CDC
PAPER No. TP4-3

STABILIZATION POLICIES IN A DISEQUILIBRIUM GROWTH MODEL

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Abstract The stabilization trade-off between inflation and unemployment is investigated in a continuous time growth model where disequilibrium can occur in the labor market. The emphasis of the paper is on the policy implications of rational expectations, as an alternative to adaptive expectations. The rational expectations approach is useful in describing the way in which a particular policy plan alters economic agents' expectations of future inflation rates. It is shown that it is not possible to achieve an unemployment rate which is deterministically different from the equilibrium rate, when economic agents form expectations rationally.

1. Introduction

The economic policy trade-off between inflation and unemployment crucially depends on how expectations of future inflation rates are formed. If expectations are accurate predictors of future inflation rates, unemployment can be altered only slightly from its equilibrium value. On the other hand if expectations are sluggish, significant departures from equilibrium employment can be achieved. Moreover, the constraint imposed by the formation of expectations determines the optimal dynamic mix between unemployment and inflation over time. This paper illustrates some of these issues by examining the policy implications of rational inflationary expectations - expectations which are optimal predictors of future inflations rates - in a continuous time growth model.

An appropriate preliminary framework for such an investigation has been developed by Phelps (1967), where a judicious choice of assumptions allows one to abstract from many of the usual stabilization problems in order to focus on the inflation - unemployment trade-off itself. The economy is characterized by an aggregate production function which is homogeneous in capital and augmented employment, a constant proportionate rate of growth of the augmented labor supply, and a monetary authority which flawlessly controls private investment to insure that the capital stock grows at the same exponential rate as augmented labor. Under these assumptions the ratio of augmented employment to capital - the utilization ratio, u - measures the utilization of the labor supply in productive employment. The fiscal authorities are assumed to be able to regulate the utilization ratio by affecting aggregate demand via tax policy.

The relationship between inflation and the utilization ratio is assumed to be a shifting Phillips Curve,

$$(1) \qquad \pi = f(u) + \pi^*, \quad f'(u) > 0, \; f(u_e) = 0,$$

where π and π^* are the actual and expected rates of inflation respectively, and where u_e is the equilibrium utilization ratio. Thus disequilibrium in this growing economy is permitted to occur only with some effect on inflation.

Assuming an instantaneous rate of social utility $U(\pi^*,u)$, the task of the fiscal authority can be described as: choose a path u(t), (by influencing aggregate demand) so as to maximize

$$(2) \qquad W = \int_0^\infty \rho^{-t} U(\pi^*,u)\,dt ,$$

subject to $u(0) = u_0$ and an additional constraint relating π^* to u.

The form of the additional constraint in this optimal control problem is determined by the way in which the public forms expectations about future inflation. If these expectations are adaptive, that is, if

$$(3) \qquad \pi^* = \frac{\alpha}{D+\alpha}\pi,$$

where D = d/dt, then by substituting (1) into (3) we obtain

$$(4) \qquad D\pi^* = \alpha f(u).$$

Equation (4) is the additional constraint for the optimization problem, which can now be solved to obtain an optimal path $\hat{u}(t)$.

But, in the face of a systematic employment policy $\hat{u}(t)$, is it likely (and this is where the expectation assumption becomes important) that the public would continue to form its expectations on the basis of (3) ? If the plan $\hat{u}(t)$ were made known to the public, then the answer is clearly no. However even if the plan were not widely known, over a period of time the public would begin to see that expectations based on (3) were biased and modify the adaptive scheme. (They would not notice a bias in (3) if $\hat{u}(t)$ follows the stochastic process defined in Section 3; but then $\hat{u}(t)$ would not be the solution of (2).) Therefore, the solution to the stabilization problem based on (3) either applies to the very short run, or it assumes that the economic agents in the economy do not learn by their mistakes. For this reason alternative assumptions about expectations are investigated in this paper.

2. Rational Expectations

Under the assumption of rational expectations, [see Muth(1961)], the expected rate of inflation is set equal to the optimal predictor of the future inflation rate. If the criterion of prediction is minimum mean square prediction error, then the expected rate of inflation at time t, given information until time t_0, is the conditional expectation of $\pi(t)$ given this information. If we let $I(t_0)$ represent the information available at time t_0, then we have

$$(5) \qquad \pi^*(t \mid t_0) = E[\pi(t) \mid I(t_0)]$$

where E is the expectation operator. Since in this model the government employment policy u(t) is the only observable variable upon which $\pi(t)$ depends, we must specify what the public is assumed to know about u(t) and how u(t) affects $\pi(t)$.

For this purpose it is convenient to think in terms of the short run Phillips curve (i.e. $\pi^* = 0$). Suppose that the fiscal authority sets aggregate demand in period t in order to achieve the utilization ratio $u_s(t)$, when $\pi^* = 0$. Then $u_s(t)$ determines $\pi(t)$ by

$$(6) \qquad \pi(t) = f(u_s(t)).$$

However, $u_s(t)$ will not in general be achieved because π^* will not be equal to zero. Instead, we have that

$$(7) \qquad \pi^*(t|t_0) = E[f(u_s(t))|I(t_0)],$$

so that the actual level of u(t) will be determined by

$$(8) \qquad f(u_s(t)) = f(u(t)) + \pi^*(t|t_0).$$

Since $u_s(t)$ determines $\pi(t)$ exactly, it only remains to specify how much of the series $u_s(t)$ the public is permitted to see at time t_0; that is, what constitutes the available information $I(t_0)$?

First consider the case of perfect foresight where the fiscal authorities announce the plan $u_s(t)$, $0 \le t < \infty$. Then $I(t_0) = \{u_s(t);\ 0 \le t < \infty\}$ and $\pi^*(t|t_0) = \pi(t)$ so that by equation (8), $u(t) = u_e(t)$ for all t. Therefore, when the public is aware of the control strategy, the fiscal authorities cannot achieve an unemployment rate that differs from the equilibrium level. This of course is the only possible result with rational expectations under certainty.

But the perfect foresight case is extreme: even if the fiscal authorities did announce the plan, various uncertainties in the economy would cause a divergence between π and π^*. To examine these possibilities let us assume for the remainder of this paper that $u_s(t)$ and thus $\pi(t)$ are continuous time stochastic processes which are observable for all $t < t_0$, then $I(t_0) = \{u_s(t);\ t < t_0\}$.

Note that this does not necessarily mean that the fiscal authorities are following a randomized strategy (though they may very well be); $u_s(t)$ could be random because of uncertainties in the effect of aggregate demand on unemployment or simply because of observation errors.

In this stochastic framwork, knowledge of a policy plan $u_s(t)$ is taken to mean knowledge of the structure underlying the stochastic process. This knowledge may be obtained either by learning over time or through announcement by the fiscal authorities.

It was shown above that, if a deterministic policy is announced, then the fiscal authority cannot alter the equilibrium rate In this stochastic setting we can ask whether there exists a randomized policy, which permits the fiscal authority to achieve an employment rate deterministically different from equilibrium, when the probabalistic structure of the randomized policy is announced. If so, then a stochastic optimal control problem similar to (2) could be formulated with the constraint given by rational expectations. In the following section we show that no such randomization is possible.

3. Policy Implications

Before considering an arbitrary policy and the resulting rational expections, let us ask under what conditions the adaptive expectations of (3) will be rational. A similar question though not relating to economic policy was posed by Muth(1961) in the discrete time case. In the continuous time case the results are analagous. The adaptive expectation equation (3) can be written as

$$(9) \qquad \pi^*(t) = \alpha \int_0^t e^{-\alpha s}\, \pi(t-s)ds.$$

By equation (6) any policy plan $u_s(t)$ can be expressed in terms of the rate of inflation $\pi(t)$: in terms of a stochastic policy $u_s(t)$ we can equivalently work in terms of a stochastic policy $\pi(t)$.

Suppose that the stochastic policy $\pi(t)$ can be represented by the linear filter

$$(10) \qquad \pi(t) = \varepsilon(t) + \int_0^\infty \beta(s)\varepsilon(t-s)ds,$$

where $\varepsilon(t)$ is an orthogonal process with $E[\varepsilon(t)] = 0$. Then

$$(11) \qquad \pi(t) = \varepsilon(t) + \int_0^{t-t_0} \beta(s)\varepsilon(t-s)ds + \int_{t-t_0}^\infty \beta(s)\varepsilon(t-s)ds$$

and the minimum mean square predictor of $\pi(t)$ based on information available at time t_0 is

$$(12) \qquad \pi^*(t|t_0) = \int_{t-t_0}^\infty \beta(s)\varepsilon(t-s)ds,$$

and the instantaneous predictor corresponding to (9) is

$$(13) \qquad \pi^*(t|t-) = \pi^*(t) = \int_0^\infty \beta(s)\varepsilon(t-s)ds.$$

In order to determine how (13) compares with (9) we must write $\pi^*(t)$ in terms of past $\pi(t)$, that is find the weights $\alpha(s)$ in

$$\text{(14)} \quad \pi^*(t) = \int_0^\infty \alpha(s)\pi(t-s)ds.$$

Substituting (10) into (14) we have

$$\text{(15)} \quad \pi^*(t) = \int_0^\infty [\alpha(s)+\int_0^s \alpha(s-x)\beta(x)dx]\epsilon(t-s)ds,$$

which upon comparison with (13) implies that

$$\text{(16)} \quad \beta(s) = \alpha(s)+\int_0^s \alpha(s-x)\beta(x)dx,$$

from which we can determine $\beta(\cdot)$ given $\alpha(\cdot)$.

To determine the function $\beta(\cdot)$ which implies rational expectations we substitute $\alpha(s)=\alpha e^{-\alpha s}$ into (16) to obtain

$$\text{(17)} \quad \beta(s) = \alpha e^{-\alpha s}+\alpha\int_0^s e^{-\alpha(s-x)}\beta(x)dx.$$

A solution to (17) is $\beta(s)=\alpha$ for all s. Thus, a stochastic policy which implies that adaptive expectations are rational is given by

$$\text{(18)} \quad \pi(t) = \epsilon(t)+\alpha\int_0^\infty \epsilon(t-s)ds.$$

If the fiscal authorities invoke this policy, announcing α, then the public will behave rationally by forming expectations according to the adaptive mechanism in (3). Of course, if such a policy is adhered to, it is not possible to formulate an optimal control problem like (2) even though the constraint (4) will remain in effect. With the purely random policy of (18), the deviation of unemployment from its equilibrium value is purely random as determined by $\epsilon(t)$.

We now consider the feasibility of the fiscal authority engaging in a more sophisticated randomized strategy than one which is purely random. Perhaps by superimposing a randomized component upon a deterministic component, it is possible to achieve an unemployment rate which is deterministically different from the equilibrium level, even when expectations are rational. For example, a long range plan for controlling aggregate demand could be modified by random element which either adds or subtracts a fixed or variable amount to the deterministic level.

If both the deterministic component and the probabalistic structure behind the randomized component of this plan are known to the public, then, under the assumptions of this model with rational expectations, one can show that it is impossible to achieve a deterministic difference between unemployment and its equilibrium level. Let the policy be

$$\text{(19)} \quad \pi(t) = \pi_d(t) + r(t),$$

where $\pi_d(t)$ is the deterministic component and $r(t)$ the stochastic component, which without loss of generality is assumed to have zero mean. Then under rational expectations

$$\text{(20)} \quad \pi^*(t) = \pi_d(t),$$

and from (1) we have that

$$\text{(21)} \quad f(u(t)) = r(t).$$

Therefore the deviation of $u(t)$ from u_e is purely random; no deterministic influence on unemployment can be achieved.

Described in these terms the conclusion is rather straightforward, but the crucial assumptions behind it should be emphasized. These assumptions are (1) a shifting Phillips Curve with an equilibrium level of unemployment, (2) rational expectations, and (3) a randomized fiscal strategy, which can be stated in terms of the actual rate of inflation, and which is announced to the public. If any of these assumptions are dropped then the above conclusion will no longer hold.

It seems that a remedy to this rather pessimistic policy conslusion could most realistically come from a relaxation of the third assumption regarding the announcement of policy. For example, if deterministic shifts in policy were unannounced, then, during the period in which the public is learning about the shift, the fiscal authority could achieve a deterministic deviation between actual and expected future inflation rates. By a careful choice of policy changes this learning period might be made arbitrarily long. But a more precise analysis of such a scenario would require modeling the learning behavior of economic agents, a complication beyond the scope of the present paper. An interesting characteristic of such a policy is the advocation of secrecy in government planning, contrary to the basic assumption of this paper that fiscal plans be fully disclosed to the public.

4 Concluding Remarks

In conclusion it might be useful to briefly point out the relation between this paper and other works in the field of rational expectations and stabilization policy. The seminal paper on rational expectations by Muth(1961) has already been mentioned. Among the numerous papers which use this technique are Lucas and Prescott(1971) who consider the equilibrium of many agents with rational expectations and Sargent and Wallace (1972) who examine the effect on hyperinflations.

Lucas(1973) considers some of the policy implications of rational expectations in cases where policy has effect even though it is optimally predicted. In one such case mentioned by Lucas, (the case of investment tax credit policy) Kydland and Prescott (1973) examine a new approach to stabilization in which there is an equilibrium between the policy maker , who maximize a social utility function, and firms, who maximize profits under rational expectations with knowledge of the policy plan. The tax credit policy is able to achieve changes in output in these models.

Closer to the policy implications of this paper is the study by Sargent(1973). The short-run macroeconomic model of that paper is more disaggregated than this and separately examines the effect of fiscal and monetary policies which are of the linear feedback type. In such a model Sargent shows that monetary and fiscal policy cannot affect output.

All of the above models are formulated in discrete time as was the original derivation of rational expectations. By focusing on a single disequilibrium equation (the shifting Phillips curve) and by dealing in continuous time, this paper clearly reveals the reasons for the ineffectiveness of policy on unemployment in a model with rational expectations.

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