

# **A Proof of the Collatz Conjecture**

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## **Abstract**

If every positive integer is able to be operated into 1 by set operational rule of the conjecture, then begin with 1, we can get all positive integers after pass infinite many operations by another operational rule on the contrary to the set operational rule. Thereby, substitute a poof of the latter proposition for some unknown proof of the former proposition, and apply mathematical induction, then this is exactly an effective way for proving Collats conjecture, thus we use it in this article justly.

## **Keywords**

Mathematical induction, Classify integers, Leftward operational rule, A bunch of integers' chains, Rightward operational rule, Operational routes.

## **Basic Concepts**

The Collatz conjecture is also known as  $3n+1$  conjecture, the Ulam conjecture, Kakutani problem, the Thwaites conjecture, Hasse's algorithm or the Syracuse problem, etc.

The Collatz conjecture asserts that take any positive integer  $n$ , if  $n$  is an even number, then divide  $n$  by 2 to obtain an integer  $n/2$ ; if  $n$  is an odd number, then multiply  $n$  by 3 and add 1 to obtain an even number  $3n+1$ .

Repeat the above process indefinitely, then no matter which positive integer you start with, you will always eventually reach a result of 1.

We consider the way of doing for aforesaid two steps as the leftward operational rule for any positive integer given. Also consider another operational rule on the contrary to the leftward operational rule as the rightward operational rule for any positive integer got. We label two such operational rules as set each other's contrary operational rules.

The rightward operational rule states that for any positive integer got, if  $n$  is an odd number, then multiply  $n$  by 2 to obtain an even number  $2n$ . If  $n$  is an even number, then, on the one hand, multiply  $n$  by 2 to obtain an even number  $2n$ . On the other hand, if the difference of  $n$  minus 1 is able to be divided by 3 and obtain an odd number, then must operate the step as such, and from here proceed to operate; if it is not so, do not the step.

We regard a segment within an operational course by aforementioned either operational rule as an operational route. If either end of the segment is positive integer  $P$ , then we term it  $P$ -operational route further.

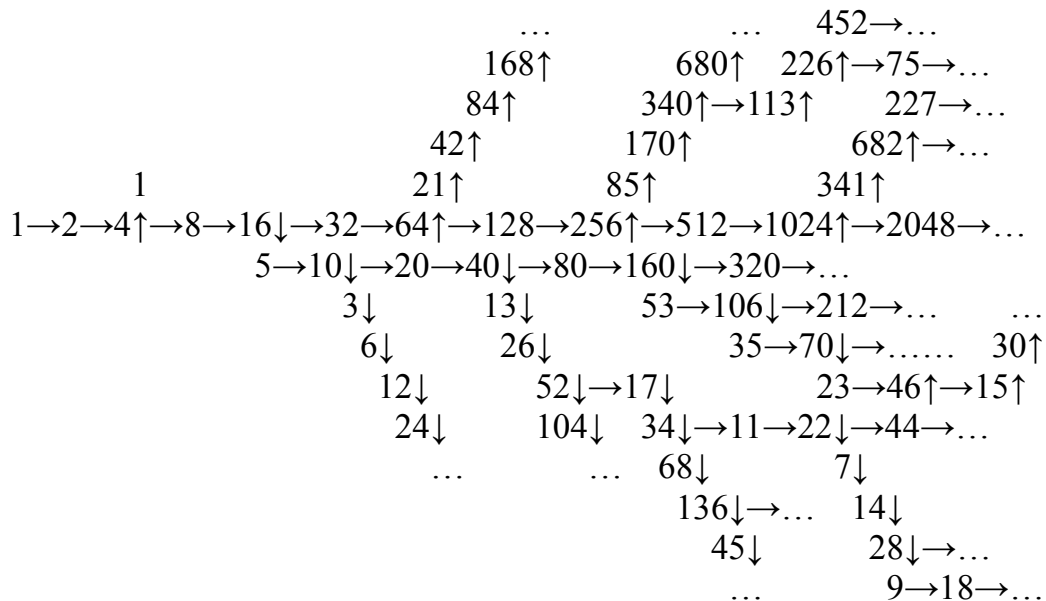
Begin with 1, proceed to operate every positive integer successively got by the rightward operational rule, then it automatically made a bunch of operational routes which consist of positive integers plus arrows. We reckon the bunch of operational routes as a bunch of integers' chains.

Since an origin of each integer in the bunch of integers' chains is unique, thus each integer therein is an oneer likewise, therefore any

segment of the bunch of integers' chains is none of the repeat.

For two adjacent odd numbers in the bunch of integers' chains, the former is always smaller than the latter. Likewise for two close even numbers in the bunch of integers' chains, the former is always smaller than the latter too. Thus, comparatively speaking, integers at the left of the bunch of integers' chains are smaller integers as compared with integers at the right, but a closest even number at the front of any odd number is greater than the odd number.

Overall, odd numbers and even numbers from left to right in the bunch of integers' chains are getting greater and greater, up to infinity. Please, see a beginning of the bunch of integers' chains as first illustration.



First Illustration

Annotation: signs  $\downarrow$  and  $\uparrow$  must rightwards tilt, but since the page is narrow.

No matter which positive integer, so long as it is able to be operated into 1 by the leftward operational rule, then it is surely in the bunch of

integers' chains; conversely, it is so too.

That is to say, positive integers got in the bunch of integers' chains by rightward operational rule and positive integers which can operate into 1 by the leftward operational rule are one-to-one correspondence.

Therefore, whether a positive integer suits the conjecture, we need merely to ascertain that whether it is in the bunch of integers' chains.

If every positive integer is able to be operated into 1 by the leftward operational rule, then, there are all positive integers in the bunch of integers' chains after pass infinite many operations by the rightward operational rule.

Consequently, we have intention to prove that the bunch of integers' chains contains all positive integers by mathematical induction.

Nevertheless, we need also to determine first an axiom as follows when use the mathematical induction to prove the aforementioned proposition.

**Axiom.** For any positive integer  $P$ , if there is a positive integer  $L < P$  at a  $P$ -operational route or at another operational route which intersects with the  $P$ -operational route, and known  $L$  suits the conjecture, then  $P$  suits the conjecture too, of course  $P$  is in the bunch of integers' chains surely.

For example, when  $P = 31 + 3^2\eta$ , and  $\eta \geq 0$ , a  $P$ -operational route is  $27 + 2^3\eta \rightarrow 82 + 3 * 2^3\eta \rightarrow 41 + 3 * 2^2\eta \rightarrow 124 + 3^2 * 2^2\eta \rightarrow 62 + 3^2 * 2\eta \rightarrow 31 + 3^2\eta > 27 + 2^3\eta$ , so  $31 + 3^2\eta$  suit the conjecture.

For example, when  $P = 5 + 12\mu$ , and  $\mu \geq 0$ , a  $P$ -operational route is

$5+12\mu \rightarrow 16+36\mu \rightarrow 8+18\mu \rightarrow 4+9\mu < 5+12\mu$ , so  $5+12\mu$  suit the conjecture.

For example, when  $P=63+3*2^8\varphi$ , and  $\varphi \geq 0$ , a P-operational route intersects with another are  $63+3*2^8\varphi \rightarrow 190+3^2*2^8\varphi \rightarrow 95+3^2*2^7\varphi \rightarrow 286+3^3*2^7\varphi \rightarrow 143+3^3*2^6\varphi \rightarrow 430+3^4*2^6\varphi \rightarrow 215+3^4*2^5\varphi \rightarrow 646+3^5*2^5\varphi \rightarrow 323+3^5*2^4\varphi \rightarrow 970+3^6*2^4\varphi \rightarrow 485+3^6*2^3\varphi \rightarrow 1456+3^7*2^3\varphi \rightarrow 728+3^7*2^2\varphi \rightarrow 364+3^7*2\varphi \rightarrow 182+3^7\varphi \rightarrow \dots$

$\uparrow 121+3^6*2\varphi \leftarrow 242+3^6*2^2\varphi \leftarrow 484+3^6*2^3\varphi \leftarrow 161+3^5*2^3\varphi \leftarrow 322+3^5*2^4\varphi \leftarrow 107+3^4*2^4\varphi \leftarrow 214+3^4*2^5\varphi \leftarrow 71+3^3*2^5\varphi \leftarrow 142+3^3*2^6\varphi \leftarrow 47+3^2*2^6\varphi < 63+3*2^8\varphi$ , so  $63+3*2^8\varphi$  suit the conjecture.

### **The Proof**

Let us set about the proof that the bunch of integers' chains contains all positive integers by mathematical induction, thereafter.

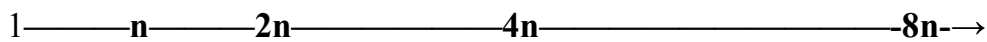
**1.** From the above-listed first illustration, you can directly see and count that begin with 1, operate positive integers got successively by the rightward operational rule, we got a part of positive integers including consecutive positive integers from 1 to 18, and they formed a beginning of the bunch of integers' chains.

**2.** Suppose that after further operate positive integers got successively by the rightward operational rule, there are consecutive positive integers  $\leq n$  within positive integers got successively in the bunch of integers' chains, where  $n$  is a natural number  $\geq 18$ .

**3.** Prove that after continue to operate positive integers successively

got by the rightward operational rule, we can get consecutive positive integers  $\leq 2n$  within positive integers successively got in the bunch of integers' chains.

Divide positive integers from left to right at the number axis into segments according to their number  $2^X n$  per segment, so as to accord with the proof, where  $X \geq 0$  from small to large. please, see second illustration.



Second Illustration

**Proof.** Since there are consecutive positive integers  $\leq n$  within positive integers got in the bunch of integers' chains, then multiply every positive integer  $\leq n$  by 2 by the rightward operational rule, we obtain all even numbers between  $n$  and  $2n+1$ , irrespective of repeated even numbers  $\leq n$ .

Since every positive integer  $\leq n$  is in the bunch of integers' chains, consequently all even numbers between  $n$  and  $2n+1$  are in the bunch of integers' chains according to the preceding axiom.

Next we must seek an origin of each kind of odd numbers between  $n$  and  $2n+1$ , whether each such origin is able to be traced to be smaller than the kind of odd numbers.

First, let us divide consecutive odd numbers between  $n$  and  $2n+1$  into two kinds, i.e.  $5+4k$  and  $7+4k$ , where  $k$  is a natural number, then any odd number between  $n$  and  $2n+1$  must belong in one of two such kinds. Now we list two such kinds of odd numbers in correspondence with  $k$ , below.

k: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, ...

5+4k: 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, ...

7+4k: 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, ...

From  $5+4k \rightarrow 16+12k \rightarrow 8+6k \rightarrow 4+3k < 5+4k$ ,  $5+4k$  suit the conjecture, and  $5+4k$  are in the bunch of integers' chains according to the preceding axiom.

For  $7+4k$ , we again divide them into three kinds, i.e.  $11+12c$ ,  $15+12c$  and  $19+12c$ , where  $c$  is a natural number or 0. From  $7+8c \rightarrow 22+24c \rightarrow 11+12c > 7+8c$ ,  $11+12c$  suit the conjecture, and  $11+12c$  are in the bunch of integers' chains according to the preceding axiom.

We list remainder two kinds of odd numbers in correspondence with  $c$  by now, next operate each kind by the leftward operational rule orderly, and reach conclusions at certain operational branches.

c: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...

15+12c: 15, 27, 39, 51, 63, 75, 87, 99, 111, 123, 135, 147, 159, ...

19+12c: 19, 31, 43, 55, 67, 79, 91, 103, 115, 127, 139, 151, 163, ...

**15+12c**  $\rightarrow 46+36c \rightarrow 23+18c \rightarrow 70+54c \rightarrow 35+27c \clubsuit$

d=2e+1: 29+27e (1)      e=2f: 142+486f  $\rightarrow$  71+243f  $\heartsuit$

$\clubsuit 35+27c \downarrow \rightarrow c=2d+1: 31+27d \uparrow \rightarrow d=2e: 94+162e \rightarrow 47+81e \uparrow \rightarrow e=2f+1: 64+81f$  (2)

c=2d: 106+162d  $\rightarrow$  53+81d  $\downarrow \rightarrow d=2e+1: 67+81e \downarrow \rightarrow e=2f+1: 74+81f$  (3)

d=2e: 160+486e  $\diamond$       e=2f: 202+486f  $\rightarrow$  101+243f  $\spadesuit$

g=2h+1: 200+243h (4)      ...

$\heartsuit 71+243f \downarrow \rightarrow f=2g+1: 157+243g \uparrow \rightarrow g=2h: 472+1458h \rightarrow 236+729h \uparrow \rightarrow \dots$

f=2g: 214+1458g  $\rightarrow$  107+729g  $\downarrow \rightarrow g=2h+1: 418+729h \downarrow \rightarrow \dots$

g=2h: 322+4374h  $\rightarrow \dots$       ...

$$\begin{array}{l}
g=2h:86+243h \text{ (5)} \\
\spadesuit 101+243f \downarrow \rightarrow f=2g+1:172+243g \uparrow \rightarrow g=2h+1:1246+1458h \rightarrow \dots \\
F=2g:304+1458g \rightarrow 152+729g \downarrow \rightarrow \dots \\
\dots \\
\dots \\
\diamond 160+486e \rightarrow 80+243e \downarrow \rightarrow e=2f+1:970+1458f \rightarrow 485+729f \uparrow \rightarrow \dots \dots \\
e=2f:40+243f \downarrow \rightarrow f=2g+1:850+1458g \rightarrow 425+729g \uparrow \rightarrow \dots \\
f=2g:20+243g \downarrow \rightarrow g=2h:10+243h \text{ (6)} \dots \\
g=2h+1:880+1458h \rightarrow 440+729h \uparrow \rightarrow \dots
\end{array}$$

Alphabet c, d, e, f, g, ... in the above-listed operational routes expresses respectively each of natural numbers or 0, similarly hereinafter. Also there are  $\clubsuit \leftrightarrow \clubsuit$ ,  $\heartsuit \leftrightarrow \heartsuit$ ,  $\spadesuit \leftrightarrow \spadesuit$ , and  $\diamond \leftrightarrow \diamond$ .

We conclude several partial conclusions as the follows from above-listed a bunch of operational routes for  $15+12c$ .

From  $c=2d+1$  and  $d=2e+1$ , get  $c=2d+1=2(2e+1)+1=4e+3$ , and  $15+12c = 15+12(4e+3) = 51+48e > 29+27e$  where mark (1), so  $15+12c$  suit the conjecture where  $c=4e+3$  according to the preceding axiom.

From  $c=2d+1$ ,  $d=2e$ , and  $e=2f+1$ , get  $c=2d+1=4e+1=4(2f+1)+1=8f+5$ , and  $15+12c=15+12(8f+5)=75+96f > 64+81f$  where mark (2), so  $15+12c$  suit the conjecture where  $c=8f+5$  according to the preceding axiom.

From  $c=2d$ ,  $d=2e+1$  and  $e=2f+1$ , get  $c=2d=4e+2=4(2f+1)+2=8f+6$ , and  $15+12c=15+12(8f+6)=87+96f > 74+81f$  where mark (3), so  $15+12c$  suit the conjecture where  $c=8f+6$  according to the preceding axiom.

From  $c=2d+1$ ,  $d=2e$ ,  $e=2f$ ,  $f=2g+1$  and  $g=2h+1$ , get  $c=2d+1=4e+1=8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25$ , and  $15+12c=15+12(32h+25) = 315+384h > 200+243h$  where mark (4), so  $15+12c$  suit the conjecture where  $c=32h+25$  according to the preceding axiom.



From  $c=2d$ ,  $d=2e+1$ ,  $e=2f$ ,  $f=2g+1$  and  $g=2h$ , get  $c=2d=2(2e+1)=4e+2=8f+2=8(2g+1)+2=16g+10=32h+10$ , and  $15+12c=15+12(32h+10)=135+384h > 86+243h$  where mark (5), so  $15+12c$  suit the conjecture where  $c=32h+10$  according to the preceding axiom.

From  $c=2d$ ,  $d=2e$ ,  $e=2f$ ,  $f=2g$  and  $g=2h$ , get  $c=2d=32h$ , and  $15+12c=15+12(32h)=15+384h > 10+243h$  where mark (6), so  $15+12c$  suit the conjecture where  $c=32h$  according to the preceding axiom.

Secondly we operate  $19+12c$  by the leftward operational rule, below.

$$19+12c \rightarrow 58+36c \rightarrow 29+18c \rightarrow 88+54c \rightarrow 44+27c \spadesuit$$

$$\begin{array}{l} d=2e: 11+27e \text{ (}\alpha\text{)} \qquad e=2f: 37+81f \text{ (}\beta\text{)} \\ \spadesuit 44+27c \downarrow \rightarrow c=2d: 22+27d \uparrow \rightarrow d=2e+1: 148+162e \rightarrow 74+81e \uparrow \rightarrow e=2f+1: 466+486f \heartsuit \\ \quad c=2d+1: 214+162d \rightarrow 107+81d \downarrow \rightarrow d=2e: 322+486e \spadesuit \\ \qquad d=2e+1: 94+81e \downarrow \rightarrow e=2f: 47+81f \text{ (}\gamma\text{)} \\ \qquad \qquad e=2f+1: 516+486f \diamond \end{array}$$

$$\begin{array}{l} g=2h: 129+243h \text{ (}\delta\text{)} \qquad \dots \\ f=2g+1: 258+243g \uparrow \rightarrow g=2h+1: 1504+1458h \rightarrow 752+729h \uparrow \rightarrow \dots \\ \heartsuit 466+486f \rightarrow 233+243f \uparrow \rightarrow f=2g: 700+1458g \rightarrow 350+729g \downarrow \rightarrow g=2h+1: 3238+4374h \downarrow \\ \qquad g=2h: 175+729h \downarrow \rightarrow \dots \dots \\ \dots \end{array}$$

$$\begin{array}{l} g=2h+1: 172+243h \text{ (}\epsilon\text{)} \\ f=2g: 101+243g \uparrow \rightarrow g=2h: 304+1458h \rightarrow \dots \\ e=2f+1: 202+243f \uparrow \rightarrow f=2g+1: 1336+1458g \rightarrow \dots \\ \spadesuit 322+486e \rightarrow 161+243e \uparrow \rightarrow e=2f: 484+1458f \rightarrow \dots \end{array}$$

$$\begin{array}{l} \diamond 516+486f \rightarrow 258+243f \downarrow \rightarrow f=2g+1: 1504+1458g \rightarrow \dots \\ \quad f=2g: 129+243g \downarrow \rightarrow g=2h: 388+1458h \rightarrow \dots \\ \qquad g=2h+1: 186+243h \text{ (}\zeta\text{)} \end{array}$$

Alphabet c, d, e, f, g, h, ... in the above-listed operational routes expresses respectively each of natural numbers or 0, similarly hereinafter. Also there are  $\clubsuit \leftrightarrow \heartsuit$ ,  $\heartsuit \leftrightarrow \spadesuit$ ,  $\spadesuit \leftrightarrow \diamond$ .

We also conclude several partial conclusions as the follows from above-listed a bunch of operational routes for  $19+12c$ .

From  $c=2d$ ,  $d=2e$ , get  $c=2d=4e$ , and  $19+12c=19+12(4e)=19+48e > 11+27e$  where mark ( $\alpha$ ), so  $19+12c$  suit the conjecture where  $c=4e$  according to the preceding axiom.

From  $c=2d$ ,  $d=2e+1$  and  $e=2f$ , get  $c=2d=2(2e+1)=4e+2=8f+2$ , and  $19+12c=19+12(8f+2)=43+96f > 37+81f$  where mark ( $\beta$ ), so  $19+12c$  suit the conjecture where  $c=8f+2$  according to the preceding axiom.

From  $c=2d+1$ ,  $d=2e$ , and  $e=2f$ , get  $c=2d+1=4e+1=8f+1$ , and  $19+12c=19+12(8f+1)=31+96f > 47+81f$  where mark ( $\gamma$ ), so  $19+12c$  suit the conjecture where  $c=8f+1$  according to the preceding axiom.

From  $c=2d$ ,  $d=2e+1$ ,  $e=2f+1$ ,  $f=2g+1$  and  $g=2h$ , get  $c=2d=2(2e+1)=4e+2=4(2f+1)+2=8f+6=8(2g+1)+6=16g+14=32h+14$ , and  $19+12c=19+12(32h+14)=187+384h > 129+243h$  where mark ( $\delta$ ), so  $19+12c$  suit the conjecture where  $c=32h+14$  according to the preceding axiom.

From  $c=2d+1$ ,  $d=2e$ ,  $e=2f+1$ ,  $f=2g$  and  $g=2h+1$ , get  $c=2d+1=4e+1=4(2f+1)+1=8f+5=16g+5=16(2h+1)+5=32h+21$ , and  $19+12c=19+12(32h+21)=271+384h > 172+243h$  where mark ( $\epsilon$ ), so  $19+12c$  suit the conjecture where  $c=32h+21$  according to the preceding axiom.

From  $c=2d+1$ ,  $d=2e+1$ ,  $e=2f+1$ ,  $f=2g$  and  $g=2h+1$ , get  $c=2d+1=2(2e+1)+1=4e+3=4(2f+1)+3=8f+7=16g+7=16(2h+1)+7=32h+23$ , and  $19+12c=19+12(32h+23)=295+384h > 186+243h$  where mark ( $\zeta$ ), so  $19+12c$  suit the conjecture where  $c=32h+23$  according to the preceding axiom.

Let  $\chi=d, e, f, g, h, \dots$  in two bunches of operational routes for  $15+12c$

and  $19+12c$ , then the operation where an unknown number equals  $2\chi$  and the operation where the unknown number equals  $2\chi+1$  synchronize at every step. Moreover begin with a greater result at every step, it will continue to operate. If a smaller result at any setp suits not the conjecture, then it must too continue to operate. If a smaller result at any setp is smaller than its beginning  $15+12c$  or  $19+12c$ , then the operational route must stop at here.

Or rather, on the one hand, two kinds of operations for an unknown number as  $2\chi+1$  plus the unknown number as  $2\chi$  are always continuously progress and branch up to infinitely progress and branch, and values of expressions which continuously operate are getting more and more, and greater and greater up to infinite many infinity. On the other hand, they uninterruptedly stop operations of part branches in the course of infinite operations because operational results at those steps arrived to suit the conjecture, and expressions which stop operate are such kinds, namely their constant terms and coefficients of  $\chi$  are always smaller positive integers as compared with expressions whose unknown number's grades are not less than now  $\chi$ .

Since for an expression which denotes positive integers, both operate it as an even number into a half of value of the expression, and operate it as an odd number into three times of value of the expression, then add 1. Thus for an incremental result and a reductive result at every step, there is

only at most one result to suit the conjecture.

Consequently operating  $15+12c$  and operating  $19+12c$  will proceed infinitely. In other words,  $15+12c$  and  $19+12c$  must be divided respectively into infinite many kinds, just enable every kind to suit the conjecture via operations at infinite many time by the leftward operational rule, yet the stop of operations of each kind of  $15+12c$  plus  $19+12c$  relies only on a kind of  $c$  to decide.

This notwithstanding, since early arisen kinds which suit the conjecture are all such expressions whose constant terms and coefficients of  $\chi$  are smaller positive integers, thus when  $\chi$  is orderly equal to  $0, 1, 2, \dots$ , such expressions are exactly smaller positive odd numbers which belong in  $15+12c$  and in  $19+12c$  respectively, for example,  $51+48e, 75+96f, 87+96f, 315+384h, 135+384h, 15+384h, 19+48e, 43+96f, 31+96f, 187+384h, 271+384h$  and  $295+384h$  in the above-listed two bunches of operational routes, etc. It is obvious that these smaller positive odd numbers contain smaller consecutive odd numbers of  $15+12c$  plus  $19+12c$  inevitably.

What we need are merely odd numbers of  $15+12c$  plus  $19+12c$  between  $n$  and  $2n+1$ , yet it is not all of  $15+12c$  plus  $19+12c$ . Evidently odd numbers of  $15+12c$  plus  $19+12c$  between  $n$  and  $2n+1$  are smaller odd numbers in kindred unproved odd numbers. Thus we can find each of them according to a certain value of  $c$  after operating  $15+12c$  and operating  $19+12c$  at finite times to suit the conjecture. Of course, when

find such  $15+12c$  and  $19+12c$ , they are just proven to suit the conjecture.

Sum up the above proof, first we have proven all even numbers between  $n$  and  $2n+1$  in the bunch of integers' chains by the rightward operational rule. After that, divide consecutive odd numbers between  $n$  and  $2n+1$  into two kinds, i.e.  $5+4k$  and  $7+4k$ , and have proven  $5+4k$  to suit the conjecture by the leftward operational rule. Next again divide  $7+4k$  into three kinds, i.e.  $11+12c$ ,  $15+12c$  and  $19+12c$ , and have proven  $11+12c$  to suit the conjecture by the leftward operational rule. For final remainder two kinds  $15+12c$  and  $19+12c$  between  $n$  and  $2n+1$ , we have proven too them to suit the conjecture via finite operations by the leftward operational rule. In a word, we have proven every odd number between  $n$  and  $2n+1$  to suit the conjecture by the leftward operational rule, so all odd number between  $n$  and  $2n+1$  are in the bunch of integers' chains according to the preceding axiom.

Altogether, we have proven that all consecutive integers from 1 to  $2n$  within integers got successively are in the bunch of integers' chains via finite operations by set each other's contrary operational rules.

Since we can get all consecutive integers between  $n$  and  $2n+1$  from consecutive integers got from 1 to  $n$  via finite operations by set each other's contrary operational rules, likewise we can get all consecutive integers between  $2n$  and  $4n+1$  from consecutive integers got from 1 to  $2n$  via finite operations by set each other's contrary operational rules,

according to the foregoing way of doing. Thus we can get consecutive integers from 1 to  $4n$ , and they all suit the conjecture, then they all are in the bunch of integers' chains.

If we divide consecutive positive integers from small to great into segments according to their number  $2^X n$  per segment, where  $X \geq 0$  from small to large, and  $n \geq 18$ , then after get consecutive integers from 1 to  $2^X n$ , can further get consecutive integers from 1 to  $2^{X+1} n$  according to the foregoing way of making the proof.

$X$  begins with 0, next it is orderly endowed with 1, 2, ... In pace with which values of  $X$  are getting greater and greater, consecutive integers from 1 to  $2^X n$  are getting more and more, and greater and greater too, when  $X$  reaches infinity, get all positive integers, and all positive integers are in the bunch of integers' chains.

That is to say, the bunch of integers' chains contains all positive integers, provided operate positive integers got progressively at infinite many times by set each other's contrary operational rules.

Consequently begin with any positive integer to operate by the leftward operational rule, eventually reach a result, and the result is always 1, according to one-to-one correspondence between positive integers in the bunch of integers' chains and positive integers which can operate into 1 by the leftward operational rule.

The proof was thus brought to a close, as a consequence, the Collatz

conjecture is tenable.