James E. Cutting
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Analyzing the Inchoate: Complex Interrelations in the Humanities and Sciences

Complexity, pictures, and parallels: An introduction to Mandelbrot

It is one thing to lead a discipline; yet another to create it, or more specifically in this instance to create and popularize a category—better still, a mode of analysis. How the leaders of this first type come to occupy their position in a field seems, for the most part, completely understandable; they have master the problems, the literature, and the methodology of a discipline that they inherit. It is as if some segment of learned society has urged the discipline into existence over some period of time, let it evolve, and simply chosen as its leader the person whose work best exemplifies its values, its questions, and its procedures of inquiry.

Those who create a field or category pose more of a problem for us. They categorize the world in a new way, be it in terms of gender or science, medieval or modern; they use what appear to be bits and pieces of unrelated facts and scholarship, and they synthesize something—not wholly new, but something whole, and by its wholeness new. Such people provide a new conceptual coherence for us where before we saw either nothing or we saw jumble and chaos. Professor Mandelbrot is a scholar of this second type.

We can call this field, and because Professor Mandelbrot created it we can legitimately call it *his* field, by the same neologism that we offers: Fractals, or fractional-dimensioned objects. In both name and content, these objects seem glaringly twentieth-century inventions; the neologism *fractal* is just about as disturbing as the concept behind it. As Professor Mandelbrot has described in the context of the earlier work by Hausdorff,[1] fractals are mathematical objects that occupy the interstices between the more accustomed dimensions of 0, 1, 2, and 3; for points, lines, planes, and solids, respectively. Fractals are disturbing because they violate an assumption that, for whatever reason, all of us have held since our youth, namely, that geometric dimensionality is confined to the cardinal numbers—1, 2, 3, and so forth. It never occurred to most of us that any object can occupy exactly 2 1/3 dimensions. This assumption is so deep that the claim for existence of any such object dimensions seems an affront to rationality.

In assessment of this new and even personal field of fractals one may reasonably ask: To what extent are fractals a solution. And, if so, to what question? Moreover and more deeply, why was this question even asked? The answers have to do with complexity, an idea on which I will give my own views. My sketch, one with which Professor Mandelbrot may or may not agree, concerns how this new category was created. I will leave aside his important contributions to information theory and statistics, and leave aside the work of others, which one would ordinarily cite as precursors to the modern idea of fractals. Instead, let me present a story in three parts: The first has to do with a fascination with complexity, the second with the use of external means such as computers to visualize the complex, and the third with the parallels between these new visualizations and current suppositions about the structure of the world around us.

Just what is complexity? Here, I have four things to say. First, complexity cannot be defined without regard to its opposite in any given time or culture, often called simplicity. This means that, if we have different ideas about what is simple, we are sure to have differing ideas about what it complex.[2] Second, what one regards as simple or complex is a product of the milieu within which one works. Thirds, what is simply is often regarded as comprehensible, and what is comprehensible is often, simply, what is familiar. That which is complex is often—incomprehensible. Thus, the various approaches to the inchoate—be it the ill formed and / or the chaotic—must necessarily grapple with the idea of comprehending what is incomprehensible and, therefor, perhaps unfamiliar. And fourth is the idea that what is simple is thought to b e good; what is foreign and complex less so, perhaps even evil.

The idea of incomprehensible complexity can be found in the mathematical milieu within which fractals evolved. One can easily say that the progress of mathematics in the nineteenth century (a century within which one can use the term ‘progress’ with some legitimacy) was one in which more and more complicated objects and concepts can to be systematized. Geometry is a case in point. One needs only look to the formation of projective geometry, to Riemann’s system of geometries or to Klein’s group-theoretic approach to them. Through this systemization much of what was previously regarded as complex eventually came to be thought of as simple. Such a conceptual transformation, redefining the complex so as to be part of the simple, is a powerful one; I would claim that it is a tool by which we can begin to sketch what it means to comprehend the incomprehensible.

Despite broad nineteenth-century attempts at systematization of various areas of mathematics, deviants remained. Like all deviants, be they in science or society, they were regarded as bizarre, illegitimate, even diseased. Some were so unacceptable that Henri Poincaré called them a “gallery of monsters”.[3] By mathematical standards these objects were malformed and horrible, and among them are the objects that first fascinated Professor Mandelbrot. If Euclid alone looked on beauty bare, Mandelbrot and his precursors looked upon a chamber of horrors. These horrors have been said to be among the most complex objects in the universe.

To end Chapter 1 of my story about fractals: The origin of Professor Mandelbrot’s synthesis of this new category named fractals is a fascination with the complex, the deviant, the ugly. Underlying this fascination is an irony, the hope that behind ugliness and complexity is a key to a beauty and a deeper simplicity. Perhaps this hope has been sustained.

Chapter 2 is about externalization. With the mid-twentieth century came a new tool, a new tool for externalizing thoughts—computers. Computers allow mathematicians and others to do many things, sometimes in new ways. Some of these are widely understood and rightly regarded as important: First, computers are both rapid an, given correctly, written programs, error-free. Second, and more pertinent to Professor Mandelbrot, some computer outputs, particularly computer graphic displays, can be used to represent visually what heretofore was not only unimaginable, but impossible. For example, what contemporary computer graphics technology allows is the possibility that each picture can portray hundreds of thousands of related mathematical solutions. Some of the striking photographs that Mandelbrot has shown us (and will show us) are just such pictures; although since fractals are solely about contour, the colors seen are irrelevant. The colors are simply like equal elevation markers on a contour map.

Thus, and as a conclusion to the second chapter of my story about fractals, Professor Mandelbrot and others have, thanks to contemporary technology, transformed mathematical solutions into pictures. Without the externalizing function of computers, these thoughts would be incomprehensible. But as pictures—in color—they might be beautiful, or analyzable, or eventually even comprehensible.

In the first two parts of my story—fascination with the complex and the use of the externalizing function of computer graphics—Professor Mandelbrot may not be the sole protagonist. Some here are Cornell, for example John Hubbard, share an indentical pedigree.[4] In the third chapter, however,. Professor Mandelbrot is almost without peer. If the thirteen-century scholar wrote: "Nothing can be known concerning the things of the world without the power of geometry,"[5] we now have a twentieth-century one telling us that "something important can be known concerning the things of the worlds through the power of fractals."

Through the use of externalized solutions to mathematical problems, Professor Mandelbrot discovered what appeared to him to be a parallel between mathematical solutions and 'natural' forms. It is possible that this parallel is one that you or I might not have seen. His parallel is not superficial; it is quite deep, and it constitutes Professor Mandelbrot's main contribution. It makes him not just a mathematician; it makes him a philosopher, a nineteenth-century physicist, even like me a twentieth-century psychologist; it makes him an aesthete and, some might have said, a magician.

Now back to the similarity between these erstwhile mathematical monsters and the 'natural' world. Professor Mandelbrot deals with repeating patterns. Many of these are, for the most part, exact and clean. They are *self-identical* at different scales; or more simply, each piece, when looked at closely, looks exactly like the whole. Like Mandelbrot's world, the natural world also shows repeating patterns. These, however, are neither exact nor clean, but the patterns repeat nonetheless. For example, the largest mountains are similar to their foothills, which are like the rocks that erupt on the surface of the foothills, which are like the pebbles surrounding the erupted rocks.

There is, however, a tension between mathematical fractals and natural fractal-like objects. That tension is this: Mathematical objects called fractals must, by definition, be infinitely iterative and infinitely self-similar. Natural objects have self-similarity only to three or four orders of magnitude; hence I call them fractal-like. Returning to the example of the mountain, there is a point below which the forces of nature at different scales make similarity with larger forms impossible. In another domain this is a lesson the great biologist D'Arcy Thompson taught us long ago.[6]

Thus, these natural similarities at different scales are different than those of the iterated patterns of, for example, the Mandelbrot set; those similarities that exist are statistical. Because of this they are often called Brown fractals, after the statistical character of Brownian motion. The natural repetitions are, nonetheless, quantifiable and specifiable; so much so that computer graphics engineers can use them to general naturalistic scenes. And so prevalent are these that it has now become difficult to open up a magazine or go to a film without being enveloped in fractals.

Professor Mandelbrot's main contribution, the, is threefold: First, he perceived that patterns repeat in both the mathematical and in the natural domain; second, he asserted that the difference between the two could be regarded as superficial; and third, he quantified them. So ends my three-part story: An ugly, mathematical complexity is externalized and found beautiful and comprehensible; comprehensibility is legitimized by parallels with the perceived structures of the natural world.

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[1] B. B. Mandelbrot, *The fractal geometry of nature*, New York: W. H. Freeman, 1973.

[2] Two background analyses are important here: A. N. Kolmogorov, "Logical basis for information theory and probability theory," *IEEE Transactions on Information Theory*, 1968, *IT-14*, 662-664, and G. J. Chaitin, "Algorithmic information theory," *IBM Journal of Research and Development*, 1977, *21*, 350-359. In engineering simplicity can be defined as the number of parameters that appear in an equation. Mandelbrot, however, shows that very simple equations can generate pictures that are indefinitely recursive, and hence among the most complex objects in the universe.

[3] See Mandelbrot, p. 9.

[4] See H.-O. Pietgen and P. H. Richter, *The beauty of fractals*. Berlin: Springer-Verlag, 1986.

[5] See R. B. Burke, *The opus majus of Roger Bacon*, Vol. 1, Philadelphia: University of Pennsylvania Press, p. 234.

[6] See D. Thompson, *On growth and form*, abridged ed. London: Cambridge University Press, 1961. Originally published in 1971.