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INFORMATION AND TIMING IN REPEATED PARTNERSHIPS

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In a repeated partnership game with imperfect monitoring, we distinguish among the effects of (1) reducing the interest rate, (2) shortening the period over which actions are held fixed, and (3) shortening the lag with which accumulated information is reported. All three changes are equivalent in games with perfect monitoring. With imperfect monitoring, reducing the interest rate always increases the possibilities for cooperation, but the other two changes always have the reverse effect when the interest rate is small.

KEYWORDS: Repeated games, partnerships, information, timing, inefficiency, folk theorem, likelihood ratio, reusable punishments.

1. INTRODUCTION

IN MANY ECONOMIC SETTINGS, the possibility of making efficient agreements is limited by the presence of *imperfect monitoring*: some agents cannot observe perfectly the actions of others. Often the economic problem of interest is modeled as involving the indefinite repetition of some fixed strategic situation. Examples include partnership problems,[2] oligopolistic coordination,[3] and principal-agent problems.[4] Because the incentives for cooperation in these repeated game models depend upon players' responding aggressively to indications that some participants are violating the agreement, anything that makes violations easier to detect and punish enlarges the set of equilibrium payoffs.[5] There is some presumption, then, that possibilities for cooperation are also enhanced when information about the players' behavior can be observed without delay and when players can respond quickly to new information. While that is exactly what happens under perfect monitoring, the presumption is entirely misleading for games with imperfect monitoring, because it omits an important effect: When information reporting is delayed or periods of fixed action are increased, the players' abilities to devise profitable cheating strategies is diminished. Frequently, this second effect more than offsets the corresponding limits on the players' ability to detect and punish the defector quickly.

We begin to model these issues in Section 2, where we develop our basic stochastic model of the Prisoners' Dilemma. There, we establish that the maximum symmetric equilibrium value is equal to the first-best value minus an incentive cost attributable to imperfect monitoring, where the incentive cost is

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[2] See, for example, Fudenberg, Levine, and Maskin (1989), Radner (1986), and Radner, Myerson, and Maskin (1986).

[3] See Green and Porter (1984), Porter (1983), and Abreu, Pearce, and Stacchetti (1986).

[4] See Fudenberg, Holmstrom, and Milgrom (1990), Fudenberg, Levine, and Maskin (1989), Holmstrom and Milgrom (1987), Malcomson and Spinnewyn (1988), Radner (1981, 1983), Rogerson (1985), Rubinstein (1979a), and Spear and Srivastava (1987).

[5] See Kandori (1991).

equal to a player's gain from defecting divided by a simple statistical measure of the power of the test used to detect and deter cheating. This formula sets the stage for our analysis of the roles of time and information in determining payoffs.

In Section 3, we enrich our model of the Prisoners' Dilemma to one in which information arrives continuously over time, the interest rate is fixed with respect to actual time (rather than periods), and the period of fixed action is a parameter. Here we find a surprise: The effects of reducing the interest rate toward zero are very different from those of making the periods of fixed action short. In the traditional model of repeated games which is the basis of the Folk Theorem,[6] it is assumed that at the end of each period of play, all the players can observe all the actions taken during that period. It then follows that both very patient players and very short periods of fixed action are represented in the model by an inter-period discount factor that is close to one, and the standard Folk Theorem correspondingly has two interpretations. To interpret the analogous limit theorem for games with imperfect monitoring in both these ways, however, would be wrong: Our analysis demonstrates that reducing the interest rates and shortening the period of fixed action *always* lead to different limits for these games. For some parameters, reducing the interest rate to zero allows asymptotically efficient equilibria, but shortening the period of fixed action destroys any possibility of cooperation.

In Section 4, we build a model of the Prisoners' Dilemma in which information arrives not continuously or every period, but only once every several periods. In the standard repeated game model in which any information that is reported is perfect, any increase in the number of periods over which information is withheld can only shrink the set of equilibrium possibilities. Once again, however, the analogy with the complete information case proves to be treacherous: When the underlying information is imperfect, delaying the release of information can allow a higher equilibrium payoff for all the players. Perhaps more surprisingly, the gains that can be achieved by delaying information are sometimes quite large: A t-period delay in revealing information multiplies the cost attributable to imperfect monitoring by a factor of $1/t$, provided the interest rate is sufficiently low. That is, if information is revealed only every four periods, then the cost is reduced to 25% of its original level.

The conclusions in Sections 2–4 were drawn in the context of Prisoners' Dilemma games. The simple two-strategy model of the Prisoners' Dilemma makes the statistical problem of detecting cheating straightforward. In Section 5, we tackle the harder technical problem of evaluating equilibrium values when there are several players each with many ways to cheat in a symmetric "partnership game." We find that the main conclusions of Section 4 as well as the supporting logic have appropriate generalizations. In particular, for a general n-player symmetric game with imperfect information, when the interest rate is sufficiently low, the equilibrium value is higher for the case with some delays in

[6] See Aumann and Shapley (1976), Rubinstein (1979b), and Fudenberg and Maskin (1986).

the release of performance information than for the case in which performance information is released as soon as it becomes available. Also, for low interest rates, a t-period delay multiplies the cost of imperfect information by a factor on the order of $1/t$.

In Section 6, we comment on the significance of these results in light of some recent developments in the theory of repeated partnerships.

2. INEFFICIENCY AND IMPERFECT MONITORING

We consider a Prisoners' Dilemma game with the following payoff matrix:

	C	D
C	π, π	$-b, \pi + g$
D	$\pi + g, -b$	$0, 0$

A defector from mutual cooperation "gains" g. We assume that π, g, and b are all strictly positive and that $\pi > g - b$. Then, (i) defection is a dominant strategy and (ii) the best symmetric outcome is (C, C) rather than a randomization between (C, D) and (D, C).

We suppose that this Prisoners' Dilemma game is played continuously over the time interval $[0, \infty)$ and interpret the payoffs in the matrix above as flow rates of payoff. If the payoff at time t is u_t, then the net present value of payoffs for the whole game is $NPV = \int e^{-rt} u_t \, dt$ or, in the equivalent mean flow terms that are customarily used in repeated game theory, $r \cdot NPV$. For example, if the cooperative strategies (C, C) are always played, then $u_t \equiv \pi$ and each player's (mean flow) payoff is π as well.

Now, suppose that information is perfect and instantaneous and that the players can adjust their actions at the end of each period of length t. It is well known that if any strategy can maintain cooperation as a perfect equilibrium outcome in this game, then the "trigger strategy" (according to which each player plays C until his competitor first defects, after which he plays D) can. Against that strategy by the other player, a player who considers defecting in some period can then expect to obtain a payoff of $(\pi + g)(1 - e^{-rt})$ over the infinite horizon, as compared to π that can be obtained by cooperating forever. The existence of an equilibrium with cooperation hinges on the comparison. The equilibrium exists if

$$(1) \qquad (\pi + g)(1 - e^{-rt}) \leqslant \pi,$$

but not otherwise. Thus, the equilibrium with cooperative play exists if and only if $rt \leqslant \ln(1 + \pi/g)$. Notice that t and r enter symmetrically in this complete information analysis.

Similarly, if information about behavior is reported to the players with a lag of t, or if performance information is reported to players only at the dates $t, 2t, 3t$, and so on, then continual cooperation is an equilibrium outcome if and only if (1) holds. Long reporting lags and long periods of fixed action have the effect of allowing a defector to profit for a considerable period of time before suffering retaliation. In repeated games with perfect monitoring, longer information reporting lags and periods of fixed actions can only inhibit cooperation.

Next, suppose that information may be imperfect. We assume that the players cannot observe past behavior directly, but instead observe a signal in each period whose distribution depends on behavior in that period. In particular, suppose that in the game in which actions are held fixed for a length of time t, the probability that the kth possible signal is realized depends on the actions taken: The probability is $p_k(t)$ if both players cooperate and $q_k(t)$ if one player defects. (The signal probabilities when both defect will be irrelevant in our analysis).[7] We assume that p is a strictly positive vector to ensure that the incentive problem is always nontrivial.[8]

With imperfect information, it may often be impossible to find any strategies that support cooperation at every stage as an equilibrium outcome, so we set about to find the symmetric pure strategy equilibrium with the highest payoff for the two players. (Throughout we use "symmetric" in the following strong sense: The equilibrium specifies symmetric actions after *all* histories.) If the players can jointly observe a public signal to use for randomizations, then there are just two possibilities. The first is that there is a unique equilibrium at which the players always play Defect and each earns an equilibrium payoff of zero. The second possibility is that the best equilibrium entails a trigger strategy as follows. The players initially play Cooperate, but after each round they randomize, switching permanently to the (D, D)-equilibrium with some probability α_k that depends on the signal realization k. Notice (i) that the (conditional) probability α_k that players who are cooperating at some date switch to the (D, D)-equilibrium at the next date depends only on the signal k observed *at that one date* and (ii) that once the players have switched to (D, D), they remain switched forever.

Trigger strategies, then, are described by a vector of probabilities $\alpha = (\alpha_1, \dots, \alpha_K)$, where $K \leqslant \infty$ is the number of possible signals. When does a vector α define an equilibrium trigger strategy with value v? In standard dynamic programming fashion, the value v is determined by a recursion: It is equal to $1 - e^{-rt}$ times the payoff earned during the initial period plus e^{-rt} times the expected continuation payoff after the initial period, both computed on the assumption that each player adopts the equilibrium strategy. For the model we have just described, the value recursion equation is:

$$v = (1 - e^{-rt})\pi + e^{-rt} \sum_{k=1}^{K} p_k(1 - \alpha_k) v. \tag{2}$$

[7] Although we have modeled the payoffs separately from the stochastic information, our model also encompasses the case in which the stochastic signal affects players' payoffs directly. In that case, the π's are to be interpreted as *expected* stage game payoffs. Analytically, our formulation helps to distinguish the separate effects of assumptions about information and payoffs.

[8] This shows up below in the conclusion that, at the best equilibrium, players are just indifferent between cheating and being honest.

What about incentives? According to the Optimality Principle of Dynamic Programming, a player has no incentive to deviate from a proposed strategy if there is no circumstance in which a one-step deviation would be profitable. Plainly, it is never profitable for the player to deviate unilaterally from the trigger strategy when that strategy calls for a play of D. It is not profitable to deviate on other occasions if

$$(3) \qquad (\pi+g)(1-e^{-rt})+e^{-rt}\sum_{k=1}^{K} q_k(1-\alpha_k)v \leqslant v.$$

The left-hand side of (3) is the expected payoff to a player who plans to defect once and then adhere to the equilibrium strategy, while the right-hand side is the equilibrium value.

Substituting (2) into (3), we obtain

$$(3') \qquad g(1-e^{-rt}) \leqslant e^{-rt}\sum_{k=1}^{K} v\alpha_k(q_k-p_k),$$

a reformulation which is sometimes convenient.

The analysis to this point has been routine. We summarize it with a Proposition.

Proposition 1: *The trigger strategy characterized by α is a symmetric equilibrium strategy with value v if and only if* (2) *and* (3) *hold and α satisfies*:

$$(4) \qquad 0 \leqslant \alpha_k \leqslant 1 \qquad \textit{for all } k.$$

Moreover, if the maximal value of a symmetric pure strategy equilibrium is positive, then it is achieved by a trigger strategy equilibrium of this form.

In view of the Proposition, the maximum symmetric equilibrium payoff v and the associated trigger strategy α can be determined from the optimal solution of the following linear programming problem:

$$(\text{LP}) \qquad \max_{v,\, v\alpha} v \qquad \text{subject to (2), (3'), and}$$

$$(4') \qquad 0 \leqslant v\alpha_k \leqslant v.$$

Notice that we have written the problem with choice variables v and $v\alpha$, rather than the more natural v and α. This change of variables makes the constraints (2), (3′), and (4′) linear in the choice variables, so that the program (LP) is a linear program and can be analyzed or numerically solved by linear programming methods. Notice, too, the feasible set (that is, the set of $(v, v\alpha)$ pairs satisfying the constraints) for this LP can be empty. In that case, according to Proposition 1, the only equilibrium of the game is the one in which the players always defect and earn payoffs of zero.

From our assumption that the probabilities p are all nonzero, we may conclude that there is no equilibrium of the game with value $v=\pi$. So, the incentive constraint (3′) in (LP) must be binding, that is, it must hold with equality at the optimal solution of (LP). This fact will prove useful in the next Proposition.

Given any feasible solution $(v, v\alpha)$ of (LP), we can determine a trigger strategy α and an associated *likelihood ratio* l defined as follows:

$$l = \left[\sum q_k \alpha_k\right] / \left[\sum p_k \alpha_k\right]. \tag{5}$$

PROPOSITION 2: *Suppose that (LP) has a nonempty feasible set. If $(v, v\alpha)$ is a feasible solution for which the incentive constraint (3′) holds with equality, then*

$$v = \pi - g/(l-1) > 0. \tag{6}$$

In particular, the payoff maximizing symmetric equilibrium satisfies (6).

PROOF: We may rewrite the definition of l in the following form:

$$l \sum p_k \alpha_k v = \sum q_k \alpha_k v. \tag{7}$$

Then, substituting into (7) from (2) (for $\sum p_k \alpha_k v$) and from (3) (for $\sum q_k \alpha_k v$) and then simplifying yields equation (6). From (3′), it is clear that $v > 0$. From Proposition 1, the optimal solution of (LP) defines a payoff maximizing symmetric equilibrium and, as discussed above, (3′) is binding at the optimal solution. *Q.E.D.*

There are two aspects of Proposition 2 that we wish to emphasize. First, the Proposition establishes that the problem of finding a payoff maximizing equilibrium is equivalent to the statistical problem of finding the maximum likelihood test which is just sufficient to deter defection, that is, which satisfies (3′) with equality. Moreover, any test which is just sufficient to deter defections and which maximizes the likelihood function corresponds to a payoff maximizing equilibrium.

Second, the Proposition provides a remarkably simple and intuitive characterization of the optimal value itself. Equation (6) expresses the optimal value as the payoff from always cooperating minus a cost c incurred even though the players behave honestly at equilibrium. Given the trigger strategy, the corresponding cost that a defector would incur is lc. The Proposition considers the situation where the extra cost $(l-1)c$ is equal to the gain g that a defector enjoys, so that $c = g/(l-1)$. The extreme cases make the interpretation of this formula clear. If there is a test to deter defections that never falsely accuses nondefectors, then $l = \infty$ and the deterrence cost at the associated trigger strategy equilibrium is zero. If defecting is statistically indistinguishable from honest behavior, then $l = 1$, and regardless of the interest rate, the gains from cooperating, or the gains from cheating (provided $g > 0$), there can never be a feasible solution to the (LP) nor can there be any equilibrium involving cooperation.

Let us now regard (LP) as being parameterized by the interest rate r and the time interval of fixed action $t > 0$. Let $\bar{v} = \bar{v}(t, r)$ be the optimal value of the (LP) and let $l = l(t, r)$ be the corresponding likelihood ratio. In this section, we consider how changes in r affect the optimal value. Section 3 introduces an information structure that yields useful comparative statics in t.

To study the limiting cases which are the subject of extensions of the Folk Theorem, it is helpful to examine the maximized likelihood ratio defined by $\bar{l}(t) = \max_k q_k(t)/p_k(t)$. In view of Proposition 2, it is clear that, regardless of the interest rate r, we must have $\bar{v}(t,r) \leqslant \pi - g/(\bar{l}(t) - 1)$, a result which is reminiscent of the asymptotic inefficiency results reported by Radner, Myerson, and Maskin (1986) and Fudenberg, Levine, and Maskin (1989). However, the role of low interest rates can be characterized more precisely.

PROPOSITION 3: *For $r > 0$, $\bar{v}(t,r)$ is monotonically decreasing in r. Furthermore,*

$$\lim_{r \downarrow 0} \bar{v}(t,r) = \pi - g/\left[\bar{l}(t) - 1\right] \qquad \text{if} \qquad \bar{l}(t) > 1 + g/\pi. \tag{8}$$

If $\bar{l}(t) \leqslant 1 + g/\pi$, then $\bar{v}(t,r) = 0$ for all $r > 0$; only defection can occur at equilibrium regardless of the interest rate.

PROOF: Let $(v', \alpha v')$ be feasible for (LP) when $r = r_1$. Consider $r_2 < r_1$ and $\alpha_k^2 = \theta \alpha_k^1$, where

$$\theta = \frac{e^{-r_1 t}/(1 - e^{-r_1 t})}{e^{-r_2 t}/(1 - e^{-r_2 t})}.$$

It may be directly checked that $(v^1, \alpha^2 v^1)$ is feasible for (LP) for $r = r_2$. This establishes monotonicity.

It is clear from Propositions 1 and 2 that $\bar{v}(t,r) \leqslant \max(0, \pi - g/[\bar{l}(t) - 1])$ and that, for all $r, \bar{v}(t,r) \geqslant 0$. So we need only show that $\lim_{r \downarrow 0} \bar{v}(t,r) \geqslant \pi - g/(\bar{l}(t) - 1)$ if the right-hand-side is strictly positive, as we shall henceforth assume. That is, we henceforth assume that $\bar{l}(t) > (\pi + g)/\pi$.

By the definition of $\bar{l}(t)$, for any sufficiently small positive ε there exists a k such that $q_k(t)/p_k(t) > \bar{l}(t) - \varepsilon > (\pi + g)/\pi$. Define $l_k = q_k(t)/p_k(t)$, $v = \pi - g/(l_k - 1) > 0$, and $\alpha_j = 0$ for $j \neq k$, and set $\alpha_k(t,r)$ to make (3) hold with equality. It is clear that as $r \downarrow 0$, $\alpha_k \downarrow 0$. In particular, for r sufficiently small, $0 \leqslant \alpha_k \leqslant 1$ and then, by Proposition 1, the values we have described are equilibrium values. Since $\bar{v}(t,r)$ is the maximal equilibrium value, we have proved that $\lim_{r \downarrow 0} \bar{v}(t,r) \geqslant \sup_k [\pi - g/(l_k - 1)] = \pi - g/(\bar{l}(t) - 1)$ as required.

Q.E.D.

In games with imperfect monitoring, lower interest rates work essentially by allowing scaled-up punishments to be used to deter cheating. This serves both to make it more likely that the feasible set is nonempty and, when interest rates are sufficiently small, to allow defection to be deterred with a test with the highest possible likelihood ratio. Nevertheless, even with very low interest rates, Proposition 2 makes clear that the efficiency of equilibrium is inevitably limited by the power of the best statistical test available to deter deviations. So long as the best test has finite power ($\bar{l}(t) < \infty$), the efficiency cost of deterrence is bounded away from zero, uniformly in the interest rate r.

3. VARYING ACTION FREQUENCY

In order to vary the period of fixed action continuously in a model where the arrival of information is exogenously fixed, we must specify some "infinitely divisible" distribution for the information process. The simplest such specification, and the one we adopt, is the Poisson process. Thus, we suppose that when the players are cooperating, signals of a certain type arrive stochastically over time at rate λ; while one player is cheating, the arrival rate is μ. We assume that λ and μ are both strictly positive and we disregard the precise arrival times of signals within a period.[9] Thus, $p_k(t) = e^{-\lambda t}(\lambda t)^k/k!$ and $q_k(t) = e^{-\mu t}(\mu t)^k/k!$.

It may be that $\mu > \lambda$, as, for example, when the events are customer complaints, product failures, or industrial accidents, which are less frequent when team members work hard and incur personal costs. Alternatively, it may be that $\lambda > \mu$, as when the events observed are sales of a product or discoveries in a laboratory which are more frequent when members of the team are working hard.

According to Proposition 3, we can identify the limiting values of these games when the interest rate is small by finding $\bar{l}(t)$. Thus,

$$\bar{l}(t) = \operatorname*{Sup}_k e^{(\lambda-\mu)t}(\mu/\lambda)^k = \begin{cases} +\infty & \text{if } \mu > \lambda, \\ e^{(\lambda-\mu)t} & \text{if } \mu \leqslant \lambda. \end{cases} \tag{9}$$

Applying Proposition 3, an immediate corollary is the following proposition:

PROPOSITION 4: *For the Poisson model*,

$$\lim_{r \downarrow 0} \bar{v}(t,r) = \begin{cases} \pi & \textit{if } \mu > \lambda, \\ 0 & \textit{if } \mu = \lambda, \\ \mathit{Max}\,(0, \pi - g/[e^{(\lambda-\mu)t} - 1]) & \textit{if } \lambda > \mu. \end{cases}$$

Notice the contrast between the efficiency obtainable for low interest rates in the "bad news" case where $\mu > \lambda$, as compared to the limited efficiency obtainable even for low interest rates when $\lambda \geqslant \mu$. In the "bad news" case, the efficient equilibria entail triggering punishments only when several signals (accidents, complaints, etc.) are observed in a period—something which is very much more likely to occur when players cheat than when they play honestly. Such a test can deter cheating while only rarely triggering punishment when players are honest: This is what makes limiting efficiency possible. In the "good news" case when $\lambda \geqslant \mu$, no such powerful statistical tests are available. Low sales, for example, are not infinitely more likely when the other team members are shirking than when they are working hard. Because no strong evidence of

[9] This is justified because the number of signals is a sufficient statistic for the Poisson parameter.

cheating can be identified in this case, payoffs are bounded away from the efficient payoff π uniformly in r.

Next, let us fix the interest rate r to see what happens when the period of fixed action t is varied. As noted in the introduction, varying r and t are equivalent exercises in the case of perfect information (that is, when $l(t,r) \equiv +\infty$). However, with imperfect monitoring, varying t changes $l(t,r)$, which introduces an effect that was absent in the perfect information case.

When r is fixed and t is small, the game may (with one minor exception) be analyzed using first-order approximations. The only events with probability of order t or greater in any period of fixed action are the event that no signal is observed and the event that one signal is observed. These occur with probabilities of approximately $1-\lambda t$ and λt, respectively, when no player cheats, and with probabilities $1-\mu t$ and μt when one player cheats. Using these probabilities, $\bar{l}(t) = \text{Max}[\mu/\lambda, (1-\mu t)/(1-\lambda t)]$ which is approximately $\text{Max}[\mu/\lambda, 1]$. It then follows from Proposition 3 that when $\lambda \geqslant \mu, \bar{v}(t,r) = 0$ for small t, so we turn our attention to the case $\mu > \lambda$. Let α_1 denote the probability that punishment is triggered when a single failure occurs. If all players adopt this trigger strategy, then the mean flow payoff is approximately

$$r \cdot \int_0^\infty \pi \cdot e^{-rt} e^{-\alpha_1 \lambda t}\, dt = \pi r/(r+\alpha_1\lambda).$$

Cheating is therefore deterred if $rg < \alpha_1(\mu-\lambda)\pi r/(r+\alpha_1\lambda)$, that is, if the gain (in mean flow terms) is less than the cost of the extra probability of punishment $\alpha_1(\mu-\lambda)$. Equilibrium is possible for some $\alpha_1 \leqslant 1$ if $g < (\mu-\lambda)\pi/(r+\lambda)$ but not if $g \geqslant (\mu-\lambda)\pi/(r+\lambda)$.[10] When equilibrium does exist, we may use the approximation $l(t,r) \approx \mu/\lambda$ and Proposition 2 to estimate the payoff function. The upshot is the following proposition.

Proposition 5: *For the Poisson model,* $\lim_{t\downarrow 0} \bar{v}(t,r) = \pi - g/(\mu/\lambda - 1)$ *if* $g < (\mu-\lambda)\pi/(r+\lambda)$.[11] *Conversely, if* $g \geqslant (\mu-\lambda)\pi/(r+\lambda)$, *then there exists* $T > 0$ *such that* $\bar{v}(t,r) = 0$ *for all* $t \in (0,T]$.

A full proof of Proposition 5 can be found in the Appendix.

Notice that r helps determine the possibility of cooperation as t grows small, but not the limit equilibrium value. It is particularly instructive to study the case where $\mu > \lambda$ but $g \geqslant (\mu-\lambda)\pi/(\hat{r}+\lambda)$. In that case, $\lim_{t\downarrow 0}\bar{v}(t,\hat{r}) = 0$ but $\lim_{r\downarrow 0}\bar{v}(t,r) = \pi$. Qualitatively, shortening the period of fixed action and reducing the discount rates have *precisely opposite* effects: Reducing t makes it

[10] The boundary case where $g = (\mu-\lambda)\pi/(r+\lambda)$ cannot be resolved by examining first-order terms alone. An examination of higher order terms shows that deterrence in this boundary case would require setting α_1 slightly greater than unity for small t, which is impossible because α_1 is a probability. Hence, cooperation cannot be achieved at equilibrium for small t in this case.

[11] One can further show that in this case, the sign of the partial derivative $\bar{v}_t(0,r)$ is the same as that of $\bar{v}(0,r)[(\mu-\lambda)/r] - 2g$. Thus, when $(\mu-\lambda)/r$ is large, shorter periods of fixed action inhibit cooperation.

impossible to provide any incentive at all for cooperation while reducing r drives the cost of such incentives to zero!

More generally, for the case where $\mu > \lambda$, reducing r always makes full cooperation possible in the limit, but reducing the period of fixed action t never does. For the case where $\mu < \lambda$, reducing r permits some (possibly substantial) cooperation for a range of parameters, but *reducing t always destroys all possibility of cooperation at equilibrium*. Taken together, these examples demolish the presumption that shorter periods of action are somehow similar to lower interest rates in a repeated game model, at least when what is being held fixed are the other real aspects of the environment (observability of outcomes and the flow payoff matrix).

4. REPORTING DELAYS IN A PRISONERS' DILEMMA MODEL

In the remainder of this paper, we develop the idea that delayed performance reports can increase the equilibrium value, provided that the interest rate is small. First, this section explores a Prisoners' Dilemma Model with a special information structure in which the equilibrium strategies and analysis take a simple form. No proofs appear here. In Section 5 we employ the same ideas to prove theorems applicable to general symmetric games.

In the present model, there is a fundamental unit of time, called a period, during which actions cannot be changed. Information signals are generated every period based on that period's actions, but they are revealed to the players only at the ends of periods $t, 2t, 3t, \dots$. For example, in periods $2t+1$ through $3t$, the players know the history of signals generated through period $2t$ only. The standard imperfect monitoring model in which signals are observed at the end of each period corresponds to the case where $t = 1$.

We assume that the signals observed in each period can be of only two types, labeled "success" and "failure." Let λ be the probability of failure if both players cooperate, and μ the probability if exactly one player cooperates. We assume $\mu > \lambda > 0$: Failure is more likely if one of the partners defects. The discount factor that applies between periods is denoted by δ.

For any fixed value of t, this set-up defines a new repeated game in which, at each "stage," t repetitions of the Prisoners' Dilemma game are played. If δ is close to one—and given certain restrictions described below on the other parameters that are necessary for some cooperation to be possible—the best symmetric equilibrium is a trigger strategy equilibrium in which there is cooperation in each of the t periods that make up the first stage of the repeated game. The optimal equilibrium strategy, as we will show, triggers noncooperation with probability α only when a "failure" occurs in *all* t periods of the stage; when there is a "success" in any of the t periods, the players continue to cooperate at the next stage. The number α is chosen to make the players just indifferent between defecting in the first period of the t-period stage and not defecting at all:

$$(1-\delta)g = \delta^t(\mu-\lambda)\lambda^{t-1}\alpha\bar{v}(t,\delta). \tag{10}$$

The left-hand side of equation (10) is the gain enjoyed from cheating for a single period, expressed in the same mean flow units as the equilibrium value $\bar{v}$. The right-hand side is the increased chance of triggering a punishment by defecting once (rather than not defecting), multiplied by the payoff lost when a punishment is triggered. We shall assume for now that there exists some $\alpha \in (0,1]$ satisfying (10); a sufficient condition for this is given in Proposition 6 below.

Notice that if a player defects during any τ of the first t periods, the likelihood that a punishment will be triggered is $\mu^{\tau}\lambda^{t-\tau}\alpha$. The ratio of this likelihood to the one when there are no defections $(\lambda^{t}\alpha)$ is $(\mu/\lambda)^{\tau}$. This ratio is also the maximum likelihood ratio that could be obtained in any test of the hypothesis that the player defected in τ specified periods against the alternative hypothesis of no defections. So, this strategy has the remarkable property that the test it uses maximizes the likelihood ratio *simultaneously against all possible single period and multi-period deviations by the players.*

We shall now show that this strategy, which by construction deters a single deviation in just the first period of the stage, actually deters *all* τ-period deviations, where $1 \leqslant \tau \leqslant t$. Notice that, due to discounting, the gain in immediate payoff to a τ-period deviation, expressed in mean flow payoff terms, is at most $(1-\delta)\tau g$. The extra cost incurred by a τ-period deviation, in terms of increased probability of punishment, is $(\mu^{\tau}\lambda^{t-\tau}-\lambda^{t})\alpha\bar{v}$, which is at least $\tau(\mu-\lambda)\lambda^{t-1}\alpha\bar{v}$.[12] Hence, a sufficient condition for all deviations to be deterred is that $(1-\delta)g \leqslant (\mu-\lambda)\lambda^{t-1}\alpha\bar{v}$, which is implied by (10).

By triggering noncooperation only when there are failures in each period, we have converted the punishment for defecting in the first period into a lottery with sufficient expected value to deter that defection. It is true that a player can affect the relevant lottery probabilities, but only adversely for himself. Consequently, the test and punishments that are used to keep players in line in period 1—and that impose welfare losses on the players—can be "reused" at no extra cost to deter defections in periods 2 through t.

As we argued in the discussion of Proposition 2 (for the case $t=1$), the cost of deterring a single deviation in the first period is $g/(l-1)$,[13] so the mean flow cost of deterring a deviation in each period is also $g/(l-1)$. As we have seen, however, for general t and δ close to one, only the incentive constraints in periods $1, t+1, 2t+1, \ldots$ are binding, so the mean flow cost of deterrence is $(1-\delta)\Sigma_{k=1}^{\infty}\delta^{tk}[g/(l-1)]$. Summing this series leads to the following Proposition.

PROPOSITION 6: *Let* $l=\mu/\lambda$. *For* $t<g/(l-1)\pi$, $\bar{v}(t,\delta)=0$ *for all* $\delta \in (0,1)$. *For any* $t>g/(l-1)\pi$, *there exists* $\delta_t \in (0,1)$ *such that for all* $\delta \geqslant \delta_t$,

$$\bar{v}(t,\delta)=\pi-\left[\delta/(\delta+\cdots+\delta^{t})\right]\cdot\left[g/(l-1)\right].$$

In this second case, $\lim_{\delta\to 1}\bar{v}(t,\delta)=\pi-(1/t)g/(l-1)$.

[12] Since $(\mu/\lambda)>1$, it follows that $(\mu/\lambda)^{\tau}-1 \geqslant \tau[(\mu/\lambda)-1]$, from which our conclusion follows.
[13] Here, $l=\mu/\lambda$.

Proposition 6 establishes that longer information lags t lead to better equilibrium outcomes, provided that δ is close enough to unity, and that lags may be necessary to achieve any cooperation at all. If we regard t as a choice variable, then the Proposition implies that for any fixed value of δ, there will be some finite optimal information lag $t^*(\delta)$, and that $t^*(\delta)$ tends to infinity as δ tends to one.

The benefits of delaying the release of information and the trigger strategy that achieves those benefits by looking for evidence of defection in each period are not special features of the Prisoners' Dilemma example. They are general properties of symmetric repeated games, as we show below.

5. SYMMETRIC GAMES WITH MULTIPLE ACTIONS AND SIGNALS

Suppose that in every period, each of n players privately chooses an action from the set $\{a_1, \dots, a_H\}$. The action profile determines a probability distribution over the finite signal space. We assume that all signal realizations have positive probability, regardless of the profile of actions played. A player's payoff in period s is the expected value of a realized reward that depends on the action he takes and possibly on the signal realization or other random variables. However, a player's only information is the public signal together with knowledge of his own past play. The game is symmetric, and we restrict attention to symmetric equilibria (that is, equilibria which specify identical behavior for all players after all histories, even off the equilibrium path). Let

π_{ij} = the single period (expected) payoff to a player who chooses action a_i when all other players choose a_j.

p_{ijk} = the probability of signal k when one player chooses a_i and everyone else chooses a_j.

A stage game of length t, where t is a positive integer, is comprised of t of the period games just described, but with the following information structure: Players remain ignorant of signal realizations within a stage until the stage ends. Thus, a pure strategy for any player specifies how to behave at each date as a function of the signals observed at the ends of the various completed past stages. It is convenient to convexify the (symmetric equilibrium) value set of the supergame $G^\infty(t, \delta)$ with stage game of length t, by including at the beginning of each stage a publicly observed random drawing from the uniform distribution on [0, 1], on which all subsequent choices by players can be conditioned.

We assume for simplicity that the stage game has a symmetric equilibrium $(a_b, \dots, a_b)$ whose payoff is normalized to zero, and consider equilibria in which play can be characterized as being in one of two states: In the "good" state players use some action a_j and in the "bad" state they use action a_b, where $\pi_{jj} > \pi_{bb} = 0$. Given that $(a_b, \dots, a_b)$ is a Nash equilibrium, the bad state can be taken to be absorbing. We limit our attention to repeated game equilibria of this kind.

We seek to compute the maximal value among equilibria of this kind along with the strategies and implicit statistical tests that support the equilibrium. Recall from Section 2 that the continuation values of the equilibrium that implements cooperation most efficiently can be obtained as the solution of a linear program with three kinds of constraints: A value recursion equation, incentive constraints, and self-generation constraints (see Abreu, Pearce, and Stacchetti (1986)). The analogous program for the present model is LP#1 below, where the choice variables $w(k)$ are continuation values following alternative t-period signal histories $k=(k_1,\dots,k_t)$.

LP#1: $\max_{v,w} v$ subject to

(i) value recursion:

$$v=(1-\delta^t)\pi_{jj}+\delta^t\sum_{k_1,\dots,k_t}\Big(\prod_s p_{jjk_s}\Big)w(k_1,\dots,k_t);$$

(ii) incentive compatibility:

$$v\geqslant\left(\delta\pi_{i_1j}+\delta^2\pi_{i_2j}+\cdots+\delta^t\pi_{i_tj}\right)\frac{1-\delta}{\delta}$$
$$+\delta^t\sum_{k_1,\dots,k_t}\Big(\prod_s p_{i_sjk_s}\Big)w(k_1,\dots,k_t)\qquad\text{for all }(i_1,i_2,\dots,i_t);$$

(iii) self-generation:

$$0\leqslant w(k_1,\dots,k_t)\leqslant v.$$

Let $\bar{v}(j,t,\delta)$ denote the maximized value of LP#1.

Given any feasible solution (v,w) of LP#1, there is a corresponding stationary symmetric trigger strategy equilibrium of $G^\infty(t,\delta)$ in which the players randomize after outcome k, continuing in the "good" state (playing j) after period t with probability $w(k)/v$ and otherwise switching to the "bad" state (playing b) where play remains forever. Similarly, any such equilibrium corresponds to a feasible solution of LP#1.

The heart of our analysis of information lags in the Prisoners' Dilemma example was our assertion that, for δ close to unity, only the first-period incentive-constraint is binding. That result was established by examining a very particular strategy that triggers punishment only if there is evidence of cheating in *every* period. A related proposition will be proved using a similar strategy in this more general framework. The connection is made using the following two linear programs. The first is obtained from LP#1 by omitting all but the first-period incentive constraints and adding "Separation Constraints:"

LP#2 $\max_{\bar{v},x_1,\dots,x_k} \bar{v}$ subject to

(i′) value recursion:

$$\bar{v}=(1-\delta^t)\pi_{jj}+\delta^t\sum p_{jjk}x_k;$$

(ii′) period one incentive compatibility:

$$\bar{v} \geqslant (1-\delta)\pi_{ij} + (\delta - \delta^t)\pi_{jj} + \delta^t \sum_k p_{ijk} x_k \qquad \text{for all } i;$$

(iii′) separation constraints:

$$\sum_k p_{jjk} x_k \geqslant \sum_k p_{ijk} x_k \qquad \text{for all } i;$$

(iv′) one-sided self-generation constraints:

$$x_k \leqslant \bar{v} \qquad \text{for all } k.$$

Notice that (iii′) is never a binding constraint in games like the Prisoners' Dilemma in which $\pi_{ij} \geqslant \pi_{jj}$ for all i, that is, in which all single period deviations increase short-run payoffs. For those games, (iii′) is implied by (i′) and (ii′).

Let $\bar{v}^2(j,t,\delta)$ denote the optimized value of LP#2.

Substituting (i′) into (ii′), we may express the period one incentive compatibility constraint as

$$\pi_{ji} - \pi_{jj} \leqslant \frac{\delta^t}{1-\delta} \sum_k (p_{jjk} - p_{ijk}) x_k \qquad \text{for all } i.$$

One can view the discrepancies between the continuation values and $\bar{v}^2(j,t,\delta)$ in LP#2 as fines whose expectation should be minimized subject to the incentive and separation constraints. In terms of the simple model of Section 2, the expected fine paid by an honest player is $g/(l-1)$ while that paid by a defector is $lg/(l-1)$. For this general model, the "fine" payable after signal k may be denoted by f_k and the expected fine paid by a player who chooses action i by F_i. The problem can then be stated as one of minimizing the expected fine paid by an honest player, subject to appropriate constraints.

LP#3 $\quad \underset{f_1,\dots,f_k}{\text{Min}} \sum_k p_{jjk} f_k \quad$ subject to

(ii″) incentive constraints:

$$\pi_{ij} - \pi_{jj} \leqslant \sum_k p_{ijk} f_k - \sum_k p_{jjk} f_k \qquad \text{for all } i;$$

(iii″) separation constraints:

$$\sum_k p_{jjk} f_k \leqslant \sum_k p_{ijk} f_k \qquad \text{for all } i,$$

(iv″) nonnegativity constraints:

$$f_k \geqslant 0 \qquad \text{for all } k.$$

Let $(f_1^*, \dots, f_k^*)$ solve LP#3 and define

$$F_i = \sum_k p_{ijk} f_k^*, \qquad \text{when LP\#3 is feasible.}$$

Then, the formal relationship between LP#2 and LP#3 is given by the next proposition.

PROPOSITION 7: *LP#2 is feasible if and only if LP#3 is feasible. When both are feasible, their optimal values are related as follows*:

$$\bar{v}^2(j,t,\delta) = \pi_{jj} - \frac{\delta}{\delta + \cdots + \delta^t} F_j \qquad \text{for all } \delta \in (0,1).$$

PROOF: Suppose that f^* is an optimal solution of LP#3, and $F_j = \Sigma_k p_{jjk} f_k^*$. Let

$$v = \pi_{jj} - \frac{\delta}{\delta + \cdots + \delta^t} F_j,$$

and

$$x_k = v - \frac{1-\delta}{\delta^t} f_k^*.$$

Noting that f^* satisfies (ii″), (iii″) and (iv″), and recalling the identity

$$\frac{1-\delta}{1-\delta^t} = \frac{\delta}{\delta + \cdots + \delta^t},$$

we see that (v, x) as defined here is feasible for LP#2. Hence

$$\bar{v}^2(j,t,\delta) \geqslant \pi_{jj} - \frac{\delta}{\delta + \cdots + \delta^t} F_j.$$

Now consider an optimal solution (v^*, x^*) of LP#2, and define

$$f_k = \frac{\delta^t}{1-\delta}(v^* - x_k^*).$$

It may be checked that the f_k's so defined are feasible for LP#3. Also

$$\begin{aligned} \bar{v}^2(j,t,\delta) = v^* &= (1-\delta^t)\pi_{jj} + \delta^t \sum_k p_{jjk} x_k^* \\ &= (1-\delta^t)\pi_{jj} + \delta^t \sum_k p_{jjk}\left(v^* - \frac{1-\delta}{\delta^t} f_k\right) \\ &\leqslant (1-\delta^t)\pi_{jj} + \delta^t v^* - (1-\delta) F_j \end{aligned}$$

or

$$\bar{v}^2(j,t,\delta) \leqslant \pi_{jj} - \frac{\delta}{\delta + \cdots + \delta^t} F_j. \qquad Q.E.D.$$

In our Prisoners' Dilemma example, we assumed that $\mu > \lambda$, that is, there exists some statistical test that can detect a single-period deviation, even though the test might have little power. For this more general analysis, we require that the following condition hold:

Distinguishability Condition: The probability vector p_{jj} does not lie in the convex hull of the set of probability vectors $\{p_{ij};\ i \neq j\}$.

The Distinguishability Condition says that no player has a mixed strategy deviation from the strategy j that exactly duplicates the distribution of signals associated with playing j. If such a deviation did exist, then it would of course be statistically indistinguishable from honest behavior, and therefore impossible to detect and deter.[14] So, the Distinguishability Condition is a minimally necessary one for our analysis.

PROPOSITION 8: *If the Distinguishability Condition is satisfied, then LP#3 is feasible.*

PROOF: If j satisfies the assumption, there exists a vector f which strictly separates p_{jj} from the vectors p_{ij}, $i \neq j$, that is,

$$\sum_k p_{jjk} f_k < \sum_k p_{ijk} f_k \qquad \text{for all } i.$$

Since the p_{ij}'s are probability vectors, f may be taken to be positive in all components;[15] furthermore, f may be scaled up so that (ii″) is satisfied. Such an f is feasible for LP#3. *Q.E.D.*

The main result of this section is Proposition 9, which provides a lower bound on the value of equilibria using action j, when the players are sufficiently patient. The bound may not be tight, because it is not always necessary to impose the separation constraints.

PROPOSITION 9: *Suppose LP#3 is feasible and that $\pi_{jj} > F_j/t$. Then there exists $\delta_t \in (0,1)$ such that for all $\delta > \delta_t$ a trigger strategy equilibrium using action j exists and*

$$\bar{v}(j,t,\delta) \geqslant \bar{v}^2(j,t,\delta) = \pi_{jj} - \frac{\delta}{\delta + \cdots + \delta^t} F_j.$$

PROOF: We use the solutions of LP#2 and LP#3 to construct a trigger strategy that is feasible for LP#1 and achieves the value specified in the Proposition.

[14] There do exist joint restrictions on payoffs and statistical information that are sufficient for our conclusions and weaker than the Distinguishability Assumption, but the distinguishability is the weakest *statistical* assumption that implies our conclusion. Distinguishability is implied, for example, by the individual full rank condition used by Fudenberg, Levine, and Maskin (1989), but does not imply that condition.

[15] Given any separating vector f, the vector $f + \alpha e$ (where e is a vector of ones) is also separating and is positive for α sufficiently large.

Let (v^*, x^*) be the optimal solution of LP#2 and let f^* be the optimal solution of LP#3 with corresponding "expected fines" F_i, as defined above. Following a t-period outcome $(k_1, \ldots, k_t)$, a switch to the punishment state is triggered with probability $\hat{\alpha} = \varepsilon \cdot \prod_{s=1}^{t} f_{k_s}$, where $\varepsilon = (1-\delta)\delta^{-t}/(vF_j^{t-1})$. That is, $w(k_1, \ldots, k_t) = [1-\hat{\alpha}]v$, where v is the value associated with the proposed trigger strategy. Note that $\hat{\alpha}$ is a probability for δ close to one, because it is nonnegative for all δ and converges to zero as δ is increased toward one. Consequently, the self-generation constraint (iii) of LP#1 is satisfied.

By the value recursion equation (i),

$$\begin{aligned} v &= (1-\delta^t)\pi_{jj} + \delta^t \sum_{k_1, \ldots, k_t} \left(\prod_{s=1}^{t} p_{jjk_s} \right) \cdot \left(1 - \varepsilon \prod_{s=1}^{t} f_{k_s} \right) \cdot v \\ &= (1-\delta^t)\pi_{jj} + \delta^t v - \varepsilon (F_j)^t v. \end{aligned}$$

Substituting for ε and solving for v, we obtain $v = v^* = \bar{v}^2(j,t,\delta) = \pi_{jj} - F_j\delta/(\delta + \cdots + \delta^t)$, as required.

Finally, we must verify the incentive compatibility constraint (ii) of LP#1, that is, that a player cannot gain by deviating from action sequence $(j, \ldots, j)$ to any action sequence $(i_1, \ldots, i_t)$ in the first t periods. The change in payoff resulting from such a deviation is:

$$\begin{aligned} &(1-\delta) \sum_{s=1}^{t} \delta^s (\pi_{i j_s} - \pi_{jj}) - \delta^t \varepsilon \left(\prod_{s=1}^{t} F_{i_s} - F_j^t \right) v^* \\ &\leqslant (1-\delta) \sum_{s=1}^{t} \operatorname{Max}(0, \pi_{i j_s} - \pi_{jj}) - \delta^t \varepsilon v^* F_j^t \left[\prod_{s=1}^{t} (F_{i_s}/F_j) - 1 \right] \\ &\leqslant (1-\delta) \sum_{s=1}^{t} \operatorname{Max}(0, \pi_{i j_s} - \pi_{jj}) - \delta^t \varepsilon v^* F_j^t \left[\sum_{s=1}^{t} (F_{i_s}/F_j - 1) \right] \\ &= (1-\delta) \sum_{s=1}^{t} \left[\operatorname{Max}(0, \pi_{i j_s} - \pi_{jj}) - (F_{i_s} - F_j) \right] \\ &\leqslant 0. \end{aligned}$$

The first inequality follows from replacing the short-run gain to cheating by the maximum of itself and zero, and then removing the discounting. The second follows from the fact that, according to the separation constraint, F_{i_s}/F_j exceeds unity. The equality on the next-to-last line follows from the definition of ε. The incentive constraint in LP#3 ensures that each summand in that line is nonpositive when $\pi_{ji_s} - \pi_{jj}$ is positive, while the separation constraint implies that the other terms are nonpositive. Together, these facts establish the final inequality.

This verifies that the proposed strategy satisfies (i)–(iii) and attains the desired value. *Q.E.D.*

Notice the similarities among Propositions 2, 6, and 9. In each case, the value is expressed as the flow payoff from cooperation minus a cost of deterring defections. In Proposition 2, the cost is the gain from cheating divided by the measure of statistical power: $(l-1)$. In Proposition 6, we found that the cost is the same expression multiplied by a factor of $\delta/(\delta+\cdots+\delta^t)$, because the cost of deterring deviations is incurred only once in each t periods. In Proposition 9, the cost is $F_j\delta/(\delta+\cdots+\delta^t)$, where the cost F_j of deterring a first period deviation while satisfying the separation constraint is obtained from the contracting problem LP#3.

The proof of Proposition 9 uses the solutions to LP#2 and LP#3 to construct a trigger strategy that deters all possible single period and multiperiod deviations in the t-period stage game. We have already seen how deterring single period deviations in the first period can deter all single period deviations and certain multiperiod deviations. Indeed, one can infer from Proposition 9 that when a player has only profitable deviations, it is never more costly to deter multiperiod deviations than to deter single period deviations—the separation constraints are never binding in that case and can be omitted. However, when a player has some unprofitable single period deviations, it may still be necessary to consider how their play affects the probability of punishment, because they could otherwise be played as part of some multiperiod deviation that is profitable in total but that uses the unprofitable deviations to reduce the probability of triggering a punishment. The separation constraints preclude this by ensuring that there is *no* single period deviation that can ever reduce the probability of punishment.

The following Corollary is a straightforward implication of Proposition 9. Note that when action a_j is Pareto efficient, the Corollary is an asymptotic efficiency result.

COROLLARY: *Suppose that $\pi_{jj}>0$ and the Distinguishability Condition holds. Then for any $\varepsilon>0$, there exist t and $\bar{\delta}\in(0,1)$ such that, for all $\delta\geqslant\bar{\delta}$,*

$$\bar{v}(t,\delta,j)\geqslant\pi_{jj}-\varepsilon.$$

6. DISCUSSION

According to one standard interpretation of the Folk Theorem, if players can react quickly to one another's choices, then the possibilities for cooperation are improved, because the discounting between periods becomes small. Our conclusions expose the limits of this sort of reasoning, showing that when monitoring is imperfect, short periods can make it costly or even impossible to provide effective incentives. Intuitively, shorter periods of fixed action and frequent performance reports multiply the ways that players can deviate from the equilibrium strategies. The need to deter these extra deviations raises the cost of deterrence. To draw an analogy to principal-agent theory, frequent performance reports in a repeated game is like performance information that arrives during a period in a principal-agent model: It helps the agent to find more effective cheating strategies but serves no beneficial purpose for the principal (see Holmstrom and Milgrom (1988)). The mathematical connection between principal-agent models and repeated partnership games is established in Propositions 7 and 9.

We have performed our analysis in the context of the symmetric equilibria of a repeated partnership game. Subsequent papers (Fudenberg, Levine, and Maskin (1989), Matsushima (1989))[16] have shown that the results for low interest rates and symmetric equilibria in these games may be misleading for a different reason: It may be possible to construct asymmetric equilibria in which the cost of punishments is much lower than at any symmetric equilibrium. Again, the powerful theorems in these papers should be interpreted as ensuring asymptotic efficiency as interest rates decline to zero, but not necessarily as periods of fixed action or information lags grow short.

Our examples and propositions clearly demonstrate several points. First, low interest rates, short reporting lags, and short periods of fixed action are distinct assumptions that merit distinct treatment in games with imperfect monitoring. Second, when monitoring is imperfect, short reporting lags and short periods of fixed action are not generally conducive to cooperation. Finally, when modeling a given partnership or oligopoly as a repeated game, one ought to consider not only the players' abilities to adjust their actions but also the timing of their information flows. An artificial identification of the periods of fixed action and fixed information can produce seriously misleading results.

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APPENDIX

PROOF OF PROPOSITION 5: Consider any sequence $\{\alpha_k(t)\}$. If it defines an equilibrium, then by Proposition 1,

$$g(e^{rt}-1) \leqslant (Q-P)v$$

where $Q=\Sigma_k \alpha_k(t) q_k(t)$ and $P=\Sigma_k \alpha_k(t) p_k(t)$. Equation (ii′) of LP#2 and (iii) of LP#1 imply

$$v \leqslant \pi - \frac{g^P}{Q-P}.$$

Together with the earlier inequality this yields

$$g(e^{rt}-1) \leqslant \pi(Q-P) - g^P.$$

[16] An early paper by Williams and Radner (1987) makes a related point in a static model.

Expanding terms and dividing through by t,

$$(*) \qquad g\left(r+\frac{r^2 t}{2}+\cdots\right) \leqslant \pi\left[\frac{\alpha_o(t)}{t}(e^{-\mu t}-e^{-\lambda t})+\alpha_1(t)(e^{-\mu t}\mu-e^{-\lambda t}\lambda) + \alpha_2(t)\left(e^{-\mu t}\frac{\mu^2 t}{2}-e^{-\lambda t}\frac{\lambda^2 t}{2}\right)+\cdots\right] - g\left[\frac{\alpha_o(t)}{t}e^{-\lambda t}+\alpha_1(t)e^{-\lambda t}\lambda+\alpha_2(t)e^{-\lambda t}\frac{\lambda^2 t}{2}+\cdots\right].$$

By Proposition 1, α defines an equilibrium if and only if α satisfies $(*)$ and (4) $0 \leqslant \alpha_k(t) \leqslant 1$ for all k. First, note that if $\lambda \geqslant \mu$, there exist no α which satisfies $(*)$ and (4) for small t. Assume that $\mu > \lambda$. Then, there exists $T' > 0$ such that for all $t \in (0, T']$, if α satisfies (4) it satisfies $(*)$ only if $\alpha_1(t) > 0$. For α^* which defines a payoff maximal equilibrium this implies that (see Proposition 2) $\alpha_0^*(t) = 0$ and $\alpha_k^*(t) = 1$, $k = 2, 3, \ldots$. Hence $\alpha_1^*(t)$ is determined by:

$$(**) \qquad g\left(r+\frac{r^2 t}{2}+\cdots\right) \leqslant \pi\left[\alpha_1^*(t)(e^{-\mu t}\mu-e^{-\lambda t}\lambda) + \left(e^{-\mu t}\frac{\mu^2 t}{2}-e^{-\lambda t}\frac{\lambda^2 t}{2}\right)+\cdots\right] - g\left[\alpha_1(t)e^{-\lambda t}\lambda+e^{-\lambda t}\frac{\lambda^2 t}{2}+\cdots\right].$$

If $(\mu - \lambda)\pi < (r + \lambda)g$, it is clear that for small enough $t, \alpha_*^1(t)$ as determined by $(**)$ exceeds 1. Except for the case $(\mu - \lambda)\pi = (r + \lambda)g$, this establishes the second half of Proposition 5. In the case of strict equality, $\alpha_1^*(0) = 1$. Differentiating $(**)$, evaluating right-hand derivatives at $t = 0$, substituting $\alpha_1^*(0) = 1$ and simplifying, we obtain

$$2\bar{v}(o)(\mu-\lambda)\left.\frac{d\alpha_1^*}{dt}\right|_{t=0} = gr^2 + (\mu-\lambda)(\lambda, \bar{v}(o) + \mu\pi),$$

where

$$\bar{v}(o) = \pi - \frac{g^{\lambda}}{\mu - \lambda} > 0$$

if $(\mu - \lambda)\pi \geqslant (r + \lambda)g$. It follows that if $(\mu - \lambda)\pi \leqslant (r + \lambda)g$, there exists $T > 0$ such that $\alpha_1^*(t) > 1$ for all $t \in (0, T]$.

Conversely, if $(\mu - \lambda)\pi > (r + \lambda)g$, then for small enough t, $\alpha_1^*(t) \in (0, 1)$ as required for an equilibrium. Also,

$$\lim_{t \downarrow 0} \bar{v}(t, r) = \pi - g - \lim_{t \downarrow 0} \frac{Q(t) - P(t)}{P(t)} = \pi - \frac{g^{\lambda}}{\mu - \lambda}.$$

This establishes Proposition 5. *Q.E.D.*

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