Paul Garabedian's Contributions to Transonic Airfoil and Wing Design

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Abstract

This note on Paul Garabedian's work on transonic airfoil and wing design is written from the perspective of aeronautical engineering as well as applied mathematics. Paul's contributions in this area had a profound and lasting impact on the way people set about designing wings in the aircraft industry.

1

Transonic flow is of great relevance to aircraft design because it is the most efficient regime for long range transport aircraft. The range of an aircraft is quite well predicted by the Breguet equation

$$R = \frac{V}{sfc}\frac{L}{D}log(\frac{W_o + W_f}{W_o}) \tag{1}$$

where V is the cruising speed, sfc is the specific fuel consumption of the engines, $\frac{L}{D}$ is the lift drag ratio, W_f is the weight of the fuel burnt and W_o is the final weight at the end of the flight. In subsonic flow the drag coefficient is given in classical aerodynamic theory as

$$C_D = C_{D_o} + \frac{{C_L}^2}{\pi AR}$$

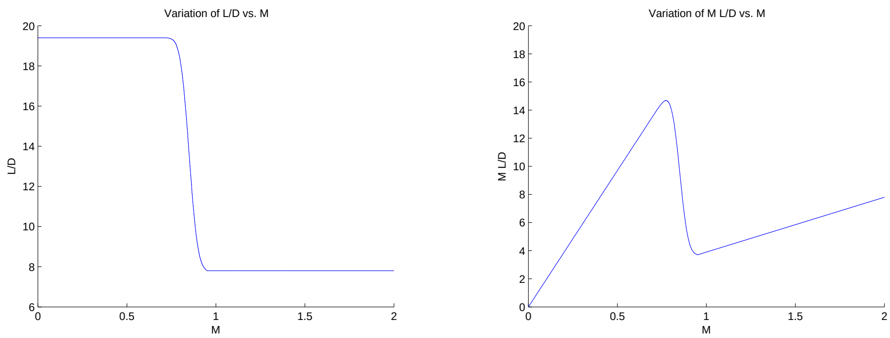
where C_{D_o} is the zero lift drag, C_L is the lift coefficient, and AR is the wing aspect ratio. Then $\frac{L}{D}$ is maximized by flying at a lift coefficient such that the two terms are equal

$$C_{L_{max.\frac{L}{D}}} = \sqrt{\pi \; AR \; C_{D_o}} \tag{2}$$

Now it is evident from the range equation (1) that one should fly at $C_{L_{max.\frac{L}{D}}}$ and one flies further by flying faster. With a wing area S and air density ρ the lift which is equal to the weight is

$$L = \frac{1}{2}\rho \; V^2 \; S \; C_L = W$$

so one increases the speed while matching the weight to the lift by flying higher where the air density is lower. This breaks down as V approaches the speed of sound because of the formation of shock waves which not only generate wave drag, but as they become stronger cause flow separation leading to a catastrophic increase in drag (about five fold at Mach 1 for a typical blunt shape). A first estimate of range efficiency is given by $M\frac{L}{D}$, and this will peak just beyond the onset of drag rise as illustrated in Figure 1.



(a) $\frac{L}{D}$ vs Mach Number (b) $M\frac{L}{D}$ vs Mach Number

Figure 1: **(a)** $\frac{L}{D}$ vs Mach Number **(b)** $M\frac{L}{D}$ vs Mach Number

At Mach 2 the maximum attainable $\frac{L}{D}$ for a feasible shape with enough volume is around 8. This is not competitive with subsonic aircraft which nowadays achieve a lift-to-drag ratio of about 20 at Mach 0.85. Thus long range transport aircraft should, and actually do, fly transonically in the speed range of Mach 0.8 to 0.85.

2

Transonic flow is also of great mathematical interest. The typical flow pattern of a two dimensional wing section is illustrated in Figure 2.

As the Mach number is increased a pocket of supersonic flow is formed on the upper side of the airfoil due to the local increase in the speed. Normally the supersonic pocket terminates in a shock wave. When the Mach number is further increased the shock strength increases to the point where it causes the viscous boundary layer to separate, leading to the full onset of drag rise, and

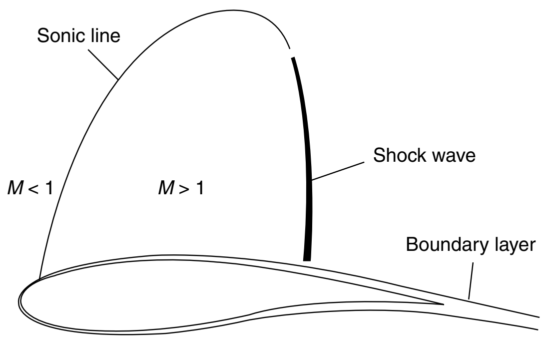


Figure 2: Schematic drawing of transonic flow over an airfoil

typically unsteady buffeting. Outside the boundary layer and wake the flow is well represented by the transonic potential flow equation

$$(a^2 - u^2)\phi_{xx} - 2uv\phi_{xy} + (a^2 - v^2)\phi_{yy} = 0 \quad (3)$$

where ϕ is the velocity potential, u and v are the velocity components

$$u = \phi_x, \ v = \phi_y$$

and a is the speed of sound, given by

$$\frac{a^2}{\gamma - 1} + \frac{u^2 + v^2}{2} = const$$

where γ is the ratio of specific heats.

This is the classical equation of mixed type, elliptic in the subsonic zone and hyperbolic in the supersonic zone, with the boundary between the zones to be determined as part of the solution. This equation proved quite intractable to analytical methods of solution. In order to reduce the drag one must look for shapes that minimize the shock strength, or even produce shock free flow. This was the problem that Paul chose to tackle. It had been established, however, by Cathleen Morawtz [1] that shock free solutions are isolated points and shocks will appear with small perturbations of the shape or the flight condition. So the problem of designing a shock free shape is not well posed.

Paul elected to pursue an inverse approach. Following earlier work by Lighthill [2] he used the hodograph transformation in which the velocity components u and v are treated as the independent variables and the coordinates x and y become the dependent variables. While this results in a linear equation of mixed type, it remains hard to find solutions in the hodograph plane which correspond to physically realizable shapes. Nieuwland had previously generated a family of hodograph solutions which resulted in airfoils that were not practically useful [3]. Paul applied the method of complex characteristics which he had successfully used to solve the supersonic blunt body problem in earlier work [4] to solve the equations in the hodograph plane. He was able to find boundary conditions and integration paths that resulted in usable shock free airfoils for a range of Mach numbers and lift coefficients. Working with his assistant Frances Bauer and his doctoral student David Korn he published the first results in the book Supercritical Wing Sections [5].

In this period he made contact with Richard Whitcomb at NASA Langley who had experimentally developed a supercritical airfoil with a flat topped shape and heavy rear camber which produced a comparatively weak shock at its design condition [6]. Paul's shock free 78-06-10 airfoil had a similar though smoother shape. This influenced Whitcomb's thinking, and he decided to fund further studies of supercritical airfoils at the Courant Institute.

Paul also made contact with R.T.Jones at NASA Ames. A pioneer of the swept wing concept, by that time Jones had come to advocate the use of an asymmetric yawed wing for supersonic flight, stemming from the realization that vortex drag due to lift varies inversely as the square of the span, while wave drag due to lift varies inversely as the square of the longitudinal extent of the wing, so that both can be minimized by extending the wing platform along a diagonal line. Paul was enthusiastic about this idea, and he obtained additional funding from Jones to pursue studies of yawed wings.

3

In 1970, as a staff engineer at Grumman, I was asked to look into the state of the art in supercritical wing technology. I soon found out that Paul's group was at the cutting edge, and managed to persuade Grumman to hire David Korn as a consultant to assist the Aerodynamics Department in designing their own supercritical wing section. The other aircraft company to take an early interest in the developments at the Courant institute was the Douglas Aircraft Division of the McDonnell-Douglas Company.

At that time the principal remaining issue was how to calculate the flows past supercritical airfoils over a range of flight conditions, because the hodograph method only provided a solution at the design point. There was an evident need to find ways of numerically solving the transonic potential flow equation (3). In a seminal paper Murman and Cole [7] had introduced their type dependent difference scheme for solving the small disturbance equation. At Grumman I started working on extending this scheme to the full potential equation, and eventually succeeded in 1971. It turned out that Garabedian and Korn had simultaneously developed an almost identical scheme. Paul then suggested that I visit the Courant Institute for 3 months early in 1972. During the period he subjected me to jumping through a series of mental hoops, such as how to conformally map a square with round corners to a circle, and finally he suggested I join his group permanently as a senior research scientist. I accepted his offer with some misgiving, as Paul had made it clear that he did not think this would lead to an academic appointment.

Paul was now working on a second book, Supercritical Wing Sections II [8], which presented an improved series of shock free airfoils, a transonic analysis method (Program H) which included a boundary layer correction, some results of experimental tests, and some preliminary results for yawed wings. My principal assignment was to write the three dimensional analysis code for yawed wings (Program J, or Flo17) which subsequently evolved into a widely used code for calculating transonic flow over swept wings (Flo22). Program H and Flo22 are still in use today for preliminary design work at Boeing.

The concept of a yawed flying wing for supersonic cruise was the subject of intensive studies at NASA fiften years later, but no viable design emerged. In the meanwhile Paul continued his studies of supercritical wing design, issuing a third book Supercritical Wing Section III in 1977 [9]. With Geoffrey McFadden he also developed a three dimensonal inverse design method [10]. By 1980 his interest has switched to magnetic containment of plasma for fusion reactors, and this remained the main focus of his research for the rest of his career.

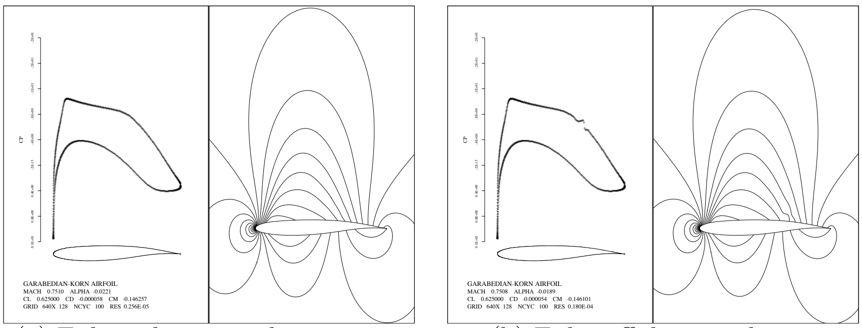
In the period I worked for him Paul was a wonderful mentor. He exposed me to broad areas of mathematics in which my knowledge was quite deficient. he would do this in a very subtle way by casually asking wasn't I aware of this, or that, for example, the Bateman variational principle. Then I would be forced to go and find out what he was talking about. Subsequently he played a major role in persuading the Courant Institute to appoint me to a faculty position in Computer Science in 1974.

He had an extraordinary youthful appearance - at age 44 one might easily have taken him to be 28. He also had a singular work style. His desk was completely bare, and he would write entries in a tiny note book. Apparently he was able to carry the complete aspects of whatever problem he was working on in his head, without any need for supporting notes.

4

To the best of my knowledge none of the airfoils listed in either of the two books was directly used in an actual aircraft, but they had a profound and lasting impact on the Aircraft Industry by showing for the first time that practically useful supercritical airfoils which are shock free or produce very weak shocks could be designed. This permanently changed the way engineers think about transonic wing design.

The 75-06-12 “Garabedian-Korn” airfoil has been widely used as a benchmark to validate new numerical methods for computational fluid dynamics (CFD). It has been a source of consternation because many CFD codes are not able to produce a shock free solution. The coordinates calculated by the hodograph method are not perfectly accurate because of integration errors. In fact I believe the Garabedian-Korn airfoil is actually shock free at Mach 0.7510 and CL 0.6250, not the originally calculated design point of Mach 0.750 at CL 0.629. This is illustrated in Figure 3a which shows an Euler solution calculated on a relatively fine mesh with 640 intervals around the profile and 128 intervals normal to the airfoil for a total of 40960 mesh points. I have verified that the solution remains shock free for a sequence of progressively finer meshes up to 5280x1280=6758400 mesh points. Figure 3b shows that there is a very small shock at Mach 0.7508 and CL 0.6250, consistent with Morawtz’s theorem. In these figures the negative pressure coefficient is plotted vertically, while the right hand window shows the Mach contours.



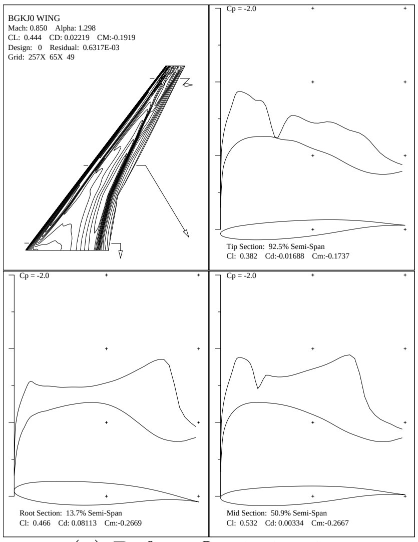
(a) Euler solution at design point (b) Euler off-design solution

Figure 3: Euler solution of transonic flow over an airfoil **(a)** at design point (shock free) **(b)** at off-design point (with small shock)

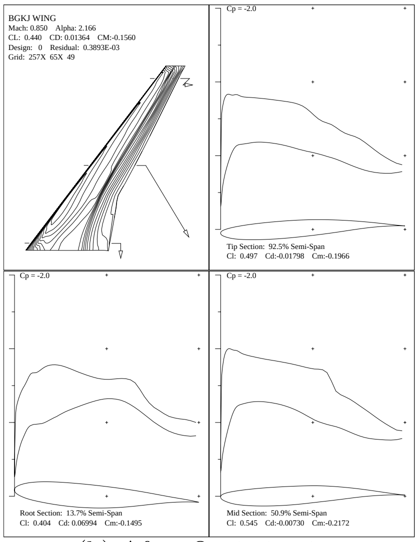
The boundary layer displacement effect would prevent the flow from being shock free in practice. In order to overcome this difficulty Paul adopted the practice of designing his airfoils with an open trailing edge, so that the estimated boundary layer displacement thickness could be subtracted. Since, however, the boundary layer thickness varies with the Reynolds number, this leads to a situation where a shape that produced shock free flow in flight would not do so in a wind tunnel, and vice-versa.

Due to three dimensional effects, particularly near the fuselage, a satisfactory swept wing cannot be designed with a fixed wing section from root to tip.

In a numerical experiment I have substituted the Garabedian-Korn section into a representative modern transonic wing design, the NASA Common Research Model (CRM), which is the test shape for the latest AIAA Drag Prediction Workshops [11]. After scaling the thickness to produce a distribution similar to the CRM, and introducing 7 degrees of twist to produce a near elliptic spanwise lift distribution, the result calculated using the Reynolds averaged Navier-Stokes equations at a design point of Mach 0.850 and CL 0.440 is as shown in Figure 4a. In this calculation the Reynolds number is 20 million based on the CRM reference chord. It can be seen that there is a very strong shock wave across the entire span. However, using an optimization method based on techniques drawn from control theory for partial differential equation [12], the wing can be redesigned to produce an essentially shock free flow as illustrated in Figure 4b. The outboard wing sections are preserved almost unchanged, but a substantial modification is required near the fuselage. The drag of the redesigned wing is slightly less than that of the CRM. While further modifications would be needed to get good performance over a range of flight conditions, this demonstrates that the Garabedian-Korn section could still be used as the starting point for a competitive wing. This calculation took 4 hours using a quad-core workstation which is about 5000 times faster than the Control Data 6600 computers at the Courant Institute in the early 70s, and has about 8000 times the memory. Evidently such a calculation would not have been feasible in that era. Nevertheless, the outcome after forty years is that all modern transonic commercial aircraft, including business jets as well as airliners, have wing sections which strongly resemble the sections designed by Paul Garabedian.



(a) Before Optimization



(b) After Optimization

Figure 4: Pressure contours for the CRM wing **(a)** before optimization (strong shock) **(b)** after optimization (with very weak shock)

References

[1] Morawetz, C.S., "On the non-existence of continuous transonic flows past profiles, I, II, and III", Comm. Pure Appl. Math., Vol. 9, pp. 45-68 (1956), Vol.10, pp. 107-131 (1957), Vol. II, pp. 129-144 (1958)

[2] Lighthill, M.J., "The hodograph transformation in transonic flow, II and III," Proc. Roy. Soc. London, Vol. A191, pp. 341-369 (1947).

[3] Nieuwland, G.Y., "Transonic poetntial flow around a family of quasi-elliptical aerofoil sections," N.L.R. Report TR.T172 (1967)

[4] Garabedian, P.R., and Lieberstein, H.M., "On the numerical calculation of detached bow shock waves in hypersonic flow," J. Aero. Sci., Vol. 25, pp. 109-118 (1958)

[5] Bauer, F., Garabedian, P., and Korn, D., "Supercritical Wing Sections I," Lecture Notes in Economics and Mathematical Systems, Vol. 66, Springer-Verlag, New York, 1972.

[6] Whitcomb, R.R., and Clark, L.R., "An airfoil shape for efficient flight at supercritical Mach numbers," NASA TM X-1109, July 1965

[7] Murman, E.M. and Cole, J.D., "Calculation of plane steady transonic flows," AIAA Journal, Vol. 9, pp.114-121 (1971)

[8] Bauer, F., Garabedian, P., Korn, D., and Jameson, A., "Supercritical Wing Sections II," Lecture Notes in Economics and Mathematical Systems, Vol. 108, Springer-Verlag, New York, 1975.

[9] Bauer, F., Garabedian, P., and Korn, D., "Supercritical Wing Sections III," Lecture Notes in Economics and Mathematical Systems, Vol. 150, Springer-Verlag, New York, 1977.

[10] Garabedian, P. and McFadden, G. "Design of Supercritical Swept Wings," AIAA Journal, Vol. 20, No. 3, 1982

[11] Vassberg, J.C., DeHann, M.A., Rivers, S.M. and Wahls, R.A. "Development of a Common Research Model for Applied CFD Validation Studies," AIAA Paper 2008-6919, Honolulu, HI, August, 2008

[12] Jameson, A. "Aerodynamic Design via Control Theory," Journal of Scientific Computing, Vol.3, No. 3, 1988