



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Mathematical Social Sciences 48 (2004) 23–35

mathematical
social
sciences

www.elsevier.com/locate/econbase

Indivisible-labor, lotteries and idiosyncratic productivity shocks

Lilia Maliar, Serguei Maliar*

*Departamento de Fundamentos del Análisis Económico, Universidad de Alicante,
Campus San Vicente del Raspeig, Ap. Correos 99, 03080 Alicante, Spain*

Received 1 January 2003; received in revised form 1 October 2003; accepted 1 November 2003

Abstract

This paper extends the indivisible-labor model by Hansen [J. Monet. Econ. 16 (1985) 309] and Rogerson [J. Monet. Econ. 21 (1988) 3] to include multiple consumers who differ in initial wealth and whose labor productivities are subject to idiosyncratic shocks. In the presence of idiosyncratic uncertainty, the optimal allocations for the individual employment probabilities are at corners: agents work with probability one (zero) when their productivities are high (low). As in Hansen [J. Monet. Econ. 16 (1985) 309], each agent in our indivisible-labor economy behaves as if her labor choice was divisible and her utility function was linear in hours worked. However, the quasi-linearity of the social preferences, established in Hansen [J. Monet. Econ. 16 (1985) 309] for the homogeneous-agent case, does not survive after the introduction of idiosyncratic shocks.

© 2004 Elsevier B.V. All rights reserved.

Keywords: Indivisible-labor; Lotteries; Idiosyncratic shocks; Neoclassical growth model; Heterogeneous agents; Quasi-linear preferences

JEL classification: J64; J65; D33

1. Introduction

In the benchmark neoclassical growth model by Kydland and Prescott (1982), the agent can dedicate any fraction of the total time endowment to work, i.e. hours worked are perfectly divisible. An important shortcoming of such a model is that it predicts too little

* Corresponding author. Tel.: +34-965-903-400x3219; fax: +34-965-903-898.

E-mail addresses: maliarl@merlin.fae.ua.es (L. Maliar), maliars@merlin.fae.ua.es (S. Maliar).

variability in hours worked, since the labor supply elasticity of “standard” CRRA type of preferences is not sufficiently large. In order to account for the variability of hours worked in the data, therefore, it is tempting to assume infinite labor supply elasticity, i.e. to assume quasi-linear preferences that are strictly concave in consumption and linear in labor. The problem is that such preferences are at odds with micro-study estimates (see [Browning et al., 1999](#), for a discussion).

There is one institutional setup in which high macro labor supply elasticity can be reconciled with micro-studies, as shown in [Hansen \(1985\)](#) and [Rogerson \(1988\)](#). If there is a continuum of identical agents who have additive utility functions, if labor is indivisible (i.e. if agents can work either a fixed number of hours or not at all), and if agents choose employment probabilities by trading lotteries, then the indivisible-labor economy behaves like a representative-agent divisible-labor economy with a linear disutility of labor.

In this paper, we study the robustness of [Hansen’s \(1985\)](#) and [Rogerson’s \(1988\)](#) result to the introduction of heterogeneity.¹ We specifically assume that agents have different endowments of wealth and that their labor productivities are subject to idiosyncratic (possibly persistent) shocks.² The fact that these two types of heterogeneity are important for understanding aggregate fluctuations and, in particular, those of the labor market in the data, has been emphasized in [Maliar and Maliar \(2003a\)](#) in the context of [Kydland and Prescott’s \(1982\)](#) divisible-labor model. Our objective is, therefore, to investigate how such heterogeneity can affect the aggregate implications of the indivisible-labor model.

Our results are as follows: In the presence of idiosyncratic productivity shocks, the optimal allocations for the individual employment probabilities are at corners (i.e. they are equal to either zero or one). In spite of the fact that our solution is not interior as in [Hansen \(1985\)](#), we still have that each agent acts as if her labor choice was divisible and her utility function was linear in labor. In our case, the equivalence between the individual behavior in the indivisible- and divisible-labor economies takes the following form: an agent in the indivisible-labor economy works with probability one (zero) if and only if the corresponding agent in the divisible-labor quasi-linear economy works a maximum possible number of hours (does not work). At the aggregate level, the quasi-linearity of preferences of the representative consumer, established in [Hansen \(1985\)](#) for the homogeneous-agent case, does not survive after the introduction of idiosyncratic productivity shocks. In our setup, the preferences of the “representative consumer” depend not only on aggregate variables but also on the heterogeneity parameters and are, in general, not quasi-linear.³ An important implication of our results is that indivisibility and lotteries

¹ The heterogeneous-agent literature which employs the assumption of indivisible-labor includes, e.g. [Cho \(1995\)](#); [Prasad \(1996\)](#); [Maliar and Maliar \(2000\)](#). A model with ex-ante identical agents by [Cho \(1995\)](#) is a particular case of our general setup.

² There is vast literature on heterogeneous agents that advocates the importance of heterogeneity in labor productivity resulting from idiosyncratic shocks, e.g. [Huggett \(1993\)](#), [Aiyagari \(1994\)](#), [Kydland \(1995\)](#), [Castañeda et al. \(1998\)](#), [Krusell and Smith \(1998\)](#), [Maliar and Maliar \(2003a,b\)](#).

³ We employ [Constantinides’s \(1982\)](#) notion of the representative consumer, which does not, in general, imply the existence of [Gorman’s \(1953\)](#) representative consumer. See [Maliar and Maliar \(2003a\)](#) for a detailed discussion and further examples.

are (a priori) no longer a sufficient “trick” for getting enough variability of hours worked in real business cycle models.

To see the intuition behind our results, we shall note that in our indivisible-labor economy, an individual expected momentary utility function is linear in the employment probability. If agents are equally productive in all periods, as in Hansen’s (1985) homogeneous-agent case, they do not care about which periods they work and in which they enjoy leisure time. As a result, we can construct a symmetric equilibrium in which all agents choose identical employment probabilities, so that there exists a representative consumer whose utility function is linear in the employment probability. However, if agents are subject to idiosyncratic productivity shocks, the symmetric equilibrium does not exist; agents, now, are no longer indifferent about which periods they work and in which they have leisure time: they work with the probability one (zero) when their productivities are high (low). The corner solutions break down the quasi-linearity of the preferences of the representative consumer.

We shall finally note that our results are of potential use in the area of international economics. To be specific, there is a body of the literature that studies international business cycles in the context of a two-country neoclassical growth model by considering a planner’s solution, e.g. Backus et al. (1992) and Baxter and Crucini (1995). If labor is indivisible and if each country is affected by a country-specific productivity shock, we can extend a two-country analysis to a multi-country case in a relatively simple fashion. All our results carry over to multi-country economies if we re-interpret heterogeneous agents as countries.

The paper is organized as follows: Section 2 describes a heterogeneous-agent variant of the indivisible-labor economy. Section 3 formulates an equivalent quasi-linear divisible-labor economy and presents the results concerning the properties of equilibrium in the indivisible-labor economy, and finally, Section 4 concludes.

2. The indivisible-labor economy

We consider a complete-market heterogeneous-agent variant of the neoclassical growth model by Hansen (1985) and Rogerson (1988). In such a model, the individual labor choice is indivisible: agents can either work a fixed number of hours (be employed) or work zero hours (be unemployed).

Time is discrete and the horizon is infinite: $t \in T$, where $T = \{0, 1, \dots, \infty\}$. The economy is populated by a continuum of infinitely-lived agents with the names on a unit interval $S \equiv [0, 1]$, an output producing firm and an insurance company. The total measure (mass) of agents is one, $\int_S ds = 1$ and, therefore, the average and aggregate values in our economy coincide. We assume two types of heterogeneity, initial endowment and labor productivity (or skills). The individual skills are subject to idiosyncratic shocks and, therefore, change with time.

We denote a labor productivity shock of agent s in period t by β_t^s and the distribution of the labor productivity shocks across agents in period t by $B_t = \{\beta_t^s\}_{s \in S} \in \mathfrak{Z} \subset R_+^S$. We assume that B_t follows a stationary first-order Markov process. Specifically, let \mathfrak{R} be the Borel σ -algebra on \mathfrak{Z} . A transition function for the distribution of shocks $\Pi: \mathfrak{Z} \times \mathfrak{R} \rightarrow [0, 1]$ is defined on the measurable space $(\mathfrak{Z}, \mathfrak{R})$ in the following way: for each $z \in \mathfrak{Z}$, $\Pi(z, \cdot)$ is a probability measure on $(\mathfrak{Z}, \mathfrak{R})$ and for each $Z \in \mathfrak{R}$, $\Pi(\cdot, Z)$ is a \mathfrak{R} -measurable

function. The function $\Pi(z, Z)$ shall be interpreted as the probability that the next-period distribution of shocks lies in the set Z , given that the current distribution of shocks is z , i.e. $\Pi(z, Z) = Pr\{B_{t+1} \in Z | B_t = z\}$. The initial distribution of shocks $B_0 \in \mathfrak{Z}$ is given. Under these assumptions, idiosyncratic shocks can be correlated across agents, so that our economy can have uncertainty at the aggregate level.

An agent s maximizes the expected lifetime utility, discounted with the factor $\delta \in (0, 1)$, by choosing consumption and the employment probability. At the beginning of each period, the agent plays an employment lottery. If the agent wins, she works a fixed number of hours, \bar{n} . In the opposite case, she does not work at all. Before playing the lottery, the agent can buy unemployment insurance, which pays one unit of consumption if the agent is unemployed and zero otherwise. Markets are complete, i.e. the agents are permitted to trade Arrow securities. The agent is endowed with one unit of time, so that leisure in the employed and unemployed states is given by $1 - \bar{n}$ and 1, respectively. The agent owns the capital stock and rents it to the firm. Capital depreciates at the rate $d \in (0, 1]$.

Therefore, the problem solved by the agent is as follows:

$$\max_{\{x_t^s\}_{t \in T}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \{ \varphi_t^s u(c_t^{s,e}, 1 - \bar{n}) + (1 - \varphi_t^s) u(c_t^{s,u}, 1) \} \right] \tag{1}$$

subject to

$$\begin{aligned} c_t^{s,e} + k_{t+1}^{s,e} + q(\varphi_t^s) y_t^s + \int_{\mathfrak{R}} p_t(Z) m_{t+1}^{s,e}(Z) dZ \\ = (1 - d + r_t) k_t^s + \bar{n} w_t \beta_t^s + m_t^s(Z_t), \end{aligned} \tag{2}$$

$$c_t^{s,u} + k_{t+1}^{s,u} + q(\varphi_t^s) y_t^s + \int_{\mathfrak{R}} p_t(Z) m_{t+1}^{s,u}(Z) dZ = (1 - d + r_t) k_t^s + y_t^s + m_t^s(B_t), \tag{3}$$

$$0 \leq \varphi_t^s \leq 1, \tag{4}$$

where $\{x_t^s\}_{t \in T} = \{\varphi_t^s, c_t^{s,j}, k_{t+1}^{s,j}, \{m_{t+1}^{s,j}(Z)\}_{Z \in \mathfrak{R}}, y_t^s\}_{t \in T}^{j \in \{e,u\}}$, and initial endowment $(k_0^s, m_0^s(Z_0))$ is given. Here, the superscript $j \in \{e, u\}$ refers to the employed and unemployed states, $c_t^{s,j}$ and $k_{t+1}^{s,j}$ denote consumption and capital in state j , y_t^s and $q(\varphi_t^s)$ are the quantity of unemployment insurance and its price, which is a function of the employment probability chosen, $\{m_{t+1}^{s,j}(Z)\}_{Z \in \mathfrak{R}}$ is the portfolio of Arrow securities, $p_t(Z)$ is the price of an Arrow security that pays one unit of consumption if $B_{t+1} \in Z$; r_t and w_t are the prices of capital and efficiency labor, respectively; and, finally, φ_t^s and $(1 - \varphi_t^s)$ are the probabilities of the employed and unemployed states, respectively. The momentary utility function, u , is continuously differentiable, strictly increasing in both arguments and strictly concave.

The production firm rents capital, k_t , and hires labor, h_t , to maximize period-by-period profits:

$$\max_{k_t, \{m_t^s\}_{s \in S}} \pi_t^F = f(k_t, h_t) - r_t k_t - w_t h_t, \tag{5}$$

where $k_t \equiv \int_S k_t^s ds$ and $h_t \equiv \bar{n} \int_S \varphi_t^s \beta_t^s ds$ are the capital and labor inputs, respectively. The production function, f , has constant returns to scale, is strictly concave, continuously

differentiable, strictly increasing with respect to both arguments and satisfies the appropriate Inada conditions.

As in Hansen (1985), we assume that the insurance company maximizes period-by-period expected profits by choosing a supply for unemployment insurances:

$$\max_{\{y_t^s\}_{s \in S}} \pi_t^{IC} = \int_S \{y_t^s q(\varphi_t^s) - (1 - \varphi_t^s)y_t^s\} ds. \tag{6}$$

In order to insure the no-arbitrage condition, we assume that the re-selling of insurance contracts between agents is not allowed.

Definition. A competitive equilibrium in the Eqs. (1)–(6) is a sequence of contingency plans for the consumers’ allocations, the firm’s allocation, the insurance company’s allocation and the prices, such that, given the prices, the allocation of each consumer solves the utility-maximization problem Eqs. (1)–(4), the allocation of the firm solves the profit-maximization problem Eq. (5), the allocation of the insurance company solves the profit-maximization problem Eq. (6); capital, labor and security markets clear, and the economy’s resource constraint (RC),

$$c_t + k_{t+1} = (1 - d)k_t + f(k_t, h_t), \tag{7}$$

is satisfied. The equilibrium quantities are to be such that $c_t^{s,j}, y_t^s, w_t, r_t \geq 0$ for all t, s . We restrict attention to a recursive Markov equilibrium. It is assumed that such an equilibrium exists and is unique.

3. The divisible-labor quasi-linear economy

In this paper, we restrict our attention to the case in which the individual momentary utility functions are identical and additive:

$$u(c, l) = v(c) + \varpi(l), \tag{8}$$

where $v' > 0, v'' < 0, \varpi' > 0$ and $\varpi'' < 0$. Note that the above utility function is not quasi-linear, as both $v(c)$ and $\varpi(l)$ are strictly concave.

It turns out that, with the above assumption of additivity, there is a direct connection between the indivisible- and divisible-labor economies. To be specific, let us consider a heterogeneous-agent variant of Kydland and Prescott’s (1982) model, where each consumer s solves the following utility-maximization problem:

$$\max_{\{\delta_t^s\}_{t \in T}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \{v(c_t^s) + An_t^s\} \right] \tag{9}$$

subject to

$$c_t^s + k_{t+1}^s + \int_{\mathbb{R}} p_t(Z) m_{t+1}^s(Z) dZ = (1 - d + r_t)k_t^s + n_t^s w_t \beta_t^s + m_t^s(B_t), \tag{10}$$

$$0 \leq n_t^s \leq \bar{n}, \tag{11}$$

where the variables in $\{\theta_t^s\}_{t \in T} = \{n_t^s, c_t^s, k_{t+1}^s, \{m_{t+1}^s(Z)\}_{Z \in \mathcal{N}}\}_{t \in T}$ are hours worked, consumption, capital and Arrow securities of agent s , respectively, and $A < 0$ is the utility-function parameter. The production side of the economy is described by the problem (5) with $h_t \equiv \int_S n_t^s \beta_t^s ds$.

We characterize the relationship between the indivisible- and divisible-labor economies with the following proposition:

Proposition 1. *Assume that agents have identical additive utility functions (8) and let $A \equiv (\varpi(1 - \bar{n}) - \varpi(1))/\bar{n}$. Then, the individual variables in the indivisible-labor economy Eqs. (1)–(6) and in the divisible-labor quasi-linear economy (9)–(11) are related by*

$$n_t^s = \bar{n} \varphi_t^s, \quad c_t^s = c_t^{s,j}, \quad k_t^s = k_t^{s,j}, \quad m_{t+1}^s(Z) = m_{t+1}^{s,j}(Z),$$

for all s, t, j, Z .

Proof. See Appendix A. \square

Thus, the agents’ behavior in the indivisible-labor economy is indistinguishable from that in the divisible-labor quasi-linear economy. Regarding the employment decisions, we have that agents in the indivisible-labor economy behave as if their labor choice was divisible and their preferences were linear in labor. The decisions of agents on consumption and savings are independent of their current employment status (i.e. the employed and unemployed have the same consumption, capital and Arrow securities), which is a result of perfect risk sharing.

Maliar and Maliar (2003b) study the properties of equilibrium in the divisible-labor quasi-linear economy Eqs. (9)–(11) and show in particular that its aggregate behavior can be described by a one-consumer model. We reproduce this one-consumer model, below, since, under the equivalence result of Proposition 1, such a model also describes the aggregate behavior of the indivisible-labor economy Eqs. (1)–(6).

Let us first formulate a planner’s economy that generates the same equilibrium allocation as the one in the decentralized quasi-linear economy Eqs. (9)–(11). We define the social utility function by

$$U(c_t, h_t, \{\lambda^s, \beta_t^s\}^{s \in S}) \equiv \max_{\{c_t^s, n_t^s\}^{s \in S}} \left\{ \int_S \lambda^s (v(c_t^s) + A n_t^s) ds \left| \begin{array}{l} \int_S c_t^s ds = c_t \\ \int_S n_t^s \beta_t^s ds = h_t \\ 0 \leq n_t^s \leq \bar{n} \end{array} \right. \right\}, \quad (12)$$

where c_t is the aggregate consumption, and $\{\lambda^s\}^{s \in S} \subset R_+^S$ is the distribution of welfare weights with its mean being normalized to one, $\int_S \lambda^s ds = 1$. We then consider the following one-consumer setup:

$$\max_{\{c_t, h_t, k_{t+1}\}_{t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t, \{\lambda^s, \beta_t^s\}^{s \in S}) | RC. \quad (13)$$

Note that the social utility function can depend not only on aggregate quantities such as c_t and h_t but also on the heterogeneity parameters $\{\lambda^s, \beta_t^s\}_{s \in S}$.⁴

The relationship between the economy Eqs. (9)–(11) and the economy Eqs. (12) and (13) is as follows:

Proposition 2. *Assume that the agents in the divisible-labor quasi-linear economy Eqs. (9)–(11) have identical additive utility functions Eq. (8). Then, the aggregate behavior of such an economy is described by the one-consumer model Eqs. (12) and (13), with an additive social utility function:*

$$U(c_t, h_t, \{\lambda^s, \beta_t^s\}_{s \in S}) = V(c_t, \{\lambda^s\}_{s \in S}) + W(h_t, \{\lambda^s, \beta_t^s\}_{s \in S});$$

where V is defined by $c_t = \int_S c_t^s ds$ and

$$c_t^s = (v')^{-1} \left(\frac{1}{\lambda^s} V_1(c_t, \{\lambda^s\}_{s \in S}) \right); \tag{14}$$

W is defined by $h_t = \int_S n_t^s \beta_t^s ds$ and

$$\lambda^s - \beta_t^s W_1(h_t, \{\lambda^s, \beta_t^s\}_{s \in S}) \begin{cases} < 0 \Rightarrow n_t^s = 0 \\ > 0 \Rightarrow n_t^s = \bar{n} \\ = 0 \Rightarrow 0 \leq n_t^s \leq \bar{n} \end{cases} ; \tag{15}$$

and where V_1 and W_1 denote the first-order partial derivatives of V and W with respect to c_t and h_t , correspondingly.

Proof. See Maliar and Maliar (2003b). \square

We now employ the results of Propositions 1 and 2 to describe some properties of the equilibrium in the indivisible-labor economy Eqs. (1)–(6).

As far as the consumption distribution is concerned, it is determined by the standard complete-market condition that the ratio of marginal utilities of any two agents is constant across time and states of nature. In fact, the implications of the indivisible-labor model for the individual consumption decisions are essentially the same as those for the divisible-labor model with strictly concave preferences considered in Maliar and Maliar (2003a). An example of an analytic construction of the subfunction V for the divisible-labor quasi-linear economy Eqs. (9)–(11) is provided in Maliar and Maliar (2003b), under the assumption that individual utility functions are given by identical power members of the Hyperbolic Absolute Risk Aversion (HARA) class (see, also, Maliar and Maliar, 2003a, for a discussion of other examples).

Regarding hours worked, we have that the property of quasi-linearity leads to corner solutions for the individual hours worked. Indeed, according to condition Eq. (15), the

⁴This construction was initially proposed by Constantinides (1982). Maliar and Maliar (2003a) use a similar planner’s problem representation to simplify the description of the equilibrium in the heterogeneous-agent version of Kydland and Prescott’s (1982) model with strictly convex individual preferences.

optimal labor behavior of an agent is to work the maximum number of hours possible, $n_t^s = \bar{n}$, when her productivity is high, to work zero hours, $n_t^s = 0$, when her productivity is low and to work any number of hours, $n_t^s \in [0, \bar{n}]$, when her productivity satisfies optimality condition Eq. (15) with equality, which corresponds to an interior equilibrium. In terms of the indivisible-labor economy, we equivalently have that agents work with probability one, $\varphi_t^s = 1$, in high-productivity states, with probability zero, $\varphi_t^s = 0$, in low-productivity states, and can choose any probability $\varphi_t^s \in [0, 1]$ in an interior equilibrium. Assuming that idiosyncratic shocks are randomly drawn from a distribution with a continuous density function, as is typically done in the literature, we obtain that the set of agents with interior optimal allocations for hours worked (equivalently, for the probability of employment) has a Lebesgue measure zero. Thus, the labor choices of almost all the agents are at corners.

In the presence of corner solutions, the standard aggregation techniques cannot be used. We, therefore, provide no results about the possibility of analytical construction of the subfunction W for a general set of welfare weights, $\{\lambda^s\}_{s \in S}$. In some cases, it might be possible to construct the subfunction W analytically, by imposing additional (very strong) restrictions, as for example, by assuming a temporary heterogeneity in productivities, which has no effect on the individual equilibrium allocations, other than working hours, as is done in [Cho \(1995\)](#) (see also [Maliar and Maliar, 2003b](#), for a discussion). In general, we can construct the subfunction W numerically, by computing a solution to optimality condition Eq. (15) for all possible sets of the aggregate working hours, h_t , and the heterogeneity parameters $\{\lambda^s, \beta_t^s\}_{s \in S}$.

We shall now discuss the relation between our results and those established by [Hansen \(1985\)](#) for the benchmark indivisible-labor model with identical (constant-productivity) consumers. Consider the following one-consumer divisible-labor quasi-linear model:

$$\max_{\{c_t, h_t, k_{t+1}\}_{t \in T}} E_0 \sum_{t=0}^{\infty} \delta^t \{v(c_t) + Ah_t\} | RC, \tag{16}$$

where $A \equiv (\varpi(1 - \bar{n}) - \varpi(1))/\bar{n}$. The following result can be shown.

Proposition 3. ([Hansen, 1985](#)). *Assume that the agents in the indivisible-labor economy Eqs. (1)–(6) have identical additive utility functions Eq. (8), identical constant productivities, $\beta_t^s = 1$ for all s, t , and identical endowments. If an equilibrium exists and it is interior, then the aggregate equilibrium behavior of this economy is described by the one-consumer divisible-labor quasi-linear model Eq. (16).*

Proof. Let us show how this result can be accommodated in our general framework. Since the agents are identical, they have identical welfare weights, $\lambda^s = 1$ for all s . From definition Eq. (14), we therefore obtain that $V = v$, up to an additive constant from integration. Furthermore, as an equilibrium is interior, condition Eq. (15) holds with equality, $W_1(h_t) = A$, and thus, we have that $W(h_t) = Ah_t$, again, up to an additive constant from integration. \square

In other words, if agents are identical in all respects, except in the realization of employment lotteries, then at the aggregate level, the indivisible-labor economy behaves as if there was a representative consumer who has a divisible-labor choice and whose

utility function is linear in hours worked (leisure). What is the intuition that underlies this result? The probabilities of employment enter the individual utility functions linearly. Furthermore, the labor productivity of agents remains constant during all periods. In an interior equilibrium, condition Eq. (15) holds with equality for all t, s , and agents are indifferent between any sequences of employment probabilities that imply the same expected amount of work. In particular, there exists a symmetric equilibrium in which all agents choose the same probability of employment, i.e. $\varphi_t^s = \varphi_t = h_t/\bar{n}$ for all t, s . Therefore, there exists a representative consumer whose lifetime utility function is linear in the average probability of employment (aggregate labor), which is precisely the result shown in Hansen (1985).⁵

The equivalence between the heterogeneous-agent versions of the indivisible- and divisible-labor economies shown in the present paper, is concerned only with individual behavior and is therefore weaker than the one established by Hansen (1985) for the homogeneous-agent case. Indeed, we show in Proposition 1 that each agent in the heterogeneous-agent indivisible-labor economy behaves as if she had a divisible-labor choice and her utility function was linear in leisure. This result does not imply, however, that the aggregate behavior of the indivisible-labor economy is described by the divisible-labor quasi-linear model Eq. (16). According to Proposition 2, it is described by the model Eqs. (13)–(15). As we have argued above, in the presence of corner solutions for the individual working hours, the social utility function in such a model is a complicated object, which depends not only on the aggregate hours worked, h_t , but also on the heterogeneity parameters $\{\lambda^s, \beta_t^s\}_{s \in S}$.

4. Conclusion

This paper studies the implications of a dynamic general-equilibrium model with indivisible-labor and heterogeneous agents. We assume that agents differ in their initial wealth and that their labor productivities are subject to idiosyncratic shocks. We show that the behavior of each agent in our indivisible-labor economy can be described by a quasi-linear utility-maximization problem. The equivalence result, which we establish for the heterogeneous-agent case, is weaker than the one shown in Hansen (1985) for the economy with identical (constant-productivity) consumers. To be specific, in Hansen's (1985) economy, we have not only that each individual behaves as if her labor choice was divisible and her utility function was quasi-linear, but also that the economy, as a whole, behaves as a one-consumer divisible-labor quasi-linear economy. In our heterogeneous-agent economy, we have equivalence only at the individual level. As regards the aggregate dynamics, the social utility function depends on both aggregate variables and heterogeneity parameters and is not, in general, quasi-linear.

⁵ An interior equilibrium in Hansen's (1985) model is not uniquely determined in the sense that there are infinitely many sequences for the individual probabilities of employment that satisfy the equilibrium conditions. However, all such sequences lead to the same aggregate equilibrium dynamics, which are described by the model Eq. (16). See Maliar and Maliar (2000, 2003b) for a discussion on this point.

Acknowledgements

We thank Jordi Caballe, Finn Kydland, Franck Portier, Michael Reiter, Xavier Sala-i-Martin, William Schworm and Andrew Scott and especially two anonymous referees for useful comments. We are grateful to Morten Ravn for his guidance. Any remaining errors are the sole responsibility of the authors. This research was supported by the Institute Valenciano de Investigaciones Económicas and the Ministerio de Ciencia y Tecnología de España, the Ramón y Cajal program and BEC 2001-0535.

Appendix A

In this section, we provide the Proof of Proposition 1.

Proof of Proposition 1. First, the profit-maximization condition of the insurance company, whose problem is stated in Eq. (6), is given by

$$q(\varphi_t^s) = 1 - \varphi_t^s. \quad (17)$$

Secondly, we derive the individual first-order conditions (FOCs) by using the value function representation of the agent's problem Eqs. (1)–(4)

$$\begin{aligned} V^s(k_t, B_t, k_t^s, \{m_t^s(Z)\}_{Z \in \mathfrak{R}}) = \max_{\{c_t^s\}} \{ & \varphi_t^s [u(c_t^{s,e}, 1 - \bar{n}) \\ & + \delta E_t V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,e}, \{m_{t+1}^{s,e}(Z)\}_{Z \in \mathfrak{R}})] \\ & + (1 - \varphi_t^s) [u(c_t^{s,u}, 1) \\ & + \delta E_t V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,u}, \{m_{t+1}^{s,u}(Z)\}_{Z \in \mathfrak{R}})] \} \end{aligned} \quad (18)$$

subject to Eqs. (2)–(4), where V^s is the value function of agent s . As a first step, we express $c_t^{s,e}$ and $c_t^{s,u}$ from the budget constraints Eqs. (2) and (3), respectively, and substitute them into the objective function of the problem Eq. (18). The FOCs with respect to the unemployment insurance holdings, the capital holdings in the two states, Arrow securities in the two states and the probability of employment, respectively, are

$$\varphi_t^s q(\varphi_t^s) u_1(c_t^{s,e}, 1 - \bar{n}) = (1 - \varphi_t^s) (1 - q(\varphi_t^s)) u_1(c_t^{s,u}, 1), \quad (19)$$

$$u_1(c_t^{s,e}, 1 - \bar{n}) = \delta E_t \left[\frac{\partial V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,e}, m_{t+1}^{s,e}(Z)_{Z \in \mathfrak{R}})}{\partial k_{t+1}^{s,e}} \right], \quad (20)$$

$$u_1(c_t^{s,u}, 1) = \delta E_t \left[\frac{\partial V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,u}, \{m_{t+1}^{s,u}(Z)\}_{Z \in \mathfrak{R}})}{\partial k_{t+1}^{s,u}} \right],$$

$$u_1(c_t^{s,e}, 1 - \bar{n}) p_t(Z) = \delta \left[\frac{\partial V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,e}, m_{t+1}^{s,e}(Z)_{Z \in \mathfrak{R}})}{\partial m_{t+1}^{s,e}(Z)} \right] \Pi(B_t, Z), \tag{21}$$

$$u_1(c_t^{s,u}, 1) p_t(Z) = \delta \left[\frac{\partial V^s(k_{t+1}, B_{t+1}, k_{t+1}^{s,u}, \{m_{t+1}^{s,u}(Z)\}_{Z \in \mathfrak{R}})}{\partial m_{t+1}^{s,u}(Z)} \right] \Pi(B_t, Z),$$

$$u(c_t^{s,e}, 1 - \bar{n}) - u(c_t^{s,u}, 1) - u_1(c_t^{s,e}, 1 - \bar{n}) q'(\varphi_t^s) y_t^s - \zeta_t^s - \zeta_t^s = 0, \tag{22}$$

$$\zeta_t^s \geq 0 \quad \text{and} \quad \zeta_t^s \varphi_t^s = 0, \tag{23}$$

$$\zeta_t^s \geq 0 \quad \text{and} \quad \zeta_t^s (1 - \varphi_t^s) = 0, \tag{24}$$

where u_1 is the derivative of u with respect to consumption, and ζ_t^s, ζ_t^s are the Lagrange multipliers associated with the restrictions $\varphi_t^s \geq 0$ and $\varphi_t^s \leq 1$, respectively.

Finally, the profit-maximization of the production firm Eq. (5) implies that the equilibrium profit is zero and that the equilibrium prices of capital and labor are equal to the respective marginal products, i.e.

$$r_t = \partial f(k_t, h_t) / \partial k_t \quad \text{and} \quad w_t = \partial f(k_t, h_t) / \partial h_t. \tag{25}$$

The result (17) together with Eq. (19) gives us the risk-sharing condition:

$$u_1(c_t^{s,e}, 1 - \bar{n}) = u_1(c_t^{s,u}, 1). \tag{26}$$

Eqs. (20), (21) and (26), therefore, imply that the holdings of capital and Arrow securities are the same in the employed and unemployed states, i.e., $k_{t+1}^{s,e} = k_{t+1}^{s,u} \equiv k_{t+1}^s$ and $m_{t+1}^{s,e}(Z) = m_{t+1}^{s,u}(Z) \equiv m_{t+1}^s(Z)$. Substituting this result into the two state-contingent constraints Eqs. (2) and (3) gives the equilibrium holdings of unemployment insurance:

$$y_t^s = \bar{n} w_t \beta_t^s - c_t^{s,e} + c_t^{s,u}. \tag{27}$$

The envelope conditions of the problem Eq. (18) are

$$\frac{\partial V^s(k_t, B_t, k_t^s, \{m_t^s(Z)\}_{Z \in \mathfrak{R}})}{\partial k_t^s} = u_1(c_t^{s,e}, 1 - \bar{n})(1 - d + r_t)$$

$$= u_1(c_t^{s,u}, 1)(1 - d + r_t), \tag{28}$$

$$\frac{\partial V^s(k_t, B_t, k_t^s, \{m_t^s(Z)\}_{Z \in \mathfrak{R}})}{\partial m_t^s(Z)} = u_1(c_t^{s,e}, 1 - \bar{n}) = u_1(c_t^{s,u}, 1). \tag{29}$$

By substituting the updated version of Eq. (28) into Eq. (20), we obtain the standard intertemporal condition:

$$v_t(c_t^{s,e}, 1 - \bar{n}) = \delta E_t[u_1(c_{t+1}^{s,j}, 1 - \bar{n})(1 - d + r_{t+1})].$$

where $j \in \{e, u\}$. Further, by using condition Eq. (27) and the result that the holdings of capital and Arrow securities do not depend on the employment status of the agent, we can replace the two state-contingent constraints Eqs. (2) and (3) by a single one

$$\begin{aligned} \varphi_t^s c_t^{s,e} + (1 - \varphi_t^s) c_t^{s,u} + k_{t+1}^s + \int_{\mathfrak{R}} p_t(Z) m_{t+1}^s(Z) dZ \\ = (1 - d + r_t) k_t^s + \varphi_t^s \bar{n} w_t \beta_t^s + m_t^s(B_t). \end{aligned} \tag{30}$$

Hence, the agent faces the same constraint Eq. (30) independently of whether she is employed or not.

Assume now that the individual utility function, u , is additive and is given by Eq. (8). Then, according to Eq. (26), consumption in the employed and unemployed states is equal, $c_t^{s,e} = c_t^{s,u} \equiv c_t^s$. This implies that the budget constraint Eq. (30) can be written as Eq. (10). By substituting the updated version of Eq. (28) into Eq. (20), we obtain the intertemporal FOC:

$$v'(c_t^s) = \delta E_t[v'(c_{t+1}^s)(1 - d + r_{t+1})]. \tag{31}$$

Similarly, by substituting the updated version of Eq. (29) into Eq. (21), we obtain the standard complete-market condition, implying that the ratio of marginal utilities of any two agents is constant across time and states of nature,

$$v'(c_t^s) p_t(Z) = \delta v'(c_{t+1}^s) \Pi(B_t, Z). \tag{32}$$

By using the fact that $c_t^{s,e} = c_t^{s,u} = c_t^s$, we can re-write Eq. (22) as follows:

$$-[\varpi(1) - \varpi(1 - \bar{n})] + v'(c_t^s) \bar{n} w_t \beta_t^s - \zeta_t^s - \zeta_t^s = 0, \tag{33}$$

where ζ_t^s and ζ_t^s satisfy restrictions Eq. (23) and Eq. (24), respectively.

As a final step, consider the recursive formulation of the individual problem in the divisible-labor quasi-linear economy Eqs. (9)–(11):

$$\begin{aligned} V^s(k_t, B_t, k_t^s, \{m_t^s(Z_t)\}_{z \in \mathfrak{R}}) = \max_{\{b_t^s\}_{t \in T}} \{v(c_t^s) + A n_t^s \\ + \delta E_t V^s(k_{t+1}, B_{t+1}, k_{t+1}^s, \{m_{t+1}^s(Z_t)\}_{z \in \mathfrak{R}})\} \end{aligned} \tag{34}$$

subject to Eqs. (10) and (11).

The solution to the problem Eq. (34) is described by FOCs Eqs. (31)–(33), where $A \equiv [\varpi(1 - \bar{n}) - \varpi(1)]/\bar{n}$ and $n_t^s \equiv \bar{n} \varphi_t^s$.

References

- Aiyagari, R., 1994. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics* 109 (3), 659–684.
- Backus, D., Kehoe, P., Kydland, F., 1992. International real business cycles. *Journal of Political Economy* 100 (4), 745–775.
- Baxter, M., Crucini, M., 1995. Business cycles and the asset structure of foreign trade. *International Economic Review* 36 (4), 821–854.
- Browning, M., Hansen, L., Heckman, J., 1999. Micro data and general equilibrium models. In: Taylor, J., Woodford, M. (Ed.), *Handbook of Macroeconomics*. Elsevier, Amsterdam, pp. 543–633.
- Castañeda, A., Díaz-Giménez, J., Ríos-Rull, J.V., 1998. Exploring the income distribution business cycle dynamics. *Journal of Monetary Economics* 42, 93–130.
- Cho, J., 1995. Ex post heterogeneity and the business cycle. *Journal of Economic Dynamics and Control* 19, 533–551.
- Constantinides, G., 1982. Intertemporal asset pricing with heterogeneous consumers and without demand aggregation. *Journal of Business* 55, 253–267.
- Gorman, W., 1953. Community preference field. *Econometrica* 21, 63–80.
- Hansen, G., 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics* 16, 309–328.
- Huggett, M., 1993. The risk free rate in heterogeneous-agent, incomplete-insurance economies. *Journal of Economic Dynamics and Control* 17, 953–969.
- Krusell, P., Smith, A., 1998. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy* 106 (5), 867–896.
- Kydland, F., 1995. Business cycles and aggregate labor market fluctuations. In: Cooley, T. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, NJ, pp. 126–157.
- Kydland, F., Prescott, E., 1982. Time to build and aggregate fluctuations. *Econometrica* 50, 1345–1370.
- Maliar, L., Maliar, S., 2000. Differential responses of labor supply across productivity groups. *Journal of Macroeconomics* 22, 85–108.
- Maliar, L., Maliar, S., 2003a. The representative consumer in the neoclassical growth model with idiosyncratic shocks. *Review of Economic Dynamics* 6, 362–380.
- Maliar, L., Maliar, S., 2003. Quasi-linear preferences in the macro-economy: resolving indeterminacy by introducing heterogeneity. *Spanish Economic Review* 5, 251–267.
- Prasad, E., 1996. Skill heterogeneity and the business cycle. *Canadian Journal of Economics* 4, 910–929.
- Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3–16.