Prediction of Daily Maximum Streamflow Based on Stochastic Approaches

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Abstract: This study analyzed daily maximum streamflow data of each month from three gauge stations on Cekerek Stream for simulation using stochastic approaches. Initially non-parametric test (Mann-Kendall) was used to identify the trend during study period. The two approaches of stochastic modeling, ARIMA and Thomas-Fiering models, were used to simulate monthly maximum data. The error estimates (RMSE and MAE) of predictions from both approaches were compared to identify the most suitable approach for reliable simulation. The two error estimates calculated for two approaches indicate that ARIMA model appear to be slightly better than Thomas-Fiering. However, both approaches were identified as appropriate method for simulating daily maximum streamflow data of each month from three gauge stations on Cekerek Stream.

Key words: Daily maximum streamflow, stochastic model, ARIMA, Thomas-Fiering

Introduction

The prediction of flood resulting from heavy rain over a catchment is one of the major problems in applied hydrology. The engineering design of hydraulic structure demands reliable information concerning the peak flow to be expected after a rainstorm of a given probability of occurrence. The estimation of design floods is, in practice, often based on small samples of data, which may cause a severe uncertainty. In this sense, the hydrologist often faced with the problem of predicting extreme flood events on basis of samples of historical flood records. Therefore, the main problem is to obtain reliable estimates of floods with given return period or, alternatively, estimates of exceedance probabilities of certain flood magnitudes. But, for many water resource studies the available streamflow records are often scarce, which implies an uncertainty of the flood prediction. In many hydrologic applications, information based on continuous discharge or flow measurements is the basis of analysis and decision-making. Haan (1977) expressed that ultimately design decisions must be based on a stochastic model or a combination of stochastic and deterministic models. This is because any system must be designed to operate in the future. Therefore, simulation is important to obtain adequate and reliable information related to hydraulic design and management of any structure.

Most of the statistical methods used in hydrologic studies are based on the assumption that the observations are independently distributed in time. The occurrence of an event is assumed to be independent of all previous events. This assumption is not always valid for hydrologic time series (Chow 1964).

Sharma et al. (1997) cited that it is very important to generate synthetic streamflow sequences to analyze alternative designs, operation policies, and rules for water resources systems, and that the dependence structure of streamflow sequences is often assumed to be Markovian, that is, dependent on only a fine set of prior values. Iturbe et al. (1972) noted that generating extreme values are the most significant in design and planning. Therefore, they compared Markovian model, fractional Gaussian noise and crossing theory in simulation studies of hydrologic record, and stated that crossing theory preserved more properties of hydrologic interest more easily than the two other models. Additional to these, McMichael and Hunter

(1972) stated that providing good forecast functions for time dependent data was a common problem.

See and Openshaw (1998) enchanced flood forecasting on the river Ouse by using ARIMA model. Hsu et al. (1995) used an ARMA model for the prediction of streamflow on a medium sized basin in Mississippi. Chaloulakou et al. (1999) forecasted the daily maximum 1-hour ozone concentrations by ARIMA model.

The work in this paper is concerned with the application of autoregressive integrated moving average and Thomas-Fiering models to simulate the daily maximum streamflow data of each month (hereafter referred to as monthly maximum data) from three gauge stations on Cekerek Stream.

Material And Method

Study Area

In this study, monthly maximum data from three gauge stations as numbered 1404, 1409 and 1424, which are managed by General Directorate of Electric Power Research Survey and Development Administration (EIE), in Cekerek Stream watershed were used as materials. The approximate locations of the gauge stations were given in Figure 1 and a summary of identification number, names and drainage areas for the gauge stations was presented in Table 1.

Cekerek Stream watershed is bounded 39° 30' and 40° 45' N latitudes, 35° 15' and 36° 15' E longitudes, covering approximately 1165440 ha which is about 1.5% of Turkey's total area. The study area is located on the north Anatolia fault line that is one of the most effective faults in the world. Therefore, tectonic movement affects this watershed area. Cekerek Stream is formed by joining together of small streams that originate from Kızık, Dinar, Calı and Kavak hills, near the Camlıbel district. Cekerek Stream joins to Yesilirmak River near Kayabası. The stream is approximately 276 km in length and water quality of the stream is C_2S_1 for irrigation (Anonymous 1970).

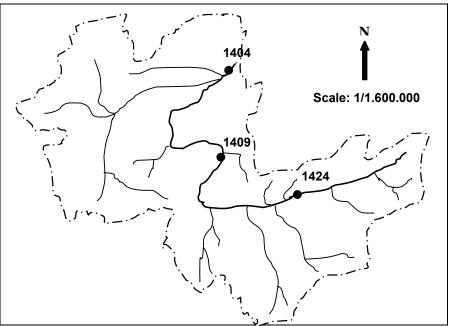


Figure 1. Location of gauge stations on Cekerek Stream

Station	Station Name	Drainage Area,	Number of years of	
Number		km ²	data	
1404	Cekerek-Kayabası	11724.0	13	
1409	Cekerek-Akcakecili	5267.6	38	
1424	Cekerek-Cırdak Bridge	1032.8	27	

Table 1. Cekerek Stream Gauge Station Identification

Time Series Analysis for Monthly Maximum Streamflow Data

In order to analyze time series for monthly maximum data from the three gauge stations, linear stochastic models known as either Box-Jenkins or ARIMA (autoregressive integrated moving average) and Thomas-Fiering were used in this study.

ARIMA Model

For fitting seasonal ARIMA model to the time series of monthly maximum streamflow data, three-stage procedure of model identification, estimation of model parameters and diagnostic checking of estimated parameters has been adopted. This seasonal ARIMA model (Hipel et al. 1977) denoted as ARIMA $(p,d,q)^*(P,D,Q)_s$ is expressed as

$$\mathcal{Q}(B)\Phi(B^{s})(w_{i} - \mu) = \theta(B)\Theta(B^{s})a_{i}$$
(1)

$$w_i = (1-B)^d (1-B^s)^D x_i$$
 (2)

In Equation 1, w_i should be taken as z_i if the series is stationary.

Identification stage is purposed to determine the differencing required to produce stationarity and also the order of both the seasonal and nonseasonal AR and MA operators for a given series. By plotting original series (monthly series), seasonality, trends in the mean and variance may be revealed (Box and Jenkins 1976). The following non-parametric test (Mann-Kendall) can be applied to decide whether trend exists in the monthly maximum data. The Mann-Kendall test recommended by Hirsch et al. (1982) is given as:

$$u_{c} = \frac{S+m}{\sqrt{V(S)}}$$
(3)

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Z_k$$
(4)

$$\begin{aligned} z_k &= 1 & \text{if } x_j > x_i \\ z_k &= 0 & \text{if } x_j = x_i \\ z_k &= -1 & \text{if } x_j < x_i \end{aligned} \tag{5}$$

$$V(S) = 18^{-1}(n^2 - n)(2n + 5) - \sum_{i=1}^{t} e_i(e_i - 1)(2e_i + 5)$$
(6)

To determine whether there is a trend, u_c statistic in Equation 3 should be compared to the z-table critical value. If the u_c statistic lies within the 5% significance interval, there is no

trend for the data set. The hypothesis of an upward or downward trend cannot be rejected at the α significance level if the absolute value of $u_c > u_{1-\alpha/2}$, where $u_{1-\alpha/2}$ is the 1- $\alpha/2$ quantile of standard normal distribution.

Autocorrelation function (ACF) and partial autocorrelation function (PACF) should be used to gather information about the seasonal and nonseasonal AR and MA operators for the monthly maximum series. Autocorrelation function measures the amount of linear dependence between observations in a time series. Therefore, the most useful device is the autocorrelation function of the time series. In this sense, the identification of the appropriate parametric time series model depends on the shape of ACF. Additional to ACF, a powerful complementary identification tool, the partial autocorrelation function (PACF) can also be used (Janacek and Swift 1993).

Estimation stage consists of using the data to estimate and to make inferences about values of the parameters conditional on the tentatively identified model. In an ARIMA model, the residuals (a_i) are assumed to be independent, homoscedastic, and usually normally distributed. However, if the constant variance and normality assumptions are not true, they are often reasonably well satisfied when the observations are transformed by a Box-Cox transformation. The transformations can be expressed as either of the following equations (Wei 1990):

$$z_{i=1}^{n} = \lambda^{-1} \left[\left(x_{i=1}^{n} + c \right)^{\lambda} - 1 \right] \qquad \lambda \neq 0$$
(8)

$$z_{i=1}^{n} = \ln(x_{i=1}^{n} + c) \qquad \lambda = 0$$
(9)

Box and Jenkins (1976) cited that the model should be parsimonious. Therefore, they recommended the need to use as few model parameters as possible so that the model fulfils all the diagnostic checks. Akaike (1974) suggests a mathematical formulation of the parsimony criterion of model building as AIC (Akaike Information Criterion) for the purpose of selecting an optimal model fits to a given data. Mathematical formulation of AIC is defined as:

AIC (M) = n
$$\ln\sigma_a^2 + 2M$$
 (10)

Where M is the number of AR and MA parameters to estimate. The model that gives the minimum AIC is selected as a parsimonious model.

Shibata (1976) has shown that the AIC criterion tends to overestimate the order of the autoregression. But, Akaike (1978, 1979) has developed a Bayesian extension of minimum AIC procedure, called as BIC. Similar to Akaike's BIC, Schwarz (1978) suggested the following Bayesian criterion for model selection, which has been called Schwarz Bayesian Criterion (SBC):

$$SBC (M) = n \ln \sigma_a^2 + M \ln n$$
(11)

Diagnostic check stage determines whether residuals are independent, homoscedastic and normally distributed. The residual autocorrelation function (RACF) should be obtained to determine whether residuals are white noise. There are two useful applications related to RACF for independence of residuals. The first one is the correlogram drawn by plotting r_k (a) against lag k. If some of the RACF are significantly different from zero, this may mean that the present model is inadequate. The second one is Q (r) statistic suggested by Ljung-Box (1978). A test of this hypothesis can be done for the model adequacy by choosing a level of significance and then comparing the value of calculated χ^2 to χ^2 -table of critical value. If the calculated χ^2 value is less than the χ^2 -table critical value, the present model is adequate on the basis of available data. The Q (r) statistic is calculated by using:

$$Q(r) = n(n+2)\sum_{k=1}^{m} (n-k)^{-1} r_k(a)^2$$
(12)

$$\mathbf{r}_{k}(\mathbf{a}) = \sum_{i=k+1}^{n} a_{i}a_{i-k} / \sum_{i=1}^{n} a_{i}^{2}$$
(13)

The following test described by Breusch and Pagan (1979) is very useful to determine whether a transformation of the data is needed. If there is a change in variance (heteroscedasticity) of residuals, a transformation is necessary for the data. For the test, the residuals from the model fitted to the data are divided into two groups. Then, residual sum of squares (ESS_F, ESS_S) for these groups are obtained. Breusch-Pagan test statistic (F_{cal}) is obtained from the following equation. If F_{cal} is smaller than F-table critical value, the residuals are assumed to be homoscedastic.

$$F_{cal} = \frac{ESS_{S}/(n_{S} - k_{p})}{ESS_{F}/(n_{F} - k_{p})} \approx F_{table} [(n_{S} - k_{p}), (n_{F} - k_{p})]$$
(14)

There are many standard tests available to check whether the residuals are normally distributed. Chow et al. (1988) cited that if historical data are normally distributed, the graph of the cumulative distribution for the data should appear as a straight line when it is plotted on normal probability paper. Haan (1977) expressed that the other way to check normality of residuals is the Kolmogorov-Smirnov (K-S) method.

Thomas-Fiering Model

Thomas-Fiering model presents a set of 12 regression equations. This linear stochastic model is used for generating synthetic monthly data. The well-known Thomas –Fiering model equation can be given as (Clarke 1984):

$$\frac{X_{i,j} - \overline{Q_j}}{S_j} = r_j \frac{X_{i,j-1} - \overline{Q_{j-1}}}{S_{j-1}} + a_{ij} \sqrt{(1 - r_j^2)}$$
(15)

Comparison of the Results

Two error estimates were taken into consideration for comparison of the results from ARIMA and Thomas-Fiering approaches (Antonopoulos et al. 2001). The first is the Root Mean Square Error (RMSE) which is given as:

RMSE =
$$\sqrt{n^{-1} \sum_{i=1}^{n} \{Q_{obs}(i) - Q_{pred}(i)\}^2}$$
 (16)

The second is the Mean Absolute Error (MAE), which is defined as

$$MAE = n^{-1} \sum_{i=1}^{n} |Q_{obs}(i) - Q_{pred}(i)|$$
(17)

Results and Discussion

To determine whether there is a trend in monthly maximum streamflow data sequences from 1404, 1409 and 1424 gauge stations, the non-parametric test (Mann-Kendal test) at 5% significance level was applied to monthly maximum data sequences. Mann-Kendal test results were given in Table 2. The Mann-Kendal statistic (u_c) values of monthly maximum data from three gauge stations were between z-table critical values (± 1.96) at 5% significant level. This suggests that there is no linear trend in monthly maximum data sequences of each mentioned gauge station.

The plots of the ACFs and PACFs drawn for monthly maximum data sequences are examined in order to identify the form of the ARIMA model. The ACFs for monthly maximum data follow an attenuating sine wave pattern that reflects the random periodicity of the data and possibly indicates the need for non-seasonal and/or seasonal AR terms in the model. For these data sequences, the cyclic seasonal component was removed by taking the seasonal differencing operator as one (1).

All the ACFs were significantly different from zero. Additional to this, Ljung-Box Q statistics were estimated. They emphasize that the ACFs obtained from monthly maximum data sequences were significantly different from zero. In other words, there was a linear dependence between monthly maximum observations. However, the ACFs did not cut off but rather damped out. This may suggest the presence of autoregressive (AR) terms. The PACFs possess significant values at some lags but rather tail off. This may imply the presence of moving average (MA) terms. The ACFs have significant values at lags that are multiples of 12. This may stress that seasonal AR terms are required but these values attenuate. There are peaks on graphs of the PACFs at lags that are multiples of 12 that may suggest seasonal MA terms, but these peaks damp out.

Alternative ARIMA models were estimated by considering the ACFs and PACFs graphs from the monthly maximum data. The SBC was taken into account for obtaining a parsimonious model among these alternatives. The model that has the minimum SBC was assumed to be parsimonious. In addition to this, model parameters were analyzed at 5% significant level by using t-test to select the best model fit to the data. If there is any parameter significant at a level 5%, it was eliminated.

Diagnostic checks were applied in order to determine whether the residuals of the selected models from the ACF and PACF graphs were independent, homoscedastic and normally distributed. A Box-Cox transformation was required for monthly maximum data for all gauge stations. By substituting λ , as -0.5 for monthly maximum data sequences from gauge stations 1409 and 1424, and as zero (0.0) for monthly maximum data sequences from gauge station 1404, and constant (c), as 1.0 for 1409 and 1424 and 0.0 for 1404 gauge station in Equations (6) and (7), a Box-Cox transformation caused the residuals to be homoscedastic and approximately normally distributed.

The models with the minimum SBC among the selected models that fulfilled all the diagnostic checks were selected as the best model for monthly maximum data sequences from the gauge stations. The selected best models for the gauge stations are presented in Table 2. The critical assumption of independence for the RACFs of the residuals was done by using the χ^2 distributed Ljung-Box Q statistic. The probabilities of Q statistics calculated for the best models were given in Table 2. Since the probabilities of Q statistics are greater than 0.05, the residuals from the best models are not significantly different from zero. Similarly, the RACF drawn for the best models indicated that the residuals were not significantly different from a white noise series at 5% significance level. Inspection of the RACF and the residual integrated periodogram (Figure 2) confirmed a strong model fit.

In Table 2, test results from Kolmogorov-Smirnov method for the normality and test results from Breusch-Pagan approach for homoscedascity of the residuals are also given. Since the normality and Breusch-Pagan test results are greater than 0.025 and 0.05, respectively, all the diagnostic checks for the residuals are fulfilled (Table 2).

Gauge	ARIMA	Model Statistics						
Station	Model	uc	AIC	SBC	LBQ/P	Norm	Homosce	σ_a^2
1404	(1,0,0)(0,1,1)	0.008	349.9	355.8	0.625	0.584	0.994	0.570
1409	(1,0,2)(0,1,1)	0.004	-219.5	-203.1	0.569	0.035	0.900	0.034
1424	(2,0,1)(0,1,1)	0.000	-34.2	-19.20	0.281	0.217	0.820	0.048

Table 2. The ARIMA models selected for Cekerek Stream gauge stations

The value (V) of the parameters associated standard errors (SEV), t-ratios and probabilities (<5%) for the standard errors are listed in Table 3. The standard errors calculated for the model parameters were rather small compared to the parameter values. Therefore, all of the parameters are significant and these parameters should be included in the models (Table 3).

Gauge Station	Model	Variables in the Model				
Gauge Station	Parameters	V	SEV	t-ratio	Probability	
1404	Ø ₁	0.519	0.068	7.69	0.000	
	Θ ₁	0.884	0.104	8.48	0.000	
1409	Ø ₁	0.874	0.049	17.71	0.000	
	θ1	0.468	0.069	6.76	0.000	
	θ_2	0.125	0.057	2.17	0.031	
	Θ ₁	0.919	0.029	32.06	0.000	
1424	Ø ₁	-0.333	0.125	-2.65	0.008	
	Ø ₂	0.545	0.069	7.94	0.000	
	θ_1	-0.881	0.129	-6.84	0.000	
	Θ ₁	0.936	0.046	20.38	0.000	

Table 3. Statistical analysis for the model parameters

Figure 3 shows the relationship between five-years of monthly maximum data at each gauge station and predicted data for the same years by using the best models from ARIMA and Thomas-Fiering approaches for each gauge station. As shown in Figure 3, the predicted data obtained from these approaches follow monthly maximum data very closely for three gauge stations on Cekerek Stream. Therefore, both models seem to be adequate for simulating monthly maximum data. Table 4 gives the error estimates obtained for monthly maximum data of the two different approaches used in the study for forecasting. The two error estimates (RMSE and MAE) obtained for two approaches indicate that ARIMA approach appear to be slightly better than Thomas-Fiering.

Table 4. Comparison of the results from different approaches

	ARIMA		Thomas-Fiering		
Gauge Station	RMSE	MAE	RMSE	MAE	
1404	0.78	0.59	0.97	0.73	
1409	0.19	0.13	0.95	0.67	
1424	0.23	0.17	0.98	0.66	

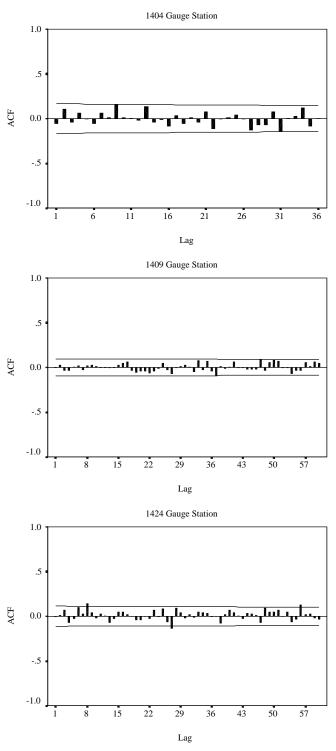


Figure 2. Residual ACF- monthly maximum flood data

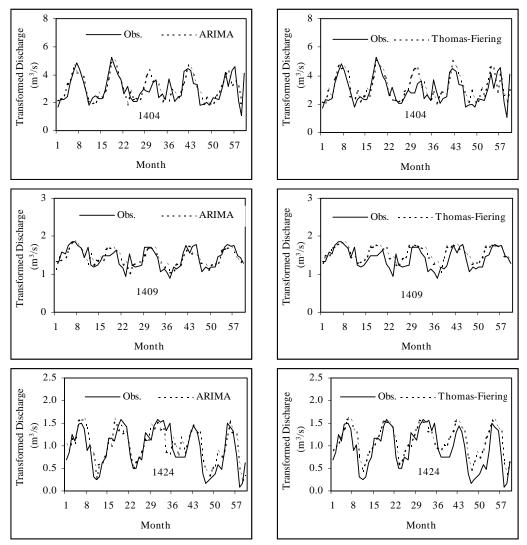


Figure 3. Comparison of observed data to predicted data using different approaches

Conclusion

In many scientific or technical applications, data are generated in the form of a time series. Therefore, time series analysis is among the major tasks in research and development. Especially, the engineering design of hydraulic structure demands reliable information concerning the peak flow to be expected after a rainstorm of a given probability of occurrence. The estimation of design floods is, in practice, often based on small samples of data, which may cause a severe uncertainty. In this sense, the hydrologist often faced with the problem of predicting extreme flood events on basis of samples of historical flood records. Therefore, the main problem is to obtain reliable estimates of floods with given return period or, alternatively, estimates of exceedance probabilities of certain flood magnitudes. But, for many water resource studies the available streamflow records are often scarce, which implies an uncertainty of the flood prediction. In many hydrologic applications, information based on continuous discharge or flow measurements is the basis of analysis and decision-making. The accuracy of time series forecasting is fundamental to many decision processes and hence research for improving the

effectiveness of forecasting models has never stopped. Generally, providing good forecast functions is a common problem.

In this study, performance of two stochastic models including ARIMA and Thomas-Fiering approaches was focused on. These models were applied to monthly maximum streamflow sequences from Cekerek Stream. The error estimates of RMSE and MAE for both approaches were taken into consideration to identify the most appropriate approach for reliable simulation. Based on the error estimates, we propose to take the ARIMA model to time series forecasting related to monthly maximum streamflows from Cekerek Stream. The ARIMA model appears to be slightly better than Thomas-Fiering. But, as the predicted data obtained from these approaches follow monthly maximum data very closely, both approaches were concluded to be able to accurately use in generating monthly maximum data.

Nomenclature

- a_i white noise time series value at time i
- a_{ij} independent standard normal variable at time i in the jth month
- B backward shift operator
- c constant for Box-Cox transformation
- d order of the nonseasonal differencing operator
- D order of the seasonal differencing operator
- e_i the number of data in the ith (tied) group
- ESS_F the residual sum of square for first group
- ESS₈ the residual sum of square for second group
- k_p degree of freedom
- LBQ/P probability for Q(r)
- n the number of observation
- n_F the number of residuals in the first group
- n_s the number of residuals in the second group
- Q_i the mean monthly discharges during month j
- Q(r) Ljung-Box statistic at lag m
- Q_{obs} observed discharge
- Q_{pred} predicted discharge
- r_j the serial correlation coefficient for discharge in the jth month from the (j-1)th month
- $r_k(a)$ ACF of a_i at lag k
- s seasonal length
- S_j the standard deviation monthly discharges during month j
- t the number of tied groups
- u_c Mann-Kendall statistic
- x_i discrete time series value at time i
- $X_{i,j}$ predicted discharge for the jth month from the (j-1)th month at time i
- wi stationary series formed by differencing the xi
- z_i transformation of x_i series

Greek Symbols

- λ exponent for Box-Cox transformation
- μ mean level of the w_i series (if D+d>0 often $\mu \approx 0$)
- Ø_i ith nonseasonal AR parameter
- Φ_i ith seasonal AR parameter
- θ_i ith nonseasonal MA parameter
- Θ_i ith seasonal MA parameter

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