S) = 0$—which yields positive net benefits. Proposition 9 states that if S_0 is chosen sufficiently close to $\underline{S}$ the value of a continuous harvesting strategy beginning at S_0 will be less than the value of the extinction plan.

To prove this result it suffices to show that the value of a continuous harvesting policy approaches 0 as the initial stock approaches $\underline{S}$. Formally,

$$\lim_{S_0 \to \underline{S}^+} V(S_0) = 0, \tag{6}$$

where $V(S_0)$ denotes the value of an optimal continuous harvesting plan beginning at S_0. To prove (6) Lewis and Schmalensee use the fact that

$$V(S_0) = \lim_{S \to \underline{S}^+} V(S) + \int_0^k V'(\xi)\, d\xi, \tag{7}$$

where $k \equiv S_0 - \underline{S}$. As they note, the second term in (7) goes to 0 as $S_0 \to \underline{S}^+$. It is also true that $V(\underline{S}) = 0$ since there no feasible continuous harvesting strategy beginning at $\underline{S}$. This, however, does not complete the proof since it is possible for

$$\lim_{S \to \underline{S}^+} V(S) > 0$$

if the value function has a jump discontinuity at $\underline{S}$.

The alternative proof of (6) given below uses the following reasoning. In the neighborhood of $\underline{S}$ the resource is growing so slowly that the time required to get from S_0 to an arbitrary stock $\tilde{S}$ can be made as large as one pleases. If $\tilde{S}$ is chosen arbitrarily, however, the profit associated with $\tilde{S}$ can be made arbitrarily small, and $V(S_0)$ can thus be made as small as one desires.

Formally, let $\theta = g_S(\underline{S})$. Then, by the strict concavity of $g(S)$, $\underline{S} \leqslant S \leqslant \bar{S}$, $\dot{S}$ can be bounded from above along a continuous harvesting path,

$$\dot{S} < g(S) < \theta(S - \underline{S}), \qquad \underline{S} < S \leqslant \bar{S}. \tag{8}$$

Let S(t) be measured as a deviation from $\underline{S}$,

$$\epsilon(t) \equiv S(t) - \underline{S}.$$

Then by (8)

$$\dot{\epsilon}(t) < \theta(t),$$

which implies

$$\epsilon(t) < \epsilon(0)e^{\theta t}, \qquad \epsilon(0) > 0.$$

By setting

$$\epsilon(0) < \eta/e^{\theta T}$$

it is clear that $\epsilon(0)$ can be chosen small enough that $\epsilon(T) < \eta$ for arbitrary $\eta > 0$ and $T > 0$. Thus one can always pick S_0 small enough that it takes an arbitrarily large time to reach an arbitrary stock $\tilde{S} \equiv \eta + \underline{S}$.

By the properties of $B(H, S)$ one can choose $\tilde{S} > \underline{S}$ such that

$$B(g(\tilde{S}), \tilde{S}) = \zeta$$

for arbitrary $\zeta > 0$. Since $H(t) < g(S(t))$ along a continuous harvesting path beginning at S_0, and since $B_H(H, S) > 0$, ζ places an upper bound on the instantaneous benefits to be earned along the path from S_0 to $\tilde{S}$.[3] Thus

$$V(S_0) < \int_0^T \zeta e^{-\rho t}\,dt + e^{-\rho t}V(S(T)),$$

where $S(T)$ denotes the stock attained at time T along an optimal continuous harvesting path. Since $V(S(T)) < \infty$ the value of an optimal continuous harvesting plan can always be made arbitrarily small by appropriate choice of ζ and T and, hence , of S_0. In particular, $V(S_0)$ can always be kept smaller than the value of an abandonment strategy beginning at $\underline{S}$. We have thus proved

PROPOSITION 1. *In the model of Section* 2 *if* $\underline{S} \geqslant \hat{S} \geqslant 0$ *then there exists an* $S_0 > \underline{S}$ *such that extinction is optimal.*

Two points are worth noting about the proposition. First, positive fixed costs are not a necessary condition for Proposition 1. Indeed, if F is large the fish will be

[3]This also assumes $g_S(S) > 0$ which is true for all S in the neighborhood of $\underline{S}$.

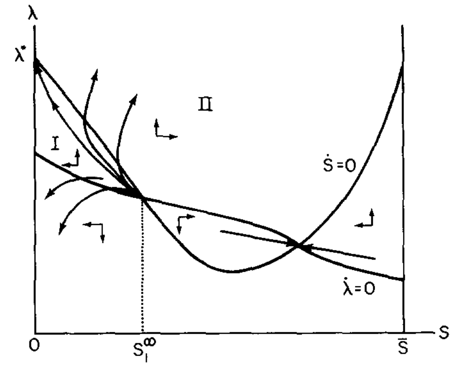

FIG. 1. Phase diagram when $\rho > g_S(\underline{S})$ and $\underline{S} = 0$.

saved from extinction since it will not be profitable to drive the stock below $\underline{S}$. Second, the proof of the proposition in no way depends on the shape of the recruitment function in the interval $(0, \underline{S})$. Thus the proof holds for the strictly concave recruitment function used by Lewis and Schmalensee, which assumes $g(0) < 0$, as well as for recruitment functions which exhibit critical depensation.

4. EXTINCTION WHEN $\underline{S} = 0$

When $\underline{S} = 0$ Proposition 1 no longer holds since the value of an extinction plan beginning at S_0 also becomes arbitrarily small as S_0 approaches $\underline{S}$. This prompts one to ask in what circumstances extinction will occur when $\underline{S} = 0$. Lewis and Schmalensee have shown that extinction is superior to continuous harvesting if fixed costs are sufficiently high. It can also be shown that extinction is optimal even if fixed costs are zero if the discount rate exceeds the growth potential of the species, $\rho > g_S(\underline{S})$.

To prove this it is convenient to examine the phase diagram of the system when $\rho > g_S(\underline{S})$ and $F = \underline{S} = 0$ (Fig. 1). When $\rho > g_S(\underline{S})$ the loci $\dot{\lambda} = 0$ and $\dot{S} = 0$ need not intersect; however, if an intersection does occur the first equilibrium will be an unstable node. To establish this, note that the λ-intercept of $\dot{S} = 0$ must occur at the λ for which $H(\lambda, 0) = 0$. The intercept of $\dot{\lambda} = 0$, however, must occur at a lower λ since (5) is satisfied only if $H > 0$. Thus $\dot{S} = 0$ must first intersect $\dot{\lambda} = 0$ from above. The directions of motion implied by (4)–(5)[4] guarantee that the first equilibrium is an unstable node [2, p. 192]. It is now possible to prove

PROPOSITION 2. *When* $\rho > g_S(\underline{S})$ *and* $\underline{S} = F = 0$, *an extinction policy is optimal for all* $0 < S_0 < S_1^\infty$.

The proof consists of showing (i) that no continuous harvesting plan beginning at $0 < S_0 < S_1^\infty$ satisfies the necessary conditions; and (ii) that an extinction plan exists for each $S_0 \in (0, S_1^\infty)$ that does satisfy the necessary conditions. By the strict concavity of the benefit and recruitment functions such a plan must be optimal [4, Theorem 1].

[4]Since $\partial G^\lambda/\partial\lambda = \rho - g_S(S) - B_{HS}H_\lambda > 0$, $\dot{\lambda} > 0$ above $\dot{\lambda} = 0$ and $\dot{\lambda} < 0$ below. Similarly $\partial G^S/\partial\lambda = -H_\lambda > 0$, guaranteeing that $\dot{S} > 0$ to the right of $\dot{S} = 0$ and $\dot{S} < 0$ to the left.

To demonstrate (i) note that any continuous harvesting path beginning in the interval $(0, S_1^\infty)$ must enter region II of the diagram where $\dot{\lambda} > 0$ and $\dot{S} > 0$. Such a path cannot be optimal by Proposition 4 of Lewis and Schmalensee since λ is a decreasing function of S along an optimal continuous harvesting path.

To show that an optimal extinction path exists consider first the case in which $\dot{S} = 0$ and $\dot{\lambda} = 0$ do not intersect. Under the assumptions of the proposition the necessary conditions imply that an optimal extinction path must terminate at the point $(0, \lambda^*)$ where $\dot{S} = 0$ intersects the λ-axis. The slope of the path satisfying (4) and (5) is infinite at this point, hence the path must approach $(0, \lambda^*)$ from below the locus $\dot{S} = 0$.[5] By continuity and the directions of motion in Fig. 1 this path must lie below $\dot{S} = 0$ and above $\dot{\lambda} = 0$ in the interval $(0, \bar{S})$. Hence for any $0 < S_0 < \bar{S}$ an extinction path satisfying the necessary conditions exists.

Now consider the case of at least one equilibrium. In this situation paths radiating from the unstable node $(S_1^\infty, \lambda_1^\infty)$ must cross the $\dot{S} = 0$ locus vertically. Since there are an infinite number of these paths there must be one which passes through the point $(0, \lambda^*)$ and thus satisfies the necessary conditions.

5. DISCUSSION

This paper has presented two sets of conditions under which the manager of a renewable resource would find it more profitable to mine the resource stock than to harvest it.

The first condition occurs when there is a critical population below which net births are negative. For initial stocks that are only slightly higher than the minimum viable population the time required to build the stock up to a level where substantial profits are earned is so great that an extinction strategy yields higher discounted benefits. The importance of this result is that extinction can be more profitable than continuous harvesting even if marginal fishing costs become infinite as the stock approaches zero and even if the discount rate is low. All that is necessary for extinction to be more profitable is that the initial stock be close to the minimum viable population and that it be profitable to harvest some members of the species below this level.

The second condition under which extinction is more profitable than continuous harvesting is when the discount rate exceeds the maximum growth rate of the species. If the resource is slow-growing and if the initial stock is small, it is not profitable to let the stock reach a steady state. This parallels Clark's finding [1; 2, p.62] that extinction is optimal at high discount rates when net benefits are a linear function of the harvest rate. The difference in the nonlinear case is that the optimality of extinction depends on the initial stock as well as on the discount rate. In Fig. 1, for example, if the initial stock is to the right of S_1^∞ the optimal path proceeds to the nearest saddlepoint and the resource is safe from extinction. A high discount rate, therefore, need not imply that extinction is optimal if it is accompanied by a high initial stock.

The general conclusion to be drawn from this paper is that in a model in which benefits are a nonlinear function of the harvest rate and in which the growth

[5]Any path satisfying (4)–(5) has slope $d\lambda/dS = G^\lambda(\lambda, S)/G^S(\lambda, S)$, which approaches infinity as $G^S(\lambda, S)$ approaches zero.

function is not purely compensatory the discount rate plays a less important role in determining the profitability of extinction than it does in a linear model with a compensatory growth function. In the model examined here it is the size of the initial stock that is crucial to the optimality of extinction. A stock that is near the minimum viable level, possibly as the result of open access harvesting, is in danger of extinction if profit maximization is the goal of resource management.

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