Symmetry-induced Dispersion and performance penalty in Coupled Resonator Optical Waveguides

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Abstract: We show that for fundamental reasons, microdisk based CROWs possess two non-degenerate transmission modes, making their long-planned use in optical communications highly challenging. Interference between these modes can lead to complete eye closure in most relevant communications scenarios.

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1. Introduction

In the past few years, much attention was devoted to slowing down the propagation speed of light and to coherently stop and store pulses of light [1-7]. In particular, significant efforts were focused on controlling the speed of light using photonic structures incorporating microcavities. Substantial delays and storage of light pulses were predicted in various coupled-cavities structures such as coupled resonator optical waveguides (CROWs) [3-6] and side-coupled integrated spaced sequence of resonators (SCISSORs) [7]. Such structures effectively slow down light by forcing it to slowly tunnel between adjacent cavities. The ability to control the speed of light in chip-scale components and realize ultra-compact optical delay lines is highly desired and much effort was invested in this direction. CROWs can be realized in two distinct technologies. In the first, the coupled resonators are defects in a photonic crystal structure, whereas in the second, light is guided through a concatenation of discrete microrings, or microdisks. The two options have always been considered as fundamentally equivalent, and thus, since fabrication of microrings, or microdiscs is usually considered easier, the latter option has been attracting more interest in the context of implementations

In this work we demonstrate that the assumed equivalence between photonic-crystal based resonator structures and microring resonator structures is fundamentally flawed. Moreover, we point out that light in microring structures propagates in two non-degenerate modes, resulting in considerable modal dispersion. As a result the microring-based CROW technology becomes prohibitive in most relevant high-speed telecomm applications. The non-degeneracy of the two propagation modes in microring-based crows is somewhat non-intuitive. In an individual ring, i.e. the unit-cell of the CROW, where one can identify the two modes as the clock-wise (CW) and counter-clock-wise (CCW) forms of propagation, the modes are perfectly degenerate. Yet, when a number of such rings are coupled together, the overall structure no longer maintains circular symmetry, and the propagation modes lose degeneracy. Note that the independent propagation modes in this case, can no longer be identified with the CW and CCW signals, but rather with their superpositions. We show in what follows that with characteristic system parameters, modal dispersion can lead to complete eye closure in a digital optical communications signal.

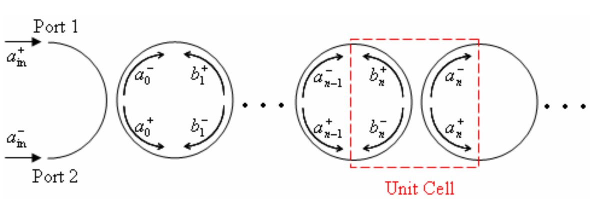

Fig. 1 – Schematic of a CROW structure and field amplitudes definitions

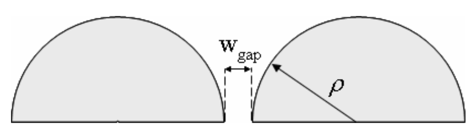

Fig 2. – Dimensions and definitions of the simulated structure.

2. Theoretical Analysis

We consider an array of coupled microdisks (Fig 1) with periodicity Λ. The propagating Bloch waves in the structure satisfy the following eigen-value equation:

$$\left(\mathbf{M} - e^{-iK\Lambda}\right)\cdot v = 0 \tag{1}$$

where $v=[a^+\ a^-\ b^+\ b^-]$ is a vector of four amplitudes of the CW and CCW propagating waves in the two half-rings constituting the unit cell, K is the Bloch wavenumber and **M** is a 4x4 transfer matrix connecting the field amplitudes in adjacent cavities. The transfer matrix **M** represents the physics of the problem, i.e. the microdisk resonances, the interaction between the adjacent cavities, etc. Intuitively, because of the degeneracy of the CW and CCW waves, it is often believed that it should be possible to represent the field in a single unit cell by a superposition of either whispering gallery modes (WGM) or standing wave solutions. Indeed, when a straightforward analysis of the CROW structure is performed, an inherent phase relation is found between the two forward propagating Bloch modes. This phase relation can be shown to be equivalent to the degeneracy of Bloch modes comprising alternating CW and CCW solutions in adjacent unit cells [8].

However, a group theoretical analysis of the CROW structure yields the four Bloch modes (two forward propagating and two backward propagating) composed of the *standing waves* solutions in each unit cell, which belong to *different* irreducible representations. Thus these Bloch modes are, in general, non-degenerate which is in contrast to the findings of previous studies.

3. Numerical simulations

To resolve the discrepancy, the Bloch modes of a microdisk based CROW were calculated using the finite difference time domain (FDTD) algorithm. The parameters of the studied structure are: disk radius $\rho = 0.7\mu m$, disk index n_{disk}=3.5, cladding index n_{clad}=1.0, separation w_{gap}=0.5μm (see Fig. 2). The resonance wavelength of an individual resonator is λres=1.565μm (f_{res}=191.6THz) with angular modal number m=7. The simulated unit cell consisted of two adjacent microdisks (to allow the appearance of WGM-like solutions, if exist) as shown in Fig. 2. Symmetric/Anti-Symmetric boundary conditions (BC) where employed in the lower part of the calculation window (-y) in order to separate between the two symmetries. Perfectly matched layers BC where used in the upper part of the calculation window (+y). Bloch boundary conditions were employed at the horizontal boundaries of the calculation windows. The objective of the calculations was to separate and determine the dispersion relations of the symmetric and anti-symmetric Bloch waves.

Figure 3 depicts the dispersion relations (DRs) of the propagating Bloch mode. An immediate result which is clearly seen, is that the DRs of the symmetric and anti-symmetric brunches *differ significantly*. Figure 3 also shows a numerical fit of the FDTD results to a generic CROW dispersion relation obtained by the tight-binding approach:

$$\Delta\omega_K = \tfrac{1}{2}\Omega\Delta\alpha - \kappa\Omega\cos(K\Lambda) \qquad (2)$$

where $\Delta\omega_K = \omega_K - \Omega$ is the difference between the optical frequency and the resonance frequency of an individual microring and where κ represents the coupling between adjacent microdisks and $\frac{1}{2}\Delta\alpha\Omega$ is the self frequency shift [3]. Excellent agreement is found between the numerical results and analytical expression with the following parameters: $f\Delta\alpha_{sym}$ = 0.6236 THz, $f\Delta\alpha_{a\text{-}sym}$ = 0.3072 THz, $f\kappa_{sym}$ = 0.1426 THz and $f\kappa_{a\text{-}sym}$ = 0.1497 THz where f is the resonance frequency of the cavity. It should be noted that while the coupling coefficients of the symmetric and anti-symmetric modes are quite similar, there is a non-negligible difference in the self frequency shifts of the two modes.

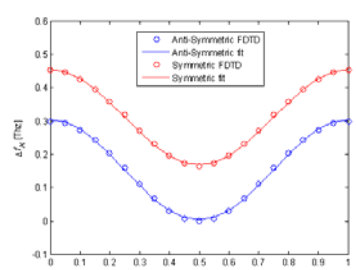

Fig 3. – Dispersion relations of the Anti-symmetric (red) and symmetric (blue) bands of a CROW for the limit of infinite number of cells.

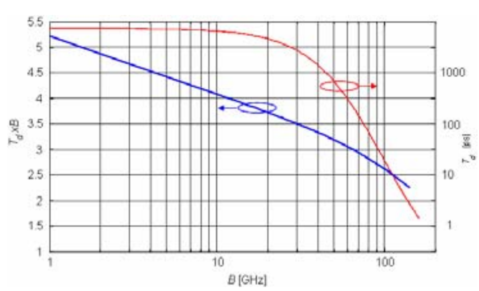

Fig 4. – Maximal delay and delay-bandwidth product as a function of the bandwidth.

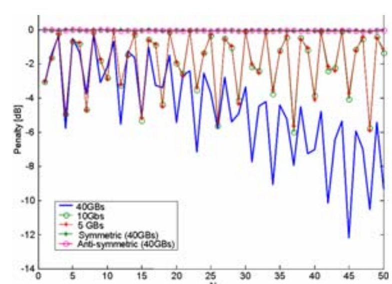

Fig 5. – Eye-penalty vs. number of resonators. B=40Gb/s (blue) and 5Gb/s (red) . Green curve is the hypothetical case, where only one mode is excited.

Thus, when light is injected into a CROW through a single input port (e.g. port 1 in Fig. 1), both dispersion branches will be equally excited and the light propagating through the CROW will experience additional modal dispersion. As shown in Fig. 3 the branches could be quite different, thus inducing a non-negligible impact on the CROW performance. Consider an RZ data-stream where the time slot allocated for each bit is $\Delta T_{sig}=B^{-1}$ where B is the bit-rate. We assume a Gaussian pulse with its FWHM equals $\frac{1}{2}\Delta T_{sig}$. The FWHM of the signal spectrum is given by $\Delta\omega_{sig}=8\ln(2)B$. The signal is launched into one of the input ports. From the dispersion relations we find the minimal and maximal time delays within the bandwidth of the RZ signal. Requiring that the maximal differential time delay does not exceed a quarter of the bit interval yields a limitation on the maximal bandwidth of the signal [10]. Fig. 4(a) depicts the maximal delay that can be provided without the above discussed distortion by the CROW, to a digital signal at a bit rate *B*. The delay is defined as the average of the delays of the symmetric and anti-symmetric branches at the carrier frequency. Below 30Gbs, the delay and the bandwidth are inversely proportional, indicating a relatively constant delay-bandwidth product. However, as the bit-rate exceeds 30Gbs the achievable delay decreases more rapidly indicating a decrease in delay-bandwidth product. This can also be seen in Fig. 4(b) which depicts the dependence of the delay-bandwidth product on the bit-rate. At low bit rates (B<10Gbs) the delay-bandwidth product of the CROW is approximately 5 but decreases rapidly reaching unity as the *B* exceeds 20Gbs. Note however, that the performance degradation shown in Fig. 4 *does not even account* for the different phase shifts accumulated in each branch, an effect which essentially dominates the degradation in the CROW performance as we explain below.

4. Impact on link performance

The impact of the CROW characteristics such as dispersion, loss, etc. on the communication link performance are often quantified in terms of the reduction in eye-opening compared to the back-to-back case (i.e. without the CROW). The eye-opening is a measure for the available noise margin around the decision threshold employed to determine the logical value of the bit. Again, we stress that the conventional injection of the input signal into one of the input ports of the device, necessarily implies excitation of both modes (and consequently of both dispersion branches). Each excited branch accumulates a different phase, and the signals at the output ports are superpositions (sum and difference) of the signals in each branch. Since the two branches accumulate a different optical phase, a strongly penalizing multi-path interference (MPI) effect follows.

Figure 5 depicts the eye penalty dependence on the number of resonators in the cases of 40Gbs and 5Gb/s signals. We assume a standard 50% duty cycle RZ format. For comparison, the figure also shows (green curve) the penalty that would prevail if only one of the modes could be excited (either symmetric, or anti-symmetric) at 40Gb/s. As shown in the figure, the impact of the branch splitting on the link performances is detrimental. While the penalty of the dispersion of an individual branch is practically negligible (less than -0.1dB for a 50 resonator CROW), the penalty when both branches are excited quickly reaches unacceptable values of several decibels. At low data-rates, the link penalty oscillates rapidly between 0 and -5dB, as a result of the above mentioned MPI, but without noticeable global degradation. At 40Gbs, in addition to the rapid variations there is also a clear overall increase in the eye-opening penalty that results from the limited delay bandwidth product.

5. Conclusion

We point out a fundamental modal dispersion mechanism in CROW delay lines. The additional dispersion stems from the symmetry properties of the structure and leads to an *inherent* limitation in the achievable delay and overall performance. This effect has not been observed experimentally yet, primarily because it becomes more prominent when the dimensions of the microdisks are decreased. An immediate consequence of the results shown here is that the practical realization of ultra-compact, coupled cavity based, integrated optical delay lines, requires much care. In particular, avoiding the additional penalty necessitates the excitation of only one of the natural Bloch modes of the CROW, a task which requires injecting the input signal into both ports simultaneously while controlling the relative phase. Finally, it should be emphasized, that although the analysis shown here is restricted to CROW, other microring based structures such as SCISSOR and SC-CROW [11] are expected to exhibit similar properties because, like the CROW, they also possess C_{2V} symmetry. In the presentation we will show the impact of the structure parameters on the dispersion curves and discuss the impact of losses and back-scattering on the CROW performance.

6. References

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