A State-Preference Approach to the Precautionary Demand for Money

By M. L. Cropper*

Much attention has been devoted in the literature to the transactions demand for money and the asset demand for money. The precautionary motive for holding cash balances, however, is seldom analyzed. A notable exception is S. C. Tsiang's inventory-theoretic analysis, which integrates precautionary demand into an inventory model of the transactions demand for cash. Precautionary demand can, however, be approached as a problem in portfolio theory using a state-preference approach, as John Hicks (1967) once suggested.[1] The states of nature in this case represent the times at which an investor may, for reasons beyond his control, have to liquidate his portfolio. It is this uncertainty regarding future cash requirements, combined with the fact that some assets are neither readily nor costlessly convertible into cash, which gives rise to the precautionary demand for money.

Consider the situation of an investor who is allocating his wealth among assets of varying degrees of liquidity to maximize expected utility of wealth at the end of his horizon. The investor believes that certain events (outside of his control) may occur which will force him to liquidate his portfolio before the end of his horizon. If liquidation is not called for, the portfolio will be held until the end of the horizon and then sold. Whether or not liquidation will actually be required is not known until the time the cash is needed. Thus, if the portfolio must be sold before the end of the period, it must be sold on short notice. This means that the investor will be able to sell his less liquid assets only at a discount, the size of the discount varying inversely with the length of notice given before the asset is sold. The individual thus has an incentive to hold liquid assets, say, cash, to reduce losses which may possibly be incurred if he must suddenly liquidate his portfolio. Money held for this purpose may be labelled precautionary cash balances. An increase in the probability that the investor will have to liquidate his portfolio in the near future should increase this demand.

The remainder of this paper investigates the precautionary demand for money in the framework outlined above. Using two concepts of liquidity, I derive conditions under which an increase in the probability of having to liquidate one's portfolio on short notice will increase the precautionary demand for money; that is, conditions are obtained under which a well-behaved precautionary demand for money, in the above sense, can be said to exist.

In Section I the concept of liquidity is discussed at some length. This discussion is necessary before liquidity can be formally incorporated into a portfolio selection model. The formal model is presented in Section II.

I

A liquid asset according to J. M. Keynes' definition is one which is "... more certainly realizable on short notice without loss" (p. 67). This is usually interpreted to mean that *il*liquid assets are either not realiz-

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[1] I have not however followed the model outlined by Hicks (1967, pp. 31–34). In that model, an illiquid asset is one which incurs high transactions cost, e.g., brokerage fees, whenever it is traded, *regardless* of the length of notice given before the transaction takes place. (This use of the term liquidity deviates somewhat from that followed by Keynes and Tsiang.) The longer the investor's horizon, the more incentive he will have to invest in illiquid assets since transactions costs will decline as a percentage of asset yield as the holding period is lengthened. Put somewhat differently, Hicks is saying that it will pay to invest in illiquid assets only if they can be held long enough to compensate for their high transactions costs. Precautionary demand enters when the planned date of realization is uncertain.

able with certainty or that they can be converted into cash on short notice, only at a loss. The size of the loss incurred is assumed to vary inversely with the length of notice given before the asset is sold. This is the common interpretation of the term liquidity (see Hicks, 1962; Tsiang); however, a little reflection indicates that this interpretation is not meaningful in all market situations. In a perfectly competitive capital market, for example, there is no obvious reason why the price at which an asset is sold should vary directly with the length of notice given before the asset is sold. Therefore, before liquidity can formally be incorporated into a portfolio selection model, some further discussion is clearly required. To give the term more meaning, I shall discuss two sources of uncertainty in capital markets.

Uncertainty in most portfolio selection models is uncertainty about which state of nature will occur at some future time t.[2] Investors are assumed to know security prices contingent on the state of the world at that time. Because an investor is uncertain exactly what the state of the world will be, he has a subjective probability distribution over asset prices. Once time t has arrived, however, the state of the world is known and there is a single known asset price. (Capital markets are assumed to be perfect in all respects other than knowledge of the future.)

In the above model the individual believes that the price of each security at time t is determined by forces external to him, for example, political conditions in the United States, the world demand for steel, etc. Once the outcome of these forces is known a unique price is determined for each security. The investor has no reason to believe that this price will be altered should he have to sell the security at time t. Thus there is no reason to believe that the investor's distribution over future asset price should be affected by the length of notice given before an asset is sold. By this reasoning, the "realizable on short notice without loss" aspect of the term liquidity really has no meaning in a portfolio model in which uncertainty is only of a state-of-nature variety. One asset can be more liquid than another in this context only if it is "more certainly realizable" regardless of the length of notice given before it is sold. Thus, in standard portfolio selection models, "more liquid" must be synonymous with "less risky," and a perfectly liquid asset must necessarily be a riskless asset.

Keynes' definition of liquidity is meaningful, however, in markets in which investors are imperfectly informed about opportunities to buy and sell securities. In markets in which information is imperfect there will usually be a distribution of prices for any security at any time, rather than a single equilibrium price.[3] In such a market an investor has a subjective distribution over asset yield (price) because he is uncertain about the actual price at which he will sell a security. The moments of the subjective distribution will generally depend on the amount of price information the individual can obtain by sampling bids of prospective buyers.

Assuming that sampling takes time, one may argue that an investor who does not have time to search out potential buyers, for example, because he must sell his assets on short notice, will most likely have to sell his assets at a lower price than an investor who has time to acquire a larger sample of bids. Reasoning in this way, one may interpret "realizable on short notice without loss" to mean that illiquid assets can be sold on short notice only at a discount. This discount arises because the individual does not have time to obtain more price information, and it should therefore vary inversely with the length of notice given before the asset is sold. This aspect of liquidity is usually re-

[2] The uncertainty in Markowitz-Tobin portfolio selection models is implicitly of the state-of-nature variety. In these models investors have probability distributions over security prices at some time in the future, which are by assumption independent of current prices. These distributions may be regarded as having been derived from a knowledge of state-contingent returns and a probability distribution over states when the number of states is infinite. It should be emphasized that state-of-nature uncertainty can exist even when capital markets are perfect, in the sense that each asset has a single known equilibrium price.

[3] The uncertainty in recent job-search models is of this variety.

ferred to as marketability.

In the model below we are interested not in the price at which an illiquid (imperfectly marketable) asset is actually sold, but in the effect that having suddenly to liquidate the asset will have on the investor's subjective distribution over asset yield. That is, we wish to know how the belief that an illiquid asset must be sold on short notice will alter the investor's subjective probability distribution over yield. If an investor believes that by increasing the size of his sample he will reduce the variability of the distribution and increase its mean, it follows that the subjective distribution should have greater variability (a more precise definition of greater variability is given below) and a lower mean if the investor must sell the security on short notice than if he has more time to sample potential buyers.[4] With this interpretation of liquidity in mind, we turn to the formal model.

II

Assume that an investor must divide his initial wealth W_0 between two securities: money, a perfectly liquid asset paying zero return with certainty, and an illiquid asset, the return on which is a random variable. One of two possible events which may be thought of as states of nature must occur before the end of the investor's horizon. The first event, which we shall call state 1, is the occurrence of some emergency which requires immediate cash payment. If state 1 occurs the investor must sell his portfolio at once. State 2 is the absence of such an emergency. If state 2 occurs the portfolio will be held to the end of the horizon and then sold.

Money, because it is a perfectly liquid asset, pays a certain return (of zero) in both states.[5] The return on the illiquid asset is r_1 in state 1 and r_2 in state 2. It is assumed that this asset is illiquid because it is imperfectly marketable. Thus the distribution over r_1 should be more variable and should have a lower mean than the distribution over r_2, since the investor will not have as much time to sample price information in state 1 as he will if state 2 occurs.[6] It is further assumed that r_1 and r_2 must lie in the interval $[-1, \infty)$. This implies that the investor's wealth can never fall below 0.[7]

The investor's objective is to set a, the fraction of W_0 invested in the illiquid asset, to maximize expected utility of wealth at the time his portfolio is liquidated. Thus the investor seeks to

$$\max_a \{pE[U(W_1)] + (1-p)E[U(W_2)]\},$$

$$W_1 = W_0(1 + ar_1) \tag{1}$$

$$W_2 = W_0(1 + ar_2)$$

where p is the subjective probability that state 1 will occur and $(1-p)$ is the subjective probability attached to state 2. The investor's utility function is assumed to be identical in both states and to obey

$$U' > 0 \quad \text{and} \quad U'' < 0 \tag{2}$$

Because we are interested only in a solution for which $0<a<1$, I shall make the following assumptions which guarantee an interior solution to (1).

$$U'(0) = \infty \tag{3}$$

$$E(r_1) > 0, \qquad E(r_2) > 0 \tag{4}$$

First-order and second-order conditions for a maximum are, respectively,

$$pW_0E[r_1U'(W_1(a))] + (1-p)W_0E[r_2U'(W_2(a))] = 0 \tag{5}$$

[4] This is a tricky point. The investor is making a judgment in the present, before any sample is taken, about what the shape of his subjective distribution will be in the future, after the sample has been taken. The belief before the sample is taken that the variability of the distribution can be reduced by taking a larger sample corresponds to what in Bayesian terminology is referred to as preposterior analysis.

[5] The analysis may be extended with little difficulty to the case where money pays a nonzero return with certainty by reinterpreting r as the yield differential between liquid and illiquid assets. As this does not significantly affect the results, I present only the case where the return on money is zero.

[6] Any other sources of uncertainty, e.g., state-of-nature uncertainty, by assumption do not differ between the two states.

[7] It also implies that the limits of integration in equations (6) and (7) below are independent of r; hence, differentiation is permissible within the integral sign.

$$\frac{da}{dp} = \frac{-W_0\{E[r_1U'(W_1(a))] - E[r_2U'(W_2(a))]\}}{pW_0^2E[r_1^2U''(W_1(a))] + (1-p)W_0^2E[r_2^2U''(W_2(a))]} \tag{7}$$

$$pW_0^2E[r_1^2U''(W_1(a))] + (1-p)W_0^2E[r_2^2U''(W_2(a))] < 0 \tag{6}$$

We now wish to see whether an increase in p, the probability that the investor must suddenly liquidate his portfolio, will increase the demand for money. This is, of course, equivalent to determining whether da/dp is in fact negative. Applying the implicit function theorem to (5), an implicit function of a and p, yields equation (7), shown above.

The denominator of (7) must be negative if the second-order condition for a maximum, equation (6), is satisfied. Thus $da/dp \gtreqless 0$ as $E[r_2U'(W_2)] \gtreqless E[r_1U'(W_1)]$. We can make statements about the sign of the numerator of (7) under alternative assumptions about the distribution functions $F_1(r_1)$ and $F_2(r_2)$. For reasons elaborated in the previous section, I argue that $F_2(r_2)$, the distribution function over the yield on the illiquid asset in state 2, should have higher mean and lower variability than $F_1(r_1)$.

We consider first the case where $r_2 = r_1 + k$, $k > 0$, so that for every possible value of r_1 there is a corresponding value of r_2 which is greater by k and has the same probability occurrence. This implies that $F_1(r_1)$ and $F_2(r_2)$ are identical distributions, but that the mean of $F_2(r_2)$ is greater than the mean of $F_1(r_1)$. In this case the sign of da/dp depends on whether the gain in utility from investing an additional dollar in the illiquid asset, $rU'[W_0(1+ar)]$, is increasing or decreasing in r.

From the assumption that $r_2 = r_1 + k$ it follows that

$$\int r_1U'[W_0(1+ar_1)]dF_1(r_1) \gtreqless \int r_2U'[W_0(1+ar_2)]dF_2(r_2) \tag{8}$$

as

$$\frac{\partial}{\partial r}\{rU'[W_0(1+ar)]\} \lesseqgtr 0 \tag{9}$$

From equation (7) we know that the sign of the inequality in (8) determines whether the fraction of initial wealth invested in the illiquid asset increases or decreases as the probability of having suddenly to liquidate the portfolio increases. Thus, assuming that the illiquid asset has lower expected return if the portfolio must be sold on short notice, the proportion of W_0 invested in this asset will increase if $\partial/\partial r\{rU'(W)\} < 0$ and will decrease if $\partial/\partial r\{rU'(W)\} > 0$. That is, an increase in the probability of having suddenly to liquidate the portfolio will cause an investor to put a smaller proportion of his wealth in the less liquid asset and will increase his precautionary demand for money, provided

$$\frac{\partial}{\partial r}\{rU'[W_0(1+ar)]\} > 0 \tag{10}$$

It is easily shown that (10) is equivalent to $U' + W_0arU'' > 0$. This condition can more meaningfully be stated in terms of the relative risk-aversion index $R = -WU''/U'$. Thus

$$\frac{\partial}{\partial r}\{rU'[W_0(1+ar)]\} = U'\left(1 - \left[\frac{ar}{1+ar}\cdot R\right]\right) \tag{11}$$

A necessary and sufficient condition for the right-hand side of (11) to be positive is that $R < (1+ar)/ar$. I argue that this is in fact likely to be the case. Since a is a fraction and r, the single-period rate of return on the risky asset, is presumably also a fraction, it is very likely for $r > 0$ that $(1+ar)/ar > 2$. Thus a sufficient condition for an increase in p to reduce the demand for the illiquid asset and to increase the demand for precautionary cash balances is that R, the relative risk-aversion index, be less than two. The literature on the theory of risk aversion suggests that this condition is indeed likely to be satisfied.[8] Thus for most utility functions an increase in the probability of having to liquidate one's portfolio on short notice should increase the precautionary demand for money if liquidity is defined in terms of differences in the means of $F_1(r_1)$ and $F_2(r_2)$.

Let us now consider a different interpretation of the term liquidity. Above it was assumed that only the mean of the subjective probability distribution was affected by forced sale of the portfolio. We now assume that the distribution over r_2 has the same mean as the distribution over r_1, but less variability. This is justified on the grounds that obtaining a larger sample can be expected to reduce the dispersion of the distribution over r.

Variability will be defined in the manner suggested by Michael Rothschild and Joseph Stiglitz (1970), viz.,

$$r_1 \underset{d}{=} r_2 + Z \tag{12}$$

where $E(Z|r_2)=0$. This should be read "r_1 has the same distribution as r_2 plus noise." Using this definition of variability we can apply the following theorem (pp. 237–38) to make statements about the sign of the numerator of equation (7).

THEOREM (Rothschild and Stiglitz): *The following statements are equivalent*:

$$Y \underset{d}{=} X + Z, \qquad E(Z|X) = 0 \tag{13}$$

$$EU(X) \geq EU(Y), \qquad U'' < 0 \tag{14}$$

As Rothschild and Stiglitz suggest (p. 67), this theorem can be applied to problems of expected utility maximization in which an agent seeks to

$$\max_{\alpha} \int U(X, \alpha) dF(X) \tag{15}$$

[8] Arrow (ch. 3) for example argues that R should be in the neighborhood of one. His argument is based on the fact that if $U(W)$ is bounded from below, then R cannot approach a limit *above* one as W approaches 0, while if $U(W)$ is bounded from above, R cannot approach a limit *below* one as W approaches infinity. From this Arrow concludes that ". . . broadly speaking, the relative risk aversion must hover around 1, being, if anything, somewhat less for low wealths and somewhat higher for high wealths" (p. 98).

The optimal α must satisfy

$$\int \frac{\partial U(X, \alpha) dF(X)}{\partial \alpha} = EU_\alpha(X, \alpha) = 0 \tag{16}$$

If U_α is monotone decreasing in α and if $U_\alpha(X, \alpha)$ is a concave function of X, then an increase in riskiness in the sense of (13) will by the theorem stated above imply

$$EU_\alpha(X, \alpha) \geq EU_\alpha(Y, \alpha) \tag{17}$$

In order to apply this result in the present case, we must see under what conditions $g(r) = rU'[W_0(1+ar)]$ is concave in r. (That $g(r)$ is a decreasing function of a is implied by equation (2).) For all utility functions for which $g(r)$ is concave, it will be true by (12) and (17) that $E[r_2U'(W_2)] \geq E[r_1U'(W_1)]$. Hence for all utility functions which imply $g(r)$ concave, $da/dp < 0$, and a well-behaved precautionary demand for money can be said to exist.

A necessary and sufficient condition for $g(r)$ concave is

$$\begin{aligned} g''(r) &= aW_0(aW_0rU''' + 2U'') \\ &= aW_0[(W - W_0)U''' + 2U''] < 0 \end{aligned} \tag{18}$$

Following Rothschild and Stiglitz, the term in brackets can be written in terms of the expressions for relative and absolute risk aversion, yielding

$$\begin{aligned} g''(r) = aW_0[&U''(1 - R + W_0A) \\ &+ U'(W_0A' - R')] < 0 \end{aligned} \tag{19}$$

where $R = -WU''/U'$ denotes relative risk aversion and $A = -U''/U'$ denotes absolute risk aversion. Equation (19) along with previous assumptions implies that a sufficient condition for $g(r)$ concave is that the investor's utility function exhibit nonincreasing absolute risk aversion, nondecreasing relative risk aversion, and a relative risk-aversion index less than $1 + W_0A$.

Arrow (ch. 3) has convincingly argued that nondecreasing relative and nonincreasing absolute risk aversion are reasonable conditions to impose on the utility function. If absolute risk aversion is decreasing, investors are willing to risk larger amounts as their wealth increases, a prediction which agrees with casual observation. Nondecreasing relative risk aversion implies that the wealth elasticity of demand for cash balances is greater than or equal to one. As Arrow notes (p. 103), this prediction agrees with empirical findings by Milton Friedman, Richard Selden and others.

The condition that R be less than $1+W_0A$ does not easily admit of an intuitive interpretation; however, it can be shown that this condition is satisfied by many of the utility functions in the constant relative and constant absolute risk aversion classes. Utility functions which satisfy this property include the logarithmic utility function, $U(W)=\log W$, and all constant relative risk-aversion utility functions, $U(W)=(1-b)W^{(1-b)}$, for which $0<b=R<(1+ar)/ar$.[9] As I have argued above for $R>0$ $(1+ar)/ar$ is very likely to be ≥ 2. Therefore all constant relative risk-aversion utility functions for which $R<2$ will most likely satisfy the requirement that $R<1+W_0A$. The constant absolute risk-aversion utility function $U(W)=-e^{-bW}$, $b>0$, satisfies the property $R<1+W_0A$ provided $b<1/arW_0$.[10] Since a is a fraction and r is likely to be a fraction also, the condition $b<1/(arW_0)$ does not appear unduly restrictive. These considerations suggest that for a large class of utility functions, the present framework leads to a well-defined precautionary demand for money when liquidity is used in the sense of "less variability."

Before concluding, note that the model developed above yields another implication which agrees with intuition. Assuming that the utility function exhibits nondecreasing relative risk aversion, it can be shown (see the Appendix) that the proportion of initial wealth invested in liquid assets for precautionary purposes should increase as wealth increases.

III

The purpose of this paper is three-fold: 1) to indicate how the precautionary demand for money can be incorporated into a portfolio selection model, albeit an extremely simplified portfolio selection model, using the notion of states of nature; 2) to give a possible interpretation to the term liquidity in the context of such a model; 3) to determine precisely what restrictions must be placed on an investor's utility function for a well-defined precautionary demand for money to exist.

I have argued that in a market in which there is a distribution of prices for any security rather than a single equilibrium price, it is reasonable to assume that an investor will perceive the distribution of prices with which he is faced as being more variable if he must sell the security on short notice than if he has more time in which to sample potential buyers. If this is the case, the investor will under the assumptions made above have a positive precautionary demand for money. Furthermore, an increase in the probability that his portfolio will have to be sold on short notice will increase the proportion of the portfolio invested in liquid assets, if the investor has a utility function which exhibits nonincreasing absolute and nondecreasing relative risk aversion and has a relative risk-aversion index which is less than $1+W_0A$. It can in addition be shown that the proportion of initial wealth invested in liquid assets for precautionary purposes should increase as wealth increases, provided that the utility function exhibits nondecreasing relative risk aversion.

Appendix

The purpose of this Appendix is to show that the proportion of initial wealth invested in liquid assets for precautionary purposes will increase as wealth increases if the investor's utility function exhibits nondecreasing relative risk aversion. To demonstrate this, it is sufficient to show that

[9] For the logarithmic utility function, $R=1$ and $A=W^{-1}$; hence $W_0A=W_0W^{-1}$. The condition $R<1+W_0A$ is always satisfied since by assumption W can never become negative. For the utility function $U(W)=(1-b)W^{(1-b)}$, $1+W_0A=1+bW_0W^{-1}$. The desired property $R<1+W_0A$ is equivalent to $b<1+bW_0W^{-1}$ in view of the fact that $R=b$. This will hold only if $b<1/(1-W_0W^{-1})$, which is equivalent to $b<(1+ar)/ar$ after substitution from $W=(1+ar)W_0$.

[10] In the case of the constant absolute risk aversion utility function $U(W)=-e^{-bW}$, $R=bW$, and $A=b$. The condition $R<1+W_0A$ is thus equivalent to $bW<1+bW_0$, which implies $b<(W-W_0)^{-1}$. Using the fact that $W=(1+ar)W_0$ this condition becomes $b<(arW_0)^{-1}$.

$$\text{(A2)}\quad \frac{da}{dW_0} = -\frac{pE[r_1U'(W_1)]+(1-p)E[r_2U'(W_2)]+pE[r_1W_1U''(W_1)]+(1-p)E[r_2W_2U''(W_2)]}{pW_0^2E[r_1^2U''(W_1)]+(1-p)W_0^2E[r_2^2U''(W_2)]}$$

$$\text{(A1)}\qquad R' \geq 0 \Rightarrow \frac{da}{dW_0} \leq 0$$

Implicit differentiation of the first-order condition for a maximum (equation (5)) with respect to a and W_0 yields equation (A2). The denominator of (A2) is negative if the second-order condition for a maximum is satisfied. The first-order condition for a maximum implies that the first two terms in the numerator of (A2) equal zero. Hence to establish (A1) it is sufficient to show that

$$\text{(A3)}\quad R' \geq 0 \Rightarrow pE[r_1W_1U''(W_1)] + (1-p)E[r_2W_2U''(W_1)] \leq 0$$

This is easily demonstrated following a method of proof due to Arrow (p. 120).

If relative risk aversion is nondecreasing then $R(W_1) \geq R(W_0)$ for $r_1 > 0$ and $W_1 = W_0(1+ar_1)$. By definition $R(W_1) = -W_1U''(W_1)/U'(W_1)$, hence

$$\text{(A4)}\quad r_1W_1U''(W_1) \leq -R(W_0)U'(W_1)r_1$$

This same inequality also holds for $r_1 \leq 0$. Taking the expectation of (A4) with respect to r_1,

$$\text{(A5)}\quad E[r_1W_1U''(W_1)] \leq -R(W_0)E[U'(W_1)r_1]$$

Because a similar inequality holds for $R(W_2)$ we have

$$\text{(A6)}\quad pE[r_1W_1U''(W_1)] + (1-p)E[r_2W_2U''(W_2)] \leq -R(W_0)\{pE[U'(W_1)r_1] + (1-p)E[U'(W_2)r_2]\}$$

But the first-order condition for a maximum implies that the term in braces is zero; hence the right-hand side of (A6) is also zero and (A3) has been shown to hold.

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