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CFD for Aerodynamic Design and Optimization: Its Evolution over the Last Three Decades

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1. Palm Springs

The AIAA First Computational Fluid Dynamics Conference, held in Palm Springs in July 1973, signified the emergence of computational fluid dynamics (CFD) as an accepted tool for airplane design. The meeting was a great success, despite the extreme heat. I have a lasting memory of the presentations of Jay Boris, who displayed the perfect advection of square waves by his flux corrected transport (FCT) algorithm,[1,2] and of Joe Thompson, who showed meshes around rocks generated by the solution of elliptic equations. As a participant in the Palm Springs meeting who has remained active in the field, I welcome the opportunity to offer some remarks on the evolution of CFD during the last three decades.

My emphasis is on the development of computational algorithms which can be used both for flow analysis and aerodynamic design. I was interested in both issues from the start of my own work in 1970. At that time we had no computational capability in fluid dynamics at all at Grumman Aerospace, where I was working, although Hess and Smith had announced their panel method several years earlier. In order to get started I wrote two computer programs for ideal two-dimensional potential flow, flo1 and syn1, both based on conformal mapping. The names were restricted to the three characters ‘flo’ and ‘syn’ because at that time fortran program names were restricted to six characters, and since I already anticipated a series of codes, I wanted to allow for three numeric digits.

Flo1 calculates the flow past a given profile by Theordorsen’s method. Syn1 solves the inverse problem of finding the profile corresponding to a specified target pressure distribution by an extension of Lighthill’s method. In developing syn1 I had the benefit of talking to Malcolm James, who had written an inverse program at McDonnell Douglas which was used by Liebeck for the design of his well known high lift airfoils. My programs were written for the IBM 1130. This was an early precursor of the class of machines which came to be called minicomputers. It was about the size of a refrigerator, and had only a few thousand words of memory. Coding was restricted to a subset of Fortran. Input was by punched cards, and output by a line printer. There was no graphics capability. The calculations took 5-10 minutes. These codes have survived, and now run on a laptop computer in about 1/50 second. Figure 1 illustrates a direct calculation by flo1 of the flow past a NACA 0012 airfoil. Figure 2 illustrates an inverse calculation by syn1 in which the Whitcomb airfoil is recovered from its subsonic pressure distribution. The conformal mapping techniques yield essentially exact results with quite a small number of mesh points, of the order of 72.

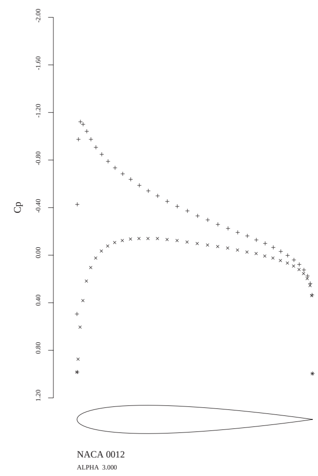

Fig. 1 Direct calculation of flow past a NACA0012 airfoil by flo 1

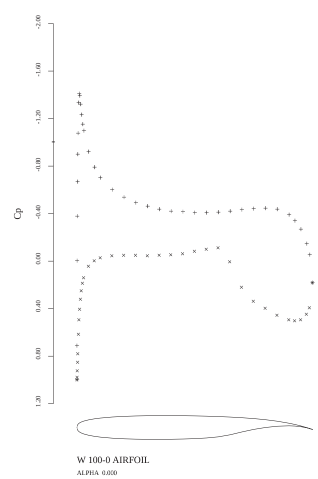

Fig. 2 Inverse calculation, recovering Whitcomb airfoil

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2. The Importance of Transonic Flow

Flo1 and syn1 were never used at Grumman. Many years later I found them very useful for the development of hydrofoils designed to delay the onset of cavitation. They were, however, a first step towards the development of methods to calculate transonic flow, which was the major challenge at that time. The compelling need both to predict transonic flow, and to gain a better understanding of its properties and character, continued to be a driving force for the development of CFD through the period 1970-1990.

In the case of military aircraft capable of supersonic flight, the high drag associated with high g maneuvers forces them to be performed in the transonic regime. In the case of commercial aircraft the importance of transonic flow stems from the Breguet range equation. This provides a good first estimate of range as

$$R = \frac{V}{sfc}\frac{L}{D}log\frac{W_0 + W_f}{W_0} \tag{1}$$

Here V is the speed, L/D is the lift to drag ratio, SFC is the specific fuel consumption of the engines, W_0 is the landing weight and W_f is the weight of the fuel burnt. The Breguet equation clearly exposes the multi-disciplinary nature of the design problem. A light weight structure is needed to minimize W_0. The specific fuel consumption is mainly the province of the engine manufacturers, and in fact the largest advances during the last thirty years have been in engine efficiency. The aerodynamic designer should try to maximize VL/D. This means that the cruising speed should be increased until the onset of drag-rise due to the formation of shock waves. Consequently the best cruising speed is the transonic regime.

3. Transonic Potential Flow

Transonic flow had proved essentially intractable to analytic methods. Garabedian and Korn had demonstrated the feasibility of designing airfoils for shock-free flow in the transonic regime by the method of complex characteristics.[3] Their method was formulated in the hodograph plane, and it required great skill to obtain solutions corresponding to physically realizable shapes. It was also known from Morawetz's theorem[4] that shock free transonic solutions are isolated points.

A major breakthrough was accomplished by Murman and Cole[5] with their development of type-dependent differencing in 1970. They obtained stable solutions by simply switching from central differencing in the subsonic zone to upwind differencing in the supersonic zone, and using a line-implicit relaxation scheme. Their discovery provided major impetus for the further development of CFD by demonstrating that solutions for steady transonic flows could be computed economically. Figure 3 taken from their landmark paper, illustrates the scaled pressure distribution on the surface of a symmetric airfoil. Efforts were now underway to extend their ideas to more general transonic flows.

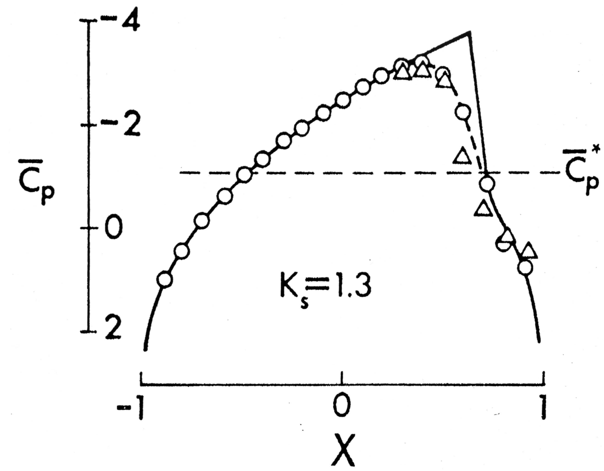

Fig. 3 Pressure distribution on the surface of a symmetric airfoil in transonic flow

In Palm Springs I presented the rotated difference scheme for the transonic potential flow equation for the first time in two papers. The first[11] was on the calculation of the flow past a yawed wing, which was then being advocated by R.T. Jones as the most efficient solution for supersonic transport aircraft. The second[12] was a joint paper with Jerry South on the calculation of axisymmetric transonic flow.

The rotated difference scheme proved to be a very robust method, and it provided the basis for flo22, developed with David Caughey during 1974-75 to predict transonic flow past swept wings. At the time we were using the CDC 6600, which had been designed by Seymour Cray, and was the world's fastest computer at its introduction, but had only 131000 words of memory. This forced the calculation to be performed one plane at a time, with multiple transfers from the disk. Flo22 was immediately put into use at McDonnell Douglas. A simplified in-core version of flo22 is still in use at Boeing Long Beach today. Figure 4, supplied by John Vassberg, shows the result of a recent calculation using flo22 of transonic flow over the wing of a proposed aircraft to fly in the Martian atmosphere. The result was obtained with 100 iterations on a 192x32x32 mesh in 7 seconds, using a typical modern workstation. John informs me that when flo22 was first introduced at Long Beach the calculations cost $3000 each. Nevertheless they found it worthwhile to use it extensively for the aerodynamic design of the C17.

By this time I had moved to the Courant Institute to work with Paul Garabedian and his group. We continued to look for more efficient and accurate methods, and to try to gain a better understanding of issues such as numerical shock structure and prediction of wave drag. This motivated the switch to equations in conservation form,[15] and also the use of multigrid tech-

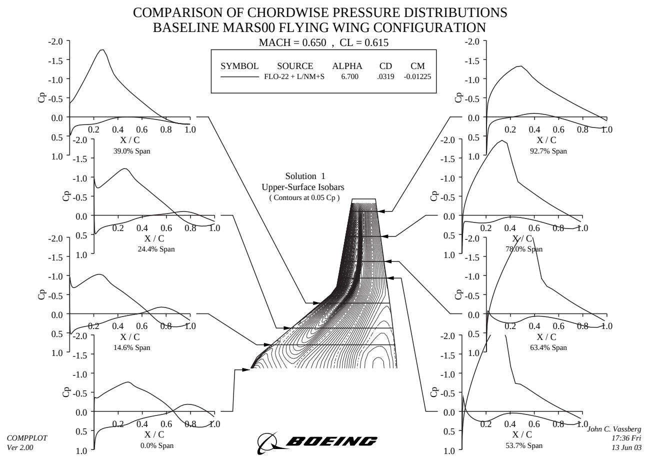

Fig. 4 Pressure distribution over the wing of a Mars Lander using FLO22

niques, which were already being advocated by Achi Brandt.[16] Many of the resulting improvements were embodied in flo36, which solves the fully conservative potential flow equations by a multigrid alternating direction method. Figure 5 shows a result for the NACA 64A410 calculated in just three multigrid cycles.

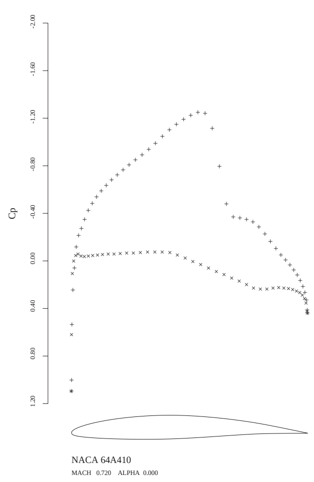

Fig. 5 Pressure distribution over NACA 64A410 in transonic flow after three multigrid cycles

David Caughey and I also developed a scheme to solve transonic potential flow on arbitrary grids.[17] The discretization formulas could be derived from the Bateman variational principle that the integral of the pressure over the domain

$$I = \int_D p d\xi$$

is stationary.[18] While we called it a finite volume scheme, it was essentially a finite element scheme using trilinear isoparametric elements, stabilized by the introduction of artificial viscosity to produce an upwind bias. The flow solvers (flo27-30) were subsequently incorporated in Boeing's A488 software, which was used in the aerodynamic design of Boeing commercial aircraft throughout the eighties.[19]

In the same period Pierre Perrier was focusing the research efforts at Dassault on the development of finite element methods using triangular and tetrahedral meshes, because he believed that if CFD software was to be really useful for aircraft design, it must be able to treat complete configurations. Although finite element methods were more computationally expensive, and mesh generation continued to present difficulties, finite element methods offered a route towards the achievement of this goal. The Dassault/INRIA group was ultimately successful, and they performed transonic potential flow calculations for complete aircraft such as the Falcon 50 in the early eighties.[20] This was a major achievement which had a significant impact on the thinking of the CFD community world wide. It placed Dassault clearly at the fore-front in the industrial application of CFD.

4. The Euler and Navier-Stokes Equations

By the eighties advances in computer hardware had made it feasible to solve the full Euler equations using software which could be cost-effective in industrial use. The idea of directly discretizing the conservation laws to produce a finite volume scheme had been introduced by MacCormack.[13] Most of the early flow solvers tended to exhibit strong pre- or post-shock oscillations. Also, in a workshop held in Stockholm in 1979,[14] it was apparent that none of the existing scheme converged to a steady state. These difficulties were resolved during the following decade.

The Jameson-Schmidt-Turkel[21] scheme, which used Runge-Kutta time stepping and a blend of second- and fourth-differences (both to control oscillations and to provide background dissipation), consistently demonstrated convergence to a steady state, with the consequence that it has remained one of the widely used methods to the present day.

A fairly complete understanding of shock capturing algorithms was achieved, stemming from the ideas of Godunov, Van Leer, Harten and Roe. The issue of oscillation control and positivity had already been addressed by Godunov in his pioneering work in the 1950s (translated into English in 1959). He had introduced the concept of representing the flow as piecewise constant in each computational cell, and solving a Riemann problem at each interface, thus obtaining a first-order accurate solution that avoids non-physical features such as expansion shocks. When this work was eventually recognized in the West, it became very influential. It was also widely recognized that numerical schemes might benefit from distinguishing the various wave speeds, and this motivated the development of characteristics-based schemes.

The earliest higher-order characteristics-based methods used flux vector splitting,[6] but suffered from oscillations near discontinuities similar to those of central difference schemes in the absence of numerical dissipation. The Monotone Upwind Scheme for Conservation Laws (MUSCL) of Van Leer[7] extended the monotonicity-preserving behavior of Godunov's scheme to higher order through the use of limiters. The use of limiters dates back to the flux-corrected transport (FCT) scheme of Boris and Book.[2] A general framework for oscillation control in the solution of non-linear problems is provided by Harten's concept of Total Variation Diminishing (TVD) schemes.

Roe's introduction of the concept of locally linearizing the equations through a mean value Jacobian[8] had a major impact. It provided valuable insight into the nature of the wave motions and also enabled the efficient implementation of Godunov-type schemes using approximate Riemann solutions. Roe's flux-difference splitting scheme has the additional benefit that it yields a single-point numerical shock structure for stationary normal shocks. Roe's and other approximate Riemann solutions, such as that due to Osher, have been incorporated in a variety of schemes of Godunov type, including Essentially Non-Oscillatory (ENO) schemes of Harten, Engquist, Osher and Chakravarthy.[9] It finally proved possible to give a rigorous justification of the JST scheme.[21]

Fast multigrid solution methods were also developed, typically using generalized Runge Kutta[24] [26] or LU[25] implicit methods with some type of preconditioning. It has recently proved possible to refine the LUSGS multigrid method to the point where steady state Euler solutions can be obtained in 3-5 cycles.[27] This allows two dimensional calculations on a 160 x 32 grid to be performed in 1/2 second on a PC with a 2GHz Pentium 4 processor, and three dimensional calculations on a 192 x 32 x 32 grid in 23 seconds. Figure 6 shows a result for the RAE 2822 airfoil.

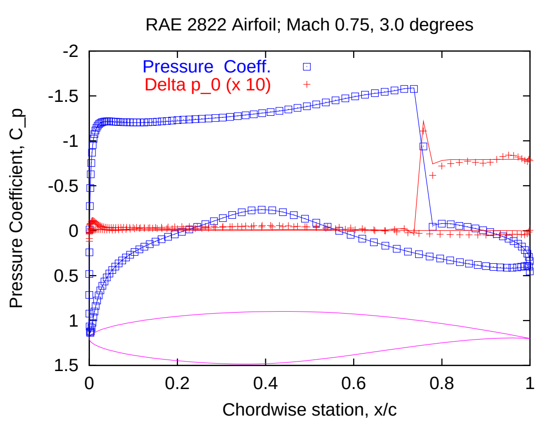

Fig. 6 Transonic flow past RAE 2822 airfoil at Mach 0.75, 3.0 degrees incidence. □ Solution with H-CUSP scheme after three multigrid cycles. Solid line (——): Fully converged solution.

In 1980 I had moved to Princeton. There, motivated

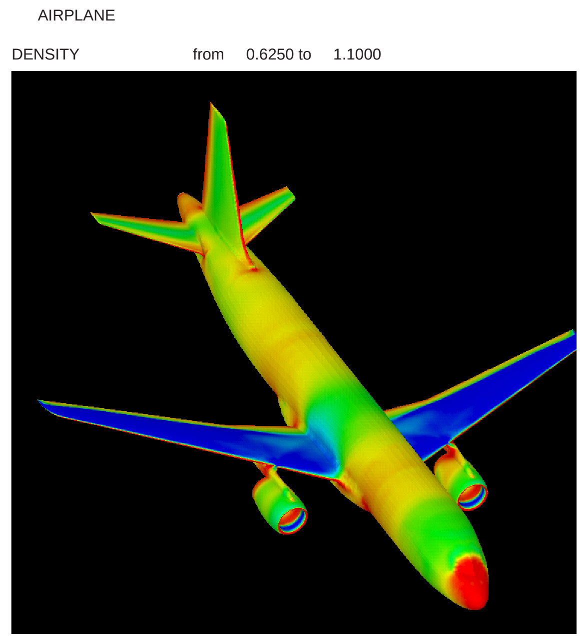

Fig. 7 Density contours for the A-320

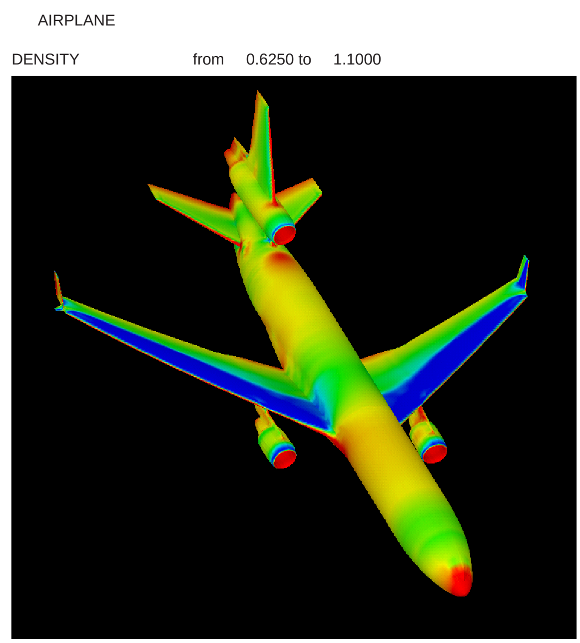

Fig. 8 Density contours for the MD-11

by the successes at Dassault, we also mounted a major effort to develop a method to solve the Euler equations on unstructured meshes, and were finally able to calculate the flow past a complete Boeing 747, including flow through the nacelles, at the end of 1985 with the "AIRPLANE" code.[28] This software was heavily used in the NASA supersonic transport program and continues to be used at the present time. Current versions use a multigrid algorithm with fully parallel operation on multiple CPUs. This enables an airplane calculation on a mesh with 2 million cells to be performed in about 30 seconds. Figures 7, 8, 9 show flow simulations of some commercial aircraft in transonic flight.

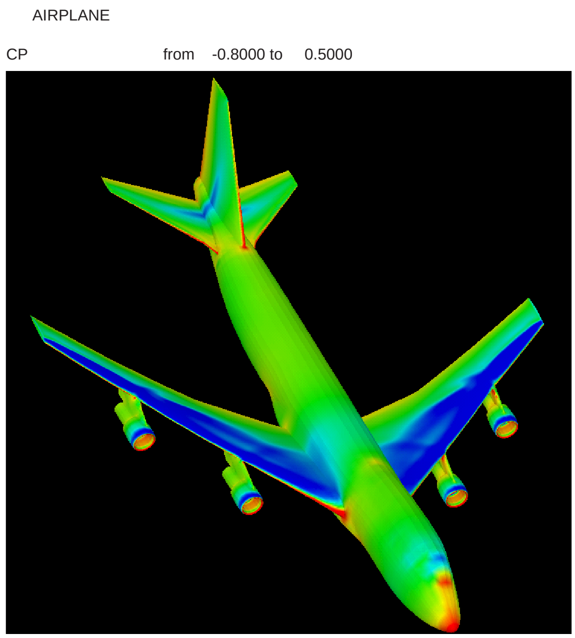

Fig. 9 Pressure contours for the Boeing 747-200

Solution methods for the Reynolds averaged Navier-Stokes (RANS) equations had been pioneered in the seventies by MacCormack and others, but at that time they were extremely expensive. By the nineties computer technology had progressed to the point where RANS simulations could be performed with manageable costs, and they began to be fairly widely used by the aircraft industry, using codes such as Buning's OVERFLOW. There were also major efforts on both sides of the Atlantic to improve the ability to predict hypersonic flow, stemming from the Hermes and NASP projects. Figures 10 and 11 shows a Hermes simulation performed with the LUSGS scheme.[25]

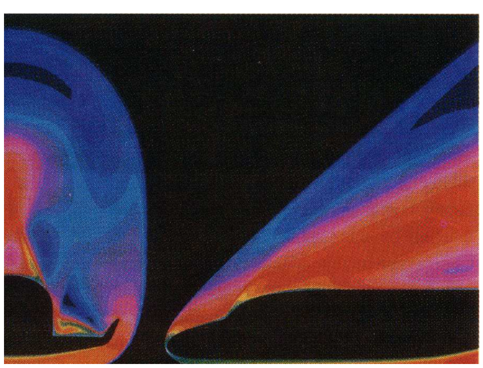

Fig. 10 CFD calculation of Hermes Spacecraft, Mach 8 and 30 degrees angle of attack, black is freestream, yellow-red the Mach number range from 3-6, and green-white the range from Mach number range from 3 to 0

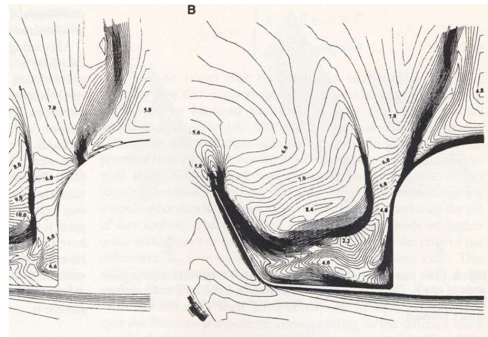

Fig. 11 CFD calculation of Hermes Spacecraft, Comparison of Mach number distribution for inviscid (A) and viscous (B) flow

5. Aerodynamic Shape Design

The effective use of CFD for design ultimately requires another level of software which can guide the designer in the search for improved aerodynamic shapes on the basis of the predicted aerodynamic performance. Hicks and Henne made a first attempt at using numerical optimization techniques in the late seventies.[29] Pironneau had also investigated the problem of optimum shape design for elliptic equations by 1984.[30]

I had revisited the issue of shape design several times since I originally wrote syn1 in 1970, and I actually wrote a program for transonic inverse design which was used by Grumman. In my first years at Princeton I supervised a thesis by John Fay[31] on inverse design using the Euler equations. In 1988 I realized that one could combine CFD with control theory to calculate optimum shapes after attending a meeting on flow control sponsored by ICASE. I was able to derive the adjoint equations for transonic potential flow and the Euler equations which allowed the extraction of the Frechet derivative (infinitely dimensional gradient) at the cost of one flow and one adjoint solution.[33]

I was certain these ideas would work and published them without attempting to demonstrate them numerically. In the following year I implemented the adjoint method for design in transonic potential flow and the first result appeared in Science.[32] This is reproduced in figure 12, which shows the redesign of the RAE 2822 airfoil to minimize its drag coefficient, subject to the constraints that the lift coefficient is held constant at approximately 1.0, and the thickness is not reduced. As can be seen, an almost shock-free profile was obtained in 5 cycles. In order to guarantee a sequence of smooth profiles, I smoothed the gradient by an implicit procedure at each step. This process, which is equivalent to redefining the gradient to correspond to an inner product in a Sobolev space[35] is a key ingredient in the success of the method.

The adjoint method has been refined over the last decade,[40 43 42 44 39 41] and extended to the Euler and Navier Stokes equations with numerous collaborators including Luigi Martinelli, James Reuther, Juan

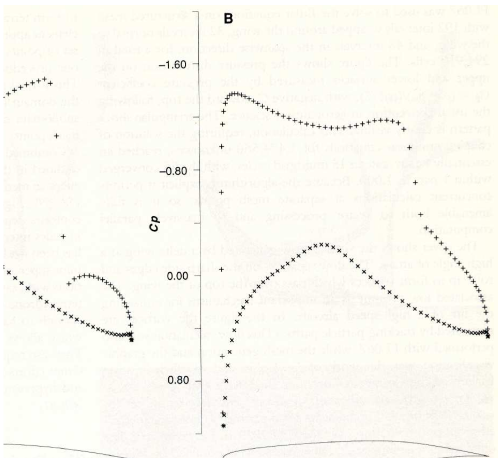

Fig. 12 Redesign of the RAE 2822 airfoil by means of control theory to reduce its shock-induced pressure drag. (A) Initial profile. Drag coefficient of 0.0175. (B) Redesigned profile after five cycles. Drag coefficient of 0.0018.

Alonso, John Vassberg, Sangho Kim, Siva Nadarajah, Kasidit Leoviriyakit and Sriram. Theoretical issues connected with the treatment of shock waves and properties of the Hessian have been addressed by Giles and Pierce,[36] Matsuzawa and Hafez[37] and Arian and Ta'asan.[38]

Control theory now provides an effective tool for wing design. Figures 13,14 show the results of Navier Stokes redesigns of the Boeing 747 wing at its present cruising Mach number of .86, and also at a higher Mach number of .90. These calculations are for the wing-fuselage combination, with shape changes restricted to the wing. In each case the planform was held fixed, while section changes was subject to the constraint of maintaining the same thickness. The lift coefficient and also the span load distribution were constrained to be fixed during the optimization, so that the root bending moment would not be increased, and the susceptibility to buffet would not be impaired due to an increase in the lift coefficient of the outboard sections.

At Mach .86 the drag coefficient is reduced from 126.9 counts (.01269) to 113.6 counts, a reduction of about 5 percent of the total drag of the aircraft. At Mach .90 it is reduced from 181.9 counts to 129.3 counts. Thus the redesigned wing has about the same drag at Mach .90 as the original wing at Mach .86, suggesting the potential for a significant increase in the cruise Mach number, provided that other problems such as engine integration could also be solved. Since both the wing thickness and span-load distribution are maintained there should be no penalty in structure weight or fuel volume. The required changes are quite subtle and there would be no hope of finding them by wind tunnel testing.

Recently, we have extended this design methodology to unstructured grids, using the same discretization scheme as the AIRPLANE code. The new software SYNPLANE has been used to redesign the Falcon business jet in the cruise condition. Figures 15, 16, 17, 18 show the density contours on the surface of the aircraft and pressure distribution at three span-wise locations on the existing wing. The results of a drag minimization that removes the shocks on the wing are shown in figures 19, 20, 21, 22. The drag has been reduced from 235 counts to 215 counts in about 8 design cycles,while the lift is held fixed at 0.4 and the thickness is maintained.

6. Reflections on the Future

Today CFD can be routinely used for the analysis of complex flows, and CFD simulation of attached flows are certainly accurate enough for performance predictions. The overall progress that has been achieved during the last 30 years was unimaginable in 1970. A major factor has been the astonishing rate of improvement of computers, so that modern laptops have a performance equivalent to the super-computers of fifteen years go. But intellectual contributions such as advances in algorithms have had a roughly equal impact.

I consider the problems of both transonic wing analysis and design to be essentially solved, although there is clearly room for improvement. In the light of the vast volume of ongoing research world-wide, we can certainly anticipate continuing advances in algorithms, particularly in the areas of higher order methods and error estimation. Higher order reconstruction methods become very complex and expensive on the general unstructured meshes which are likely to be needed to treat very complex geometries. Consequently the discontinuous Galerkin method is currently attracting a lot of interest as a way to achieve high order accuracy with a compact discretization stencil. Methods based on kinetic gas models such as the lattice Boltzmann method may also offer advantages for the treatment of some complex flows.

There are also numerous engineering applications that have yet to be adequately solved. These include three dimensional high lift systems, the flow through a helicopter rotor in forward flight, internal flows through jet engines (including compressor, combustor, turbine, and cooling flows), and the external aerodynamics of automobiles. These flows are particularly challenging because they are generally unsteady (at least in the smaller scales), and involve transition, turbulence and separation.

As computers continue to become more powerful, it is likely that there will be a shift to the wider use of Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS) methods for turbulent flows. It may be hard, however, for engineers to interpret the huge volumes of data generated by these methods in a way that will provide them with the insights needed to enable better designs. It also remains an open question whether more rational turbulence modeling procedures can be devised.

In choosing a direction of research I believe that it is generally useful to consider four main criteria. The research should be generic, not limited to a single special case. It should be intellectually challenging. It should be feasible, and it should be useful. Viewed in this light I think it is evident that shape optimization procedures based on control theory can be applied to a variety of important engineering problems (for example, reduction of the resistance of a ship hull, or radar and sonar signatures). The general aerodynamic shape optimization problem is hard, presenting a true intellectual challenge, but by now it has been clearly demonstrated that it is feasible. In fact wing redesigns using the Euler equations can be accomplished in 5 minutes on a laptop computer. If it is effectively exploited in the design process, I believe that aerodynamic shape optimization can be really useful.

The accumulated experience of the last decade suggests that most existing aircraft which cruise at transonic speeds are amenable to a drag reduction of the order of 3-5 percent, or an increase in the drag-rise Mach number of at least 0.02. The potential economic benefits are substantial, considering the fuel costs of the entire airplane fleet. Moreover, if one were to take full advantage of the improvement in the lift to drag ratio during the design process, a smaller aircraft could be designed to perform the same task, with consequent further cost reductions. It seems inevitable that some method of this type will provide a basis for aerodynamic designs in the future.

Acknowledgment

The results presented here were the outcome of collaborations with many colleagues and friends both in universities and in industry. The author's research during the last ten years on optimum aerodynamic shape design has also benefited greatly from the continuing support of the Air Force Office of Scientific research under a series of grants. This paper has been prepared with the assistance of Kasidit Leoviriyakit and Sriram.

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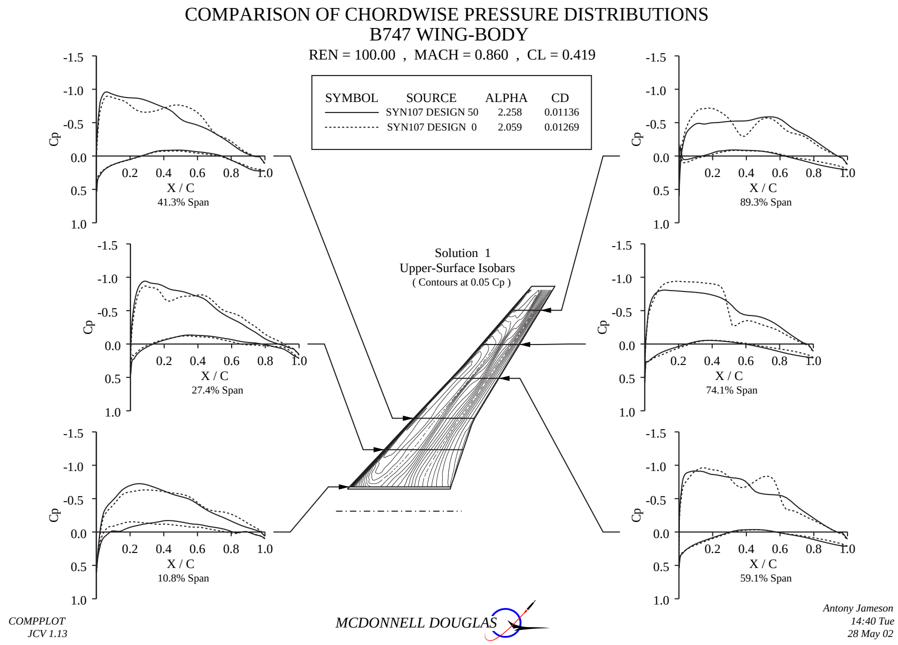

Fig. 13 Comparison of Chordwise pressure distributions before and after redesign, Re=100 million, Mach=0.86, CL=0.42

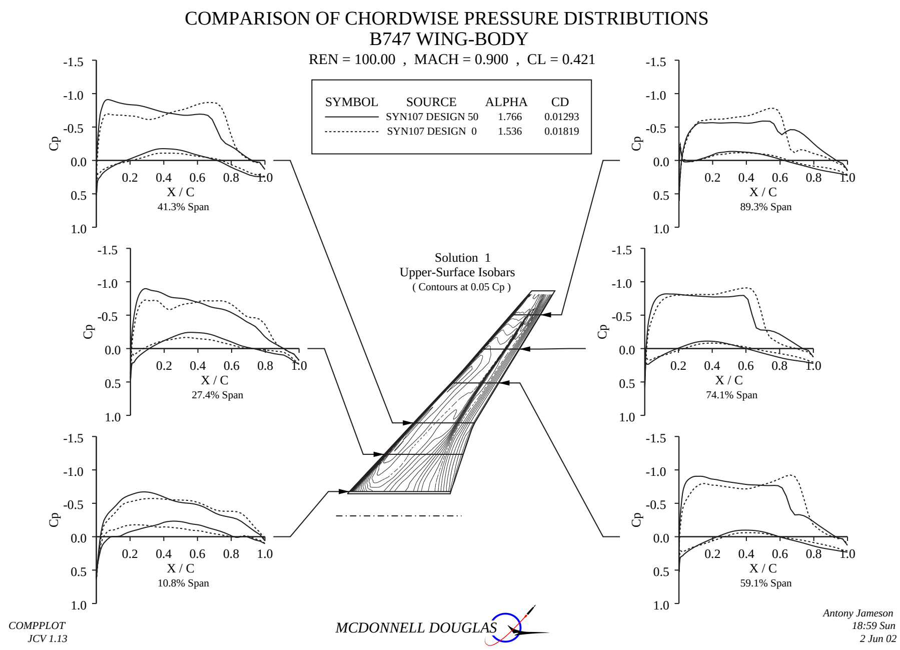

Fig. 14 Comparison of Chordwise pressure distributions before and after redesign, Re=100 million, Mach=0.90, CL=0.42

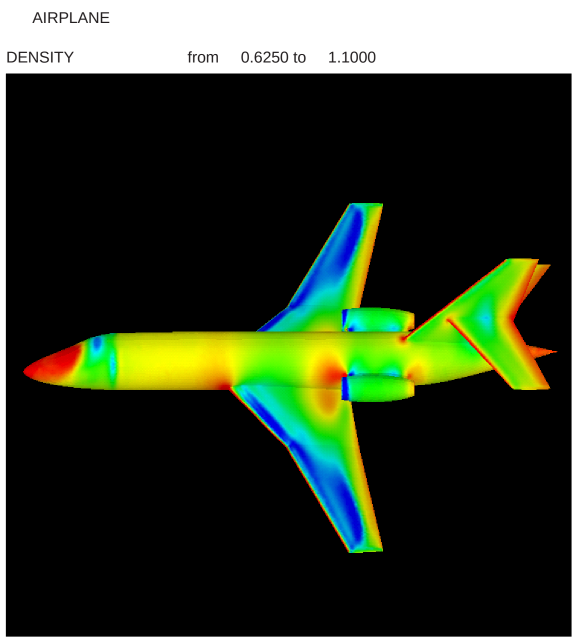

Fig. 15 Density contours for a business jet at $M = 0.8$, $\alpha = 2$

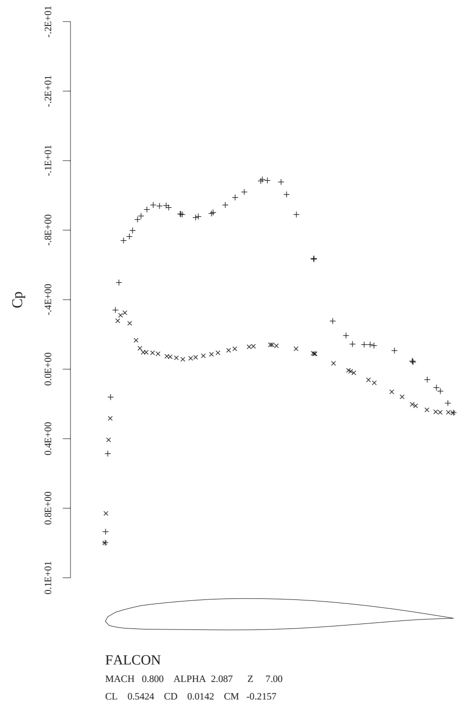

Fig. 17 Pressure distribution at 77 % wing span

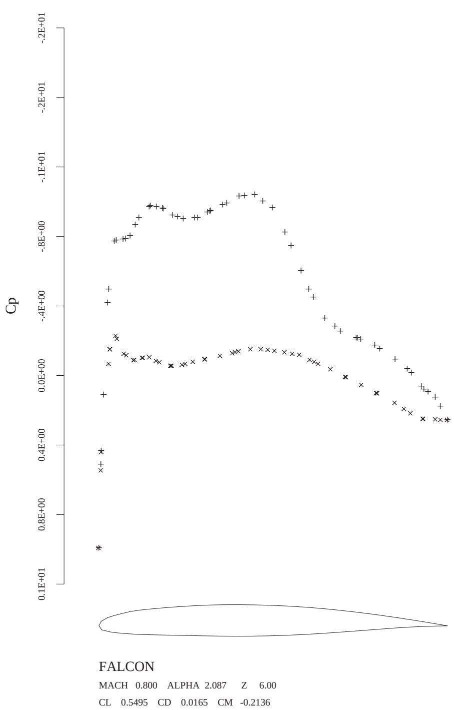

Fig. 16 Pressure distribution at 66 % wing span

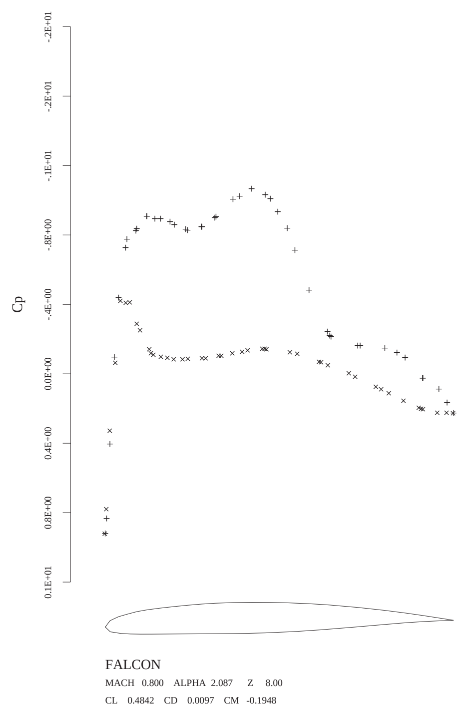

Fig. 18 Pressure distribution at 88 % wing span

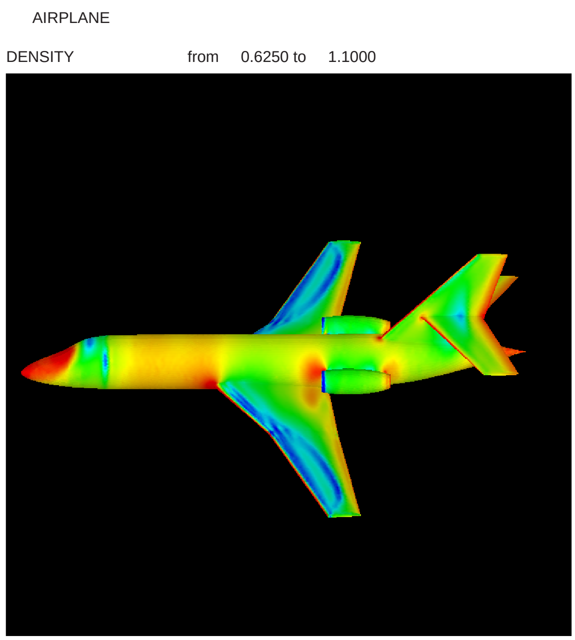

Fig. 19 Density contours for a business jet at $M = 0.8$, $\alpha = 2.3$, after redesign

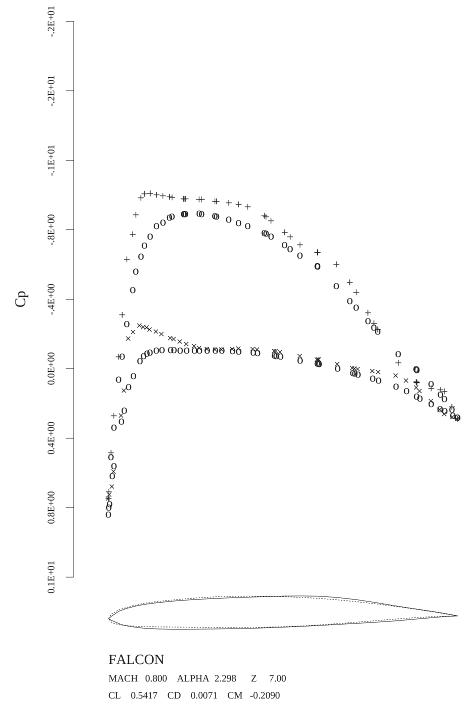

Fig. 21 Pressure distribution at 77 % wing span, after redesign, Dashed line: original geometry, solid line: redesigned geometry

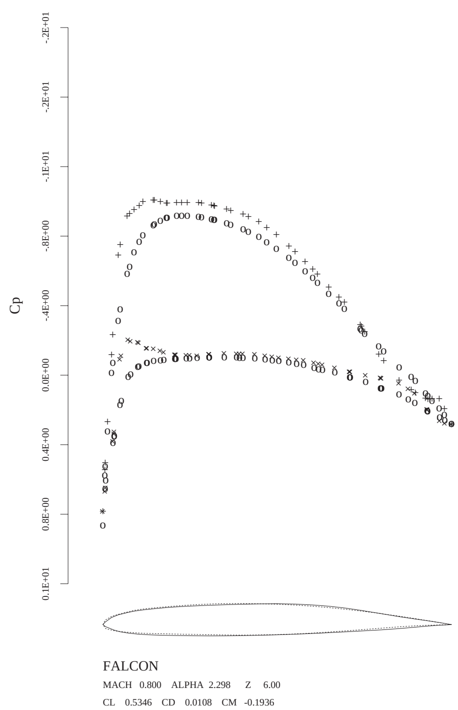

Fig. 20 Pressure distribution at 66 % wing span, after redesign, Dashed line: original geometry, solid line: redesigned geometry

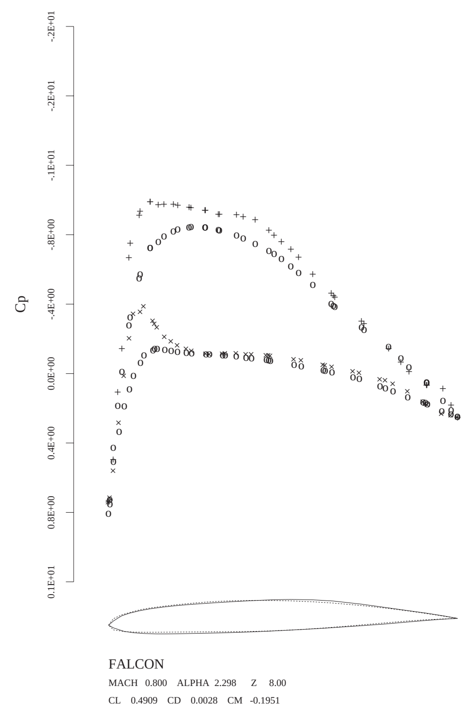

Fig. 22 Pressure distribution at 88 % wing span, after redesign, Dashed line: original geometry, solid line: redesigned geometry