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Motion as manipulation: Implementation of motion and force analogies by event-file binding and action planning

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Abstract:

Tool improvisation analogies are a special case of motion and force analogies that appear to be implemented pre-conceptually, in many species, by event-file binding and action planning. A detailed reconstruction of the analogical reasoning steps involved in Rutherford's and Bohr's development of the first quantized-orbit model of atomic structure is used to show that human motion and force analogies generally can be implemented by the event-file binding and action planning mechanism. Predictions that distinguish this model from competing concept-level models of analogy are discussed, available data pertaining to them are reviewed, and further experimental tests are proposed.

Keywords: Structure mapping; Tool improvisation; Rutherford-atom analogy; Mirror-neuron system; Cognitive impenetrability; Conceptual reasoning

Introduction

The ability to recognize similarities at the level of relational structure between remembered and novel situations, and hence to reason by structure-mapping analogy, is foundational to human intelligence (reviewed by Markman & Gentner, 2001; Gentner, 2003; Holyoak, 2005), and to the practice of science in particular (reviewed by Holyoak & Thagard, 1995; Feist & Gorman, 1998). Structure-mapping is widely viewed as an algorithmic operation on concepts, and hence as dependent on the ability to conceptualize relations in a natural language (Gentner, 2003; 2005; Gentner & Christie, 2008) or a "language of thought" accessible to awareness (Fodor, 2000; Penn *et al.*, 2008). As alternatives to this dominant view of analogy as a distinct form of human-specific, domain-general verbal reasoning, it has also been proposed that analogy is an outcome of lower-level processes including concept-recognition priming (Bar, 2008; Leech et al., 2008) or perceptual-motor simulation (Gallese and Lakoff, 2005; Barsalou, 2008). It is unclear, however, how such lower-level processes could implement structure mapping and hence produce systematic analogies (Gentner, 2005; Holyoak, 2005).

Although over three decades of work on algorithmic models of analogy have produced increasingly-realistic models of structure-mapping (reviewed by French 2002; Holyoak, 2005), significant characteristics of human analogical reasoning, including the ubiquity of conceptual change as an outcome and possibly an obligate intermediary process (Dietrich, 2000; 2010; Blanchette & Dunbar, 2002) have only started to be addressed by such models. Experimental investigation of the neurocognitive implementation of structure mapping in humans has thus far yielded primarily low-resolution localization data (reviewed by Bar, 2008). The recognition of relational structures is known to involve the frontal-parietal working memory (WM) network (Waltz *et al*., 2000; Green *et al.*, 2006) that is generally involved in making long-distance semantic connections (Jung-Beeman *et al*., 2004; Kounios *et al*., 2007; Bar, 2008; Sandkuhler & Bhattacharya, 2008). The mapping step involves regions of polar or rostral prefrontal cortex (Bunge *et al.*, 2005; Morrison *et al.*, 2005; Green *et al*., 2006) that are also implicated in multi-tasking (Dreher *et al*., 2008) and allocating attention between externally-driven perception and internal imaginative processes (Gilbert *et al.*, 2005; Burgess *et al.*, 2007). As do other forms of externally-directed problem solving, efficient analogy formation involves default-network deactivation (Buckner *et al.*, 2008; Kounios & Beeman, 2009). No structures or pathways specific to analogy have yet been characterized.

The present paper extends previous work (Fields, 2011a) showing that a specific class of analogies involving motions and forces, tool-improvisation analogies, can be implemented by event-file binding and action planning systems that are structurally homologous across mammals and appear to be shared as functional systems by mammals and birds. Using the well-known Rutherford-atom analogy as a demonstration case, it shows that the mechanisms proposed to implement tool-improvisation analogies can implement motion and force analogies in general, provided that it is assumed that the manipulations involved in tool use underpin the general concepts of motions and forces.

It then reviews evidence from developmental, cognitive, and neurocognitive studies suggesting that this key assumption is correct. The model of motion and force analogies proposed makes the surprising prediction that motion and force analogies will often, and in the theoretical sciences typically, conflict with conceptual understanding. This prediction strongly distinguishes the proposed model from the dominant model that analogies are implemented by structure mappings between relational concepts. Evidence bearing on this and other predictions of the proposed model is reviewed, and experimental designs that would test the model are discussed.

Background: The Rutherford-atom analogy

While a number of motion and force analogies have been used in cognitive studies of analogical reasoning, one has particular prominence and can be regarded as a canonical example: the Rutherford-atom analogy electrons:nucleus::planets:sun (e.g. Gentner & Wolff, 2000; Green *et al.*, 2006; Dietrich, 2010). This analogy appears straightforward and obvious in a culture in which the Rutherford representation of the atom is a ubiquitous and iconic motif, and it is often treated as such. It is useful, however, to review the role this analogy played in its historical context, and the consequences that followed from it. Reconstruction of well-documented historical analogies has previously been used to probe expert use of analogical problem solving in a natural setting as an alternative to experimental studies of non-experts in contrived settings (Gentner *et al.*, 1997).

Ernest Rutherford’s 1911 model of the atom as consisting of a small central nucleus surrounded by electrons was proposed to account for the results of experiments in which gold atoms were bombarded by high-energy alpha particles. Most of the alpha particles went straight through the gold foil target as expected, but others were deflected at large angles, suggesting collisions with a small, dense central object and thoroughly contradicting the then-dominant Thompson or “plum pudding” model of atoms as spheres containing a uniform mixture of positively-charged material and electrons (Rutherford (1911); Randall (2005) briefly reviews the relevant history from a physicist’s perspective; Mehra and Rechenberg (1982) provide a more detailed historical review). Rutherford’s model was revolutionary in that it proposed an atom consisting mostly of empty space, in which the positive charges were concentrated in the center and the negative charges (the electrons) occupied the distant periphery.

The naïve version of the Rutherford analogy, that electrons orbit nuclei as planets orbit the sun, is not very informative; its only testable (and true) prediction is that electrons orbit at different distances from the nucleus just as planets orbit at different distances from the sun. Although depictions of Rutherford-model atoms sometimes are used as heuristics in chemistry texts, the Rutherford analogy tells us nothing about chemical bonding – planets do not share orbits, and no other solar systems are near enough to ours to exchange or share planets. To a physicist, however, the Rutherford analogy is not merely the claim that atoms are like solar systems; it is the theoretically much more interesting claim that the electrostatic force binding the electrons to the nucleus is analogous to the gravitational force binding the planets to the sun. It is difficult to over-emphasize the importance of this claim for 20th century physics. The fundamental dis-analogy of classical electromagnetic and gravitational forces – orbiting classical electrons would lose energy by electromagnetic radiation and their orbits would collapse within nanoseconds – led to Bohr's postulation of electron orbit quantization and the development of the quantum theory of the atom. The obvious theoretical response to this dis-analogy – why shouldn't planetary orbits also radiate energy? – led to Einstein's prediction of gravitational radiation as part of the general theory of relativity. The relationship between electromagnetic and gravitational forces remains a central problem in theoretical physics (e.g. Randall, 2005).

What is now called the "Rutherford-atom analogy" was not the first analogy that Rutherford employed in trying to understand the astonishing experimental result of alpha particles being deflected backwards by a gold foil. To colleagues, he described the result as analogous to an artillery shell being fired at a piece of tissue paper and bouncing back (quoted in Randall (2005), p. 127). His focus on the trajectory of the alpha particles through the foil target is confirmed by his frequent use, in his 1911 publication analyzing the experiment, of the analogy of a "pencil" of particles passing through a solid material. Neither of these analogies has explanatory power. Artillery shells do not bounce off tissue paper, and pencils are not deflected at large angles when passing through materials. Rutherford was aware of a hypothetical analogy between electrons arranged around an atomic core and the rings arranged around Saturn that had been advanced by Hantaro Nagaoka in 1904, but this notion of atoms as disk-shaped was unhelpful for explaining Rutherford's backscattering data. A different analogy was needed to provide a theoretically-productive picture for the structure of the atom.

All physicists are familiar with one largely-empty object through which smaller, fast-moving objects routinely pass and which contains a dense mass at its center that deflects the trajectories of objects that pass near it. That object is the solar system. Rutherford would have recently been reminded of the passage of fast-moving objects through the solar system; Halley's comet passed very near the earth in 1910. An atom as described by the Thompson "plum pudding" model is, however, nothing like the solar system. It is a solid object in which negatively-charged electrons are uniformly distributed within a (then-uncharacterized) positively-charged material. To maintain consistency with classical electrodynamics, the most mathematically-sophisticated theory of the time, the electrons in the Thompson model did not move. In classical electrodynamics, moving charges continuously emit electromagnetic radiation; electrons moving within the confines of a Thompson atom would be expected to radiate energy continuously, and no such radiation had been observed. Hence the electrons:nucleus::planets:sun analogy was not in any sense obvious to Rutherford. The idea that the positive and negative charges would be separated in an atom strongly suggests that they would also be moving, and the idea that electrons would move within an atom violated the very theory that described the motion of electrons.

Rutherford was well aware, however, of the motion patterns observed when electrons are scattered from an atom. The cumulative electrostatic force of the many electrons bound within the atom repels incoming single electrons, causing deflection of their trajectories. The analogy Rutherford employed to calculate the observed alpha-particle deflections using classical electrodynamics was alpha-particle:X::electron:atom, where X was an uncharacterized point-like charged object at the center of the gold atom (Rutherford, 1911). Rutherford treated the mass of X as negligible and estimated its electrostatic charge to be about 100 times that of an electron (the actual charge of a gold nucleus is +79). Rutherford did not, in his 1911 paper, specify whether the charge at the center of the gold atom was positive or negative, use the term "nucleus" to refer to the central charge, or consider the electrons in the atom to be moving in orbits. While Rutherford may have employed the analogy alpha-particle:X::comet:sun suggested by the passage of Halley's comet in his private reasoning (Gentner & Wolff, 2000), neither it nor the "Rutherford-atom analogy" appear in or are even suggested by his published analysis.

This question of how the electrons are arranged within an atom with a central charge was addressed by Bohr (1913), who showed that electrons could orbit the nucleus only if their orbits were quantized, through some unknown mechanism, to prevent the continuous radiation of their orbital energy and subsequent collapse of their orbits. Bohr's analysis, like Rutherford's, was motivated by an experimental result. Electrons in atoms were known to emit radiation, but only in discrete amounts, and only if excited by being irradiated themselves. Rutherford's characterization of the central point-like charge (i.e. the nucleus) provided an electrostatic field in which electrons could move, but the motion of the electrons and why they would radiate only at discrete energies remained mysterious. Bohr was faced with conflicting facts: electrons could emit radiation only by moving, but would emit radiation constantly if they moved in classical orbits like planets around the sun, on planar orbits like the disks of Saturn, or on any other classical trajectories. His response was the novel postulate of orbital quantization, and the radical notion that electrons in quantized "orbits" do not move, or at any rate do not move in any way that would count as "motion" in classical electrodynamics. Hence the electrons:nucleus::planets:sun analogy not only does not appear in Bohr's theory; Bohr's theory renders it false.

Besides the scholarly conundrum of whether anyone involved ever seriously proposed or believed it, this thumbnail history of the Rutherford-atom analogy raises two intriguing questions. The first, clearly, is how all of the analogical reasoning that did occur was implemented in the brains of the scientists who performed it. The second question is why, given that it is wrong, the Rutherford-atom analogy has such staying power. The model proposed in the next section attempts to answer both of these questions. It makes the surprising prediction that, at least in the force-and-motion domain, the analogies with the greatest staying power will *typically* be wrong.

Implementing the Rutherford-atom analogy by event-file binding and action planning

A very broad range of creatures routinely implement structure-mapping analogies of a particular kind: those enabling the improvisation of tools (Fields, 2011a). All such analogies involve motions and the muscular and mechanical forces required to achieve them. Aside from humans, the creatures that implement these analogies lack language, and if the preponderance of evidence is to be believed, lack domain-general imaginative planning and conceptual reasoning (reviewed by Penn & Povinelli, 2007; Suddendorf & Corballis, 2007). Tool-improvisation analogies must, therefore, be implemented by systems that do not depend on language or on concepts that require manipulation with imaginative awareness.

Representations of objects and object-directed actions by event files (Hommel, 2004) and manipulations of event files by the pre-motor action-planning system appear to be sufficient to implement tool-improvisation analogies (Fields, 2011a). The primary hypothesis of the present paper is that these systems are sufficient to implement all motion and force analogies, even those in abstract domains such as atomic physics. This hypothesis encompasses both the construction of such analogies by experts in abstract motion and force domains, and the comprehension of such analogies by non-experts. An "abstract" motion and force domain, for the present purposes, is one in which at least some concepts, such as "electron", do not derive from everyday experience and hence are not concepts of an intuitive "folk physics" (e.g. Pinker, 1997; Gentner, 2002). If this hypothesis is correct, then at least in motion and force domains, analogical reasoning is not a species of verbal reasoning; it is implemented by processes that do not depend on concepts expressed in natural languages or in a "language of thought" accessible to awareness. On this model, the formalized conceptual reasoning that Rutherford or Bohr used to refine their models and represent them mathematically is distinct from, and in the present case subsequent to, analogical reasoning. Abstract analogies such as electrons:nucleus::planets:sun are not, therefore, compelling even though they may be technically incorrect because they employ intuitive but essentially Aristotelian concepts from a pre-theoretical folk physics; there are no concepts of atomic structure in folk physics. Abstract analogies such as electrons:nucleus::planets:sun are compelling for a deeper reason: because they are based on qualitative force-motion relations that are encoded as a result of experiences with body parts and tools, and that are at least approximately cognitively impenetrable in the sense defined by Pylyshyn (1986), i.e. resistant if not immune to revision in real time by top-down conceptual knowledge.

The alpha-particle:X::comet:sun analogy that Rutherford might have employed (Gentner & Wolff, 2000) and its relationship to the alpha-particle:X::electron:atom analogy that he did employ in 1911 is reconstructed using the model and notation developed for tool-improvisation analogies (Fields, 2011a) in Fig. 1. The tool-improvisation model postulates that motion and force analogies are computed in two distinct steps separated by an experimental test. In the first step, an event file representing an initial state, a desired goal state and a motion connecting them retrieves an action instance from memory based on matches to the motions encoded by the action instance. For example, the goal of driving a tent stake into the ground can retrieve a remembered action instance of driving a nail into wood (c.f. Fig. 1b of Fields, 2011a). This first step produces an intermediate, instantiated action plan that relates the desired motion to an action, in this case driving an object into another object with a hammer. This action plan is then tested, by either overt action or simulation. Such a test fails for a back-yard camper equipped only with a light carpenter's hammer, or a backpacker equipped with no hammer at all. In the case of failure, a second round of memory search is initiated, this time for an action plan with related motions but different tools, and hence different applied forces. The process of searching for actions involving forces that will produce the desired motion is continued until an action using appropriate force is found; for example, until the backpacker recalls the forces required by the action of picking up a good solid rock. If no action involving a force that meets the test of actual use is retrieved, the analogy fails.

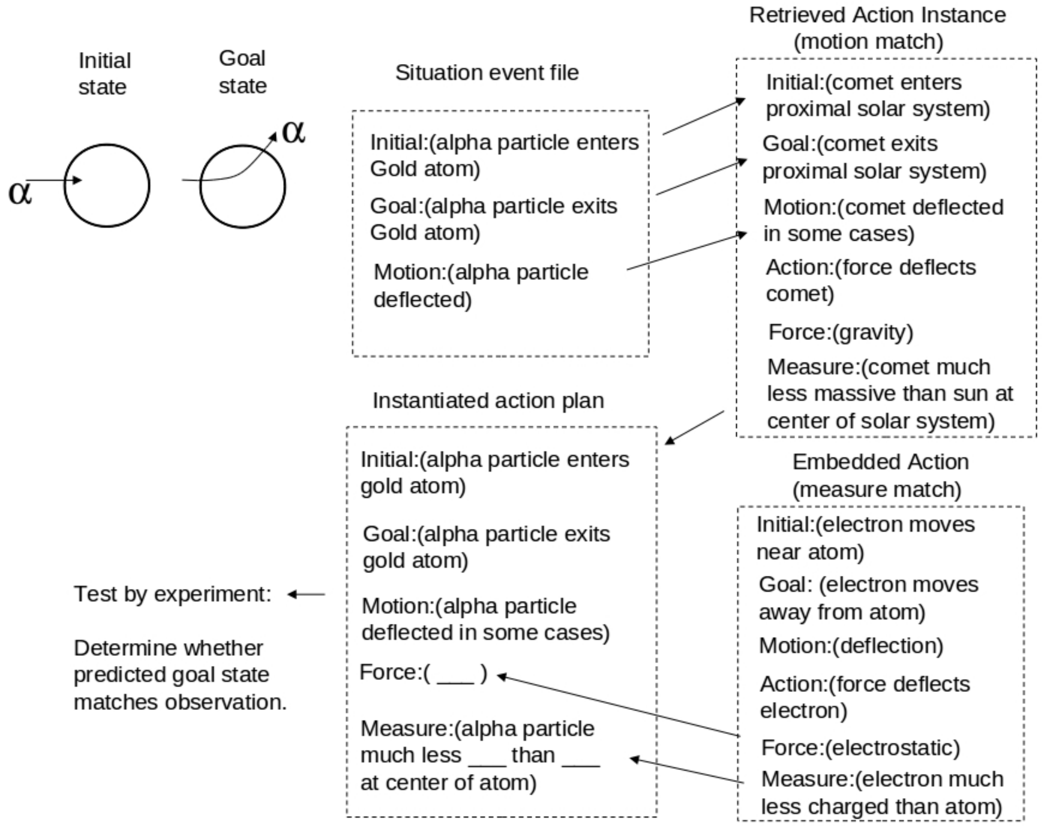

Fig. 1. Frame-based representation of structure mapping steps in Rutherford's analogical inference that atoms contain a central concentration of electric charge using the tool-improvisation model of Fields (2011a). Arrows represent mappings. The use of frames is heuristic only and is not meant to imply that the representations implemented by the fronto-parietal praxis system encode concepts expressible in either public language or an internally comprehended language of thought.

Extending this model to analogies about atomic structure is formally straightforward, but involves two significant empirical assumptions. The first is that the motion executed by the alpha particle, the initial state to goal state transition in Fig. 1, is represented as the result of an *action*. Actions such as pounding nails with hammers or tent stakes with rocks involve an experience of applying force; the assumption that the alpha particle's motion is represented as an action is therefore the assumption that the representation includes as a component a motor memory of an experienced force. Evidence that human beings represent the motions of apparently self-propelled objects as actions is discussed below, but it is interesting to note that physicists routinely talk about objects “feeling” forces (e.g. Randall, 2005), and refer to the product force × distance (or energy × time) as “action”. In Rutherford's case, the issue at hand was to discover *what* applied force caused the observed motion. If it is assumed that the motions of apparently self-propelled objects are represented as actions, it becomes reasonable also to assume that familiar instances of the motions of apparently self-propelled objects are represented as action schema, or in the context of the pre-motor system, action plans. Such action plans encode instances of employing a specific set of motor commands, with an accompanying experienced force, to achieve a specific motion. The tool-improvisation model thus conceives of Rutherford's task as retrieving an action plan that identified the force that the alpha particle would “feel” - the force that Rutherford would feel if he were moving the alpha particle through the gold foil.

With these two assumptions, the tool-improvisation model predicts that an event file representing the surprising motion of the alpha particle would initiate a memory search for action plans encoding similar motions, and associating them with forces. As a physicist, Rutherford would be familiar with a variety of possible motions of planets through the solar system, and would by assumption encode instances of such motions as action plans. Such a search could be expected to retrieve an action plan representing the deflection of a comet passing through the solar system. This action plan relies on an imposed force, the force of gravity. Hence the initial instantiated action plan for the alpha-particle:X::comet:sun analogy would specify gravity as the force responsible for the deflection. A non-physicist, on the other hand, would have no reason to encode action plans based on observations of comets. There is no reason to believe that such a person, given the alpha-particle backscattering result and told that it was surprising, would retrieve any stored action plan at all.

It is at this point that a distinguishing characteristic of motion and force analogies becomes clear. Motion and force analogies must actually work, or they are no good. This distinguishes them from analogies for which the criterion of systematicity (Gentner, 2005; Holyoak, 2005) boils down to plausibility. One can argue, for example, that Saddam's invasion of Kuwait is analogous to Hitler's invasion of Poland (Holyoak & Thagard, 1997; Holyoak, 2005) without worrying about the force ratios of the armies involved or their levels of resources or technical sophistication. If one is proposing stone:tent-stake::hammer:nail, however, the details of the stone matter: a good-sized chunk of flint or granite is good, but a similar piece of shale or pumice is not. This is not a matter of theory, but of experience; chimpanzees make this distinction (Carvalho *et al.*, 2008; Brill *et al.*, 2009). In motion and force analogies, systematicity becomes a requirement for coherent scaling between the forces applied and the motions achieved (Fields, 2011a). The alpha-particle:X::comet:sun analogy with gravity as the applied force is conceptually coherent and perfectly plausible; indeed a gravitational interaction between the incoming alpha particle and the gold atom, with a resulting deflection of the alpha-particle trajectory, is required by the laws of physics. The problem with the alpha-particle:X::comet:sun analogy is not its formal structure or conceptual coherence, it is that gravity is, in point of fact, far too weak a force to produce the large deflections that were so surprising to Rutherford. A completely naïve physicist would have to determine this by numerical calculation using the relevant force constants or by experimental test in the laboratory, i.e. by non-analogical means. An experienced physicist would know it intuitively; Rutherford dismissed the gravitational interaction between the alpha particle and the gold atom as irrelevant at the very outset of his 1911 paper, and clearly expected all readers of his paper to accept this dismissal without question.

The tool-improvisation model of analogy is, therefore, distinct from models in which all or some of the quantitative details of the base case are abstracted away prior to structure mapping (e.g. Dietrich, 2010). This distinction is motivated by two facts. First, tool improvisation is learned by observation combined with trial and error, not only in humans, but also in other tool-using species (Fields, 2011a). Good tool-improvisation analogies are learned by watching and experiencing qualitatively plausible but quantitatively incorrect ones fail. Second, the quantitative information gained from such experiences is encoded in the system that makes the analogies. Success and failure are associated with distinct experienced forces. The pre-motor planning system is a quantitative motion and force computer, with a dynamic range that is continually being expanded by experience with the reaching, prying, twisting, pulling, pounding and breaking capabilities of novel tools. Encoding quantitative force-motion relations is what motor learning is all about.

Once it is clear that the alpha-particle:X::comet:sun analogy does not work with gravity as the applied force, a second search of action plans to find a different, stronger force is initiated. This search for a heavier stone, a stouter stick, a stiffer spine is observed across tool-using species (Fields, 2011a). In Rutherford's case, recent electron-scattering experiments had demonstrated the strength of the electrostatic interaction acting outside of atoms. Familiarity gave Rutherford an action plan encoding this stronger force. By substituting the electrostatic force for gravity and charge for mass, Rutherford obtained an analogy, alpha-particle:X::electron:atom, that did work. This analogy produced Rutherford's critical insight into atomic structure: that an *electric charge* was concentrated at the center of the atom (Rutherford, 1911).

This reconstruction of Rutherford's reasoning on the model of tool improvisation rests, as noted above, on the assumptions that the motions of apparently self-propelled objects are represented as actions and remembered as schema that function, in the pre-motor context, as action plans. This second assumption has the corollary, evident in Fig. 1, that the forces experienced by moving objects are represented as manipulations, i.e. as applications of force by an agent. Evidence from developmental, cognitive, and neurocognitive studies supports the plausibility of these assumptions. Infants divide the world into animate beings and inanimate objects that only move if caused to do so by an agent (Saxe *et al.*, 2007), but with appropriate experience distinguish a class of self-propelled inanimate objects (Luo *et al.*, 2009). By the pre-school years, children are familiar with mechanical motion and able to attribute it to hidden, internal causes (Sobel *et al.*, 2007). Children become proficient solvers of simple mechanical problems long before they encounter formalized education in physics (reviewed by Karmiloff-Smith, 1995; Gopnik & Schulz, 2004). They are, moreover, enthusiastic if often less than proficient improvisers and wielders of a variety of force-amplifying tools. These abilities develop during a period of rapid elaboration of the frontal-parietal praxis network but limited pre-frontal development (Casey *et al.*, 2005); in particular, they develop well before abstract reasoning capabilities or the "relational shift" to adult-like analogical reasoning capabilities (Rattermann & Gentner, 1998; Gentner, 2005). These observations all suggest the transfer of non- or minimally conceptualized understandings of the typical kinematics and dynamics of different kinds of entities across situations either without or with only minimal explicit conceptual reasoning. The conceptual reasoning that children do display is, moreover, highly biased toward the attribution of agency to the causally-active entity in any situation, whether or not it is otherwise regarded as "inanimate" (Kelemen, 2004).

The ability to solve mechanical-reasoning problems without formal training in mechanics, but without the ability to retrospectively explain the details of the reasoning process, persists into adulthood (reviewed by Hegarty, 2004). Adults engaging in unformalized, intuitive mechanical problem solving appear to represent forces qualitatively as vectors originating at the object that exerts a force and pointing in the direction that the force is acting (Wolff, 2007; 2008), just as physics texts would instruct them to do. However, naïve adults perceive forces asymmetrically: in violation of Newton's third law, they see an object that "causes" an effect as exerting a force on a "passive" object that may offer "resistance", but does not act back on the "active" object with an equal and opposite force, and predict outcomes in accord with this perception (White, 2009). As is well known, the "folk physics" that these abilities to perceive forces and predict mechanical outcomes reflect is more Aristotelian than Newtonian, and incorporates informal constructs such as "curvilinear momentum" that have no counterpart in formalized classical mechanics (e.g. Pinker, 1997; Gentner, 2002). While the structure of folk physics is often attributed to the general observable behavior of everyday objects, both specific notions such as centrifugal force or curvilinear momentum and the general impression of forces as asymmetrical actions that may be met by resistance are easily understandable as proprioceptive images accompanying tool use (Fields, 2011a). From this perspective, the over-attribution of intentionality characteristic of the naïve human perception of causation (Scholl & Tremoulet, 2000; Atran & Norenzayan, 2004; Rosset, 2008) may reflect our proprioceptive imagination as much as our social imagination.

Proprioceptive imagination in both social imitation and tool use is implemented by the fronto-parietal mirror neuron system (MNS; reviewed by Puce & Perrett, 2003; Rizzolatti & Craighero, 2004; Culham & Valyear, 2006). Proprioceptive images of tool use can be generated by seeing tools or pictures of tools, hearing tools being used, grasping unseen tools, or seeing or hearing names of tools (reviewed by Johnson-Frey, 2004; Johnson-Frey *et al.*, 2005; Lewis, 2006; Martin, 2007). While mirror neurons were originally identified in monkeys based on their "mirroring" of actions performed by conspecifics, human mirror neurons also respond to human-like motions executed by robots and to non-biological motions (Schubotz and von Cramen, 2004; Engel et al., 2007). Although few experiments have specifically targeted non-human-like actions, the systematic human over-attribution of intentionality to causation suggests that the human MNS responds to all or at least most motions resulting in causal consequences. Recent data indicate that the specificities of mirror neurons can be reprogrammed by associative learning (Catmur et al., 2007; 2008; 2009; reviewed by Heyes, 2010), providing a mechanism for experience-dependent proprioceptive imaginations of and abilities to imitate non-human actions. Consistent with broad imitative functionality and experience-dependent modulation of specificity, MNS and related areas of the frontal-parietal praxis network are activated by mental rotation tasks (Formisano et al., 2002; Vingerhoets et al, 2002), approximate numerical comparisons of non-symbolic arrays (Cantlon et al., 2006) and algebraic equation solving (Qin et al., 2004). All of these activities involve imagined manipulations of imagined objects; hence MNS activation during these tasks appears comparable to MNS activation during imagined tool use (Frey *et al.*, 2005; Lewis, 2006).

While they do not establish that humans represent all motions with causal consequences as actions and remember them as action plans, the developmental, cognitive and neurocognitive observations reviewed above suggest that humans tend to represent motions with causal consequences as actions, that such representations are implemented by patterns of MNS activation, and that these patterns can be employed to organize imagined manipulations in both concrete and abstract domains. They therefore support the plausibility of the tool-improvisation model of motion and force. They suggest, moreover, that such analogies will typically be implemented without conscious executive control, and as in the attribution of intentional action to simple geometrical shapes (Scholl & Tremoulet, 2000), independently of explicit conceptual knowledge or even explicit goals to the contrary. It has been demonstrated that adults can draw correct analogical inferences without awareness of doing so (Day & Gentner, 2007; Day & Goldstone, 2011), consistent with the history from Archimedes to Kekulé of significant analogical inferences occurring without conscious executive control.

It can be objected that while non-experts might implement motion and force analogies the way they implement tool-improvisation analogies, experts in highly technical domains such as atomic physics would not represent abstract motion concepts in terms of action plans, but would employ strictly conceptual analogies. An expert-novice distinction along these lines is, however, highly implausible. First, the analogies made by experts are analogies from the strange to the commonplace: they relate unfamiliar abstract concepts to familiar everyday concepts. Invisible atoms are related to solar systems, invisible forces are related to tossing a ball back and forth, invisible particles sharing abstract mathematical properties are related to strings being shaken at different frequencies. Such analogies would not be made were it not easier to reason about the everyday concepts in their native representation. Second, such analogies demonstrably help both experts (Holyoak & Thagard, 1995; Feist & Gorman, 1998) and novices (Podolefsky & Finkelstein, 2006) solve motion and force problems, indicating that the native representation of the everyday concepts targeted by the relevant analogies is in fact easer to manipulate than the abstract, conceptual representation in which the problems are framed. Third, automaticity increases as expertise increases (Ericsson & Lehmann, 1996). It is the naïve who struggle with abstract, formal representations; experts solve problems by experience-based intuition. The continuing search for simple, physical assumptions to replace the abstract mathematical postulates of quantum mechanics (e.g. Fuchs, 2003; Fields, 2011b) is a search for analogies that permit the use of experience-based physical intuitions in place of cumbersome, formalized conceptual reasoning.

The actual methods either experts or novices employ to solve motion and force problems can, however, only be demonstrated conclusively by experimental investigation. While "good" analogies have been studied extensively in the laboratory, failures of analogical reasoning have received less attention. An adequate model of analogy must account not only for cases in which an analogy cannot be made, but also for cases in which structurally coherent, systematic and plausible analogies conflict. As noted above, Rutherford's successful alpha-particle:X::electron:atom analogy provoked such a conflict by concentrating an electrical charge in the center of the atom and hence implying that electrons occupying the periphery of the atom might be moving with respect to this central charge. This conflict was addressed by Bohr (1913). Assuming that Bohr's reasoning involved analogy, it can be reconstructed within the tool-improvisation model as shown in Fig. 2. Bohr knew that the force acting on the electrons in an atom had to be electrostatic, but did not know what motion on the part of the electrons could cause them to emit radiation only in discrete amounts; his task was, therefore, to find a motion that was compatible with both a force and an observed outcome. He assumed an atom comprising a positive nucleus surrounded by orbiting electrons, i.e. he assumed electrons:nucleus::planets:sun, and attributed this model to Rutherford (the two were evidently in regular communication, as Rutherford submitted Bohr's paper to the *Philosophical Magazine*). He then showed that this analogy could not be right: *any* continuous motion on the part of the electrons would be expected to generate continuous radiation, and the electrons would be expected to collapse into the center of the atom. It is ironic that the classical analogies upon which the mathematical representation of a continuously-radiating electron was based – those of moving objects creating visible waves in water or sound waves in air – had been proven false by the Michaelson-Morley experiment 26 years before Bohr's paper. Physicists still, however, think of electromagnetic radiation in terms of ripples in a "field" that pervades space-time; thinking of waves that are not waves *in* anything appears to be impossible.

Whether Bohr experienced his insight into atomic structure – the insight that classical electron *motion* within an atom was impossible – as a conflict between analogies can perhaps never be established. What is clear is that despite his efforts, the electrons:nucleus::planets:sun analogy is alive and well, not only in the academic analogy literature but as an iconic symbol of 20th century culture. Its coherence as a structure mapping renders it plausible, its systematicity gives it explanatory power, and its pre-motor implementation renders it resistant to revision by explicit knowledge. It exists, moreover, within a cultural context in which the explicit knowledge with which it conflicts is held only by a small minority of specialists: other than that they are small and somehow used in bombs, electrons:nucleus::planets:sun may well be all that most people "know" about atoms.

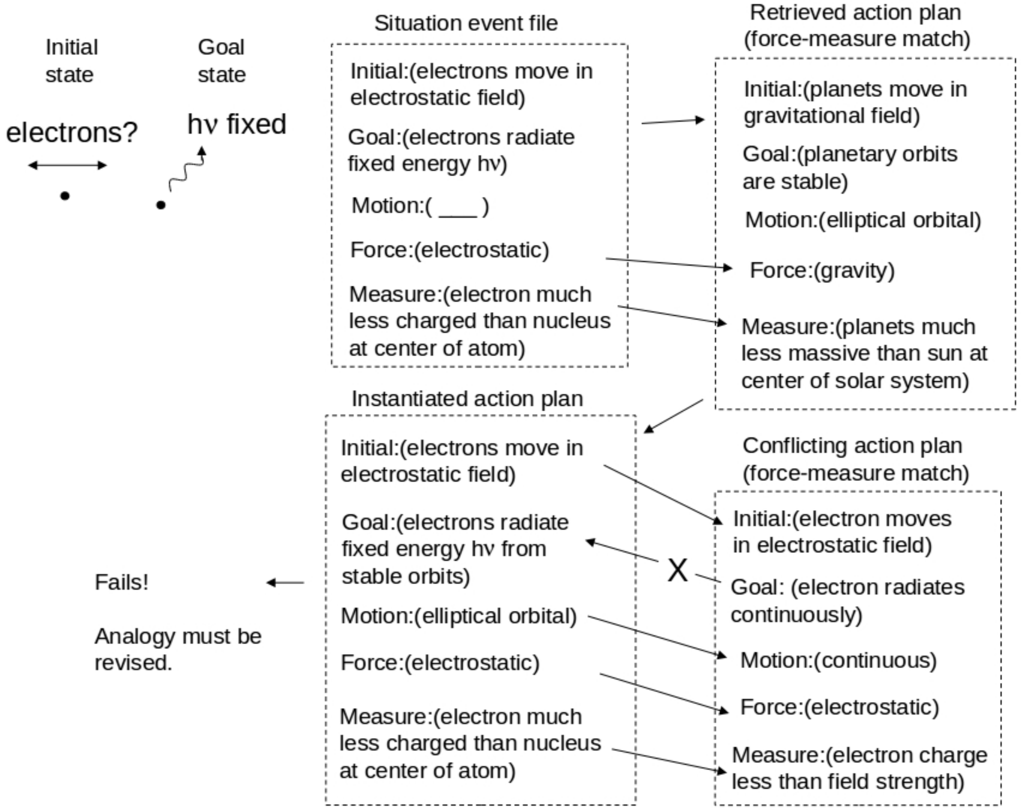

Fig. 2. Frame-based representation of analogical conflict in Bohr's inference that the electrons in atoms cannot undergo classical motion. Arrows represent mappings. The use of frames is heuristic only and is not meant to imply that the representations implemented by the fronto-parietal praxis system encode concepts expressible in either public language or an internally comprehended language of thought.

Predictions of the tool-improvisation model of motion and force analogies

The tool-improvisation model of abstract motion and force analogies such as those of Rutherford and Bohr is *prima facie* implausible: such open-ended scientific reasoning is the very paradigm of what Fodor (2000), for example, assigns to fully-conceptual “central cognition”. The model is rendered somewhat plausible by the developmental, cognitive, and neurocognitive considerations reviewed above: if human beings systematically interpret concrete, perceptible motions with causal consequences as actions, the interpretation of motions as actions in abstract domains may simply reflect an ability to represent abstract motions as causal. Some further plausibility is added by evolutionary considerations: it is more parsimonious, at least, if structure mapping in general motion and force analogies is derivative of structure mapping in the special case of tool use, as opposed to having been re-invented following the acquisition of conceptualization and language. The question of how structure mapping is implemented, in these cases or any others, however remains open. The available functional localization data indicate a two-step process in the fronto-parietal network (Waltz *et al.*, 2000; Bunge *et al.*, 2005; Morrison *et al.*, 2005; Green *et al.*, 2006), but do not settle the question.

The tool-improvisation model clearly does not imply that either the statements of abstract motion and force problems – such as Bohr's problem of how electrons could emit discrete packets of radiation – or their reportable solutions are independent of conceptual awareness or natural languages. It implies only that the implementation of structure mapping is independent of conceptual awareness and natural languages. It is fully consistent with, and in fact an implication of, the tool-improvisation model that an interface must exist between concepts reportable in natural languages and the event-files and action plans implicated in structure mapping by the model. Such an interface clearly exists in the case of tools, and it is clearly multi-modal (Frey *et al.*, 2005; Lewis, 2006). The further claim that abstracted event files or action plans implement the corresponding concepts (Gallese and Lakoff, 2005; Barsalou, 2008) is consistent with but not required by the tool-improvisation model.

The tool-improvisation model generates predictions in four areas that differentiate it from the dominate view that analogy is a form of high-level, domain-general verbal reasoning (Gentner, 2005; Holyoak, 2005) and the alternative view that it is an outcome of concept-level priming (Leech *et al.*, 2008). First, it predicts that abstraction of the problem statement occurs as a component of event-file construction, and that searches for both motion and force matches are carried out at multiple levels of abstraction simultaneously. As noted earlier, this distinguishes the tool-improvisation model from all models in which retrieved structures are partially abstracted along specific dimensions. Second, it predicts that practice, and particularly practice involving overt or simulated visuo-motor manipulations, is critical to the incorporation of motion and force analogies as usable knowledge. This prediction distinguishes the tool-improvisation model from all models in which analogical reasoning is implemented over either natural-language or “language-of-thought” concepts, as visuo-motor manipulation plays no role in such models. Third, it predicts that motion and force analogies are, in general, refractory to revision by conceptual knowledge that is not encoded by visuo-motor representations. Hence it predicts that even scientists who “know better” will employ motion and force analogies, such as the Rutherford-atom analogy, that are demonstrably incorrect. Purely conceptual models of analogy could only explain such an effect by appeals to “familiarity” or some other preference ranking defined over concepts, which would itself have to be explained on a case-by-case or at least culture-by-culture basis. Finally, the tool-improvisation model predicts that “systemizing” or “mechanizing” as a problem-solving orientation (Baron-Cohen, 2002; 2008; Crespi & Badcock, 2008) is distinguished from “mentalizing” or “empathizing” not by the representation of causation in terms of abstract forces as opposed to manipulative actions, but by a systematic suppression of associations with intentionality in the representation of manipulations. It therefore predicts a dissociation, in systemizers but not mentalizers, between MNS activity and social emotions and default-network activity associated with agency.

The first prediction of the tool-improvisation model, that abstraction is carried out as a component of event-file construction, can be tested by designs that determine what is retrieved by analogical matching. As discussed above, the model predicts that fully-concrete event files will retrieve fully-concrete action plans, including specific, not abstracted, force representations. It is known, however, that event files are constructed hierarchically (Colzato *et al.*, 2006); hence the model also predicts that abstracted action plans will be retrieved in a search against memory. However, abstracted plans should not be retrieved preferentially. A design in which both concrete and abstracted matches and mismatches are used as primes and distinct, equally-abstractable concrete cases are used as probes could test this prediction. Visual presentation of primes, as employed by Day & Goldstone (2011), would guard against interference from potentially confounding word-association effects. Designs that assess recall or practical application of concrete versus abstracted analogies would also test this prediction. A design in which multiple concrete analogies were employed in undergraduate physics instruction has already shown that students remember and preferentially apply the concrete characteristics of the analogies with which they are taught, indicating a preference for concrete over abstract analogical retrieval (Podolefsky & Finkelstein, 2006). This latter result is consistent with priming models, but inconsistent with models that require partial abstraction.

The second prediction, that visuo-motor practice is a specific enabler of motion and force analogy learning, is suggested by the extensive use of diagrams, simulations, and hands-on experiments in physics and engineering pedagogy. It has been shown that undergraduate physics students perform better in an instructional setting that requires the students themselves to integrate multiple representations of the material being learned, including diagrams and simulations, while presentation of multiple representations by an instructor has little effect (Lasry & Aulls, 2007). Extending such pedagogical experiments from the classroom to the teaching laboratory, and hence the opportunity for students to perform hand-on experiments, would more thoroughly test this prediction. Cognitive designs that compared the effectiveness as primes for analogical transfer to novel situations of reading descriptions of motion and force analogies between systems, watching videos depicting the kinematics of the analogous systems, and manipulating simulations of the systems with both visual and proprioceptive feedback would provide a strong experimental test of this prediction. Conceptual priming models would predict either no distinction, or greater effectiveness of verbal primes in this situation; the dominant concept-level structure-mapping model would also predict greater effectiveness for concept-level primes.

The third prediction, that motion and force analogies will typically be refractory to revision by conceptual knowledge, is tested on a grand scale by the Rutherford-atom analogy. It is not unique. The Feynman diagrams used by physicists to represent interactions between elementary particles are effectively analogies: they depict well-defined objects moving along well-defined trajectories, and exchanging smaller objects at well-defined points (Randall, 2005 provides several examples). They are far easier to understand and manipulate than the mathematical formalism that they represent and hence are valuable theoretical tools, but if taken literally they contradict quantum mechanics. Papers reminding physicists not to take notions such as “particle” or “interaction” literally are staples of the foundations of physics literature (e.g. Zeh, 2009). In the public sphere, the persistence of compelling but misleading analogies can have real consequences; political opponents of global climate analysis, for example, have been enormously aided by exploitable dis-analogies between the atmosphere and a greenhouse.

Concept-level models of analogy would predict that concept-level input indicating that an analogy was false would extinguish the analogy. The tool-improvisation model predicts that motion and force analogies are executed by a cognitively impenetrable mechanism, and hence that coherent, systematic and plausible motion and force analogies will persist even in populations that are aware that they are false. It predicts that explicit knowledge that an analogy does not work will initiate a search for a better analogy as illustrated in Fig. 1, but that if this search fails, the original analogy will not be abandoned. It will continue to be executed essentially automatically, requiring the contradicting information to be employed explicitly, each time the analogy is used, to prevent false inferences. In this sense, the construction of motion and force analogies is analogous to the perception of illusory motion, and experimental paradigms that evaluate illusory motion perception under conditions in which the subject knows that the perceived motion is illusory may be adaptable to test for the persistence of motion and force analogies that are known to be false.

The situation illustrated in Fig. 2, in which equally systematic and plausible motion and force analogies conflict, is predicted to be rare on any model. Humans experience the motions and forces of a causally-coherent world; hence distinct motions and forces can be expected to abstract upwards in a coherent way. Flowing water and moving crowds of people are, for example, commonly studied as analogies for electrical current. These analogies have different particulars, but abstract upwards to a single intuitive understanding of continuous flow. Similarly, sound waves and traveling waves on a string (or in water) involve different physics and hence induce differently understandings of waves on an abstract electromagnetic field (Podolefsky & Finkelstein, 2006), but abstract upwards to a common intuition of wave-like motion. An object moving in a well-defined orbit and an object continuously losing energy by creating waves in the medium through which it is moving do not, however, abstract upwards to a single concept: one motion is stable, the other is not. The wave and particle “pictures” of objects that are equally compatible with quantum mechanics similarly do not abstract upwards into a single intuitive concept. Abstract theoretical domains may prove to be the only domains in which a conflicting-analogy mechanism of analogy failure can be investigated. An unambiguous demonstration of this mechanism with analogies that were independently shown to be cognitively impenetrable would provide significant support for the tool-improvisation model.

The final prediction of the tool-improvisation model concerns the neurocognitive locus of the distinction between systemizing and mentalizing (Baron-Cohen, 2002; 2008; Crespi & Badcock, 2008). Systemizing and mentalizing are defined in terms of abilities in formal, mechanistic, or algorithmic reasoning on the one hand and theory-of-mind, social cognition, and empathy on the other, but are typically assayed in non-clinical situations by surveys that probe orientation toward mechanical or social reasoning (Baron-Cohen et al., 2003). It is widely assumed that problem-solving orientation and capability are strongly correlated, but this has not been demonstrated experimentally in unbiased populations. Individual cases, moreover, suggest that any such correlation is not exact. Albert Einstein, for example, was clearly an enormously capable systemizer. His social skills have been considered sufficiently poor to warrant a retrospective diagnosis of Asperger's syndrome (Fitzgerald & O'Brien, 2007) . He was, however, highly skilled in adopting different points of view; his theories of relativity are based on postulates of invariance across points of view, and his “thought experiment” methodology of imagining events as seen from the perspective of an observer in an technologically-unachievable situation is now a commonplace of theoretical physics. Imagining different points of view, however, is a canonical mentalizing ability. Hence cases such as Einstein's raise the question of how systemizing and mentalizing are distinguished at the level of neurocognitive function.

By treating all motion and force representations as action representations, the tool-improvisation model locates the distinction between systemizing and mentalizing at the interface between the MNS and the limbic and default-network pathways that implement social emotions and attributions of intentionality (reviewed by Adolphs, 2003; Frith, 2007; Buckner *et al.*, 2008). Systemizers, in the model, tend not to associate actions (i.e. MNS activation) with social emotions and intentionality, or to associate actions by only some kinds of actors with social emotions and intentionality, while mentalizers tend to associate actions with social emotions and intentionality more broadly. This locus for the systemizing – mentalizing distinction is consistent with a causal dependence of systemizing on default-network deactivation (reviewed by Fields, 2011c). A search for specific dissociations between MNS activation and default-network activation during action observation, in systemizers but not mentalizers, would test this prediction of the tool-improvisation model.

Conclusion

Tool improvisation requires structure mapping, and animals without language or conceptual reasoning perform tool improvisation. Comparative neurofunctional and neurocognitive evidence suggests that structure mapping in tool improvisation is implemented by the event-file binding and action-planning systems (Fields, 2011a). The present paper extends this model of tool-improvisation analogies to all motion and force analogies. If the model proposed here is correct, a single mechanism enables structure mapping not only across phylogenetic orders, but also across applications ranging from food gathering to theoretical physics. This mechanism is reliable, however, only to the extent that its results can be tested. In the case of tool improvisation, such testing is typically performed *in situ* and in real time. In the case of theoretical sciences, it must be performed *ex situ* in laboratories or by calculation using simplified model systems. These latter tests require an interface between the event files and action plans that implement structure mapping and the conceptual memory and natural-language terminology that enable group problem solving that is distant in both time and space from the task environment in which the problem arose. Natural language terms that refer to visual or other modal imaginative representations are a critical component of this interface. If the model presented here is correct, the well-established dependence of analogical reasoning outside of the narrow domain of tool improvisation on the language of relational concepts (Gentner, 2005) appears to lie in this interface, not in the mechanism of structure mapping.

The model presented here suggests that an answer to Gentner's (2003) question of “whether there are other relations, besides same/different, that might be implicitly present in humans prior to language learning” (p. 226) is that motion and force relations are implicitly present prior to language learning, whether they are innate (Baillargeon, 2008) or learned from general experience (Rakison & Lupyan, 2008). Language learning builds an interface between these implicit relations and explicit concepts. The tool-improvisation model of motion and force analogies thus provides a mechanism by which the idea that perceptual-motor simulation implements natural-language concepts (Gallese & Lakoff, 2005) can be made consistent, in at least one significant domain, with structure mapping as an algorithm defined over such concepts.

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