Proceedings of Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 2004.

A Distributed Algorithm for Maximum Lifetime Routing in Ad Hoc Wireless Networks

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1 Introduction

An ad hoc network of nodes with wireless transceiver capabilities is considered. We propose a distributed algorithm to compute an optimal routing scheme that maximizes the time at which the first node in the network drains out of energy. This problem is formulated as a linear programming problem and a subgradient algorithm is used to solve it in a distributed manner. The resulting algorithm has low computational complexity and is guaranteed to converge to an optimal routing scheme that maximizes the network lifetime. The algorithm is illustrated by an example in which an optimal flow is computed for a network of randomly distributed nodes.

1.1 Problem Definition

We consider a static wireless ad-hoc network modeled as an undirected graph $G(N, A)$ where N is the set of nodes, and A is the set of links. Two nodes i and j are connected by a link if they can transmit a packet to each other with a transmission power less than the maximum transmission power at each node. Thus all links are assumed to be bi-directional. This assumption is not necessary for the convergence of our distributed algorithm; however it makes the presentation clearer. We denote by N_i the set of nodes that are connected to node i by a link.

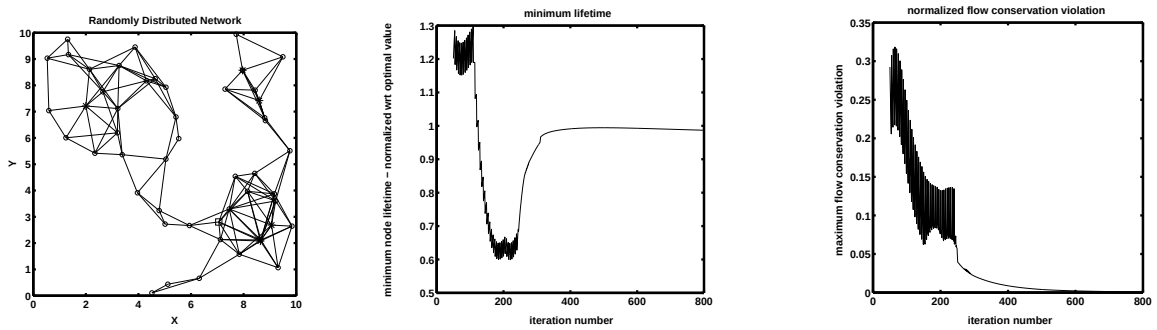
Each node i is assumed to have an initial battery energy E_i. Let R_i be the rate at which information is generated at node i; this information needs to be communicated to the sink node which we denote as s. We will write $R_s = -\sum_{i \in N, i \neq s} R_i$. For a network flow r, let r_{ij} denote the rate of information flow from node i to node j. Assume that the energy spent by node i to transmit a unit of information to node j is e_{ij}. Then the lifetime of node i under flow $r = \{r_{ij}\}$ is given by $T_i(r) = \frac{E_i}{\sum_{j \in N_i} e_{ij} r_{ij}}$. We define the *system lifetime* T_{sys} under flow r to be the time until the first node runs out of energy, i.e. $T_{\text{sys}}(r) = \min_{i \in N} T_i(r)$. The goal is to find a distributed algorithm to compute a flow $r = \{r_{ij}\}$ that maximizes the system lifetime. The model considered in this paper, as well as the formulation of the *lifetime maximization problem* is similar to that in [1–3]. Our contribution is the proposal of a *distributed algorithm* to compute the optimal flow that maximizes the system lifetime. Our algorithm is guaranteed to converge to the optimal solution, unlike the algorithm described in [1, 2].

1.2 Linear Programming Formulation

It can be shown that maximizing the system lifetime, subject to the flow conservation constraints, is equivalent to solving the following linear program.

$$\begin{array}{lrcl} \text{min.} & q & & \\ \text{s.t.} & \sum_{j \in N_i} (r_{ij} - r_{ji}) & = & R_i \quad \forall i \in N \\ & r_{ij} & \geq & 0 \quad \forall i \in N, \forall j \in N_i \\ & \sum_{j \in N_i} e_{ij} r_{ij} & \leq & q E_i \quad \forall i \in N \end{array}$$

The optimal value of q gives the inverse of the maximum system lifetime.



(a) connectivity graph - 50 nodes (b) minimum node lifetime (c) violation of flow conservation equations

Figure 1. Computation of optimal flows over a 50 node network

2 Distributed Algorithm

For small values of ϵ, it can be shown that the following *subgradient algorithm* [4] solves the dual of the linear program to an arbitrary degree of accuracy.

$$\begin{aligned} q^{(k)}(\lambda,\nu) &= \arg\min_{0\le q\le Q}\left(q^2 - q\textstyle\sum_{i\in N}\lambda_i^{(k)}E_i\right) & \lambda_i^{(k+1)} &= \left(\lambda_i^{(k)} + \alpha_k h_i(\lambda^{(k)},\nu^{(k)})\right)_+ \\ r_{ij}^{(k)}(\lambda,\nu) &= \arg\min_{0\le r_{ij}\le R}\left(\varepsilon r_{ij}^2 + r_{ij}\left(\lambda_i^{(k)}e_{ij} + \nu_i^{(k)} - \nu_j^{(k)}\right)\right) & \nu_i^{(k+1)} &= \nu_i^{(k)} + \alpha_k f_i(\lambda^{(k)},\nu^{(k)}) \end{aligned} \tag{1}$$

Here λ and ν are the dual variables, and h, f are the corresponding subgradient components of the dual function. One simple condition for convergence is $\alpha_k \to 0$, and $\sum_{k=1}^{\infty}\alpha_k = \infty$. All the computations in the above equation need very low computational power. Note that we need a central node to update the variable q during each iteration. This node needs to collect the Lagrange multipliers λ_i from each node, and broadcast the updated value of q to all the nodes. All other communication is only between a node and its neighbors. Another equivalent linear programming formulation can be used to obtain a *fully distributed algorithm*; the details are omitted due to lack of space.

2.1 Simulation Results

The subgradient algorithm described in Section 3 was simulated over a network of 50 nodes randomly distributed over a square of side 10 units. Each node was assumed to have a maximum communication radius of 2 units. The energy consumption for sending a packet from node i to node j was taken to be $e_{ij} = c_1 + c_2 d^4$, with $c_2 = 0.1c_1$. Five nodes were selected as source nodes; these nodes generated information packets which were routed to the sink node. All the nodes were assumed to have equal initial energy. The results of the simulation are shown in Figure 1. The maximum violation of flow conservation is normalized with respect to the total flow in the network. The algorithm converges after about 600 iterations.

Acknowledgments

The authors would like to thank Stephen Boyd, Lin Xiao and Z.-Q. Luo for helpful discussions. Both authors were partially supported by the Stanford URI *Architectures for Secure and Robust Distributed Infrastructures*, AFOSR DoD award number 49620-01-1-0365. Ritesh Madan is also supported by a *Sequoia Capital Stanford Graduate Fellowship.*

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