The distributive law of primes and the proof of Goldbach hypothesis

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Abstract. This paper concerns a graph of $(3\leqslant x\leqslant 2500, N)$ orthogonal coordinate system.

1. Distributive law of primes,

   $(x/\log x)\log e<\pi(x)\leqslant(x/\log x)\log 19^{9/19}$, $(3\leqslant x<\infty)$;

2. ①. Folding expression of odd numbers, $(x=2n-1)$,

   $| x,\cdots,\cdots,\cdots,2x-1 |$

   $| x,\cdots,\cdots,\cdots, 1 |$,

   Number of odd number in pairs $=(x+1)/2$;

   ②. Folding expression of odd numbers, $(x=2n)$,

   $| x+1,\cdots,\cdots,\cdots,2x-1 |$

   $| x-1,\cdots,\cdots,\cdots, 1 |$,

   Number of odd number in pairs $=x/2$;

3. Arithmetic average of N,

   $MN=(\pi(2x-1)-\pi(x-1))(\pi(x)-\pi(0))/((x+1)/2)$, $(x=2n-1)$;

   $MN=(\pi(2x-1)-\pi(x))(\pi(x-1)-\pi(0))/(x/2)$, $(x=2n)$;

4. Infimum of N,

   $N\geqslant[k(x)]+1$, $(5\leqslant x=2n-1<\infty)$;

   $N\geqslant[f(x)]+1$, $(8\leqslant x=2n<\infty)$;

5. Uniformly continuous,

   Union formula, $N\geqslant[k(x)]+1$, $(5\leqslant x<\infty)$,

   Critical point;

6. Monotone increasing,

   $N>1$, $N\geqslant[k(x)]+1\geqslant 1$, $(5\leqslant x<\infty)$;

   $N\geqslant 1$, $(1\leqslant x<\infty)$;

7. Supremum of N,

   $N^*\leqslant\pi(2x-1)-\pi(x-1)=h(x)$, $(3\leqslant x=2n-1<\infty)$;

   $N^*\leqslant\pi(2x-1)-\pi(x)=g(x)$, $(4\leqslant x=2n<\infty)$;

   Critical points.

Key words. Goldbach hypothesis, supremum, infimum, prime in pairs p_1+p_2.

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Theorem 1. The distributive law of primes in the natural numbers,

$$(x/\log x)\,\alpha<\pi(x)\leqslant(x/\log x)\log 19^{9/19},\ (3\leqslant x<\infty). \qquad (1)$$

Proof. If $y=x^{\pi(x)/x}$, then $\pi(x)=(x/\log x)\log y$,

$\because$ $\lim\ \pi(x)/x=\lim 1/\log x$, $(x\to\infty)$. [1]

We have $\lim x^{\pi(x)/x}=\lim\ x^{1/\log x}$, $(x\to\infty)$.

$\because$ $x^{1/\log x}=e$. $\lim x^{\pi(x)/x}=e=y_{min}$, $(x\to\infty)$. $\log\ y_{min}=\log e=\alpha$.

When $x\geqslant 3$, we have $y_{min}<y\leqslant y_{max}$,

$\pi(1)=1$, $\log\ y_{max}=\log\ 19^{9/19}$.

(1) is obtained. Theorem 1 is proved.

Theorem 2. $N\geqslant$ the infimum of N,

$$N\geqslant[(2x-1)\alpha/\log(2x-1)-(x-1)\log 19^{9/19}/\log(x-1))(x\alpha/\log x)/((x+1)/2)]+1$$
$$=[k(x)]+1\geqslant 1,\ (5\leqslant x<\infty). \qquad (2)$$

Proof. Let N be the number of prime in pairs p_1+p_2 which suit $2x=p_1+p_2,(2<p_1\leqslant p_2)$

when the natural number $x \geqslant 3$ is given.

All points (x, N), ($3 \leqslant x \leqslant 2500$) form the graph showing the number of ways an even number can be written as the sum of two primes.

$\because\ 2 < p_1 \leqslant p_2,\quad 4 < 2p_1 \leqslant p_2 + p_1 = 2x,\quad \therefore\ 2 < p_1 \leqslant x.$

$N = \Sigma(\pi(p_2) - \pi(p_2 - 1)),\quad (2 < p_1 \leqslant p_2 = 2x - p_1),$

$= \Sigma(\pi(2x - p_1) - \pi(2x - p_1 - 1)),\quad (2 < p_1 \leqslant x).$ (3)

①. When $x = 2n - 1$, given a folding expression of odd number of the interval [1,2x −1] as the following:

$|\ x, \cdots, \cdots, \cdots, 2x - 1\ |$

$|\ x, \cdots, \cdots, \cdots,\ 1\ |$,

The sum of two meeting numbers $= 2x$.

Upper row contains p_2, lower row contains p_1,

The number of odd number in pairs = $(x+1)/2$.

The arithmetic average of N,

$MN = (\pi(2x - 1) - \pi(x - 1))(\pi(x) - \pi(0)) / ((x+1)/2).$ (4)

By (1), transforming (4) into the infimum of N,

$N \geqslant [((2x-1)\alpha/\log(2x-1) - (x-1)\log 19^{9/19}/\log(x-1))(x\alpha/\log x)/((x+1)/2)] + 1$

$= [k(x)] + 1,\quad (5 \leqslant x = 2n - 1 < \infty).$ (G)

②. When $x = 2n$, given a folding expression of odd number of the interval [1,2x −1] as the following:

$|\ x+1, \cdots, \cdots, \cdots, 2x - 1\ |$

$|\ x-1, \cdots, \cdots, \cdots,\ 1\ |$,

The sum of two meeting numbers $= 2x$.

upper row contains p_2, lower row contains p_1,

The number of odd number in pairs = $x/2$.

The arithmetic average of N,

$MN = (\pi(2x - 1) - \pi(x))(\pi(x - 1) - \pi(0))/(x/2).$ (5)

By (1), transforming (5) into the infimum of N,

$N \geqslant [((2x-1)\alpha/\log(2x-1) - x\log 19^{9/19}/\log x)((x-1)\alpha/\log(x-1))/(x/2)] + 1$

$= [f(x)] + 1,\quad (8 \leqslant x = 2n < \infty).$ (H)

From (G),(H), when $n \geqslant 3$, $[k(x)] + 1 \geqslant [f(x)] + 1 \geqslant 1$,

For choosing the greatest lower bound, we delete [f(x)],

$N \geqslant [((2x-1)\alpha/\log(2x-1) - (x-1)\log 19^{9/19}/\log(x-1))(x\alpha/\log x)/((x+1)/2)] + 1$

$= [k(x)] + 1,\quad (5 \leqslant x = 2n - 1 < \infty).$

The characteristcs of the infimum of N,

ⅰ. uniformly continuous.

k(x) is an elementary function, its interval of definition [5, x] is closed, thus, k(x), [k(x)]+1 are uniformly continuous. [2]

$\therefore\ N \geqslant [k(x)] + 1,\quad (5 \leqslant x = 2n - 1 \text{ or } 2n < \infty).$

When $x \geqslant 5$, $N \geqslant [k(x)] + 1 \geqslant 1$,

When $x = 5 \sim 18$, $[k(x)] + 1 = 1$,

When $x = 19 \sim 50$, $[k(x)] + 1 = 2$,

When $x = 51 \sim 89$, $[k(x)] + 1 = 3$,

……

This infimum of N is a ladder line.

When $x=34$, $N=[k(x)]+1=2$, critical point,

ii. monotone increasing

Differentiating the function k(x):

$k'(x) = (A(BC+DE) - BD)F, \ (5\leqslant x<\infty)$.

$A = x+1>0$,

$B = (2x-1)\alpha/\log(2x-1) - (x-1)\log 19^{9/19}/\log(x-1)>0$,

$C = (\log x - 1)\alpha/(\log x)^2>0$,

$D = x\alpha/\log x>0$,

$E = 2(\log(2x-1)-1)\alpha/(\log(2x-1))^2 - (\log(x-1)-1)\log 19^{9/19}/(\log(x-1))^2>0$,

$F = 2/(x+1)^2>0$,

When $x\geqslant 5$, $A(BC+DE) - BD>F$, $(x^2(\log x)^{-2}>x^{-2})$.

$k'(x)>0, (5\leqslant x<\infty)$. k(x) is monotone increasing in [5, x].

$N>1$, $N\geqslant[k(x)]+1\geqslant 1, (5\leqslant x<\infty)$.

(2) is obtained. Theorem 2 is proved.

Theorem 3. All even numbers $2x\geqslant 2$ can be expressed as the sum of two primes.

Proof. From Theorem 2, $N>1$, $(5\leqslant x<\infty)$.

From (3), $N\geqslant 1$, $(3\leqslant x\leqslant 5)$.

Now, let N^* be the number of prime in pairs p_1+p_2 which suit

$2x=p_1+p_2, (1\leqslant p_1\leqslant p_2)$. $N^*\geqslant N$,

∵ $\pi(1) = 1$, 1 is a prime, when $x = 2$, $2x = 1+3$; when $x = 1$, $2x = 1+1$,

∴ $N^*\geqslant N\geqslant 1$, $(1\leqslant x<\infty)$.

Theorem 4 is proved. Goldbach hypothesis is proved.

Theorem 4. $N^*\leqslant$ the supremum of N^*,

$N^*\leqslant \pi(2x-1) - \pi(x-1) = h(x), (3\leqslant x=2n-1)$. (6)

$N^*\leqslant \pi(2x-1) - \pi(x) = g(x), \quad (4\leqslant x=2n)$. (7)

Proof. From Theorem 2, | x,…,…,…,2x - 1 |

| x,…,…,…, 1 | ,

$N^*\leqslant \pi(2x-1) - \pi(x-1) = h(x), \quad (3\leqslant x=2n-1)$.

From Theorem 2, | x+1,…,…,…,2x - 1 |

| x−1,…,…,…, 1 | ,

$N^*\leqslant \pi(2x-1) - \pi(x) = g(x), \quad (4\leqslant x=2n)$.

When $x=3,4,5,6$, $N^*=h(x)=g(x)=2$, critical points.

When $x=7,9$, $N^*=h(x)=3$, critical points.

When $x=12,15,18$, $N^*= h(x)=g(x)=4$, critical points.

When $x=21$, $N^*=h(x)=5$, critical point.

When $x=24$, $N^*=g(x)=6$, critical point.

When $x=30$, $N^*=g(x)=7$, critical point.

When $x=45$, $N^*=h(x)=10$, critical point.

When $x=105$, $N^*=h(x)=19$, critical point.

(6), (7), are obtained. Theorem 4 is proved.

References

[1]. Hadamard & De La Vall′ee Poussin, Prime number Theorem. 1896.

[2]. Cantor, Cantor Theorem about uniformly continuous. 1872.