

Proof of Riemann hypothesis

(This paper refuses kindly the premium from Clay)

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Abstract. Let $\text{Re}(s) = \pi(n) / n, (n=2)$, and $\text{Im}(s) = ((\pi(n) - \pi(2)) \log n) i, (n>2)$, Riemann hypothesis is proved.

Key words: even prime number, odd prime number, non-trivial zero point.

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1. The prime numbers consists of even prime number and odd prime numbers.
2. German mathematician Riemann considered that the distribution of primes is closely related to the zeros of the zeta function $\zeta(s)$, i.e. $\text{Re}(s) = f_1(\pi(n))$. Let the density of even prime number $\pi(n) / n, (n=2)$ be the $f_1(\pi(n))$. Thus, in 1859, Riemann wrote “ $s = 1/2 + ti$ ” in his article which was referred to the Academy of Sciences of Berlin. [1]
3. Riemann considered that the distribution of primes is closely related to the zeros of the zeta function $\zeta(s)$, i.e. $\text{Im}(s) = f_2(\pi(n))$, which can be got only from the odd prime numbers, but Riemann had not explained. Don't worry! Let the divergent factors of odd prime numbers $((\pi(n) - \pi(2)) \log n) i, (n>2)$ be the $f_2(\pi(n))$.

Thus, we can write all true non-trivial zero points of the Riemann zeta function $\zeta(s)$ when $n>2$. Example:

n	σ	t	$s = \sigma + ti$	
1	0	0	$s = 0$	
2	1/2	0	$s = 1/2 + 0i$	↑ trivial zero point
3	1/2	1 log 3	$s = 1/2 + 1.0986 i$	↓ non-trivial zero point
4	1/2	1 log 4	$s = 1/2 + 1.3862 i$	
5	1/2	2 log 5	$s = 1/2 + 3.2188 i$	
6	1/2	2 log 6	$s = 1/2 + 3.5835 i$	
7	1/2	3 log 7	$s = 1/2 + 5.8377 i$	
8	1/2	3 log 8	$s = 1/2 + 6.2383 i$	
9	1/2	3 log 9	$s = 1/2 + 6.5916 i$	
10	1/2	3 log 10	$s = 1/2 + 6.9077 i$	
11	1/2	4 log 11	$s = 1/2 + 9.5915 i$	
12	1/2	4 log 12	$s = 1/2 + 9.9396 i$	
13	1/2	5 log 13	$s = 1/2 + 12.8247 i$	

4. Given an exponential function of the density of the odd prime number,

$$y = n^{(\pi(n) - \pi(2)) / n}, \quad \text{i.e. } \pi(n) - \pi(2) = (n / \log n) \log y.$$

$$\text{When } n>2, \quad \log y_{\min} = \log 4^{1/4} = \lambda, \quad \log y_{\max} = \log 113^{29/113} = \mu,$$

$$(n / \log n) \lambda \leq \pi(n) - \pi(2) \leq (n / \log n) \mu, \quad (2 < n < \infty).$$

$$n \lambda \leq t \leq n \mu, \quad (2 < n < \infty).$$

$$\pi(2) + (n / \log n) \lambda \leq \pi(n) \leq \pi(2) + (n / \log n) \mu, \quad (2 < n < \infty).$$

This is the distributive law of primes in the natural numbers.

∴ Riemann hypothesis is proved.

References

- [1]. Riemann. 《Über die Anzahl der Primzahlen unter einer gegebenen Größe》, 1859.