Information Environments and the Impact of Competition on Information Provision

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Abstract

We study symmetric information games where a number of senders choose what information to communicate. We show that the impact of competition on information revelation is ambiguous in general. We then identify a condition on the information environment (i.e., the set of signals available to each sender) that is necessary and sufficient for equilibrium outcomes to be no less informative than the collusive outcome, regardless of preferences. The same condition also provides an easy way to characterize the equilibrium set and governs whether introducing additional senders or decreasing the alignment of senders' preferences necessarily increases the amount of information revealed.

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1 Introduction

Does competition increase the amount of information revealed? A long tradition in political and legal thought holds that the answer is yes. This view has motivated protection of freedom of speech and freedom of the press, media ownership regulation, the adversarial judicial system, and many other policies.[1]

Economic theory suggests several mechanisms by which competition can increase information revelation. Milgrom and Roberts (1986) study verifiable message games and point out that if there is some sender in each state who wishes that state to be known, then full revelation is the unique equilibrium. Shin (1998) shows that two adversarial senders who convey verifiable messages about their independent signals always generate more information than a single signal directly observed by the receiver. In a cheap talk setting, Battaglini (2002) establishes that it is generically possible to construct a full revelation equilibrium when there are two senders and uncertainty is multidimensional. These results, however, do not imply that competition need increase information in all settings, as the following example makes clear.

Example. There are two pharmaceutical companies $j = 1,2$ each of which produces a drug, and unit mass of potential consumers indexed by i. For a given consumer i, drug j may have either high or low efficacy, which we represent by $\omega_{ij} \in \{l,h\}$, with $Pr(\omega_{ij} = h) = 0.2$ distributed independently across consumers and drugs. All consumers prefer high efficacy and are otherwise indifferent between the drugs, but they differ in their outside options: half always buy whichever drug has the higher expected efficacy, while the other half buys the better drug only if its $Pr(\omega_{ij} = h)$ is greater than 0.5. The share of these two types is independent of ω_{ij}.

Each firm j simultaneously chooses one of two disclosure policies: a completely uninformative signal ($null$), or a fully informative one ($reveal_j$) that allows each consumer i to determine her ω_{ij} for that firm's drug. The firms maximize the share of consumers buying their drug. We can represent this situation as the following normal form game:

	$null$	$reveal_2$
$null$	$.25,.25$	$.40,.20$
$reveal_1$	$.20,.40$	$.34,.34$

This is a Prisoner's Dilemma. Revealing information is beneficial for the firms' joint profits as it increases

[1] See Gentzkow and Shapiro (2008) and references cited therein.

expected profits from the consumers who buy only if $Pr(\omega_{ij} = h) > 0.5$. (Those consumers never buy unless given information.) But revealing information is unilaterally unattractive since it disadvantages the revealing firm in the competition for the consumers who always buy. The uninformative signal is thus a dominant strategy and the unique equilibrium yields no information. In contrast, if the firms were to collude and maximize the sum of their payoffs, they would choose $(reveal_1, reveal_2)$.

In this example, competition between the firms decreases information revelation and lowers consumer welfare. Note that the situation would be very different if firms could disclose information not only about their own drug, but also about their competitor's drug—i.e., if each firm could choose $reveal_1$ and/or $reveal_2$. In this case, each firm would prefer to unilaterally disclose the efficacy of their competitor's drug, and so full revelation would be an equilibrium.

The main contribution of this paper is to identify a necessary and sufficient condition on the information environment (i.e., the set of signals available to each sender) under which competition cannot decrease information revelation. We consider a setting where senders with a common prior simultaneously conduct costless, publicly observed experiments about an unknown state of the world. The information revealed by these experiments can be succinctly summarized by the induced distribution of posterior beliefs (Blackwell 1953). We refer to this distribution of beliefs as the *outcome* of the game. We allow senders to have arbitrary utility functions over outcomes.

We say that an information environment is *Blackwell-connected* if for any profile of others' strategies, each sender has a signal available that allows her to unilaterally deviate to any feasible outcome that is more informative. Note that the environment in the example above is not Blackwell-connected, because starting from $(null, null)$, firm 1 cannot unilaterally deviate to induce the more informative outcome produced by $(null, reveal_2)$. The modified game where firms can disclose information about their competitors is Blackwell-connected. In general, the key implication of Blackwell-connectedness is that any individual sender must be able to generate as much information as the senders can do jointly.

Our main result shows that no pure-strategy equilibrium outcome can be less informative than the collusive outcome (regardless of preferences) if and only if the information environment is Blackwell-connected. Moreover, when the environment is Blackwell-connected, we show there is a simple way to characterize the set of equilibrium outcomes without solving a fixed point problem.

For a concrete illustration of Blackwell-connectedness, suppose we wish to know whether a merger between two pharmaceutical companies would result in consumers becoming more or less informed about the quality of the firms' drugs. If each firm can commission a range of clinical trials about the efficacy of both drugs, the environment will be Blackwell-connected, and our result implies that the merger could only reduce consumers' information. In contrast, if each firm can conduct clinical trials only about its own drug, the information environment is not Blackwell-connected, so there are some demand systems for which the merger will make consumers more informed. We develop this example in more detail below.

We also analyze two other notions of increased competition: presence of additional senders and a decrease in the alignment of senders' preferences. When considering these additional comparative statics, we restrict our attention to situations where each sender has access to the same set of signals and we focus on minimally informative, or *minimal,* equilibria. (These equilibria have some desirable properties we discuss below.) We find that if the environment is Blackwell-connected, neither introducing additional senders nor increasing preference misalignment can decrease the informativeness of minimal equilibria.

To simplify our main comparative statics, we assume the collusive outcome is unique and focus on minimal equilibria when we vary the number of senders or preference alignment. In Section 7, we drop these assumptions and extend our comparative statics using set comparisons. We also briefly discuss mixed strategy equilibria, non-Blackwell information orders, and costly signals.

Our work connects to several strands of existing literature. First, our analysis relates to the work on multi-sender communication (e.g., Milgrom and Roberts 1986; Krishna and Morgan 2001; Battaglini 2002).[2] Our model differs from this literature in three ways. First, senders' information in our model is endogenous, but we abstract from incentive compatibility issues in disclosure. Second, we allow for arbitrary preferences and characterize the full set of equilibria whereas most existing papers focus on identifying specific preferences under which full revelation is possible. Finally, we consider a richer set of comparative statics than most existing papers — comparing competition to collusion, varying the set of senders, and varying the alignment of senders' preferences.

Second, our work relates to research on advocacy. Dewatripont and Tirole (1999) consider a principal

[2]Bhattacharya and Mukherjee (2013) analyze multiple-sender persuasion games when there is uncertainty about whether each sender is informed. Under the assumption that senders' preferences are single-peaked and symmetric, they geometrically characterize the equilibrium strategies. They establish that receiver's payoff may be maximized when senders have identical, extreme preferences rather than opposed ones. Chen and Olszewski (2014) analyze a model of debate in which two senders with opposed preferences try to convince a receiver. They take senders' information as exogenous and do not consider comparative statics with respect to the extent of competition. Banerjee and Somanathan (2001) show how the relative homogeneity of preferences within a group affects the ability of individuals to communicate information to a leader.

who employs agents to gather costly information. Effort exerted to gather information is unobservable and experts' wages are contingent only on the principal's decision. The authors establish that employing two advocates with opposed interests is preferable to employing a single unbiased agent; it either generates more information or yields less rent to the employee(s). Shin (1998) develops a related result in a model with exogenous information. In his setting, two advocates get two independent draws of a signal whereas an unbiased investigator gets a single draw. Our analysis differs from these papers in showing necessary and sufficient conditions for competition to be beneficial, and analyzing a broader set of environments, including those where competition does not change the set of feasible signals.

Third, our analysis is related to the nascent literature that examines situations with *ex ante* symmetric information with multiple senders. Brocas *et al.* (2012) and Gul and Pesendorfer (2012) examine settings where two senders with exactly opposed interests provide costly signals about a binary state of the world. In contrast to these papers, we consider an arbitrary state space, arbitrary preferences, and arbitrary signal structures. Moreover, neither Brocas *et al.* (2012) nor Gul and Pesendorfer (2012) examine comparative statics with respect to the extent of competition.[3] Finally, Gentzkow and Kamenica (2012) analyze a special case of the current where every sender can conduct any experiment whatsoever, including those that are arbitrarily correlated with the outcomes of other senders' experiments. That is a far more restrictive environment than the one we consider in this paper but it yields a richer characterization of the equilibrium set.

2 Model

2.1 Setup

Let Ω be the state space, with a typical state denoted ω. There are n senders indexed by i who share a common prior μ_0. The senders play a simultaneous-move game in which each sender i chooses a signal $\pi_i \in \Pi_i$. Throughout the paper we focus on pure-strategy equilibria. We denote a strategy profile by $\pi = (\pi_1, ..., \pi_n)$. Let $\Pi \equiv \times \Pi_i$ and $\Pi_{-i} \equiv \times_{j \neq i} \Pi_j$. We refer to Π as the *information environment*.

For some results, we will focus on situations where *each sender has access to the same signals*, i.e.,

[3] Ostrovsky and Schwarz (2010) examine a model where schools choose how much information to provide about their students' abilities so as to maximize the students' job placement. They focus on a different question than we do – they examine whether the amount of information revealed depends on how students' abilities are distributed across schools.

where $\Pi_i = \Pi_j$ for any pair i and j.

A *signal* is a random variable whose distribution may depend on ω. Given a set of signals P, let $\langle P \rangle$ denote the distribution of beliefs of a Bayesian with prior μ_0 who observes the realization of all signals in P.[4] We say that $\pi = \pi'$ if $\langle \{\pi, \pi'\} \rangle = \langle \pi \rangle = \langle \pi' \rangle$. Note that in this notation, saying that $\pi = \pi'$ does not merely mean that observing π yields as much information as observing π'; rather $\pi = \pi'$ means that the two signals provide exactly the same, mutually redundant information.

A sender's payoff is a function of the aggregate information revealed jointly by all the signals. We summarize this information by the distribution of beliefs $\langle \pi \rangle$. Sender i's payoff given distribution of beliefs $\tau = \langle \pi \rangle$ is denoted by $v_i(\tau)$. In Subsection 2.2, we discuss a range of models nested by this specification.

If π is a Nash equilibrium, we say $\langle \pi \rangle$ is an *equilibrium outcome.* If π^c solves $max_{\pi \in \Pi} \sum_i v_i(\langle \pi \rangle)$, we say that $\langle \pi^c \rangle$ is a *collusive outcome*. All of our results remain true (with near-identical proofs) if we define collusive outcomes based on other aggregations of senders' preferences.[5] We say an outcome τ is *feasible* if there exists $\pi \in \Pi$ such that $\tau = \langle \pi \rangle$.

Let $\succeq$ denote the Blackwell (1953) order on distributions of beliefs. That is, $\tau \succeq \tau'$ if τ is a mean preserving spread of τ', in which case we say that τ is *more informative* than τ'. It is immediate that if $P' \subseteq P$, we have $\langle P \rangle \succeq \langle P' \rangle$. If $\tau \not\succeq \tau'$, we say that τ' is *no less informative* than τ. If $\tau \succeq \tau'$ or $\tau' \succeq \tau$, we say that τ and τ' are *comparable*.

We assume that no sender is forced to provide information, so that each Π_i includes the *null signal* $\underline{\pi}$ s.t. $\langle P \cup \underline{\pi} \rangle = \langle P \rangle$ for any P. We also make a technical assumption that the set of equilibrium outcomes is non-empty and compact. This could be guaranteed by assuming that each $v_i(\langle \pi \rangle)$ is jointly continuous in all components of π and that each Π_i satisfies weak regularity conditions.[6]

Finally, to ease exposition we assume that the collusive outcome is unique. This will be true generically.[7] When we say a result holds "regardless of preferences," we mean that it holds for any preferences consistent

[4]We will slightly abuse notation by writing $\langle \pi \rangle$ for $\langle \{\pi\} \rangle$ and $\langle \pi \rangle$ for $\langle \{\pi_i\}_{i=1}^n \rangle$.

[5]Consider any function V such that if $v_i(\tau') \geq v_i(\tau'')$ for all i and at least one inequality is strict, then $V(\tau') > V(\tau'')$. If we define a collusive outcome to be a τ that maximizes V, all of our results remain true. For example, we could assume that $V(\tau) = \sum \lambda_i v_i(\tau)$ to reflect that some senders' have more influence in determining the collusive outcome. Or, we could assume that $V(\tau) = \prod max\left(v_i(\tau) - v_i^d, 0\right)$ to reflect that senders reach the collusive outcome through Nash bargaining where sender i's outside option is v_i^d.

In Section 7 where we drop the assumption that the collusive outcome is unique, we could also simply define a collusive outcome as any Pareto-undominated outcome. That definition, however, would be problematic under the assumption that the collusive outcome is unique.

[6]Namely, that it is a non-empty compact subset of a locally compact Hausdorff topological vector space.

[7]If we perturb any set of preferences replacing $v_i(\tau)$ with $v_i(\tau) + \varepsilon$ where ε has an atomless distribution, the collusive outcome will be unique almost surely.

with the uniqueness of the collusive outcome. In Section 7 we relax the uniqueness assumption and state our results using orders on sets.

2.2 Interpretation

Our model applies to circumstances where a number of senders wish to influence the action $a \in A$ of some receiver with a utility function $u(a, \omega)$. In that case we have

$$v_i(\tau) = E_\tau \left[E_\mu \tilde{v}_i(a^*(\mu), \omega)\right] \tag{1}$$

where $a^*(\mu)$ is the action chosen by receiver given belief μ and $\tilde{v}_i(a, \omega)$ is sender i's primitive preference over receiver's action and the state.[8] The general formulation with an arbitrary v_i, however, also allows for circumstances where receiver's action depends on τ as well as μ. Suppose, for instance, that sender i is a seller and receiver is a potential buyer who must pay a cost to visit the seller's store. The seller chooses information π_i that will be revealed about ω, the buyer's value for the seller's good. If receiver observes the choice of π_i (but not its realization) before deciding whether to visit the store, then a^* depends on τ as well as μ.

Our model also applies to settings with multiple receivers who may engage in strategic interaction with each other. For instance, suppose sender i is an auctioneer and there are a number of bidders. The sender chooses information π_i that will be revealed about ω, the bidders' values for the good (e.g., as in Milgrom and Weber 1982). Our model covers both the case of common values (where ω is one-dimensional) and the case of private values (where ω is the vector of valuations). It also applies both to the case where the bidders observe the same signal realizations (in which case $a^*(\mu)$ is the equilibrium vector of actions given the commonly held posterior μ) and the case where each bidder observes an independent draw of the signal realizations (in which case the equilibrium actions may be a function of τ as well as μ).

[8] This microfoundation puts some restrictions on the induced preferences over informational outcomes: not every v_i can be generated by choosing a suitable u and $\tilde{v}_i$. This is particularly important to keep in mind when interpreting the results that a given property of the information environment is necessary for comparative statics to hold for all possible v_i. It is an open question of whether there is a weaker condition on the information environment that is necessary for comparative statics to hold for any v_i generated by some u and $\tilde{v}_i$. Also, note that continuity of u and $\tilde{v}_i$ are not sufficient to guarantee continuity of v_i when a^* is not single-valued.

2.3 Discussion

Our model makes several strong assumptions. First, we assume that the information generated by each sender is directly observed by others. This assumption distinguishes our setting from both cheap talk (e.g., Crawford and Sobel 1982) and persuasion games with verifiable information (e.g., Grossman 1981; Milgrom 1981; Milgrom and Roberts 1986; Bull and Watson 2004; Kartik and Tercieux 2012). In Kamenica and Gentzkow (2011) we discuss the variety of real-world settings where this assumption is suitable.

Second, an important feature of our model is that senders do not have any private information at the time they choose their signal. If they did, the choice of the signal could convey information even conditional on the signal realization, which would substantially complicate the analysis. Perez-Richet and Prady (2012) and Rosar (2014) consider models where privately informed senders choose publicly observable signals about their type.

Third, we assume that all signals available to a sender are equally costly. This is clearly a restrictive assumption, but the fact that we allow for arbitrary Π_i's means that our framework does allow for cases where some signals are prohibitively costly to generate. The possibility that Π_i can vary across senders means that some senders can have a comparative advantage in accessing certain kinds of information.

Finally, our model implicitly assumes that no sender can drown out the information provided by others, say by sending many useless messages. This is the basic import of the fact that $P' \subseteq P$ implies that $\langle P \rangle \succeq \langle P' \rangle$. One interpretation of this assumption is that receiver is a classical Bayesian who can costlessly process all information she receives. This means that, from receiver's point of view, the worst thing that any sender can do is to provide no information.

3 The information environment

Definition. Π is *Blackwell-connected* if for all i, $\pi \in \Pi$, $\pi_{-i} \in \Pi_{-i}$ such that $\langle \pi \rangle \succeq \langle \pi_{-i} \rangle$, there exists a $\pi_i \in \Pi_i$ such that $\langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$.

In other words, an information environment is Blackwell-connected if, given any strategy profile, each sender can unilaterally deviate to induce any feasible outcome that is more informative. We illustrate this definition with several examples.

Example 1. (*Number of draws*) Given a signal π, each sender i chooses the number of independent draws from π to generate. Signals from distinct senders are uncorrelated so if sender i chooses n_i draws, aggregate information is simply $\sum_i n_i$ independent draws from π.

Example 2. (*Precisions*) Suppose $\Omega \subset \mathbb{R}$ and let π^h be a normal signal with precision h, i.e., π^h generates a signal realization s with distribution $\mathcal{N}\left(\omega, \frac{1}{h}\right)$. Each sender chooses the precision $h_i \in \mathbb{R}_+$ of her signal. (We interpret the signal with zero precision as the uninformative signal $\underline{\pi}$.) Signals from distinct senders are uncorrelated so aggregate information is a normal signal with precision $\sum_i h_i$.

Example 3. (*Partitions*) Each sender chooses a partition of Ω. Observing the realization of a signal means learning which element of the partition the state is in (as in Aumann 1976).

Example 4. (*Facts*) There is a set F of facts about ω and revealing any one of these facts generates an $i.i.d.$ signal. Each sender i chooses a subset $F_i \subset F$ of facts to reveal. The outcome is determined by the total number of facts that are revealed, i.e., by the cardinality of $\cup F_i$.

Example 5. (*All-or-nothing*) Each sender has access to only two signals, $\underline{\pi}$ that reveals nothing and $\overline{\pi}$ that fully reveals the state of the world.

All of the information environments above are Blackwell-connected.

The most substantive implication of an environment being Blackwell-connected is that an individual sender can unilaterally provide as much information as several senders can do jointly. Say that *individual feasibility equals aggregate feasibility* if for any sender i, $\{\langle\pi\rangle | \pi \in \Pi_i\} = \{\langle\pi\rangle | \pi \in \Pi\}$.[9]

Remark 1. If the information environment is Blackwell-connected, then individual feasibility equals aggregate feasibility.

This follows from the observation that for any $\pi \in \Pi$ and any sender i, Π being Blackwell-connected means there is a $\pi_i \in \Pi_i$ s.t. $\langle \pi_i \cup \underline{\pi} \rangle = \langle \pi \rangle$ where $\underline{\pi} = (\underline{\pi}, ..., \underline{\pi}) \in \Pi_{-i}$. It implies that the information environment from the example in the introduction cannot be Blackwell-connected because each firm can generate information only about the quality of its own drug. Also, consider the *Number of draws* environment, but modify it so that each sender can generate no more than some fixed number of draws. Remark 1 implies that this modified information environment is not Blackwell-connected.

[9]Note that if each sender has access to the same set of signals, then individual feasibility necessarily equals aggregate feasibility, but the converse does not hold. In the *Number of draws* and *Precisions* environments, for example, individual feasibility equals aggregate feasibility, but it is not the case that each sender has access to the same set of signals. In contrast, *Partitions, Facts* and *All-or-nothing* are environments where each sender has access to the same set of signals.

Having individual feasibility equal aggregate feasibility is thus a necessary condition for Blackwell-connectedness, but it is not quite sufficient. An information environment can also fail to be Blackwell-connected if the sets Π_i are too "coarse." Suppose, for example, that we modify the *Number of draws* environment so that each sender must generate at least two draws (unless she sends the null signal). Even though each sender can unilaterally generate any feasible outcome, this information environment is not Blackwell-connected: the outcome with three total draws is feasible and more informative than the outcome with two total draws, but if π_{-i} induces the outcome with two total draws, there is no π_i that sender i can choose such that $\pi_{-i} \cup \pi_i$ induces the outcome with three.

4 Competition versus collusion

4.1 Main result

When does competition increase information revelation? In this section, we consider this question by comparing equilibrium outcomes to the collusive outcome. Since this comparison is meaningful only when there are at least two senders, we assume that $n \geq 2$. Recall that we maintain the assumption that the collusive outcome is unique. Our main result is the following:

Proposition 1. *Suppose $n \geq 2$. Every equilibrium outcome is no less informative than the collusive outcome (regardless of preferences) if and only if the information environment is Blackwell-connected.*

The basic intuition behind this proposition is the following. Let τ^* be some equilibrium outcome and let τ^c denote the collusive outcome. Suppose contrary to the proposition that $\tau^c \succ \tau^*$. It must be the case that for at least one sender i we have $v_i(\tau^c) > v_i(\tau^*)$; otherwise, it could not be the case that τ^c is collusive and τ^* is not. But, since the environment is Blackwell-connected this sender i could deviate from the strategy profile that induces τ^* and induce τ^c instead. Hence, τ^* could not be an equilibrium.

The proof of the converse is constructive. If the environment is not Blackwell-connected, there is some strategy profile π^c and some $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \succeq \langle \pi'_{-i} \rangle$ but player i cannot induce π^c when others are playing π'_{-i}. Consider a strategy profile π^* where i sends the null signal and others play π'_{-i}. If sender i strictly prefers π^c to π^* and other senders are indifferent, then π^c is collusive, π^* is an equilibrium, and yet $\langle \pi^c \rangle \succ \langle \pi^* \rangle$.

The environment being Blackwell-connected does not by itself ensure that an equilibrium outcome is comparable to the collusive outcome in the Blackwell order. It might be the case that the collusive outcome yields information that is more relevant for a particular decision maker than an equilibrium outcome does. However, an argument closely related to the proof of Proposition 1 can be used to show that every equilibrium outcome is more informative than the collusive outcome if and only if the information environment is Blackwell-connected and all feasible outcomes are comparable.

4.2 Illustrations

In this subsection, we illustrate the implications of Proposition 1 with a few examples.

4.2.1 Clinical trials on competitors' drug

Suppose we wish to know whether a merger between two pharmaceutical companies would result in consumers being more or less informed about the firms' drugs. Suppose further that each firm can commission a third-party to conduct a clinical trial and that each additional group of subjects provides an i.i.d signal about the quality of both drugs. This environment is Blackwell-connected (and moreover, any two outcomes are comparable.) Hence, Proposition 1 tells us that regardless of the demand structure – the extent of differentiation between the firm's products, consumers' outside options, etc.—the merger will reduce consumers' information about both firms' drugs.

In contrast, suppose that each firm has a comparative advantage in generating certain type of information. For instance, it might be the case that a firm is not allowed to conduct clinical trials about its competitor's drug. Remark 1 tells us that whenever such comparative advantage exists, the information environment is not Blackwell-connected. By Proposition 1, whether a merger leads to more or less information will depend on the demand structure. For some demand structures, such as the one in the introductory example, a merger can make consumers more informed.

4.2.2 Educating consumers about a new technology

There are two firms, G and N in a food industry. Firm G's food contains genetically modified organisms (GMOs) whereas firm N's food does not. Each firm can conduct an investigation into safety of GMOs. An investigation by firm i generates an i.i.d normal signal about safety with precision $h_i \in [0,H]$ where

$H \in [0,\infty]$. Precision of 0 is equivalent to the null signal $\underline{\pi}$. Aggregate information τ_h is determined by $h \equiv h_G + h_N$. Uncertainty about the safety of GMOs reduces the demand for firm G's product on average and somewhat increases the demand for firm N's product, but by substantially less. Consequently, $v_G(\tau_h)$ and $v_G(\tau_h) + v_N(\tau_h)$ are increasing in h while $v_N(\tau_h)$ is decreasing in h.

First consider the case where $H = \infty$, i.e., each firm can convincingly reveal the safety of GMOs on its own. In that case, the collusive outcome is full revelation (τ_∞). Moreover, the environment is Blackwell-connected so the equilibrium outcome cannot be any less informative: full revelation is also the unique equilibrium outcome. (Clearly, firm G chooses the fully revealing signal.)

In contrast, suppose that H takes on some finite value. This implies that no amount of available evidence will completely eliminate consumers' uncertainty but more importantly it also means that firm G cannot unilaterally reveal as much information as firms G and N can do together. Hence, the environment is not Blackwell-connected and we cannot be certain that equilibria will be more informative than the collusive outcome. In fact, the unique equilibrium profile is $h_G = H$ and $h_N = 0$, strictly less informative than the collusive $h_G = h_N = H$. Firm G would like to bribe firm N to reveal information but contractual incompleteness prevents it from doing so. This can happen whenever the environment is not Blackwell-connected.

4.2.3 Dislike of partial information

There are two facts about the world θ and γ which jointly determine the state of the world. There are 4 possible outcomes, determined by which facts are revealed: $\tau_{\{\theta,\gamma\}}$, $\tau_{\{\theta\}}$, $\tau_{\{\gamma\}}$, and $\tau_\emptyset$ where τ_S is the outcome when facts in S are revealed. Each firm i can reveal facts $F_i \subset \{\theta,\gamma\}$. All firms have the same preference over the informational outcome: $v_i\left(\tau_{\{\theta,\gamma\}}\right) > v_i(\tau_\emptyset) > v_i\left(\tau_{\{\theta\}}\right), v_i\left(\tau_{\{\gamma\}}\right)$. In other words, all firms would most rather have a fully informed public, but partial information is worse than no information.

First consider the Blackwell-connected environment where $F_i = \{\theta,\gamma\}$, i.e., any firm can reveal both facts. In this case, the collusive outcome and the unique equilibrium coincide at full information.

In contrast, suppose that both facts can be revealed ($\cup F_i = \{\theta,\gamma\}$), but each individual firm can reveal at most one fact ($|F_i| \leq 1$). In this case, the environment is not Blackwell-connected and there is an equilibrium outcome $\tau_\emptyset$ where nothing is revealed. Hence, competition is bad for information. This example illustrates how equilibrium miscoordination can lead to less information being revealed than would take place under collusion, but it only shows that this can only happen when the environment is not Blackwell-connected.

5 Characterizing equilibrium outcomes

Ascertaining whether a particular τ is an equilibrium outcome requires identifying whether there is a strategy profile π such that π induces τ and no sender has a profitable deviation from π. In general, the set of deviations that are feasible for sender i depends on the entire strategy profile of the other players. Hence, identifying equilibrium outcomes requires solving a fixed point problem.

When the information environment is Blackwell-connected, however, the set of feasible deviations becomes easy to identify: sender i can deviate to induce a feasible outcome τ if and only if $\tau \succeq \langle \pi_{-i} \rangle$. The "only if" part of this claim is trivial. The "if" part is equivalent to the environment being Blackwell-connected. Moreover, the "if" part imposes an important restriction on equilibrium outcomes. Say that an outcome τ is *unimprovable for sender i* if for any feasible $\tau' \succeq \tau$, we have $v_i(\tau') \leq v_i(\tau)$. If Π is Blackwell-connected, any equilibrium outcome must be unimprovable for all senders. Moreover, if each sender has access to the same set of signals and there are at least two senders, this condition is not only necessary but also sufficient for a given τ to be an equilibrium outcome:

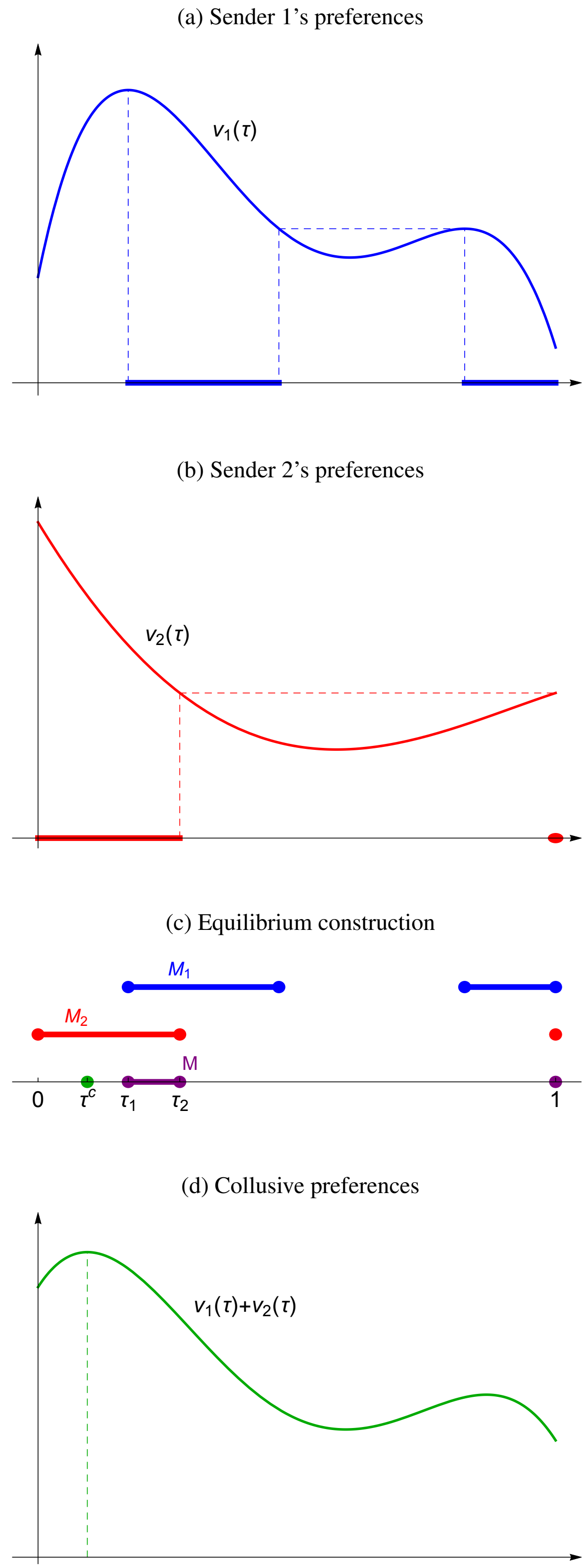
Proposition 2. *Suppose each sender has access to the same set of signals, the information environment is Blackwell-connected, and $n \geq 2$. A feasible outcome is an equilibrium outcome if and only if it is unimprovable for each sender.*

As we already mentioned, the "only if" part of this result follows directly from the environment being Blackwell-connected. The "if" part relies on the assumptions that $n \geq 2$ and that each sender has access to the same set of signals. Given an outcome τ that is unimprovable for each sender, there must be a π such that $\langle \pi \rangle = \tau$ and then $(\pi, .., \pi)$ is an equilibrium: the only deviations possible are those that yield more information but such deviations cannot be profitable.

Proposition 2 can be quite useful as it provides an easy way to determine the set of equilibrium outcomes. In particular, we can identify this set simply by taking an intersection rather than by solving a fixed point problem. We illustrate this in Figure 1.

We consider a *Facts* environment with a unit measure of facts. Each of the two senders chooses a set of facts $F_i \subset [0,1]$ to uncover and the outcome is determined by the overall share of facts that are revealed, i.e., by the measure of $F_1 \cup F_2$. Hence, we can represent each feasible τ simply as a real number in the unit interval.

Figure 1: Characterizing equilibrium outcomes



Panel (a) shows the graph of a hypothetical $v_1(\tau)$ as well as the outcomes that are unimprovable for sender 1 given those preferences (the thick lines on the x-axis). As the figure shows, this set of unimprovable outcomes is a union of two intervals. The dashed lines show how the set of unimprovable outcomes is identified. Panel (b) depicts the same information for sender 2, with another hypothetical utility function $v_2(\tau)$. In this case, the set of unimprovable outcomes is the union of a single interval and the fully revealing outcome.

Panel (c) depicts how to construct the equilibrium set. The set of unimprovable outcomes for sender 1 (M_1) is shown right above the unimprovable outcomes for sender 2 (M_2). Their intersection, $M = [\tau_1, \tau_2] \cup \{1\}$, is the set of equilibrium outcomes. In equilibrium either all facts will be revealed, or some share of facts between τ_1 and τ_2.

Finally, we can also illustrate our main comparative statics result in Figure 1. Panel (d) shows collusive preferences, $v_1(\tau) + v_2(\tau)$. The dashed line depicts its argmax, i.e., the collusive outcome τ^c. Panel (c) shows that τ^c reveals fewer facts than any equilibrium outcome, consistent with Proposition 1.

6 Other notions of increased competition

In this section we consider two other notions of increased competition. First, we analyze the impact of introducing additional senders. This counterfactual could be relevant, for example, to the impact of barriers to entry. Second, we consider increasing the misalignment of senders' preferences. This analysis could inform adversarial judicial systems and advocacy.

To simplify these comparative statics, we initially focus on a particular class of equilibria that eliminates pure miscoordination among senders. When the antecedent of Proposition 2 is satisfied and τ is an equilibrium outcome, any feasible $\tau' \succeq \tau$ is also an equilibrium outcome. In particular, if the set of feasible outcomes has a maximum element $\bar{\tau}$, this is an equilibrium outcome regardless of senders' preferences. Hence, even if all senders have the exact same preferences and dislike providing any information, revealing all information is still an equilibrium. Such "excessively informative" equilibria clearly rely on miscoordination between senders. More broadly, for any two comparable equilibrium outcomes, the less informative one must be preferred by all of the senders:

Remark 2. Suppose the information environment is Blackwell-connected. If π and π' are equilibria and

$\langle \pi \rangle \succeq \langle \pi' \rangle$, then for each sender i, $v_i(\langle \pi' \rangle) \geq v_i(\langle \pi \rangle)$.

To see this, note that Blackwell-connectedness implies sender i can unilaterally deviate from π' to π, so if $v_i(\langle \pi' \rangle)$ were strictly less than $v_i(\langle \pi \rangle)$, π' could not be an equilibrium.

Given this Pareto ranking of equilibria by senders, minimally informative equilibria are of particular interest. We say that π is a *minimal equilibrium* if π is an equilibrium and there is no equilibrium π' such that $\langle \pi \rangle \succ \langle \pi' \rangle$. If π is a minimal equilibrium, we say that $\langle \pi \rangle$ is a *minimal equilibrium outcome*. Throughout this section we restrict our attention to minimally informative equilibria. In Section 7 we consider comparative statics on the full equilibrium set.

6.1 Adding senders

In this subsection we consider the difference between outcomes when the set of senders is J and when the set of senders is some $J' \subset J$. Note that if the information environment is Blackwell-connected when the set of senders is J, it is also Blackwell-connected when the set of senders is $J' \subset J$. Hence, when we say that Π satisfies this property, we mean that it does so when the set of senders is J.

We begin the analysis by noting that having a Blackwell-connected environment is no longer sufficient for unambiguous comparative statics. For example, suppose we are in the *Number of draws* environment and let τ_m denote the outcome if a total of m independent draws are generated. Suppose sender 1's preferences satisfy $v_1(\tau_0) > v_1(\tau_2) > v_1(\tau_m)\ \forall m \notin \{0,2\}$. Sender 2's preferences satisfy $v_2(\tau_1) > v_2(\tau_2) > v_2(\tau_m)\ \forall m \notin \{1,2\}$. If senders 1 and 2 are the only senders, any equilibrium outcome must generate strictly more than 2 independent draws.[10] But, if we introduce a third sender who is indifferent across all outcomes, we can now support τ_2 as an equilibrium by having senders 1 and 2 generate no draws and sender 3 generate 2 draws. No sender can then profitably deviate. This is an example of a more general principle that introducing additional senders can *reduce* the set of possible deviations available to the existing senders. Hence, additional senders can expand the set of equilibrium outcomes and make minimal equilibria less informative.

That said, when we restrict our attention to settings where each sender has access to the same set of signals, the environment being Blackwell-connected does imply that the presence of additional senders

[10] Suppose τ_0 is an equilibrium. Then, sender 2 has a profitable deviation by increasing her number of draws by 1. Suppose τ_1 is an equilibrium. Then, sender 1 has a profitable deviation by increasing her number of draws by 1. Suppose τ_2 is an equilibrium. It must be the case that sender 2 does not generate any draws; otherwise, she has a profitable deviation by lowering her number of draws by 1. Hence, it must be that sender 1 generates both draws, but then she has a profitable deviation to τ_0.

cannot lead to less information. (Recall that access to the same set of signals means not only that $\langle\Pi_i\rangle = \langle\Pi\rangle$ as in the *Number of draws* environment, but that $\Pi_i = \Pi$.)

Proposition 3. *Suppose each sender has access to the same set of signals. If the information environment is Blackwell-connected, then (regardless of preferences) any minimal equilibrium outcome when the set of senders is J is no less informative than any minimal equilibrium outcome when the set of senders is $J' \subset J$.*

When $|J'| \geq 2$, this result is related to Proposition 2: loosely speaking, adding senders "shrinks" the set of equilibrium outcomes and thus makes minimal equilibria more informative. When $|J'| = 1$, a different argument, more closely related to the proof of Proposition 1, establishes the result.

Finally, it is easy to show that the environment being Blackwell-connected is sufficient but not necessary for the result. For an extreme example, suppose that all senders in J' can only send the null signal: $\Pi_j = \{\underline{\pi}\}$ $\forall j \in J'$. Then, additional senders cannot make the equilibrium outcome less informative regardless of whether the informational environment is Blackwell-connected or not.

6.2 Increasing misalignment of senders' preferences

Given that senders can have any arbitrary state-dependent utility functions, the extent of preference alignment among senders is not easy to parametrize in general. Hence, we consider a specific form of preference alignment. Suppose there are two functions f and g, and two senders j and k, with preferences of the form

$$\begin{aligned} v_j(\tau) &= f(\tau) + bg(\tau) \\ v_k(\tau) &= f(\tau) - bg(\tau) \end{aligned}$$

while preferences of other senders are independent of b. The parameter $b \geq 0$ thus captures the extent of preference misalignment between two of the senders.

Proposition 4. *Suppose each sender has access to the same set of signals. If the information environment is Blackwell-connected, then any minimal equilibrium outcome when the level of misalignment is b is no less informative than any minimal equilibrium outcome when the level of misalignment is some $b' < b$.*

In other words, increasing the level of preference misalignment increases the informativeness of (minimal) equilibria. A detailed proof is in the Appendix. The basic intuition behind Proposition 4 is the following. When preferences are less aligned, there are fewer outcomes such that none of the senders wishes to deviate from the outcome. Hence, the set of equilibrium outcomes shrinks and minimal equilibria become more informative. As in the previous subsection, this comparative static only applies to settings where each sender has access to the same set of signals. Otherwise, it is possible for increased misalignment to increase the set of equilibrium outcomes and make minimal equilibria less informative.

7 Extensions

7.1 Comparisons of sets of outcomes

So far in the paper we made simplifying assumptions that allowed us to avoid the issue of comparative statics on sets. Specifically, we assumed that the collusive outcome is unique and in Section 6 we focused on minimal equilibria. In this subsection we will drop these assumptions and analyze comparative statics on sets of outcomes.

Note that it would be too much to expect for *any* equilibrium outcome to be more informative than *any* collusive outcome simply because the two sets may overlap. For example, if all senders are indifferent across all outcomes, every outcome is both an equilibrium outcome and a collusive outcome; hence, it is not the case that every equilibrium is more informative than every collusive outcome.

Topkis (1998) defines two orders on subsets of a lattice. Given two subsets Y and Y' of a lattice $(\mathscr{Y}, \geq)$, Y is strongly above Y' if for any $y \in Y$ and $y' \in Y'$ we have $y \vee y' \in Y$ and $y \wedge y' \in Y'$. Set Y is weakly above Y' if for any $y \in Y$ and $y' \in Y'$, $\exists \hat{y} \in Y : \hat{y} \geq y'$ and $\exists \hat{y}' \in Y' : y \geq \hat{y}'$.

We cannot apply Topkis' definitions directly, however, because the set of distributions of posteriors is not always a lattice under the Blackwell order if there are more than two states (Müller and Scarsini 2006). Accordingly, throughout this section we assume that any two feasible outcomes are comparable.[11] The strong order then reduces to the following: we say that T *is strongly more informative than* T' if for any $\tau \in T$ and $\tau' \in T'$ such that $\tau' \succeq \tau$ we have $\tau \in T'$ and $\tau' \in T$. We say T *is weakly more informative than* T' if T is weakly above T' under $\succeq$.

We might hope that Proposition 1 generalizes to the claim that the set of equilibrium outcomes is strongly (or weakly) more informative than the set of collusive outcomes if and only if the information environment is Blackwell-connected. This, however, turns out not to be true in general. Accordingly, we restrict our

[11] An alternative would be to weaken the notion of the strong order and require to hold only along chains of the partially ordered set. See discussion in Gentzkow and Kamenica (2012).

attention to two natural classes of information environments for which the analogue of Proposition 1 does hold.

Say that *signals are independent* if given any sender i and any $\pi_a, \pi_b \in \Pi_i$ we have that for all $\pi_{-i} \in \Pi_{-i}$, $\langle \pi_a \rangle \succeq \langle \pi_b \rangle \Leftrightarrow \langle \pi_a \cup \pi_{-i} \rangle \succeq \langle \pi_b \cup \pi_{-i} \rangle$. In other words, when signals are independent, whether one signal is more informative than another does not depend on what other information is being provided. This will be true, in particular, if the signals are statistically independent.

At the other extreme is a situation where each sender has access to the same set of signals so one of the senders can always provide information that makes the other's information completely redundant. Note that the *Number of draws* and *Precisions* environment are independent while *Facts* and *All-or-nothing* are environments where each sender has access to the same set of signals. Restricting our attention to these types of environments, we establish the key result of this subsection:

Proposition 5. *Suppose any two feasible outcomes are comparable. Suppose that each sender has access to the same set of signals or that signals are independent. The set of equilibrium outcomes is strongly more informative than the set of collusive outcomes (regardless of preferences) if and only if the information environment is Blackwell-connected.*

Consider some collusive outcome τ^c and some equilibrium outcome $\tau^* = \langle \pi^* \rangle$ with $\tau^c \succeq \tau^*$. To establish Proposition 5, we need to show that, if the environment is Blackwell-connected, it must be the case that τ^* is also a collusive outcome and τ^c is also an equilibrium outcome. The argument for the first part is closely analogous to the proof of Proposition 1. Since $\tau^c \succeq \tau^*$, any sender can deviate from τ^* to τ^c, so it must be the case that $v_i(\tau^*) \geq v_i(\tau^c)$ for each sender i. Since τ^c is a collusive outcome, this implies that τ^* must also be a collusive outcome. The second part of the argument, which establishes that τ^c must be an equilibrium, is more involved, and relies on the assumption that each sender has access to the same set of signals or that signals are independent. As the proof in the Appendix shows, in these two classes of environments there must be some strategy profile π^c such that $\tau^c = \langle \pi^c \rangle$ and any outcome that sender i can deviate to from π^c, she can also deviate to from π^*. Therefore, since π^* is an equilibrium, π^c must be one as well.

We also derive full-equilibrium-set analogues of our comparative statics results on adding senders and increasing preference misalignment, though they only hold in the weak set order, when each sender has access to the same set of signals, and when the set of feasible outcomes has a maximum element, i.e., there exists a feasible $\overline{\tau}$ s.t. for all $\pi \in \Pi$ we have $\overline{\tau} \succeq \langle \pi \rangle$.

Proposition 6. *Suppose any two feasible outcomes are comparable. Suppose each sender has access to the same set of signals and the set of feasible outcomes has a maximum element. If the information environment is Blackwell-connected, then*

(1) The set of equilibrium outcomes when the set of senders is some J is weakly more informative than the set of equilibrium outcomes when the set of senders is some $J' \subset J$.

(2) The set of equilibrium outcomes when the level of misalignment is b is weakly more informative than the set of equilibrium outcomes when the level of misalignment is $b' < b$.

The basic idea behind Proposition 6 is the following. Since each sender has access to the same set of signals and the environment is Blackwell-connected, the set of equilibrium outcomes is the intersection of unimprovable outcomes for each sender. Hence, the set of equilibrium outcomes shrinks when we add senders or increase misalignment of their preferences. But, Proposition 2 also implies that the maximum element always remains an equilibrium outcome. Hence, loosely speaking, adding senders shrinks the equilibrium set "toward" the most informative equilibrium. The argument is somewhat different when $|J'| = 1$.

7.2 Preferences over own signals

We have assumed that each sender only cares about the overall amount of information revealed rather than about her own chosen signal.[12] This precludes both situations where generating informative signals is privately costly (in which case a sender prefers her signal to be less informative given the outcome) and situations where a sender obtains "good will" from being the source of information (in which case a sender prefers her signal to be more informative given the outcome).

It is easy to see why in the former case we cannot expect competition to generally increase information revelation. The provision of costly information creates a classic public goods problem. Suppose for instance that all senders have the same preferences $v_i(\tau) \equiv v(\tau)$ and prefer more information to less: $\tau \succ \tau' \implies v(\tau) > v(\tau')$. Then, if generating information is privately costly, non-cooperative strategic behavior and the presence of additional players would both reduce provision of information relative to social optimum.

[12]This connects our model to the literature on aggregate games (e.g., Martimort and Stole 2012), though the preferences we consider are even starker than in that literature as we assume each sender cares only about the aggregate and not about her own contribution to it.

7.3 Mixed strategies

Throughout the paper we focus on pure strategy equilibria. This focus has substantive consequences for our results. If senders choose mixed strategies from $\Delta(\Pi_i)$, the information environment may cease to be Blackwell-connected. For example, in most of the information environments we discuss in Section 3, namely *Number of draws, Precisions, Partitions,* and *Facts* we can construct a feasible outcome τ and a mixed strategy profile $\tilde{\pi}_{-i}$ such that $\tau \succeq \langle \tilde{\pi}_{-i} \rangle$ but there exists no $\pi_i \in \Pi_i$ such that $\tau = \langle \tilde{\pi}_{-i} \cup \pi_i \rangle$. Without knowing which signal others will generate, sender i cannot "add" a suitable amount of information to induce a particular outcome. Consequently, by Proposition 1 we know that in all of these cases we can construct preferences such that the collusive outcome is strictly more informative than some mixed strategy equilibrium outcome.

7.4 Non-Blackwell orders

While the Blackwell order is a natural way to present our comparative statics, none of our results rely on use of this particular order. Consider any partial order $\geq$ on the set of outcomes. Say that the information environment is $\geq$*-connected* if for all i, $\pi \in \Pi$, $\pi_{-i} \in \Pi_{-i}$ such that $\langle \pi \rangle \geq \langle \pi_{-i} \rangle$, there exists a $\pi_i \in \Pi_i$ such that $\langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$. By the exact same arguments as before, we can conclude that every equilibrium outcome is no less informative (under the $\geq$ order) than the unique collusive outcome regardless of preferences if and only if the environment is $\geq$*-connected.* Similar analogues apply to the characterization result and the other comparative statics.

8 Conclusion

A large body of policy and legal precedent has been built on the view that competition in the "marketplace of ideas" will ultimately lead more truth to be revealed. Existing models of strategic communication with multiple senders have focused on settings such as cheap talk or disclosure where the key issue is the credibility with which senders can communicate what they know, and shown some conditions under which this intuition is valid. The strategic complexity of these settings, however, means that they stop short of full characterizations and consider a limited range of comparative statics (Sobel 2013).

We depart from the literature in setting aside incentive compatibility in communication and focusing instead on senders' incentives to gather information, assuming that they can commit to communicate it truthfully. In this setting, we show that the impact of competition is ambiguous in general, and that Blackwell-connectedness is the key condition separating cases where competition is guaranteed to be beneficial from those where it is not.

9 Appendix

9.1 Proof of Proposition 1

Proof. Suppose that Π is Blackwell-connected. Suppose τ^* is an equilibrium outcome and τ^c is the collusive outcome. Let π^* be the strategy profile that induces τ^* and π^c the strategy profile that induces τ^c. Suppose contrary to the claim that $\tau^c \succ \tau^*$. Since τ^c is the unique collusive outcome, there is some sender i s.t. $v_i(\tau^c) > v_i(\tau^*)$. Consider $\pi_{-i} = \left(\pi_1^*, ..., \pi_{i-1}^*, \pi_{i+1}^*, ..., \pi_n^*\right) \in \Pi_{-i}$. We have $\langle \pi^c \rangle \succeq \langle \pi^* \rangle \succeq \langle \pi_{-i} \rangle$, so the fact that Π is Blackwell-connected implies that there exists a signal $\pi' \in \Pi$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \langle \left(\pi_1^*, ..., \pi_{i-1}^*, \pi', \pi_{i+1}^*, ..., \pi_n^*\right) \rangle = \tau^c$. Hence, sender i has a profitable deviation which contradicts the claim that π^* is an equilibrium.

Suppose that Π is not Blackwell-connected. This means that there exist some i, $\pi^c \in \Pi$ and $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \succeq \langle \pi'_{-i} \rangle$ but for all $\pi \in \Pi_i$ we have $\langle \pi^c \rangle \neq \langle \pi'_{-i} \cup \pi \rangle$. Moreover, since $\underline{\pi} \in \Pi_i$, we have $\langle \pi^c \rangle \succ \langle \pi'_{-i} \rangle$. Let $\pi^* = \left(\pi'_1, ..., \pi'_{i-1}, \underline{\pi}, \pi'_{i+1}, \pi'_n\right)$. For all $j \neq i$, let v_j be constant. Let $v_i(\langle \pi^c \rangle) = 1$, $v_i(\langle \pi^* \rangle) = 0$, and $v_i(\tau) = -1$ for $\tau \notin \{\langle \pi^c \rangle, \langle \pi^* \rangle\}$. Then we have that $\langle \pi^c \rangle$ is the unique collusive outcome, $\langle \pi^* \rangle$ is an equilibrium outcome and yet $\langle \pi^c \rangle \succ \langle \pi^* \rangle$. □

9.2 Proof of Proposition 2

Proof. Suppose each sender has access to the same set of signals, Π is Blackwell-connected, and $n \geq 2$. Let $\mathscr{P}$ denote the set of signals available to each sender.

We first show that every equilibrium outcome is unimprovable for every sender. Consider a feasible outcome τ that is improvable for some sender i. Let π^* be a strategy profile that induces τ. Since τ is improvable for sender i, there is a π s.t. $\langle \pi \rangle \succeq \tau$ and $v_i(\langle \pi \rangle) > v_i(\tau)$. Consider $\pi_{-i}^* = \left(\pi_1^*, ..., \pi_{i-1}^*, \pi_{i+1}^*, ..., \pi_n^*\right) \in \Pi_{-i}$. Since $\langle \pi \rangle \succeq \tau = \langle \pi^* \rangle \succeq \langle \pi_{-i}^* \rangle$ and Π is Blackwell-connected, there exists a signal $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i}^* \cup \pi' \rangle = \langle \pi \rangle$. Hence, π' is a profitable deviation for sender i from π^*, so π^* is not an equilibrium.

Conversely, suppose that some feasible outcome τ is unimprovable for each sender. Let π' be a strategy profile that induces τ. Consider $\underline{\pi} = (\underline{\pi}, ..., \underline{\pi}) \in \mathscr{P}^{n-1}$. Since $\langle \pi' \rangle \succeq \langle \underline{\pi} \rangle$ and Π is Blackwell-connected, there exists some $\pi^* \in \mathscr{P}$ s.t. $\langle \pi' \rangle = \langle \pi^* \rangle$. Consider a strategy profile $\pi^* = (\pi^*, ..., \pi^*)$. Since $n \geq 2$, no sender can deviate except to a more informative outcome. Since τ is unimprovable for each sender, no such deviation is profitable. □

9.3 Proof of Proposition 3

Proof. First consider the case where $|J'| = 1$. Let i be the sender in J' and let τ^i be a minimally informative outcome that is unilaterally optimal for sender i. Let τ^* be a minimal equilibrium outcome when the set of senders is J. Suppose that $\tau^i \succ \tau^*$. Since τ^i is a minimally informative outcome that is unilaterally optimal for sender i, τ^* must not be unilaterally optimal for sender i. (By Remark 1, we know $\tau^* \in \{\langle\pi\rangle | \pi \in \Pi_i\}$ because Π is Blackwell-connected). In other words, $v_i\left(\tau^i\right) > v_i\left(\tau^*\right)$. But, since Π is Blackwell-connected, given any strategy profile π^* that induces τ^*, there exists a signal $\pi' \in \Pi_i$ that allows sender i to deviate from π^* to induce τ^i. Hence, π^* cannot be an equilibrium.

Now consider the case where $|J'| > 1$. Let τ' be a minimal equilibrium outcome when the set of senders is J' and let τ^* be a minimal equilibrium outcome when the set of senders is J. Suppose that $\tau' \succ \tau^*$. But, by Proposition 2, the set of equilibrium outcomes when the set of senders is J is a subset of the set of equilibrium outcomes when the set of senders is J'. Hence, τ^* is an equilibrium outcome when the set of senders is J'. Therefore, $\tau' \succ \tau^*$ contradicts the fact that τ' is a minimal equilibrium outcome. □

9.4 Proof of Proposition 4

We begin the proof with the following Lemma (which will also be useful for the proof of Proposition 6).

Lemma 1. *Suppose each sender has access to the same set of signals. Let T^m be the set of equilibrium outcomes when the level of misalignment is $m \in \{b, b'\}$. If $b > b'$ then $T^b \subset T^{b'}$.*

Proof. Let T_i^m be the set of feasible outcomes that are unimprovable for sender i when the level of misalignment is $m \in \{b, b'\}$. By Proposition 2, $T^m = \cap_i T_i^m$. For $i \notin \{j, k\}$ we have $T_i^b = T_i^{b'}$. Hence, it will suffice to show that $T_j^b \cap T_k^b \subset T_j^{b'} \cap T_k^{b'}$. Consider any $\tau \in T_j^b \cap T_k^b$. Let T' be the set of feasible outcomes that are more informative than τ. Since $\tau \in T_j^b$ we have that $f(\tau) + bg(\tau) \geq f(\tau') + bg(\tau')$ for all $\tau' \in T'$. Since $\tau \in T_k^b$ we have that $f(\tau) - bg(\tau) \geq f(\tau') - bg(\tau')$ for all $\tau' \in T'$. Combining these two inequalities we get $f(\tau) - f(\tau') \geq b\,|g(\tau) - g(\tau')|\ \forall \tau' \in T'$ which in turn implies that $f(\tau) - f(\tau') \geq b'\,|g(\tau) - g(\tau')|$ $\forall \tau' \in T'$. This last inequality implies $f(\tau) + b'g(\tau) \geq f(\tau') + b'g(\tau')$ and $f(\tau) - b'g(\tau) \geq f(\tau') - b'g(\tau')$ $\forall \tau' \in T'$, so we have $\tau \in T_j^{b'} \cap T_k^{b'}$. □

With this Lemma, the proof of Proposition 4 follows easily:

Proof. Let τ' be a minimal equilibrium outcome when the level of misalignment is b' and let τ be a minimal equilibrium outcome when the level of misalignment is b. Suppose $\tau' \succ \tau$. By Lemma 1, the set of equilibrium outcomes when the level of misalignment is b is a subset of the set of equilibrium outcomes when the level of misalignment is b'. Hence, τ is an equilibrium outcome when the level of misalignment is b'. Therefore, $\tau' \succ \tau^*$ contradicts the fact that τ' is a minimal equilibrium outcome when the level of misalignment is b'. $\square$

9.5 Proof of Proposition 5

We begin by establishing a key property of settings where each sender has access to the same set of signals or signals are independent. Say that τ' is an *i-feasible deviation from* π if there exists a $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \tau'$. Say that Π is *simple* if given any $\pi \in \Pi$ and any feasible $\tau' \succeq \langle \pi \rangle$ there exists a π' s.t. $\langle \pi' \rangle = \tau'$ and for any sender i, the set of i-feasible deviations from π' is a subset of i-feasible deviations from π. Say that Π is *incrementable* if given any $\pi \in \Pi$ and any feasible $\tau' \succeq \langle \pi \rangle$ there exists a $\pi' \in \Pi$ s.t. $\langle \pi' \rangle = \tau'$ and $\pi'_i \succeq \pi_i$ for all i.

Lemma 2. *If* Π *is Blackwell-connected and each sender has access to the same set of signals, then* Π *is simple.*

Proof. If $n = 1$, every environment is simple so suppose $n \geq 2$. Suppose Π is Blackwell-connected and each sender has access to the same set of signals. Let $\mathscr{P}$ denote the set of signals available to each sender. Consider some $\pi \in \Pi$ and some feasible $\tau' \succeq \langle \pi \rangle$. Since Π is Blackwell-connected, individual feasibility equals aggregate feasibility (by Remark 1) which implies there exists a $\pi' \in \mathscr{P}$ such that $\langle \pi' \rangle = \tau'$. Let $\pi' = (\pi', ..., \pi')$. Consider some τ'', an i-feasible deviation from π'. We must have $\tau'' \succeq \tau' \succeq \langle \pi \rangle$. Since Π is Blackwell-connected, τ'' must be i-feasible from π. $\square$

Lemma 3. *If* Π *is Blackwell-connected and signals are independent, then* Π *is simple.*

Proof. Suppose the environment is Blackwell-connected and independent. We first show that Π is incrementable. Consider some $\pi \in \Pi$ and a feasible $\tau' \succeq \langle \pi \rangle$. Pick any sender i and consider π_{-i}. We have that $\tau' \succeq \langle \pi \rangle \succeq \langle \pi_{-i} \rangle$. Since the environment is Blackwell-connected, there is a $\pi' \in \Pi_i$ s.t. $\langle \pi_{-i} \cup \pi' \rangle = \tau'$. Let $\pi' = (\pi_{-i}, \pi')$. By construction $\pi'_j = \pi_j$ for $j \neq i$ so $\pi'_j \succeq \pi_j$ for $j \neq i$. By independence, $\langle \pi_{-i} \cup \pi' \rangle = \tau' \succeq \langle \pi \rangle = \langle \pi_{-i} \cup \pi_i \rangle$ implies that $\pi' = \pi'_i \succeq \pi_i$.

Next we show that Π is simple. Given a $\pi \in \Pi$ and a feasible $\tau' \succeq \langle \pi \rangle$ consider a $\pi' \in \Pi$ s.t. $\langle \pi' \rangle = \tau'$ and $\pi'_j \succeq \pi_j$ for all j. (Such a profile exists since the environment is incrementable.) Suppose that some outcome τ^d is an i-feasible deviation from π'. That means that $\tau^d \succeq \pi'_{-i} \succeq \pi_{-i}$. The last inequality follows from the fact that $\pi'_j \succeq \pi_j$ for all $j \neq i$ and the fact that the environment is independent. Since $\tau^d \succeq \pi_{-i}$ and the environment is Blackwell-connected, we know that τ^d is an i-feasible deviation from π. □

We are now ready to turn to the proof of Proposition 5.

Proof. Suppose that each sender has access to the same set of signals or that signals are independent. Suppose the information environment is Blackwell-connected and any two feasible outcomes are comparable. By Lemmata 2 and 3, we know the environment is simple. Suppose there is some collusive outcome τ^c and some equilibrium outcome τ^*such that $\tau^c \succeq \tau^*$. We need to show that τ^* is a collusive outcome and τ^c is an equilibrium outcome. Let π^* be the strategy profile that induces τ^*. Since $\tau^c \succeq \tau^*$ and Π is Blackwell-connected, any sender can deviate from π^* to induce τ^c, so it must be the case that $v_i(\tau^*) \geq v_i(\tau^c)$ for each sender i. Since τ^c is a collusive outcome, this implies that τ^* is also a collusive outcome. Moreover, since the environment is simple, there exists a strategy profile π^c such that $\tau^c = \langle \pi^c \rangle$ and any outcome that is i-feasible from π^c is also i-feasible from π^*, for every sender i. But then the fact that π^* is an equilibrium implies that π^c is also an equilibrium. That completes the "if" part of the proof.

We now turn to the "only-if" part. Suppose Π is not Blackwell-connected. There exist some i, $\pi^c \in \Pi$ and $\pi'_{-i} \in \Pi_{-i}$ such that $\langle \pi^c \rangle \succeq \langle \pi'_{-i} \rangle$ and for all $\pi \in \Pi_i$ we have $\langle \pi^c \rangle \neq \langle \pi'_{-i} \cup \pi \rangle$. Let $\pi^* = (\pi'_1, ..., \pi_{i-1}, \underline{\pi}, \pi_{i+1}, \pi'_n)$. For all $j \neq i$, let v_j be constant. Let $v_i(\langle \pi^c \rangle) = 1$, $v_i(\langle \pi^* \rangle) = 0$, and $v_i(\langle \pi \rangle) = -1$ for $\pi \notin \{\pi^c, \pi^*\}$. Then we have that $\langle \pi^c \rangle$ is a collusive outcome, $\langle \pi^* \rangle$ is a non-collusive equilibrium outcome and $\langle \pi^c \rangle \succeq \langle \pi^* \rangle$. □

9.6 Proof of Proposition 6

Proof. We first prove part (1). Let τ^* be an equilibrium outcome when the set of senders is some set J and τ' an equilibrium outcome when the set of senders is some $J' \subset J$. We need to show that: (i) there is an equilibrium outcome when the set of senders is J that is more informative than τ' and (ii) there is an equilibrium outcome when the set of senders is J' that is less informative than τ^*. Consider the case where $|J'| = 1$. If $J = J'$, the proposition is trivially true, so suppose that $|J| > 1$. By Proposition 2, we know

$\overline{\tau}$ is an equilibrium outcome when the set of senders is J and by definition $\overline{\tau} \succeq \tau'$. That establishes claim (i). Now, suppose contrary to the second claim that there is no outcome $\tau'' \preceq \tau^*$ that is unilaterally optimal for the sender in J'. Since any two feasible outcomes are comparable, this implies that $\tau' \succ \tau^*$ and that τ^* is not optimal for the sender in J'. But since Π is Blackwell-connected, the sender in J' can deviate from the strategy profile that induces τ^* and induce τ' instead so τ^* cannot be an equilibrium outcome. That establishes claim (ii).

Now consider the case where $|J'| > 1$. By Proposition 2, we know $\overline{\tau}$ is an equilibrium when the set of senders is J and by definition $\overline{\tau} \succeq \tau'$. That establishes claim (i). Moreover, by Proposition 2 we know τ^* must be an equilibrium when the set of senders is J', which establishes claim (ii). This completes the proof of part (1).

We now turn to part (2) of the proposition. Let τ^* be an equilibrium outcome when the the level of misalignment is some b and τ' an equilibrium outcome when the level of misalignment is some $b' < b$. By Proposition 2, we know $\overline{\tau}$ is an equilibrium when the the level of misalignment is b and by definition $\overline{\tau} \succeq \tau'$. By Lemma 1, we know that τ^* is also an equilibrium when the level of misalignment is b'. □

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