Dissipation in few-photon waveguide transport [Invited]

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We develop a formulation of few-photon Fock-space waveguide transport that includes dissipation in the form of reservoir coupling. We develop the formalism for the case of a two-level atom and then show that our formalism leads to a simple rule that allows one to obtain the dissipative description of a system from the nondissipative case.

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1. INTRODUCTION

The topic of dissipation in quantum optics is both important and subtle. Dissipation is important because many quantum optical systems are open; therefore tracking all of the out flowing energy is often not possible, but accounting for it is often quite crucial. Dissipation is subtle because energy in quantum systems cannot simply disappear, and naively describing dissipation with phenomenological damping factors in the system's equations of motion is formally incorrect, leading to a non-Hermitian Hamiltonian, violation of commutation relations [1], and time-reversal invariance [2].

A correct treatment of dissipation entails coupling the system under study, which usually contains discrete energy levels, to a reservoir that is modeled as having a continuum of energy states [1]. When this is done, the presence of dissipation in the system is accompanied by a source of fluctuation from the reservoir, enabling two-way energy exchange.

In quantum optics, the system–reservoir model is treated with many different approaches. In the Schrödinger picture, this model has been treated in the master equation approach, where multiple-time averages are calculated via the quantum regression theorem [2]. Alternatively in the Schrödinger picture one can use the stochastic approach of the Monte Carlo wave function [3], which involves evolving the wave function of the system with a non-Hermitian Hamiltonian term. In the Heisenberg picture, the input–output formalism [4] is widely used. In typical use of the input–output formalism, one assumes that the input is a weak coherent state. The nonlinear response is then often treated approximately in the weak excitation limit [5]. The input–output formalism is attractive because the external degrees of freedom, such as those of the reservoir, are integrated out, allowing one to focus on the dynamics of the system alone.

Recently, there has been great interest in describing the properties of photons propagating in a waveguide coupled to quantum multilevel atoms either directly [6–16] or through the use of a microcavity [17,18]. The one-dimensional nature of waveguide propagation allows incident and scattered photons to interfere coherently, which can give rise to significant capabilities in controlling transport [6,7,11,16,19–23], switching [17,24], and amplification [9,15,25] processes for few-photon states.

A formulation of input–output formalism has been recently developed that enables analytical calculation of few-photon scattering matrices in these systems of waveguide coupled to quantum multilevel atoms. In this paper we extend this formalism to include dissipation. We model loss by introducing an auxiliary reservoir that transports energy away from the atom.

In applying the input–output formalism to treat two-photon transport [8,18], a key step is to insert a complete set of basis states for the single-excitation Hilbert space. Here we show that, to obtain the correct result for two-photon transport in the presence of dissipation, such a basis set needs to include the states of the reservoir.

The paper is organized as follows. In Section 2 we present the Hamiltonian describing a two-level atom with reservoir coupling. In Section 3 we obtain the one and two-photon S-matrices of the system using the input–output formalism, showing that excluding the reservoir from the system description leads to violation of outgoing particle exchange symmetry. Finally, in Section 4 we conclude with a general recipe for the inclusion of dissipation in a general class of important and relevant systems.

2. SYSTEM HAMILTONIAN

We consider a two-level atom side-coupled to a waveguide with right and left moving photons, as described by the Hamiltonian

$$\begin{aligned} H_{\text{wvg+atom}} = \int \mathrm{d}x\Big(&-iv_g c_R^\dagger(x)\frac{d}{dx}c_R(x) + iv_g c_L^\dagger(x)\frac{d}{dx}c_L(x) \\ &+ \sqrt{\kappa}\delta(x)\Big[c_R^\dagger(x)\sigma_- + \sigma_+ c_R(x) + c_L^\dagger(x)\sigma_- + \sigma_+ c_L(x)\Big]\Big) \\ &+ \frac{\Omega}{2}\sigma_z. \end{aligned} \tag{1}$$

Here $c_R^\dagger(x)[c_R(x)]$ and $c_L^\dagger(x)[c_L(x)]$ create (annihilate) a right or left moving photon with group velocity v_g, respectively. κ is the coupling rate between the atom and waveguide. The form of the interaction here assumes the rotating-wave approximation. $\sigma_+(\sigma_-)$ raises (lowers) the state of the two-level atom whose transition frequency is Ω. We note that the waveguide model in $H_{\text{wvg+atom}}$ can describe any single-mode dielectric or plasmonic waveguide in the optical regime [13] or a transmission line in the microwave regime [26]. Since the atomic linewidth is typically quite narrow, a linear approximation of the waveguide dispersion relation, which we adopt here, is generally sufficient to describe the waveguide. We further note that the two-level atom in $H_{\text{wvg+atom}}$ can describe either a real atom or an artificial atom, such as a quantum dot [13] or superconducting qubit [15].

To model dissipation we introduce reservoir–atom coupling as described by H_r and consider the composite system $H = H_{\text{wvg+atm}} + H_r$ as shown schematically in Fig. 1, where

$$H_r = \int \mathrm{d}x\left(-iv_r b^\dagger(x)\frac{d}{dx}b(x) + \sqrt{\gamma}\delta(x)[b^\dagger(x)\sigma_- + \sigma_+ b(x)]\right). \tag{2}$$

Here $b(x)$ and $b^\dagger(x)$ are the reservoir photon operators and γ is the atom–reservoir coupling rate. We assume the reservoir has a continuum of modes centered around the atomic transition frequency Ω. We further assume that within the spectral range of interest the dispersion of these reservoir modes is well described by a group velocity v_r. The reservoir will initially be set to the vacuum state. As a result of interaction with the atom, photons originating in the waveguide may leak back into the waveguide, described in Eq. (1), or they may leak into the reservoir as described in Eq. (2), resulting in loss.

To proceed, we exploit the spatial inversion symmetry of H and decompose it to even and odd subspaces $H = H_e + H_o$, where $[H_e, H_o] = 0$. We transform from right- and left-moving waveguide photon operators to even and odd photon operators. We omit from here on the subscripts of the even mode operators:

$$a(x) = \frac{a_R(x) + a_L(-x)}{\sqrt{2}}, \tag{3a}$$

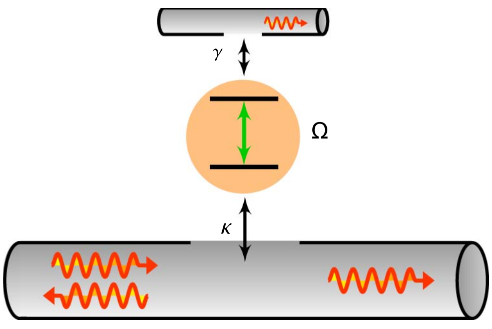

Fig. 1. Schematic of sample system considered. A waveguide side coupled with rate κ to a two-level atom with transition frequency Ω. The atom is additionally coupled with rate γ to a reservoir. The initial state in the waveguide is either a one- or a two-photon Fock state, whereas the auxiliary waveguide is initially in the vacuum state.

$$a_o(x) = \frac{a_R(x) - a_L(-x)}{\sqrt{2}}, \tag{3b}$$

with the above operators, the even subspace Hamiltonian H_e and the odd subspace Hamiltonian H_o follow

$$H_e = \int \mathrm{d}x\Big(-iv_g a^\dagger(x)\frac{d}{dx}a(x) - iv_r b^\dagger(x)\frac{d}{dx}b(x) + V\delta(x)[a^\dagger(x)\sigma_- + \sigma_+ a(x)] + V_r\delta(x)[b^\dagger(x)\sigma_- + \sigma_+ b(x)]\Big) + \frac{1}{2}\Omega\sigma_z,$$

$$H_o = -iv_g\int \mathrm{d}x a_o^\dagger(x)\frac{d}{dx}a_o(x).$$

Here H_e describes a one-way propagating chiral waveguide mode that interacts with the atom. H_o describes a one-way propagating waveguide mode without any interaction with the atom. Once the solutions for H_e are known, the solution to $H_{\text{wvg+atm}} + H_r$ can be straightforwardly obtained by using Eqs. (3a) and (3b). In what follows we consider only the even Hamiltonian, since it contains all of the nontrivial physics of the system.

The Hamiltonian H_e can be recast in k-space by defining momentum-space photon operators $a_k \equiv (1/\sqrt{2\pi})\int \mathrm{d}x a(x)e^{-ikx}$ and $b_k \equiv (1/\sqrt{2\pi})\int \mathrm{d}x b(x)e^{-ikx}$,

$$H_e = \int \mathrm{d}k\left(kv_g a_k^\dagger a_k + kv_r b_k^\dagger b_k + \sqrt{\frac{\kappa}{2\pi}}[a_k^\dagger\sigma_- + \sigma_+ a_k] + \sqrt{\frac{\gamma}{2\pi}}[b_k^\dagger\sigma_- + \sigma_+ b_k]\right) + \frac{1}{2}\Omega\sigma_z.$$

We note that the total number of excitations $N_{\text{exc}} = \int \mathrm{d}k a_k^\dagger a_k + \int \mathrm{d}k b_k^\dagger b_k + \sigma_+\sigma_-$ commutes with H_e, H_o, allowing for the solution to proceed independently in each excitation number manifold.

3. INPUT–OUTPUT FORMALISM

In the absence of dissipation, the one- and two-photon scattering matrices of this system were solved in [10], to which we refer the reader for a detailed derivation of the formalism we use. Our focus here is on the addition of dissipation to the formalism.

By defining input and output field operators for the waveguide,

$$a_{\text{in}}(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}k a_k(t_0)e^{-ik(t-t_0)},$$

$$a_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}k a_k(t_1)e^{-ik(t-t_1)},$$

and reservoir,

$$b_{\text{in}}(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}k b_k(t_0)e^{-ik(t-t_0)},$$

$$b_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}k b_k(t_1)e^{-ik(t-t_1)},$$

we obtain (in the limits $t_0 \to -\infty$ and $t_1 \to \infty$) the input–output equations,

$$a_{\mathrm{out}}(t) = a_{\mathrm{in}}(t) - i\sqrt{\kappa}\sigma_-(t), \tag{4}$$

$$b_{\mathrm{out}}(t) = b_{\mathrm{in}}(t) - i\sqrt{\gamma}\sigma_-(t), \tag{5}$$

$$\frac{d\sigma_-(t)}{dt} = -i\left(\Omega - i\frac{\kappa}{2} - i\frac{\gamma}{2}\right)\sigma_-(t) + i\sqrt{\kappa}\sigma_z(t)a_{\mathrm{in}}(t) + i\sqrt{\gamma}\sigma_z(t)b_{\mathrm{in}}(t). \tag{6}$$

We see that the presence of dissipation in Eq. (6) in the form of damping rate γ is accompanied by the input field operator $i\sqrt{\gamma}\sigma_z(t)b_{\mathrm{in}}(t)$ from the reservoir. Below we use Eqs. (4)–(6) to obtain one and two-photon S-matrices.

A. One-Photon S-Matrix

The one-photon S-matrix relates an incoming free-photon state $|k\rangle$ with momentum k to an outgoing free-photon state $|p\rangle$ with momentum p and is given by $\langle p|S|k\rangle = \langle p^-|k^+\rangle$, where S is the scattering operator and $|k^+\rangle \equiv a_{\mathrm{in}}^\dagger(k)|0\rangle$ is an incoming one-excitation interacting eigenstate that evolved from a free-photon state $|k\rangle$ in the distant past. $|p^-\rangle \equiv a_{\mathrm{out}}^\dagger(p)|0\rangle$ is an outgoing one-excitation interacting eigenstate that will evolve into a free photon state $|p\rangle$ in the distant future. The one-photon S-matrix can be reexpressed as follows:

$$\langle p^-|k^+\rangle = \langle 0|a_{\mathrm{out}}(p)a_{\mathrm{in}}^\dagger(k)|0\rangle = \delta(k-p) - i\sqrt{\kappa}\langle 0|\sigma_-(p)|k+\rangle,$$

where we have used the input–output relation in Eq. (4) and $\langle 0|a_{\mathrm{in}}(p)|k^+\rangle = \delta(k-p)$ [10]. To obtain the matrix element $\langle 0|\sigma_-(p)|k^+\rangle$, we solve for its time-domain counterpart $\langle 0|\sigma_-(t)|k^+\rangle$ using Eq. (6), noting that $\langle 0|\sigma_z(t)b_{\mathrm{in}}(t)|k^+\rangle = 0$ and $\langle 0|\sigma_z(t)a_{\mathrm{in}}(t)|k^+\rangle = -(1/\sqrt{2\pi})e^{-ikt}$, where $|k^+\rangle_r = b_{\mathrm{in}}^\dagger(k)|0\rangle$ is an incoming reservoir eigenstate. The resulting one-photon transport matrix elements are

$$\langle 0|\sigma_-(p)|k^+\rangle = \frac{\sqrt{\kappa}\delta(k-p)}{k-\Omega+i\frac{\kappa}{2}+i\frac{\gamma}{2}} \equiv s_k\delta(k-p),$$

$$\langle 0|\sigma_-(p)|k^+\rangle_r = \frac{\sqrt{\gamma}\delta(k-p)}{k-\Omega+i\frac{\kappa}{2}+i\frac{\gamma}{2}}\delta(k-p) \equiv s_k^{(r)}\delta(k-p),$$

$$\langle p^-|k^+\rangle = \frac{k-\Omega-i\left(\frac{\kappa}{2}-\frac{\gamma}{2}\right)}{k-\Omega+i\left(\frac{\kappa}{2}+\frac{\gamma}{2}\right)}\delta(k-p) = t_k\delta(k-p),$$

$$_r\langle p^-|k^+\rangle = \frac{-i\sqrt{\kappa\gamma}\delta(k-p)}{k-\Omega+i\left(\frac{\kappa}{2}+\frac{\gamma}{2}\right)} \equiv t_k^{(r)}\delta(k-p).$$

We note that, as a result of reservoir damping, $s_k \neq t_k s_k^*$. This seemingly innocuous difference between the dissipative case and the dissipation-free case stems from the fact that the set $|k^+\rangle$ is not complete in the presence of dissipation. We will see the crucial importance of this fact in the two-photon calculation.

B. Two-Photon S-Matrix

The transport of two waveguide photons is given by the two-photon S-matrix, which describes the probability amplitude of incoming photons with momenta k_1 and k_2 to scatter into two outgoing photons with momenta p_1 and p_2 and can be expressed as

$$\langle p_1^-, p_2^-|k_1^+, k_2^+\rangle = \langle 0|a_{\mathrm{out}}(p_2)a_{\mathrm{out}}(p_1)a_{\mathrm{in}}^\dagger(k_1)a_{\mathrm{in}}^\dagger(k_2)|0\rangle.$$

To proceed, we insert the resolution of the identity in the single-excitation subspace

$$\hat{I}_1 = \int \mathrm{d}k|k^+\rangle\langle k^+| + \int \mathrm{d}k|k^+\rangle_{rr}\langle k^+|,$$

which comprises of a sum of both waveguide and reservoir interacting eigenstates outer products

$$\begin{aligned}\langle p_1^-, p_2^-|k_1^+, k_2^+\rangle &= \int \mathrm{d}k\langle p_1^-|k^+\rangle\langle k^+|a_{\mathrm{out}}(p_2)a_{\mathrm{in}}^\dagger(k_1)a_{\mathrm{in}}^\dagger(k_2)|0\rangle \\ &+ \int \mathrm{d}k\langle p_1^-|k^+\rangle_{rr}\langle k^+|a_{\mathrm{out}}(p_2)a_{\mathrm{in}}^\dagger(k_1)a_{\mathrm{in}}^\dagger(k_2)|0\rangle \\ &= t_{p_1}\langle p_1^+|a_{\mathrm{out}}(p_2)|k_1^+, k_2^+\rangle + t_{p_1}^{(r)}{}_r\langle p_1^+|a_{\mathrm{out}}(p_2)|k_1^+, k_2^+\rangle,\end{aligned}$$

where we have used the definitions of the one-photon scattering amplitudes $\langle p_1^-|k^+\rangle = t_k\delta(k-p)$ and $_r\langle p_1^-|k^+\rangle = \langle p_1^-|k^+\rangle_r = t_k^{(r)}\delta(k-p)$. Inserting the Fourier transform of Eq. (4) and noting that $_r\langle p_1^+|a_{\mathrm{in}}(p_2)|k_1^+, k_2^+\rangle = 0$, we have

$$\begin{aligned}&= t_{p_1}[\delta(k_1-p_1)\delta(k_2-p_2) + \delta(k_1-p_2)\delta(k_2-p_1)] \\ &\quad - i\sqrt{\kappa}t_{p_1}\langle p_1^+|\sigma_-(p_2)|k_1^+, k_2^+\rangle - i\sqrt{\kappa}t_{p_1}^{(r)}{}_r\langle p_1^+|\sigma_-(p_2)|k_1^+, k_2^+\rangle.\end{aligned} \tag{7}$$

We now need to solve for the matrix elements $\langle p_1^+|\sigma_-(p_2)|k_1^+, k_2^+\rangle$ and $_r\langle p_1^+|\sigma_-(p_2)|k_1^+, k_2^+\rangle$. To do so we sandwich Eq. (6) between the appropriate states, getting the equations

$$\frac{d}{dt}\langle p_1|\sigma_-(t)|k_1^+, k_2^+\rangle = -i\left(\Omega - i\frac{\kappa}{2} - i\frac{\gamma}{2}\right)\langle p_1^+|\sigma_-(t)|k_1^+, k_2^+\rangle + i\sqrt{\kappa}\langle p_1^+|\sigma_z(t)a_{\mathrm{in}}(t)|k_1^+, k_2^+\rangle, \tag{8}$$

$$\frac{d}{dt}{}_r\langle p_1|\sigma_-(t)|k_1^+, k_2^+\rangle = -i\left(\Omega - i\frac{\kappa}{2} - i\frac{\gamma}{2}\right)_r\langle p_1^+|\sigma_-(t)|k_1^+, k_2^+\rangle + i\sqrt{\kappa}{}_r\langle p_1^+|\sigma_z(t)a_{\mathrm{in}}(t)|k_1^+, k_2^+\rangle, \tag{9}$$

where we note that $\langle p_1^+|\sigma_z(t)b_{\mathrm{in}}(t)|k_1^+, k_2^+\rangle = 0$ and $_r\langle p_1^+|\sigma_z(t)b_{\mathrm{in}}(t)|k_1^+, k_2^+\rangle = 0$. To proceed, we solve for the source terms in Eqs. (8) and (9) by using $\sigma_z = 2\sigma_+\sigma_- - 1$ and inserting a resolution of the identity $\hat{I}_0 = |0\rangle\langle 0|$ in the zero-excitation Hilbert space,

$$\langle p_1^+|\sigma_z(t)a_{\rm in}(t)|k_1^+,k_2^+\rangle = \frac{e^{-ik_2t}}{\sqrt{2\pi}}\left[\frac{1}{\pi}s_{p_1}^* s_{k_2}e^{i(p_1-k_2)t} - \delta(p_1-k_2)\right] + \frac{e^{-ik_1t}}{\sqrt{2\pi}}\left[\frac{1}{\pi}s_{p_1}^* s_{k_1}e^{i(p_1-k_1)t} - \delta(p_1-k_1)\right],$$

$$_r\langle p_1^+|\sigma_z(t)a_{\rm in}(t)|k_1^+,k_2^+\rangle = \frac{e^{-ik_2t}}{\sqrt{2\pi}}\frac{1}{\pi}s_{p_1}^{*(r)} s_{k_2}e^{i(p_1-k_2)t} + \frac{e^{-ik_1t}}{\sqrt{2\pi}}\frac{1}{\pi}s_{p_1}^{*(r)} s_{k_1}e^{i(p_1-k_1)t}.$$

Having calculated the source terms, we may now solve Eqs. (8) and (9) and Fourier transform their solution to obtain

$$_r\langle p_1^+|\sigma_-(p_2)|k_1^+,k_2^+\rangle = -\frac{1}{\pi}s_{p_2}s_{p_1}^{*(r)}[s_{k_1}+s_{k_2}]\delta(p_1+p_2-k_1-k_2), \quad (10)$$

$$\langle p_1^+|\sigma_-(p_2)|k_1^+,k_2^+\rangle = s_{p_2}\{[\delta(p_2-k_1)\delta(p_1-k_2)+\delta(p_2-k_2)\delta(p_1-k_1)] - \frac{1}{\pi}s_{p_1}^*[s_{k_1}+s_{k_2}]\delta(p_1+p_2-k_1-k_2)\}. \quad (11)$$

Finally, we insert Eqs. (10) and (11) into Eq. (7) to get the two-photon S-matrix,

$$\langle p_1^-,p_2^-|k_1^+,k_2^+\rangle = t_{p_1}t_{p_2}[\delta(p_2-k_1)\delta(p_1-k_2)+\delta(p_2-k_2)\delta(p_1-k_1)] + i\frac{\sqrt{\kappa}}{\pi}s_{p_1}s_{p_2}[s_{k_1}+s_{k_2}]\delta(p_1+p_2-k_1-k_2). \quad (12)$$

We note that the two-photon S-matrix is invariant under exchange of incoming and outgoing momenta $k_1,k_2\leftrightarrow p_1,p_2$ as required by time-reversal symmetry, and also with respect to exchange of incoming or outgoing momenta $k_1\leftrightarrow k_2$ and $p_1\leftrightarrow p_2$, as required by the Bose statistics of photons.

Instead of the correct approach outlined above, one might instead consider an *ad hoc* approach where an imaginary part is added to the frequency of the atom, resulting in the following Hamiltonian:

$$H_{\text{ad-hoc}} = \int {\rm d}k\left(kv_g a_k^\dagger a_k + \sqrt{\frac{\kappa}{2\pi}}[a_k^\dagger\sigma_- + \sigma_+ a_k]\right) + \frac{1}{2}\left(\Omega - i\frac{\gamma}{2}\right)\sigma_z.$$

From this *ad hoc* Hamiltonian, one then follows the steps in [10]. With this approach, the Hilbert space consists of only waveguide and atom states—with no reservoir. For one photon, this approach yields the correct S-matrix. For two photons, the term of Eq. (10) would have been missing, and the resulting incorrect two-photon S-matrix would have been

$$\langle p_1^-,p_2^-|k_1^+,k_2^+\rangle = t_{p_1}t_{p_2}[\delta(p_2-k_1)\delta(p_1-k_2)+\delta(p_2-k_2)\delta(p_1-k_1)] + i\frac{1}{\pi}\sqrt{\kappa}t_{p_1}s_{p_1}^*s_{p_2}[s_{k_1}+s_{k_2}]\delta(p_1+p_2-k_1-k_2). \quad (13)$$

This form of the S-matrix violates the particle exchange symmetries mentioned above.

If we examine the form of the S-matrix in Eq. (13), it might seem odd that, even though the result is expressible by single-photon scattering amplitudes, it violates exchange symmetry of outgoing particle momenta while the one-photon S-matrix does not. This is explained as follows. The physical origin of the symmetry violation is in the scattering amplitude $\langle p_1^+|\sigma_+|0\rangle$. This amplitude is proportional to the Fourier coefficient $s_{p_1}^*$ of the atom's spontaneous emission amplitude into the waveguide. However, in the presence of dissipation, the atom may also spontaneously emit a photon into the reservoir, corresponding to the amplitude $_r\langle p_1|\sigma_+|0\rangle$. This amplitude is provided by the term in Eq. (10) and originates in our inclusion of the reservoir when inserting the two-excitation resolution of the identity $\hat{I}_2 = \int {\rm d}k|k^+\rangle\langle k^+| + \int {\rm d}k|k^+\rangle_{rr}\langle k^+|$.

4. A GENERAL RULE FOR DISSIPATION

When examining the correct form of the two-photon S-matrix in Eq. (12), we see that the presence of dissipation is seen as the addition of a $i(\gamma/2)$ term in the one-excitation transmission and atomic excitation amplitudes. It follows that the one- and two-photon S-matrices of a two-level atom with reservoir coupling rate γ, as described by $H_{\rm wvg+atm}+H_r$, can be obtained from the dissipation-free S-matrices by making the replacement $\Omega' = \Omega - i(\gamma/2)$. However, we stress that this substitution should be made in the S-matrix and not in the Hamiltonian, since, as described above, doing so would leads to an incorrect result in the two-photon case.

This result is even more general. In the presence of N reservoir dissipation channels with coupling rates $\gamma_1,\gamma_2,\ldots,\gamma_N$, the resulting S-matrix is obtained from the dissipation-free S-matrix by making the substitution $\Omega' = \Omega - i\sum_{n=1}^N \gamma_n/2$. Moreover, this result holds for a general quantum multilevel system as long as the Hamiltonian light-matter interaction term can be described in the rotating-wave approximation. Furthermore, the quantum multilevel system can be either in a cavity or directly coupled to a waveguide.

Finally, the eigenstate of the *ad hoc* Hamiltonian turns out to be the same as the true scattering eigenstate—other than the reservoir degree of freedom, which it is missing. This fact allows a correct real-space interacting eigenstate solution [6,7,27,28] for the nonreservoir dynamics that account for the loss of energy that accompanies reservoir coupling, in complete agreement with the solutions presented here. In particular, the existence of two-photon bound states, which is one of the important predictions made by using the wave-function method, is still valid in the presence of dissipation. In conclusion, the method described in this paper provides a straightforward and correct construction of the S-matrix in the presence of dissipation while working with the full Hermitian Hamiltonian of the system in the context of the input–output formalism.

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