Simultaneous Rate and Power Control in Multirate Multimedia CDMA Systems

Sunil Kandukuri and Stephen Boyd

Department of Electrical Engineering

Stanford University

Stanford, CA 94305

e-mail: {sunilkn,boyd}@stanford.edu

Abstract — We consider a wireless multimedia CDMA system, where the different mobiles transmit at different rates. We devise a method of simultaneously optimizing the power and rate at which the different terminals transmit for achieving their required Quality of Service (QoS). The QoS is defined to be the effective data rate which is different from the transmission rate. Optimization of power enhances battery life and optimization of data transmission rates facilitates in building cheaper and power efficient mobile systems. The joint optimization problem is formulated so that for a specified QoS, the total power transmitted by all the mobiles, and the sum of the transmitting rates of different mobiles can be minimized. The optimization problem is shown to be a non-linear and non-convex problem, but is solved to get a globally optimal solution using geometric programming. Results show that with optimized rates and powers, we can obtain better QoS than that obtained by present systems which use higher power.

I. INTRODUCTION

As the wireless channel crosses its traditional use for voice into the realms of multimedia and data traffic, there increases a need for it to support subscribers having different QoS. In this paper, we consider multirate DS/CDMA (Direct-Sequence Code Division Multiple Access) systems which support multimedia traffic. There are different ways to design a DS/CDMA multi-rate system [3]. They include spreading all the signals independent of the bit rate to the same bandwidth, altering the chip rate, and changing the modulation format [14, 15, 5]. In this paper, we consider systems which keep the chip rate constant and spread to the same bandwidth irrespective of their bit rates. Thus, users transmitting a low bit rate have a high processing gain. This allows those users to transmit at a lower power, thus making inappropriate to use the constant received power scheme [16] in a multirate system. Thus, we need to use more sophisticated power control schemes to achieve diverse QoS [4].

Different power control schemes have been proposed for multimedia CDMA systems. The scheme proposed in [17] has the objective of attaining the highest possible system capacity while guaranteeing the minimum required voice quality. However, their approach considers only two traffic types and has no generalization to multi-media traffic. In [5], a distributed algorithm for power control for integrated traffic is proposed, which is based on heuristic reasoning. In [18], a power control algorithm for bursty data sources is given,

where the spreading gains of the users are static. An optimal power control strategy which can be extended to include delay sensitive traffic is proposed in [4]. The relationship between data rate, QoS, and transmission power is nicely developed in that paper.

Some of the approaches and algorithms considered above are heuristics and are not proved to be globally optimum. Other algorithms like [5, 6, 7, 4] only optimize the power, and arbitrarily fix the transmission rate at which the different mobiles transmit. Thus, as they completely overlook one dimension of system optimization, they are not globally optimum. Optimizing over both power and rate results in lower transmission rates as well as lower power. These are both desirable features, because: i) mobile systems with lower power require smaller batteries and, ii) systems using lower rates are power efficient as they have their digital signal processing circuitry running at lower rates [8], and are also simpler and cheaper to build [2]. In this paper, we describe an efficient method for simultaneously optimizing both the powers and transmission rates of the mobiles roaming about in the wireless system.

The combined rate and power control problem is formulated such that the total power used by all the mobiles can be minimized while simultaneously imposing constraints on the rates at which the mobiles can transmit. Not surprisingly, the optimization problem which facilitates both the power and the transmission rate to be modeled as optimization parameters turns out to be non-linear, and non-convex. But, we show that it is a special kind of non-convex optimization problem called geometric programming [1], which enables us to find the global optimum.

The two characteristics of geometric programming; ability to find the global optimum and the efficiency with which it can be obtained, have very important practical implications. The fact that geometric programs can be globally solved also means that sets of infeasible specifications are unambiguously recognized. We attain either the global optimum or a proof that the set of specifications is infeasible. The speed and efficiency with which geometric programs can be solved facilitates for robust designs, i.e., designs guaranteed to meet multiple sets of specifications. Most of the algorithms and designs available in the literature today assume quasistationarity of the interference and design for a single set of parameters. In the quick fading wireless channels where the link gains change in the order of milliseconds, it might not always be possible to track the changing link gains and redesign the system parameters in time. Even if it were possible, it would take a lot of processor power and time which results in nonuseful power consumption [8]. As a result, if we design the system parameters to satisfy multiple sets of specifications, the system will be

able to operate in rapidly changing environments. Thus, our proposed method which enables us to optimize both power and transmission rate, and facilitates design for robustness, results in minimal power consumption for a given QoS.

In Section II, we briefly describe geometric programming. In Section III, we describe in detail how the multi-rate DS/CDMA system is modeled. In section IV, we formulate the problem and solve it. In section V, we present the simulation results. We finish with the conclusions in section VI.

II. GEOMETRIC PROGRAMMING

Geometric programming is a special type of non-convex optimization problem which can be cast as a convex optimization problem. Convex optimization problems are those in which the objective and constraint functions are all convex. The practical advantages of convex optimization have been brought into the fore due to the extremely powerful interior-point methods developed initially for linear programming by Karmarkar [10] and extended to *nonlinear* convex optimization problems by Nesterov and Nemirovsky [11]. These methods can solve large problems, with thousands of variables and tens of thousands of constraints, very efficiently. More information about interior point methods can be obtained from [12, 13].

Let $x_1, \ldots, x_n$ be n real, positive variables, and x denote the vector of these n variables. A function f is called a *posynomial* function of x if it has the form

$$f(x_1, \ldots, x_n) = \sum_{k=1}^{t} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \ldots x_n^{\alpha_{nk}}$$

where $c_k \geq 0$ and $\alpha_{ij} \in \mathcal{R}$. Only the coefficients c_k must be nonnegative, but the exponents α_{ij} can be any negative or fractional number. f is called a *monomial* function if $t = 1$. A *geometric program* is an optimization problem of the form

$$\begin{array}{ll}
\text{minimize} & f_0(\mathbf{x}) \\
\text{subject to} & f_i(\mathbf{x}) \leq 1, \quad i = 1, \ldots, m, \\
& g_i(\mathbf{x}) = 1, \quad i = 1, \ldots, p, \\
& x_i > 0, \quad i = 1, \ldots, n,
\end{array} \quad (1)$$

where $f_1, \ldots, f_m$ are posynomial functions and $g_1, \ldots, g_p$ are monomial functions. A geometric program can be reformulated as a *convex optimization problem*, i.e., the problem of minimizing a convex function subject to convex inequality constraints and linear equality constraints. This major simplification is effected by the following change of variables. Defining $y_i = log(x_i)$, and taking the logarithm of posynomial f, we get

$$h(y) = log(f(e^{y_1}, \ldots, e^{y_n})) = log(\sum_{k=1}^{t} e^{a_k^T y + b_k})$$

where $a_k^T = [\alpha_{1k} \ldots \alpha_{nk}]$ and $b_k = log(c_k)$. It can be easily seen that h is a convex function of the new variable y, by showing that for all $y, z \in \mathcal{R}^n$ and $0 \leq \lambda \leq 1$ we have

$$h(\lambda y + (1 - \lambda)z) \leq \lambda h(y) + (1 - \lambda)h(z).$$

We can convert the standard geometric program (1) into a convex program by expressing it as

$$\begin{array}{ll}
\text{minimize} & log(f_0(e^{y_1}, \ldots, e^{y_n})) \\
\text{subject to} & log(f_i(e^{y_1}, \ldots, e^{y_n})) \leq 0, \quad i = 1, \ldots, m, \\
& log(g_i(e^{y_1}, \ldots, e^{y_n})) = 0, \quad i = 1, \ldots, p.
\end{array} \quad (2)$$

This is the exponential form of the geometric program (1). As mentioned before, since the exponential form geometric program (2) is a convex optimization problem, it can be solved globally and very efficiently, and there is also a complete and useful duality and sensitivity theory for them [1].

III. SYSTEM MODEL

We consider a single-cell CDMA system with N mobiles. The power transmitted by each terminal i is denoted by P_i and the raw data rate at which it transmits is denoted by R_i. The signals of all users are spread to the same bandwidth, W. Since each terminal has different transmission rate, the spreading gain is different for each of them. Only the uplink channel is considered in this paper. In a CDMA system, different mobiles transmit using different signature sequences which are ideally orthogonal to each other. However, due to the effects of fading channels and asynchrony present in wireless systems, they are seldom orthogonal. The interfering effect of the power transmitted by one mobile on another mobile can be captured by the cross correlations C_{ij} between signature sequences. If we denote the gains of the paths between the mobiles and the base station by L_i, then the total interfering effect caused by mobile j to mobile i is given by $G_{ij} = L_i C_{ij}$. In particular, the power $G_{ij}P_j$ received at the base station represents the interference by mobile j in the signal space of mobile i. Thus, G_{ij} takes into account both the effects of distance, shadowing and fading, as well as cross correlations between signature sequences. The power $G_{ii}P_i$ is the signal power of terminal i received at the base station where G_{ii} represents the total signal gain (link gain L_i multiplied with autocorrelation C_{ii}) between mobile i and the base station. Thus, the signal-to-interference ratio (SIR) Γ_i of mobile i can be written as

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j \neq i} G_{ij}P_j + \eta} \quad (3)$$

where η is the power of the receiver noise. The bit energy-to-interference spectral density ratio, E_b/I_0, is related to the SIR by the formula [4]

$$(\frac{E_b}{I_0})_i = \frac{\Gamma_i W}{R_i}. \quad (4)$$

In digital and wireless communications, it is well known that the bit error rate P_e is a function of E_b/I_0. As terminal i uses the channel R_i times per unit time, the information rate through the channel is given by [4],

$$R_i[1 - P_e(\frac{\Gamma_i W}{R_i})] \quad (5)$$

This is called the effective data rate of user i. In this paper, when user i specifies his QoS requirements γ_i, it is assumed that he is specifying his effective data rate, because it is his effective date rate and not the raw data rate which determines the quality of service he receives. For flat fading wireless channels, the probability of error depends on the modulation scheme and is given by the following formulas [9, 19]. Denoting $\beta = \frac{E_b}{I_0}$, we have

- $P_{e,PSK} = \frac{1}{2}[1 - \sqrt{\frac{\beta}{1+\beta}}]$ (coherent binary PSK)

- $P_{e,FSK} = \frac{1}{2}[1 - \sqrt{\frac{\beta}{2+\beta}}]$ (coherent binary FSK)

- $P_{e,DPSK} = \frac{1}{2(1+\beta)}$ (differential binary FSK)

- $P_{e,NCFSK} = \frac{1}{2+\beta}$ (noncoherent FSK)

For large values of E_b/I_0, the error probability equations may be simplified as

$$P_e = \frac{1}{c\beta} \qquad (6)$$

for all the modulation schemes, and the nonnegative constant c differs according to the modulation scheme. We will be using the above expression for P_e in this paper. Since the optimum design aims for the minimum possible data rate for reasons listed in Section I, our optimization results in the maximum possible E_b/I_0 for a given QoS (γ). The above reason and the fact that any well designed system has a high E_b/I_0 justify the use of the assumption made for simplifying the expresssion for P_e.

Given the above problem setup and modelling of the system, we see that we can solve various optimization problems having different objectives and constraints. In particular some of the optimization problems that can be solved are:

1. Minimize the sum of powers of all transmitters subject to the requirement that each mobile attains its requested QoS (or effective data rate) γ_i.

2. Minimize the sum of raw data rates subject to the requirement that each mobile attains its target QoS γ_i.

In this paper, we assume that we know the path gains between all the terminals and the base station. Eventhough, this is an uplink power control problem, as the base station passes information to all the mobiles on the control channel, this is a very realistic assumption.

IV. Optimization: Power and Rate Control

We first consider the problem of minimizing the sum of powers of all transmitters subject to the QoS requirements of all the mobiles, assuming every mobile arriving into the system specifies its required QoS γ_i. In addition we require the raw data rate R_i to be less than some threshold R_{thresh}. The optimization variables are the transmitter powers P_i and transmitter data rates R_i of different mobiles. The above problem can be posed as follows:

$$
\begin{aligned}
\text{minimize} \quad & 1^T P \quad \text{(sum of powers)} \\
\text{subject to} \quad & R_i[1 - P_e(\tfrac{\Gamma_i W}{R_i})] \geq \gamma_i \\
& R_i \leq R_{thresh} \\
& P \succ 0
\end{aligned}
\qquad (7)
$$

where 1^T is a vector with all elements equal to 1 and P is the vector of P_i. All the constraints except the first one are linear constraints and can be easily handled. We simplify the first constraint in the following way,

$$R_i[1 - P_e(\frac{\Gamma_i W}{R_i})] \geq \gamma_i$$

$$R_i[1 - \frac{R_i}{c\Gamma_i W}] \geq \gamma_i$$

$$R_i[1 - \frac{R_i(\sum_{j \neq i} G_j P_j + \eta)}{cWG_i P_i}] \geq \gamma_i$$

$$1 \geq \frac{\gamma_i}{R_i} + \frac{R_i(\sum_{j \neq i} G_j P_j)}{cWG_i P_i} + \frac{R_i \eta}{cWG_i P_i}$$

The term on the right hand side is a posynomial, as it is the sum of monomials. Thus, the above optimization problem reduces to a geometric programming problem and can be solved
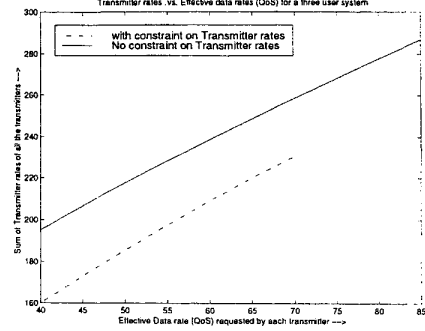

Figure 1: Sum of Raw data rates .vs. QoS for a 3 user system

for a global optimum, if feasible. To the above problem, we can apply the bisection algorithm [1] to solve for the minimum possible R_i (R_{thresh}). Thus, we can find the globally optimal parameters R_i and P_i for all the mobiles in the multimedia CDMA system. Moreover, we can have additional constraints on the minimum and maximum admissible rates and powers of each mobiles. In most practical systems, there is always a minimum power limit as the circuitry needs to be able to detect the signal, and a maximum power limit again limited by the circuit components used. These constraints can be easily accomodated in the optimization problem by observing that

$$P_i \geq P_{min}$$

$$P_i \leq P_{max}$$

are monomial constraints. Similarly, we can solve the other optimization problem of minimizing the sum of raw data rates used by all the mobiles by changing the objective function in the above problem to be

$$\text{minimize} \quad 1^T R \quad \text{(sum of rates)}$$

and imposing a constraint on the maximum and minimum powers that can be transmitted by the mobiles, where R is the vector of raw data rates of all the transmitters. We could alternately optimize for the maximum γ_i, again by using the bisection algorithm.

V. Simulation Results

We consider a single cell CDMA system with three mobiles operating in a wireless flat fading channel. We will assume that the modulation scheme used is NCFSK. Fig. 1 depicts the scenario when we optimize for the sum of powers of all the transmitters for a given QoS (γ). It compares the case, when the optimization is done without having any constraints on the raw data rates, with the case when the optimization is performed by constraining the data transmission rates to be less than some constant R_{thresh}.

The values of $W = 100$, $R_{thresh} = 80$ and $\gamma = 70$ were used. By constraining the data rates to be less than some threshold, we attained the same QoS using lower raw data rates. It was also observed that the total power used was identical in both cases. Thus, our optimization scheme allows us to attain a specified QoS at the minimum possible power and raw data
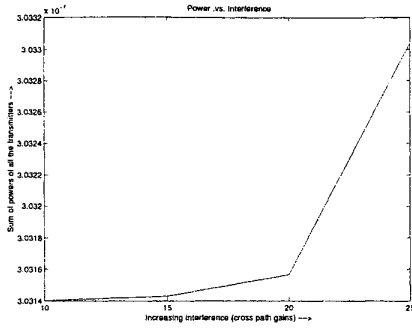
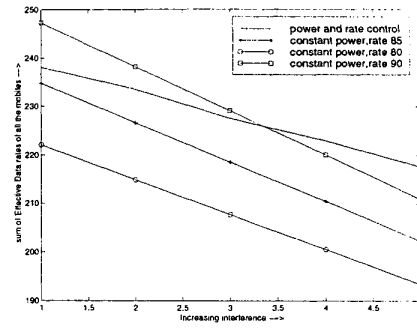Figure 2: Sum of Powers Transmitted .vs. interference

Figure 3: QoS .vs. interference

rate. A close look at the equation for the QoS or the effective data rate, $R_i[1 - P_e(\frac{\Gamma_i W}{R_i})]$, reveals that P_i has only a second order effect on the QoS, and that QoS is mainly controlled by the raw data rate R_i. Thus by constraining the raw data rates to be below a threshold, some values of QoS become infeasible as shown in the graph. The reason for no significant increase in the power transmitted can also be attributed to the same cause. As mentioned in Section III, the G_{ij} represent cross interference gains, which fluctuate with the fading channel conditions. Fig. 2 shows how the total power that needs to be transmitted increases with the increasing cross interference gains for the same raw data rates and target QoS.

We next compare how the simultaneous rate and power control algorithm for the multimedia CDMA systems compares with those used in some present day systems, wherein all the mobiles transmit a fixed power of the order of several milli watts and transmit at a same rate. Fig. 3 shows how the effective data rate (QoS) varies with increasing cross interference gains. The power resulting from our optimization routine was of the order of micro watts, which is clearly a couple of orders less than that used by the present day systems. We see that in the presence of large interference, our combined power and rate control scheme does much better than other schemes even while using less power. The optimization procedure chose the raw data rates to transmit which would result in the maximum QoS, rather than using an arbitrary data rate to transmit.

Eventhough in this particular example, the QoS increases with the raw data rate, in general this might not be true as a higher raw data rate implies a lower spreading gain and might result in lower QoS. Thus, the flexibility of the combined power and rate control and the ease of the optimization procedure helps us find the optimum parameters for the given set of requirements, whether they be the minimum raw data rates for a given QoS or the maximum QoS for a given set of system parameters G_{ij}.

VI. CONCLUSION

In this paper, we have come up with a scheme which jointly determines the optimum rate and power at which each mobile should transmit for the best system performance. Our methodology also enables designing robust systems capable of operating in fast fading channels (adding a new set of constraints in the optimization problem). We have hinted at

how doing simultaneous power and rate control in a multirate CDMA system might be efficient for the whole system than doing just power control. We have also come up with a method wherein each of the mobiles in the multirate system can specify their own QoS. However, our work needs to be extended to the wireless channels when the expression for the probability of error is just not a simple inverse function (6) of the bit energy-to-interference ratio.

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