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Kriging of Groundwater Levels – A Case Study

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Abstract

In this paper, application of the spatial statistical technique, kriging, for the spatial analysis of groundwater levels is shown. The data set consists of groundwater levels measured at about 60 points (the number of points vary from year to year) twice in a year (June and September) for six years (1985-1990) in an area of 2100 sq km in part of the canal command area of Indira Gandhi Nahar Pariyojana (IGNP) in Rajasthan, India. With the use of measured elevations of the water table, experimental semivariograms were constructed that characterises the spatial variability of the measured groundwater levels. Spherical, exponential and gaussian semivariogram models were fitted to the experimental semivariograms. The finally selected models were used to estimate the groundwater levels and estimation variance (which express the accuracy of the estimated groundwater levels) at the nodes of a square grid of 5km x 5km and to develop corresponding contour maps. Groundwater levels were also interpolated by generally used Inverse Square Distance (ISD) method and it was found that ISD method resulted in higher errors as compared to kriging method. The kriged groundwater table maps were compared with the groundwater table maps prepared using the ISD method.

Keywords: Geostatistics, Groundwater levels, Semivariogram, Kriging, India

Introduction

Groundwater is one of the major sources of water. Management of this resource is very important to meet the increasing demand of water for domestic, agricultural and industrial use. Various management measures need to know the spatial and temporal behaviour of groundwater. Observed groundwater levels serve as one of the main input data in studies related to groundwater simulation for various purposes as required in water balance studies, estimation of groundwater recharge potential, in the design of drainage structures etc. However, the measurement of groundwater levels are generally carried out at spatially random locations in the field, whereas, most of the groundwater models requires these measurement at a pre-specified grid. Some interpolation method is generally employed to get these values at grid nodes. The accuracy with which this interpolation can be carried out affects the accuracy of the model output.

Kriging is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the semivariogram and the initial set of data values (David 1977). Kriging takes into consideration the spatial structure of the parameter and hence score over other methods like arithmetic mean method, nearest neighbour method, distance weighted method, and polynomial interpolation. Also, kriging provides the estimation variance at every estimated point, which is an indicator of the accuracy of the estimated value. This is considered as the major advantage of kriging over other estimation techniques.

Basic concepts of the kriging technique and its application to natural phenomenon have been reviewed by the ASCE Task Committee (1990a, b). Kriging has been used in soil science (Burgess

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and Webster 1980; Vieria et al. 1981; Berndtsson and Chen 1994; Bardossy and Lehmann 1998); hydrology (Creutin and Obled 1982; Storm et al. 1988; Ahmed and de Marsily 1989; Germann and Joss 2001; Araghinejad and Burn 2005); and atmosphere science (Bilonick 1988; Casado et al. 1994; Merino et al. 2001). Kriging of groundwater levels was carried out by Delhomme (1978); Volpi and Gambolati (1978); Aboufirassi and Marino (1983); Virdee and Kottegoda (1984); Kumar (1996) and Kumar and Ahmed (2003). In this paper, application of kriging to interpolate the groundwater levels, as observed in the part of canal command area of Indira Gandhi Nahar Pariyojana (IGNP), Rajasthan, India, has been shown.

Methodology

Although details on the kriging techniques are well documented (Journal and Huijbregts 1978; Isaaks and Srivastava 1989), a brief account of the relevant methods used is prescribed here. The first step in kriging is to calculate the experimental semivariogram using the following equation.

$$\gamma^*(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2 \tag{1}$$

where $\gamma^*(h)$ = estimated value of the semivariance for lag h; N(h) is the number of experimental pairs separated by vector h; $z(x_i)$ and $z(x_i+h)$ = values of variable z at x_i and x_i+h, respectively; x_i and x_i+h = position in two dimensions. Experimental semivariograms were calculated for June and September period from the year 1985 to 1990 using the computer program (in FORTRAN language) written by Kumar (1996). A lag distance of 5km and a tolerance of 2.5km were used for the calculation of semivariogram.

The experimental semivariograms were fitted with various theoretical models like spherical, exponential, gaussian, linear and power by the weighted least square method. The theoretical model that gave minimum standard error is chosen for further analysis. The adequacy of the fitted models was checked on the basis of validation tests. In this method, known as jackknifing procedure, kriging is performed at all the data points, ignoring, in turn, each one of them one by one. Differences between estimated and observed values are summarised using the cross-validation statistics (de Marsily and Ahmed 1987): mean error (ME), mean squared error (MSE), and kriged reduced mean error (KRME), and kriged reduced mean square error (KRMSE). If the semivariogram model and kriging procedure adequately reproduce the observed value, the error should satisfy the following criteria.

$$ME = \frac{1}{N} \sum_{i=1}^{N} (z^*(x_i) - z(x_i)) \cong 0 \tag{2}$$

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (z^*(x_i) - z(x_i))^2 \text{ minimum} \tag{3}$$

$$KRME = \frac{1}{N} \sum_{i=1}^{N} \left[(z^*(x_i) - z(x_i)) / \sigma_{ki}\right] \cong 0 \tag{4}$$

$$KRMSE = \frac{1}{N} \sum_{i=1}^{N} \left[(z^*(x_i) - z(x_i))^2 / \sigma_{ki}^2\right] \cong 1 \tag{5}$$

where, $z^*(x_i)$, $z(x_i)$ and σ_{ki}^2 are the estimated value, observed value and estimation variance, respectively, at points x_i. N is the sample size. As a practical rule, the MSE should be less than the variance of the sample values and KRMSE should be in the range $1 \pm 2\sqrt{2/N}$.

In all interpolation techniques, interpolated value of z at any point x_0 is given as the weighted sum of the measured values i.e.

$$z^*(x_0) = \sum_{i=1}^{N} \lambda_i z(x_i) \quad i = 1,2,3,\ldots\ldots,N \tag{6}$$

where, λ_i is the weight for the observation z at location x_i. In kriging, the weights λ_i are calculated by equation (7) so that $z^*(x_0)$ is unbiased and optimal (minimum squared error of estimation).

$$\begin{cases} \sum_{j=1}^{N} \lambda_j \gamma(x_i, x_j) + \mu = \gamma(x_i, x_0) & i = 1,2,3,\ldots\ldots, N \\ \sum_{j=1}^{N} \lambda_j = 1 \end{cases} \tag{7}$$

where,

μ = Lagrange multiplier

$\gamma(x_i, x_j)$ = semivariogram between two points x_i and x_j

The minimum squared error estimation is also a measure for the accuracy of estimates, which is known as estimation variance, or kriging variance, and is given by

$$\sigma_k^2(x_0) = \sum_{i=1}^{N} \lambda_i \gamma(x_i, x_0) + \mu \tag{8}$$

where, μ is the Langrange multiplier.

Inverse Square Distance (ISD) method, widely used in geohydrology, was also employed to interpolate the groundwater level data. In this method, the weights λ_i are inversely proportional to the square of distance from the estimation point as:

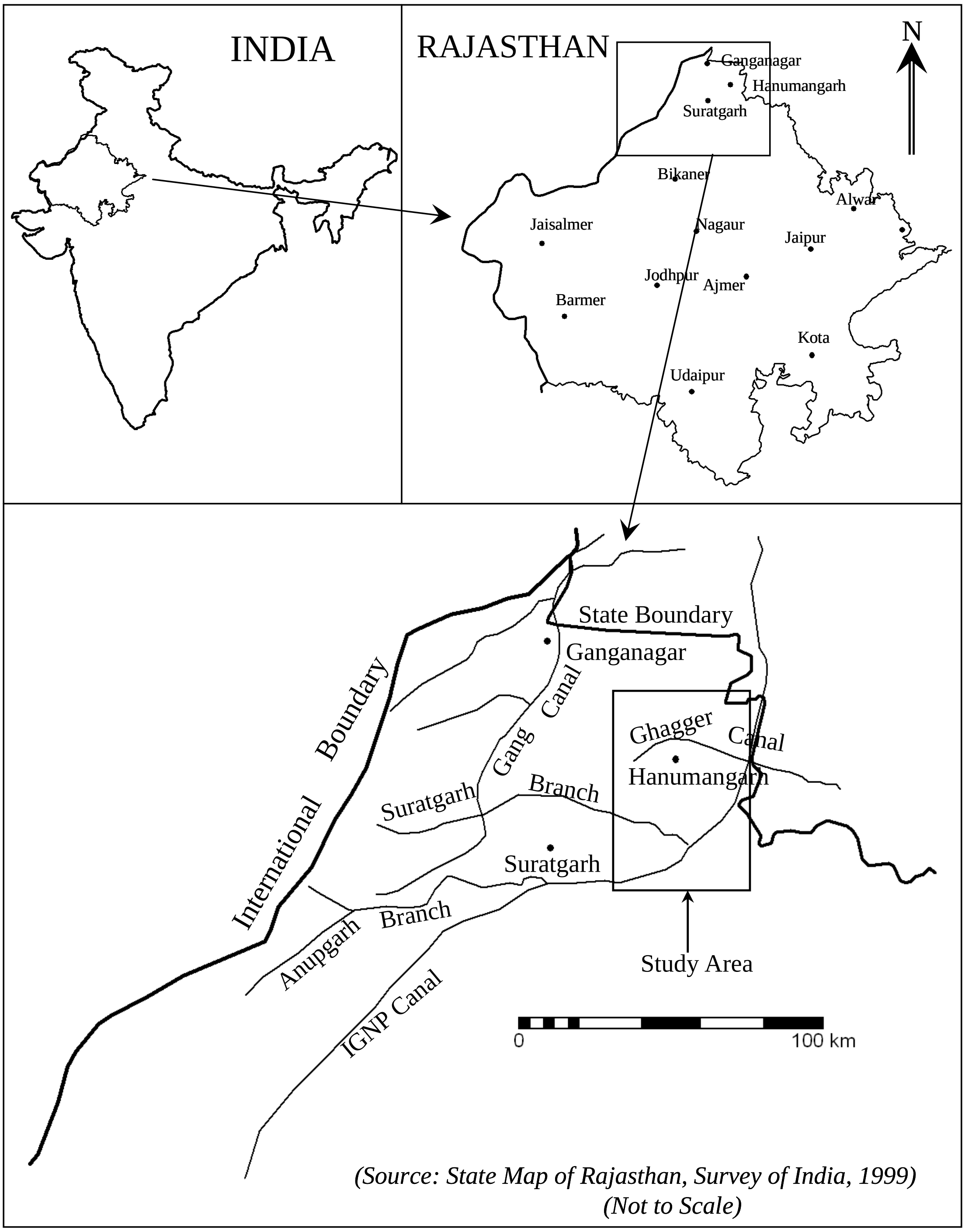
$$\lambda_i = \frac{\dfrac{1}{(d_{0i})^2}}{\sum_{i=1}^{N} \dfrac{1}{(d_{0i})^2}} \tag{9}$$

where , d_{oi} is the distance between the sample point and the estimated point.

Study Area and Data Used

The study area (Fig. 1) is located in the north-western part in the state of Rajasthan, India. The study area forms a part of the vast expanse of the Great Indian Desert, the Thar, and is part of the command area of Indira Gandhi Nahar Pariyojana (IGNP). The climate of the area is arid with extremes of temperature (maximum upto 50°C and minimum upto 1°C), low erratic rainfall (annual rainfall of about 250mm, of which 90% is received during south-west monsoon in the months of June to September) and very high potential evapotranspiration (Ramakrishna and Rao, 1991). The main soil types of the study area are deep and calcareous flood plain soils and sand dunes. The geology of the area is marked by aeolian sand and alluvium of quaternary age which forms extensive sandy plains. Alluvium is mostly fluviate in origin and comprises of unconsolidated to loosely consolidated sediments, consisting of an alternate sequence of sand, silt and clay with frequent lens of silty clays and kankar with occasional gravel horizons. Groundwater occurs in these alluvial sediments under water table conditions. Groundwater is generally saline in most part of the study area. The important components of groundwater recharge in the area are IGNP canal system and their distributaries, Ghaggar diversional channel (constructed to divert the flood water of Ghagger river to inter-dunal depressions) and inter-dunal depressions south of Suratgarh. A substantial part of recharge is contributed by return flow of irrigation water and some by annual precipitation. The groundwater level in the area is rising since the commencement of canal irrigation leading to waterlogging in the area (Ground Water Dept. 1985). This high rise in groundwater levels has led to systematic groundwater level monitoring from the year 1981-82.

For this study, groundwater level data pertaining to pre-monsoon (June) and post-monsoon (September) seasons over the years from 1985 to 1990 covering an area of 2100 sq. km (Fig.1), were selected. Fig. 2 shows plan of existing canal network and the location of observation wells. The descriptive statistics of the observed groundwater levels are shown in Table 1. Mean values of groundwater levels indicate rise in groundwater levels in post-monsoon season in 3 years, no change in one year and decrease in remaining one year. There is very small change but that is due to as mean values are provided. Also study area receives very little rainfall.



(Source: State Map of Rajasthan, Survey of India, 1999)
(Not to Scale)

Figure 1. Location map of study area

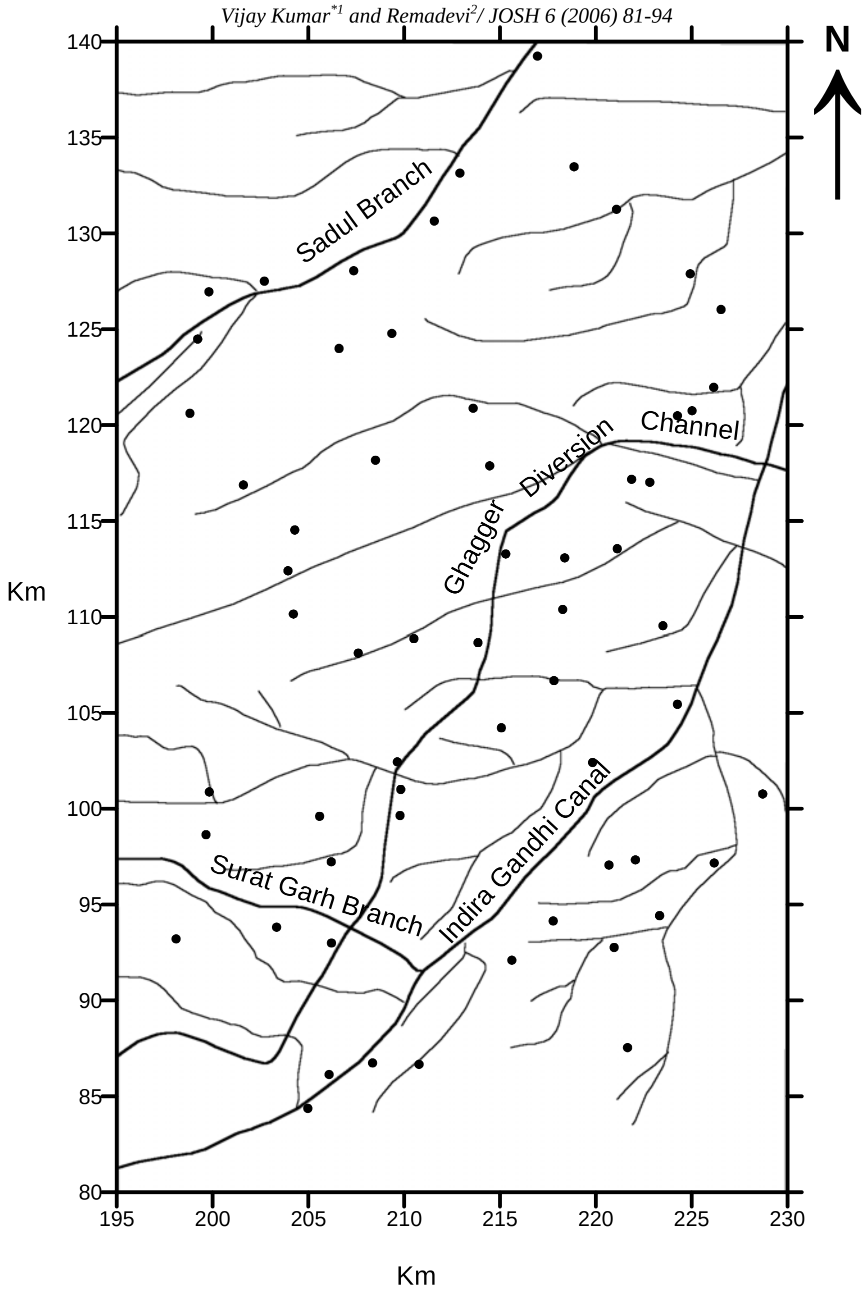


Fig. 2. Plan of canal network and location of observation wells

Results and Discussion

The experimental semivariograms and the best-fitted theoretical model for all the data sets are shown in Fig. 3. In all the data sets, Gaussian model resulted in the minimum standard error and so considered the best-fit model. The theoretical fitted gaussian semivariogram for September 1990 data is of the form:

$$\gamma(h) = 2.39 + 73.48\left[1 - \exp\left(-\left(\frac{h}{17.4}\right)^2\right)\right] \qquad (10)$$

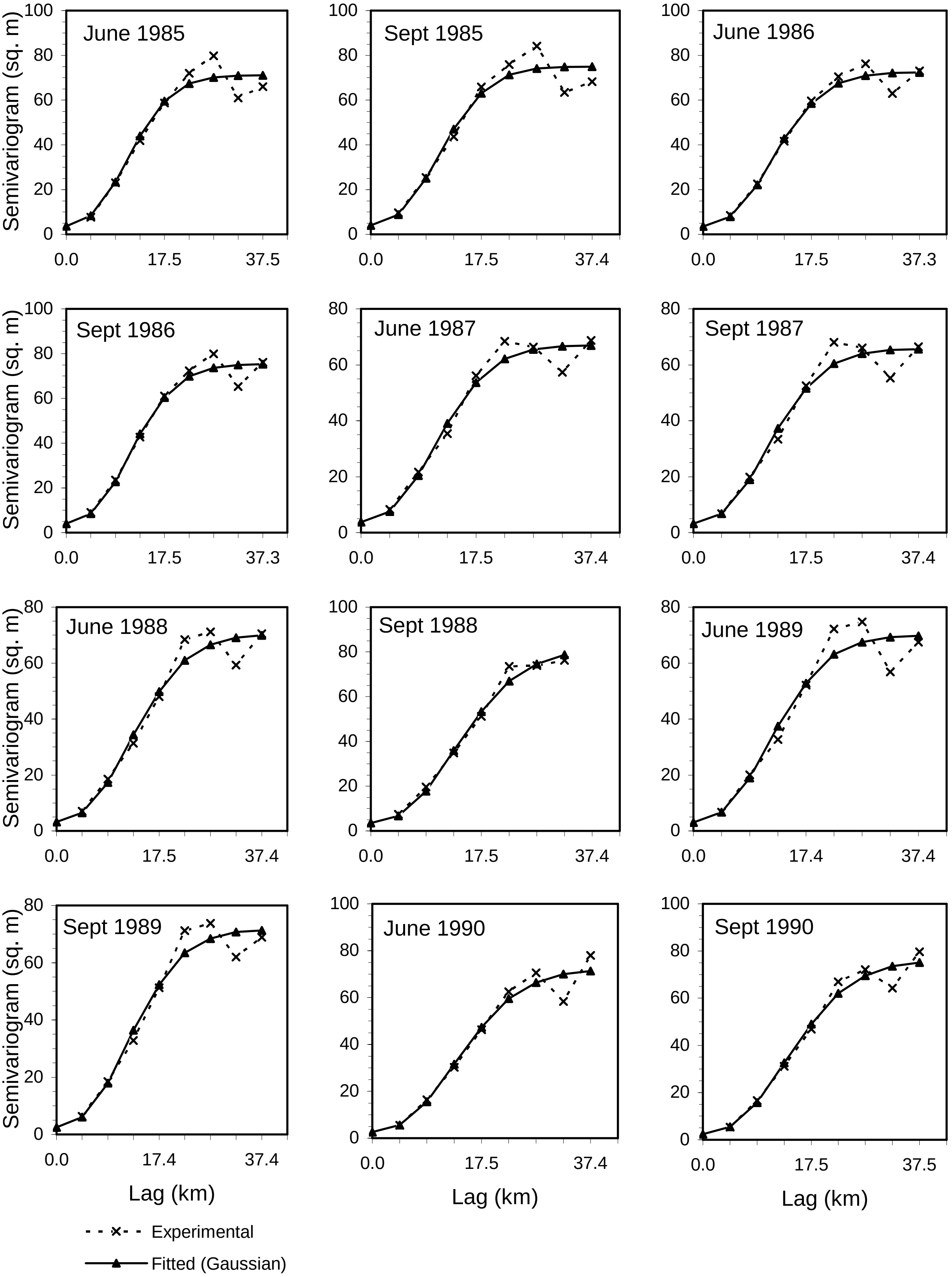


Figure 3. Experimental and fitted semivariogram for different data sets

Table 1 Basic statistics of the data set

S. No.	Data		No. of Wells	Mean (m)	Vari[*1] (m^2)	Coeff of Vari	Max. RL (m)	Min. RL (m)
	Year	Month						
1.	1985	June	61	177.25	53.20	0.041	187.63	161.93
		Sept.	57	177.26	56.84	0.042	188.12	162.15
2.	1986	June	62	177.68	53.42	0.041	187.75	162.12
		Sept.	60	177.96	55.13	0.042	188.13	162.48
3.	1987	June	59	178.08	49.88	0.040	188.21	162.73
		Sept.	60	178.16	48.21	0.039	188.16	162.64
4.	1988	June	63	176.95	48.49	0.039	187.68	161.77
		Sept.	48	176.77	52.68	0.041	188.54	161.69
5.	1989	June	65	177.68	50.03	0.040	188.55	161.71
		Sept.	65	177.67	50.22	0.040	188.43	162.01
6.	1990	June	65	178.18	46.37	0.038	187.94	161.76
		Sept.	68	178.39	48.39	0.039	188.58	161.83

[*1] Variance

The parameters of the best-fit gaussian model for 1985 to 1990 data set are given in Table 2. An important feature which has emerged from the best fit models (Table 2) is that while the gaussian model is the best fit for all the data set, the parameters, namely, nugget effect (C_0), intercept between sill and nugget effect (C) and range (a), have changed over the years. Nugget effect shows random change between 2.39 and 4.0 in September and between 2.64 and 3.75 in June and in both the seasons, there is a general decreasing trend in nugget value. The range in which the intercept C lies between 63.2 and 69.35 for June and it is between 62.44 and 77.40 for September. The range `a' exhibits constant increase through both the season and over the years (except a deviation in year 1989).

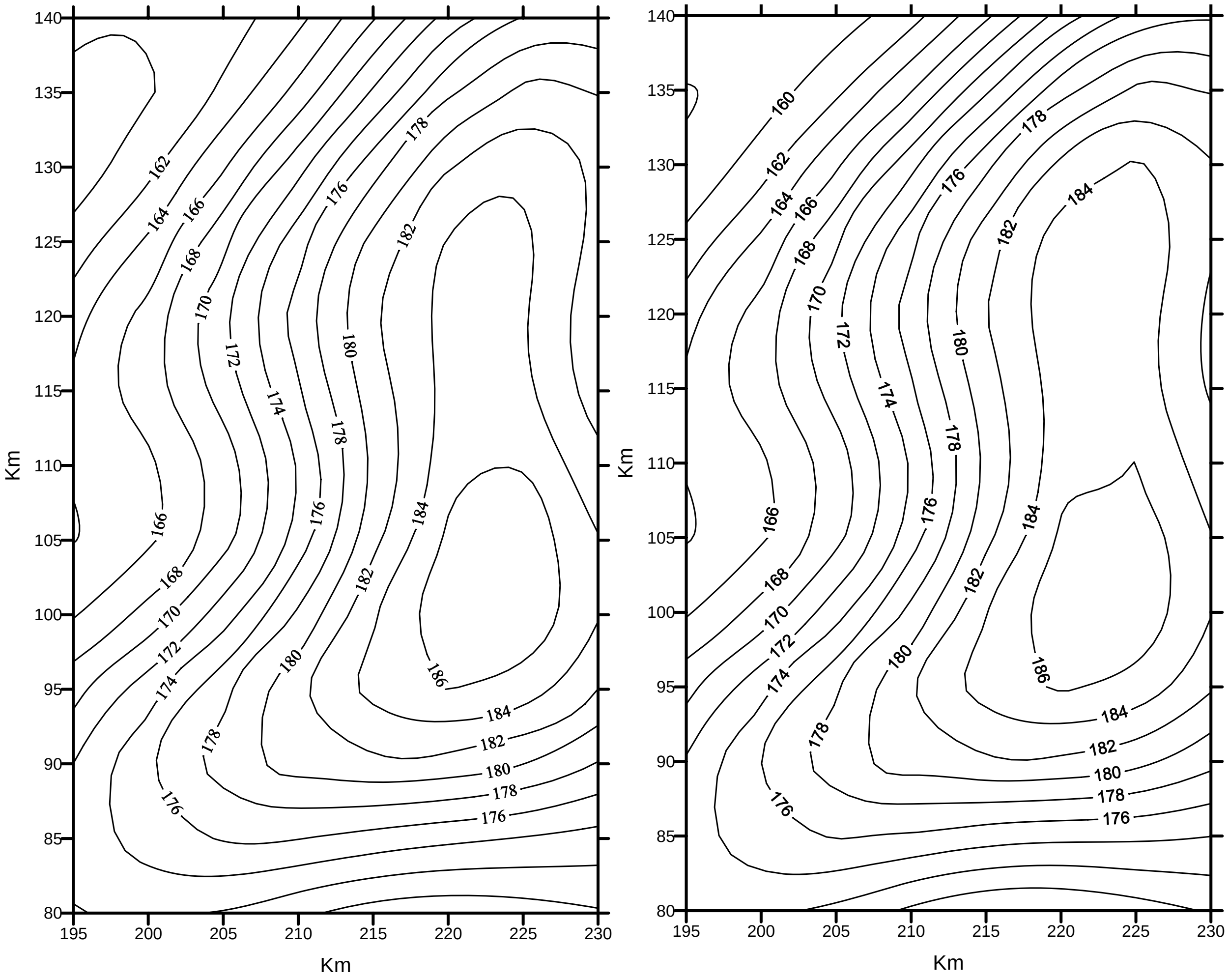
Table 2: Summary details of fitted gaussian models

S. No.	Data		C_0[*1] (m^2)	C[*2] (m^2)	a[*3] (km)
	Year	Month			
1.	1985	June	3.60	67.50	13.20
		Sept.	4.00	71.00	13.10
2.	1986	June	3.50	68.85	13.80
		Sept.	4.00	71.30	14.00
3.	1987	June	3.75	63.20	14.00
		Sept.	3.22	62.44	14.30
4.	1988	June	3.20	67.00	16.00
		Sept.	3.50	77.40	17.20
5.	1989	June	3.05	66.82	14.90
		Sept.	2.52	68.97	15.40
6.	1990	June	2.64	69.35	17.20
		Sept.	2.39	73.48	17.40

[*1] = Nugget effect [*2] = Sill - Nugget effect [*3] = Range

The cross validation results for 1985 to 1990 are shown in Table 3. Results of Jackknifing procedure for September 1990 data with the fitted gaussian model resulted in a mean error (ME) of -0.053, (which is very near to zero), mean square error (MSE) of 3.59, (which is very low as compared to the variance of the data), kriged reduced mean square error (KRMSE) of 1.088, (which is very near to 1) and a kriged reduced mean error (KRME) of -0.009, (which is near to zero). Here the bracketed quantities refer to the requirements to consider a model as adequate. The above cross validation results show that the chosen model and its parameters are adequate.

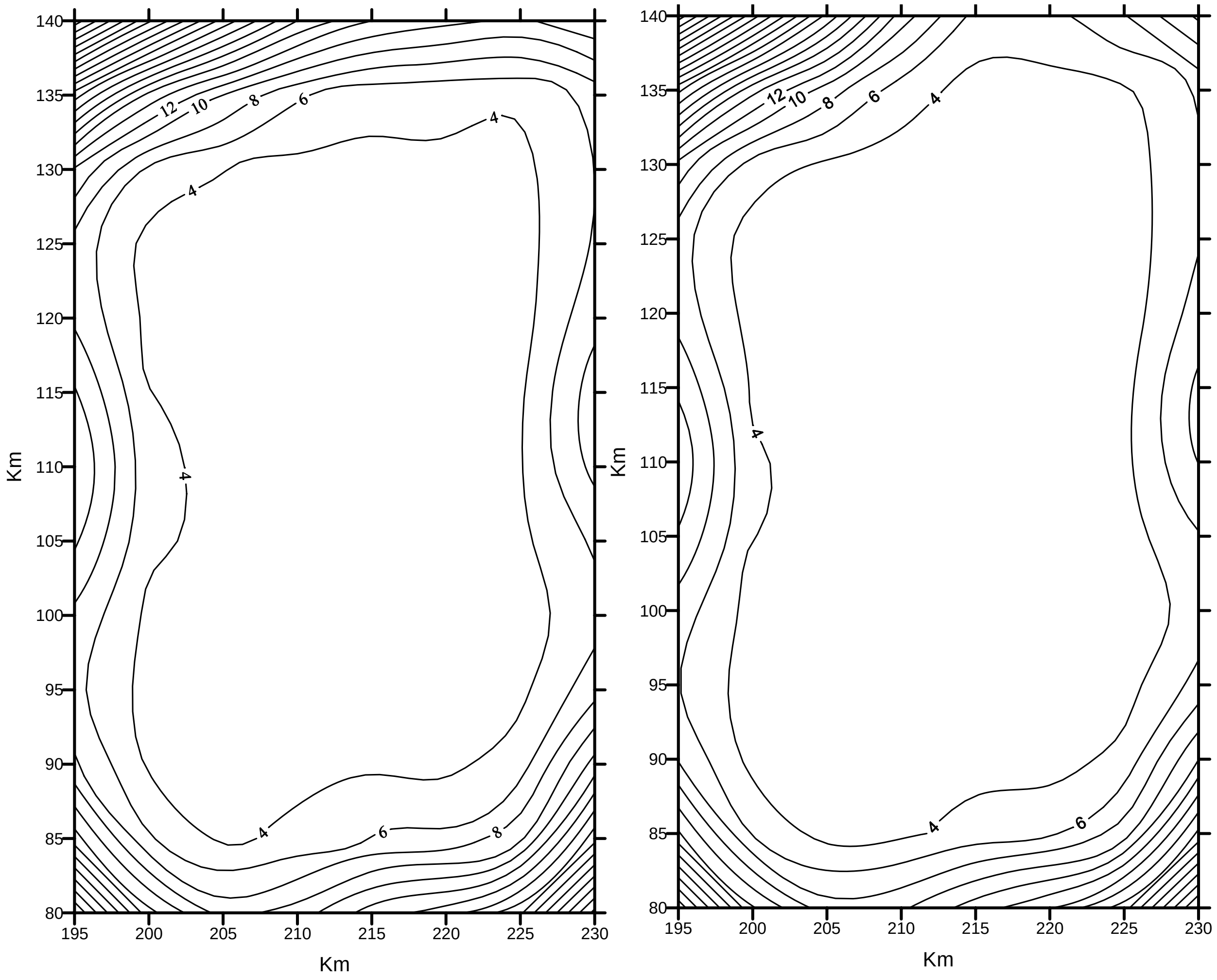
Groundwater levels and estimation variances were calculated by kriging at the nodes of a square grid of 5km x 5km for June and September months of 1985-1990. These estimated level values are used with the SURFER software to draw the contour maps of groundwater levels and estimation variance. The contour maps of the groundwater levels and estimation variance obtained for June and September 1990 are shown in Fig. 4 and 5. Fig. 5 can be interpreted as the map of the reliability of the kriged ground water level in Fig. 4. As seen from the Fig. 5, the estimation variance is low at 4m^2 in the middle of the study area (where most of the observation points are located) and increase rapidly towards the boundaries, where no observation well is located. It indicates that the estimated groundwater level are highly reliable in the middle of the study area and at or near the boundary, these are not reliable to the same extent.



June 1990

September 1990

Figure 4. Groundwater level contours (m) by Kriging Method



June 1990

September 1990

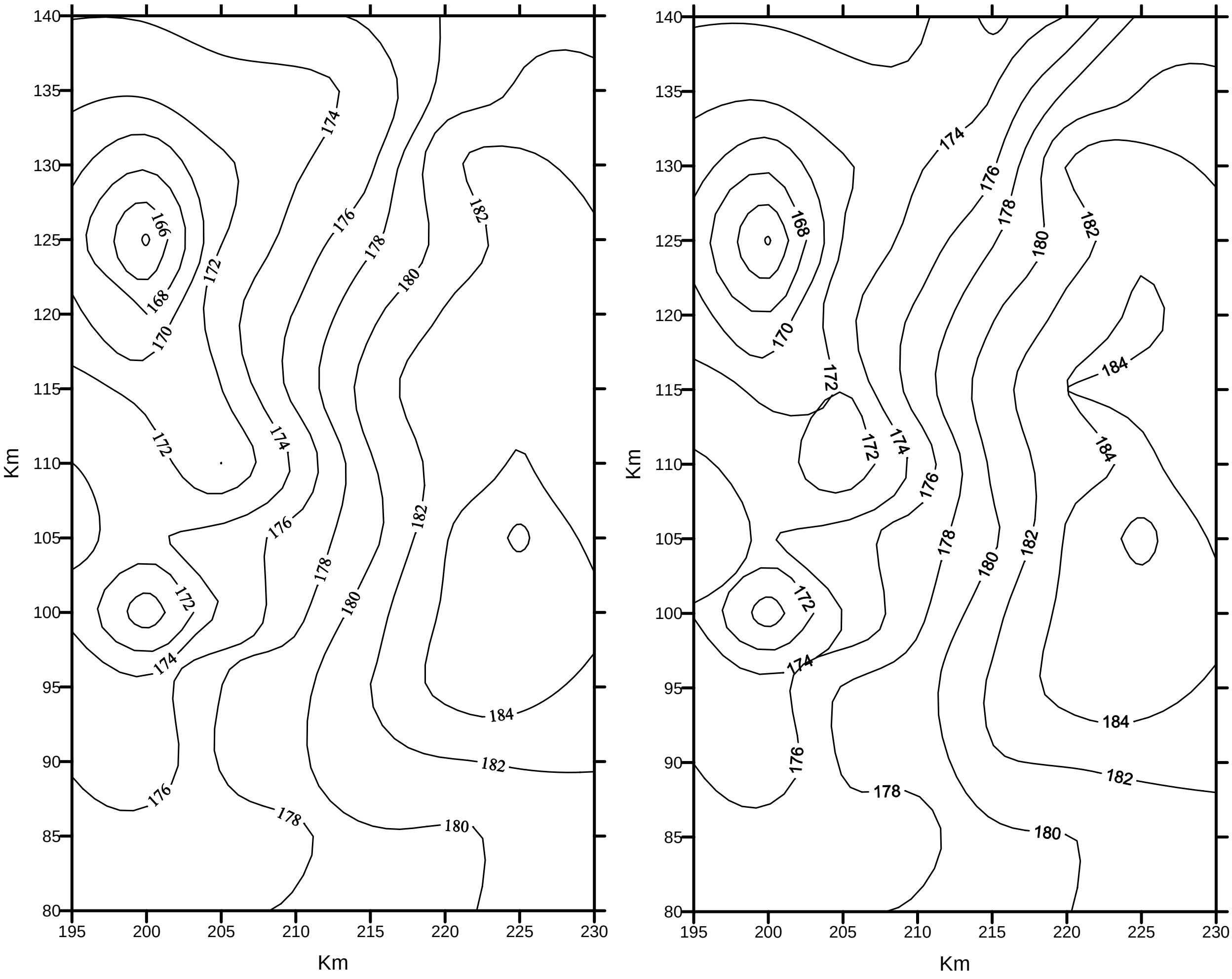
Figure 5. Estimation variance (sq. m) by Kriging

Table 3: Cross validation results with gaussian model

S. No.	Data		ME[*1] (m)	MSE[*2] (m^2)	KRME[*3]	KRMSE[*4]
	Year	Month				
1.	1985	June	-0.171	7.94	-0.029	1.203
		Sept.	-0.236	6.46	-0.037	0.984
2.	1986	June	-0.196	5.14	-0.034	0.924
		Sept.	-0.194	5.53	-0.032	0.882
3.	1987	June	-0.110	5.29	-0.021	0.922
		Sept.	-0.123	5.15	-0.024	1.046
4.	1988	June	-0.096	4.17	-0.018	0.915
		Sept.	-0.040	4.29	-0.006	0.899
5.	1989	June	-0.096	4.28	-0.019	0.950
		Sept.	-0.069	4.37	-0.013	1.172
6.	1990	June	-0.082	3.92	-0.015	1.032
		Sept.	-0.053	3.59	-0.009	1.088

[*1] = Mean error, [*2] = Mean sq error, [*3] = Kriged reduced mean error,
[*4] = Kriged reduced mean sq error

The ground water level contour obtained by inverse square distance (ISD) method for June and September 1990 are given in Fig. 6. The contour map provided by two interpolation methods (Fig. 4 and 6) are different as kriging takes into consideration the spatial structure of the parameter and ISD method consider only distance between estimated and observed points. The comparison of ISD map with the map obtained by kriging (Fig. 4) indicated that kriging has resulted in smoother map. More quantitative comparison of these two techniques was obtained by comparing the ME and MSE obtained by jackknifing procedure (Table 4). ISD resulted in a ME of –0.08m to -0.79m whereas kriging gave a ME of –0.04m to -0.24m. Similarly, ISD gave a MSE of 10.9m^2 to 23.0 m^2 and kriging 3.6 m^2 to 7.9 m^2. It is concluded that for this study, kriging performed better than the inverse square distance method and more importantly, the degree of difference between the kriged values and the estimates using ISD are significantly high. Also, kriging out performs ISD in giving reliability indices and in the present study the reliability of the estimates is high as indicated by low level of variance.



June 1990

September 1990

Figure 6. Groundwater level contours (m) by Inverse Square Distance Method

Table 4: Comparison of errors of two interpolation methods

S. No	Data		ME[*1](m)		MSE[*2] (m^2)	
	Year	Month	ISD[*3]	K[*4]	ISD	K
1.	1985	June	-0.79	-0.17	21.2	7.9
		Sept.	-0.76	-0.24	23.0	6.5
2.	1986	June	-0.65	-0.20	18.5	5.1
		Sept.	-0.70	-0.19	18.5	5.5
3.	1987	June	-0.49	-0.11	17.6	5.3
		Sept.	-0.57	-0.12	16.0	5.2
4.	1988	June	-0.27	-0.10	13.0	4.2
		Sept.	-0.08	-0.04	11.6	4.3
5.	1989	June	-0.31	-0.09	13.0	4.3
		Sept.	-0.26	-0.07	12.3	4.4
6.	1990	June	-0.22	-0.08	10.9	3.9
		Sept.	-0.36	-0.05	11.6	3.6

[*1] Mean error [*2] Mean square error

[*3] Inverse square distance [*4] Kriging

Conclusions

In this study, kriging, a type of geostatistical techniques, is applied to the groundwater level data of pre-monsoon (June) and post-monsoon (September) in the part of the canal command area of IGNP in Rajasthan, India, over a period of six years (1985-1990). The gaussian model is found to the best model representing the spatial variability of groundwater level data over the years. However, its parameters, namely, nugget effect, sill and range, have changed over the years. The groundwater levels are found to be auto-correlated upto a distance varying from 13.1km to 17.4km in the study area. The modeling results indicate that the kriged groundwater levels satisfactorily matched the observed groundwater levels.

Estimation errors from this analysis can provide guidance for the selection of new observation sites to reduce estimation errors. The kriged map provided a more regular gradient of the groundwater table, which seems more likely than the mound and valley combination provided by the inverse square distance method.

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