The distributive law of primes in the natural numbers

Even numbers 2x⩾6 can be expressed as the sum of two primes

Zhang xi-wen (CHINA)

Abstract. Find out the distributive law of primes in the natural numbers.

By means of the distributive law of primes we show that even numbers 2x⩾6 can be expressed as the sum of two primes.

Key words. Goldbach hypothesis; prime; infimum.

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Theorem 1. The distributive law of primes in the natural numbers,

$$\left(\frac{x}{\log x}\right)\cdot 1 < \pi(x) \leqslant \left(\frac{x}{\log x}\right)\log 113^{\frac{30}{113}},\ (11\leqslant x<\infty). \qquad (1)$$

Proof. To found an exponential function of density of primes, $y = x^{\frac{\pi(x)}{x}}$,

then $\pi(x) = \left(\frac{x}{\log x}\right)\log y.$

$$\because \lim_{x\to\infty}\frac{\pi(x)}{x} = \lim_{x\to\infty}\frac{1}{\log x}, \qquad [1]$$

$$\therefore \lim_{x\to\infty} x^{\frac{\pi(x)}{x}} = \lim_{x\to\infty} x^{\frac{1}{\log x}},$$

$$\because x^{\frac{1}{\log x}} = e.$$

$$\therefore \lim_{x\to\infty} x^{\frac{\pi(x)}{x}} = e = y_{min},$$

$$\log y_{min} = 1,$$

when $x\geqslant 11$, $y_{min} < y \leqslant y_{max}$,

$$\log y_{max} = \log 113^{\frac{30}{113}}.$$

(1) is obtained.

Theorem 1 is proved.

Theorem 2. The number of Goldbach partition for even numbers and its infimum.

$$D \geqslant \left[\frac{\left(\frac{2x-3}{\log(2x-3)} - \frac{2}{\log 2}\log 113^{\frac{30}{113}}\right)^2}{2(2x-5)}\right]+1$$

$$=[k(x)]+1\geqslant 1,\quad (11\leqslant x<\infty). \qquad (2)$$

Proof. 1. Let D be the discrete number of prime in pairs p_1, p_2 which suit $2x=p_1+p_2$, $(3\leqslant p_1\leqslant p_2\leqslant 2x-3)$ from the interval $[3,\ 2x-3]$ when the natural number $x\geqslant 3$ is given.

$\because$ $p_1\leqslant p_2\leqslant 2x-3$, $p_1+p_2\leqslant 2p_2\leqslant 2(2x-3)$, $\therefore$ $x\leqslant p_2\leqslant 2x-3$,

$$D = \sum_{p_2 \ge x}^{2x-3} (\pi(2x - p_2) - \pi(2x - p_2 - 1)),$$

$\because$ $3 \leqslant p_1 \leqslant p_2$, $6 \leqslant 2p_1 \leqslant p_2 + p_1$, $\therefore$ $3 \leqslant p_1 \leqslant x$,

$$D = \sum_{p_1 \ge 3}^{x} (\pi(2x - p_1) - \pi(2x - p_1 - 1)), \qquad (3)$$

D is the number of Goldbach partition for even numbers.

In the rectangular coordinate system, we have innumerable discrete points (x, D).

2. Given an expression of two opposed directional sequences 3～2x－3 as the following:

| 3 , …, …, …, 2x－3 |
| 2x－3 , …, …, …, 3 |

The sum of two meeting numbers upper and lower＝2x.

The number of natural number in pairs＝2x－5.

From two opposed directional sequences, we may obtain double D .

Let C be the continuous number of prime in pairs p_1, p_2 which suit $2x = p_1 + p_2$, ($3 \leqslant p_1 \leqslant p_2 \leqslant 2x-3$) from above expression when the natural number $x \geqslant 3$ is given.

$$C = \left[\frac{(\pi(2x-3) - \pi(2))^2}{2(2x-5)} \right] + 1, \quad (3 \leqslant x < \infty). \qquad (4)$$

In the same rectangular coordinate system, we have a rising oblique line C consisting of all points (x, C) across the innumerable discrete points (x, D).

The innumerable discrete points (x, D) are partitioned by the line C into three parts.

ⅰ. Above the line C, there are innumerable discrete points (x, D), D＞C,

Ex. x＝5, D＝2, C＝1,
x＝105, D＝19, C＝5,
x＝2500, D＝76, C＝45,

ⅱ. On the line C, there are some discrete points (x, D), D＝C,

Ex. x＝3, D＝1, C＝1,
x＝124, D＝6, C＝6,
x＝1468, D＝31, C＝31,

ⅲ. Under the line C, there are some discrete points (x,D), D＜C,

Ex. x＝34, D＝2, C＝3,
x＝64, D＝3, C＝4,
x＝166, D＝6, C＝7,
x＝199, D＝7, C＝8,
x＝316, D＝10, C＝11,
x＝346, D＝11, C＝12,
…

3. By (1), transforming (4) into the infimum of C,
(decreasing minuend, increasing subtrahend)

$$C_{inf}=\left[\frac{\left(\frac{2x-3}{\log(2x-3)}-\frac{2}{\log 2}\log 113^{\frac{30}{113}}\right)^2}{2(2x-5)}\right]+1$$

$=[k(x)]+1$, $(11\leqslant x<\infty)$. (5)

4. By means of some discrete points under the oblique line C, we will prove the infimum of C = the infimum of D,

when $x\geqslant 11$, $[k(x)]+1\geqslant 1$,

when $x=34$, $D=2$, $[k(x)]+1=2$, (34, 2) is a critical point.

when $x=64$, $D=3$, $[k(x)]+1=3$, (64, 3) is a critical point.

when $x=166$, $D=6$, $[k(x)]+1=5$, difference = 1.

when $x=199$, $D=7$, $[k(x)]+1=5$, difference = 2.

when $x=316$, $D=10$, $[k(x)]+1=8$, difference = 2.

when $x=346$, $D=11$, $[k(x)]+1=8$, difference = 3.

…

∴ $[k(x)]+1$ also is the infimum of D .

$C_{inf}=D_{inf}$,

$$D_{inf}=\left[\frac{\left(\frac{2x-3}{\log(2x-3)}-\frac{2}{\log 2}\log 113^{\frac{30}{113}}\right)^2}{2(2x-5)}\right]+1$$

$=[k(x)]+1\geqslant 1$, $(11\leqslant x<\infty)$.

$x=11\sim 30$, $D_{inf}=1$;

$x=31\sim 63$, $D_{inf}=2$;

$x=64\sim 104$, $D_{inf}=3$;

$x=105\sim 150$, $D_{inf}=4$;

$x=151\sim 201$, $D_{inf}=5$;

$x=202\sim 255$, $D_{inf}=6$;

…

In the same rectangular coordinate system, we have a rising ladder-like line D_{inf} under all discrete points (x, D).

The characteristics of $[k(x)]+1$,

①. Uniformly continuous,

k(x) is an elementary function, its interval of definition [11, x] is closed, thus, k(x), $[k(x)]+1$ are uniformly continuous. [2]

②. Monotone increasing,.

Differentiating the function k(x):

$k'(x)=(EFG-FF)H$, $(11\leqslant x<\infty)$.

$E=2x-5>0$,

$$F=\frac{2x-3}{\log(2x-3)}-\frac{2}{\log 2}\log 113^{\frac{30}{113}}>0,$$

$$G=\frac{2(\log(2x-3)-1)}{(\log(2x-3))^2}>0,$$

$$H = \frac{1}{(2x-5)^2} > 0,$$

When $x \geqslant 11$, $EFG - FF > H > 0$,

$k'(x) > 0$, $(11 \leqslant x < \infty)$.

$k(x)$ is monotone increasing in $[11, x]$.

$D \geqslant D_{\inf} = [k(x)] + 1 \geqslant 1$, $(11 \leqslant x < \infty)$.

(2) is obtained.

Theorem 2 is proved.

Theorem 3. Even numbers $2x \geqslant 6$ can be expressed as the sum of two primes.

Proof. From Theorem 2, $D \geqslant 1$, $(11 \leqslant x < \infty)$.

From (3), $D \geqslant 1$, $(3 \leqslant x \leqslant 11)$.

$\therefore$ $D \geqslant 1$, $(3 \leqslant x < \infty)$.

Theorem 3 is proved.

Theorem 4. The number of Goldbach partition for even numbers and its supremum.

$D \leqslant \pi(2x-3) - \pi(x-1)$, $(3 \leqslant x = 2n-1 < \infty)$. (6)

$D \leqslant \pi(2x-3) - \pi(x)$, $(4 \leqslant x = 2n < \infty)$. (7)

Proof. Given a folding expression of $[3, 2x-3]$ as the following:

When $x = 2n-1$,

$|\ x, \cdots, \cdots, \cdots, 2x-3|$

$|\ x, \cdots, \cdots, \cdots, \quad 3\ |$

Upper row contains p_2, lower row contains p_1,

$D \leqslant$ the number of p_2.

$D \leqslant D_{sup} = \pi(2x-3) - \pi(x-1) = h(x)$, $(3 \leqslant x = 2n-1 < \infty)$.

When $x = 2n$,

$|\ x+1, \cdots, \cdots, \cdots, 2x-3|$

$|\ x-1, \cdots, \cdots, \cdots, \quad 3\ |$

Upper row contains p_2, lower row contains p_1,

$D \leqslant$ the number of p_2.

$D \leqslant D_{sup} = \pi(2x-3) - \pi(x) = g(x)$, $(4 \leqslant x = 2n < \infty)$.

$x = 3, 4, (6)$, $D_{sup} = h(x) = g(x) = 1$, critical point.

$x = 5, 7, 8, 9$, $D_{sup} = h(x) = g(x) = 2$, critical point.

$x = 12, 15$, $D_{sup} = h(x) = g(x) = 3$, critical point.

$x = 18, 21$, $D_{sup} = h(x) = g(x) = 4$, critical point.

$x = 24$, $D_{sup} = g(x) = 5$, critical point.

$x = 30$, $D_{sup} = g(x) = 6$, critical point.

$x = 45$, $D_{sup} = h(x) = 9$, critical point.

$x = 105$, $D_{sup} = h(x) = 19$, critical point.

In the same rectangular coordinate system, we have a rising ladder-like line D_{sup} above all discrete points (x, D).

(6), (7) are obtained. Theorem 4 is proved.

References

[1]. Hadamard & De La Vall′ee Poussin, Prime number Theorem.1896.

[2]. Cantor, Cantor Theorem about uniformly continuous. 1872.