第五章

狭义相对论

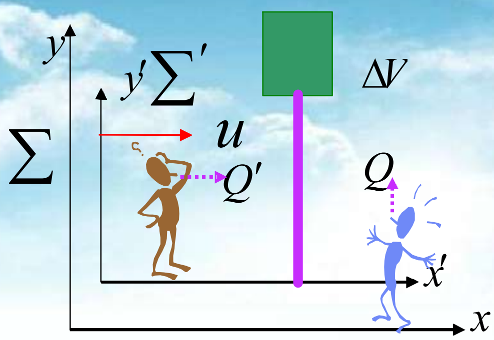
5.5电动力学的相对论不变性

5.5.1四维电流密度矢量

1、电荷密度的可变性

$Q = Q'$ 电荷是四维标量

Σ' 系观察



电荷静止时 $\rho_0 = \lim\limits_{\Delta v \to 0} = \dfrac{\Delta Q_0}{\Delta x_0 \Delta y_0 \Delta z_0}$

尺缩效应 $\Delta x = \Delta x_0 \sqrt{1 - \dfrac{u^2}{c^2}}$

$$\gamma_u = \frac{1}{\sqrt{1 - \dfrac{u^2}{c^2}}}$$

在Σ，电荷运动

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta x \Delta y \Delta z} = \lim_{\Delta v_0 \to 0} \frac{\Delta Q_0}{\Delta x_0 \Delta y_0 \Delta z_0 \sqrt{1 - \dfrac{u^2}{c^2}}}$$

$$\rho = \rho_0 \gamma_u$$

电荷密度不是四维标量，也不是四维矢量

2、四维电流分布矢量

3维电流密度 $\vec{J} = \rho\vec{u} = \gamma_u \rho_0 \vec{u}$

3维标量电荷密度与3维速度之积。

4维电流密度矢定义为:

4维标量电荷密度与4维速度之积。

$$J_\mu = \rho_0 U_\mu \qquad J_\mu = (\vec{J},\ ic\rho)$$

$$J_i = \rho_0 U_i = \gamma_u \rho_0 u_i = \rho u_i$$

$$U_i = u_i \gamma_u$$

$$J_4 = \rho_0 U_4 = ic\gamma_u \rho_0$$

$$U_4 = ic\gamma_u$$

显然它是四维矢量，它将 $\rho, \vec{J}$ 统一为整体，满足洛伦兹变换

$$J'_\mu = a_{\mu\nu} J_\nu$$

$$\begin{cases} J'_1 = \gamma(J_1 - v\rho) \\ J'_2 = J_2 \\ J'_3 = J_3 \\ \rho' = \gamma(\rho - \dfrac{v}{c^2} J_1) \end{cases}$$

四维电流密度矢量

$$J_\mu = (\vec{J},\ ic\rho)$$

电流密度

$$J_i = \rho_0 U_i = \gamma_u \rho_0 u_i$$

空间分量

$$J_4 = \rho_0 U_4 = ic\gamma_u \rho_0$$

时维分量

电荷密度

$$J'_\mu = a_{\mu\nu} J_\nu$$

$$\begin{pmatrix} J'_x \\ J'_y \\ J'_z \\ ic\rho' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} J_x \\ J_y \\ J_z \\ ic\rho \end{pmatrix} \Rightarrow \begin{cases} J'_x = \gamma(J_x - v\rho) \\ J'_y = Jy \\ J'_z = J_z \\ \rho' = \gamma(\rho - \dfrac{v}{c^2} J_x) \end{cases}$$

3、电荷守恒定律的四维形式

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial ic\rho}{\partial ict} = 0$$

$$J_4 = ic\rho, \quad x_4 = ict$$

$$\frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial J_4}{\partial x_4} = 0 \Rightarrow \frac{\partial J_\mu}{\partial x_\mu} = 0$$

为洛伦兹标量式，因此在洛伦兹变换下形式不变。

此方程具有协变性。
（数学上可理解为四维散度）

5.5.2四维势矢量

1、达朗伯方程的协变性

引入算符： $\Box=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

$x_4=ict$

$$\Box=\frac{\partial^2}{\partial x_1{}^2}+\frac{\partial^2}{\partial x_2{}^2}+\frac{\partial^2}{\partial x_3{}^2}+\frac{\partial^2}{\partial (ict)^2}=\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu}$$

洛伦兹标量算符

达朗伯方程

$$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{J}$$ → $$\Box\vec{A}=-\mu_0\vec{J}$$

$$\Box A_i=-\mu_0 J_i$$

分量式

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0}$$ → $$\Box\varphi=-\frac{\rho}{\varepsilon_0}$$

$$\Box\frac{i}{c}\varphi=-i\mu_0c\rho$$

$$\Box A_i = -\mu_0 J_i$$

$$\Box \frac{i}{c}\varphi = -i\mu_0 c\rho$$

定义四维矢势

$$A_\mu = (\vec{A}, i\frac{\varphi}{c})$$

$$\Box A_\mu = -\mu_0 J_\mu$$

其具有协变性

矢势与标势合为一个四维矢量，表明电场与磁场实为一体。

2、洛仑兹条件的协变性

$$\nabla\cdot\boldsymbol{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0 \;\rightarrow\; \frac{\partial A_i}{\partial x_i}+\frac{\partial i\frac{\varphi}{c}}{\partial ict}=0 \;\rightarrow\; \frac{\partial A_\mu}{\partial x_\mu}=0$$

其具有协变性

3.四维矢势的变换关系

$$A_\mu = (\boldsymbol{A}, i\frac{\varphi}{c})$$

$$A'_\mu = a_{\mu\nu} A_\nu$$

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \\ i\dfrac{\varphi'}{c} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \\ i\dfrac{\varphi}{c} \end{pmatrix} \Rightarrow \begin{aligned} A'_x &= \gamma(A_x - \frac{v}{c^2}\varphi) \\ A'_y &= A_y \\ A'_z &= A_z \\ \varphi' &= \gamma(\varphi - vA_x) \end{aligned}$$

电磁场四维矢量

$$V'_\mu = a_{\mu\nu} V_\nu$$

$$x_\mu = (\vec{\boldsymbol{x}}, ict)$$

$$U_\mu = (\vec{\boldsymbol{u}}, ic)\gamma_u$$

$$k_\mu = (\vec{\boldsymbol{k}}, i\frac{\omega}{c})$$

$$J_u = (\vec{\boldsymbol{J}}, ic\rho)$$

$$A_\mu = (\vec{\boldsymbol{A}}, i\frac{\varphi}{c})$$

5.5.3. 电磁场张量 麦克斯韦方程的协变性

1.电磁场张量

$$\vec{B}=\nabla\times\vec{A} \qquad \vec{E}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}=ic\nabla\frac{i\varphi}{c}-ic\frac{\partial\vec{A}}{\partial ict}$$

$$B_i=\frac{\partial A_k}{\partial x_j}-\frac{\partial A_j}{\partial x_k} \qquad -\frac{i}{c}E_j=\left(\frac{\partial A_4}{\partial x_j}-\frac{\partial A_j}{\partial x_4}\right) \qquad -\frac{i}{c}\vec{\boldsymbol{E}}=\left(\nabla A_4-\frac{\partial\vec{\boldsymbol{A}}}{\partial x_4}\right)$$

$$F_{\mu\gamma}=\frac{\partial A_\gamma}{\partial x_\mu}-\frac{\partial A_\mu}{\partial x_\gamma}$$

引入反对称电磁场张量

$$F=\begin{bmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{bmatrix}$$

电场和磁场为同一个物理量的不同分量，表明电磁场是一个不可分割的整体。

2.麦克斯韦方程的协变性

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = \mu_0 J_\mu$$

$\mu = 4$ → $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$\mu = 1,2,3$ → $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$

$$\frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} + \frac{2F_{\lambda\mu}}{\partial x_\nu} = 0$$

$\mu,\nu,\lambda = 1,2,3$ → $\nabla \cdot \vec{B} = 0$

$4,2,3 - -4,3,1$
$4,1,2$ → $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

因电场与时间维相联系，磁场与空间维相联系，所以关于电场的散度是μ=4,。磁场的旋度μ=1，2，3。关于磁场的散度是μ，ν，λ=1，2，3，所以关于电场的旋度是μ=4开头。

麦克斯韦方程在电磁场张量的表示下,由四个方程合并为两个方程，相当于对张量场进行散度和旋度分析。

可以证明其具有协变性

3.电磁场的变换关系

$$F'_{\mu\nu} = a_{\mu\lambda}a_{\nu\tau}F_{\lambda\tau}$$

非零项

$$F'_{24} = a_{2\lambda}a_{4\tau}F_{\lambda\tau} = a_{21}a_{4\tau}F_{1\tau} + a_{22}a_{4\tau}F_{2\tau} + a_{23}a_{4\tau}F_{3\tau} + a_{24}a_{4\tau}F_{4\tau}$$

$=$ [redacted] $+ a_{42}F_{22} + a_{43}F_{23} +$ [redacted] $= -i\beta\gamma F_{21} + \gamma F_{24}$

非零项

$$-\frac{i}{c}E'_2 = -i\frac{v}{c}\gamma(-B_3) - \gamma\frac{i}{c}E_2$$

$$E'_2 = \gamma(E_2 - vB_3)$$

$$a = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$F = \begin{bmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{bmatrix}$$

同理可得其它分量的变换关系

练习求出B_3变换式

练习求出B_3变换式 $$F'_{\mu\nu}=a_{\mu\lambda}a_{\nu\tau}F_{\lambda\tau}$$

$$F'_{12}=a_{1\lambda}a_{2\tau}F_{\lambda\tau}=a_{11}a_{2\tau}F_{1\tau}+\underline{a_{12}a_{2\tau}F_{2\tau}+a_{13}a_{2\tau}F_{3\tau}}+a_{14}a_{2\tau}F_{4\tau}$$

$$=\gamma a_{2\tau}F_{1\tau}+i\beta\gamma a_{2\tau}F_{4\tau}=\gamma a_{22}F_{12}+i\beta\gamma a_{22}F_{42}$$

$$B'_3=\gamma B_3+i\beta\gamma\left(\frac{i}{c}E_2\right)$$

$$B'_3=\gamma(B_3-\frac{v}{c^2}E_2)$$

$$a=\begin{pmatrix}\gamma & 0 & 0 & i\beta\gamma\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\beta\gamma & 0 & 0 & \gamma\end{pmatrix}$$

$$F=\begin{bmatrix}0 & B_3 & -B_2 & -\frac{i}{c}E_1\\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2\\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3\\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0\end{bmatrix}$$

电磁场各分量的变换关系

$$E_1' = E_1 \qquad E_2' = \gamma(E_2 - vB_3) \qquad E_3' = \gamma(E_3 + vB_2)$$

$$B_1' = B_1 \qquad B_2' = \gamma(B_2 + \frac{v}{c^2}E_3) \qquad B_3' = \gamma(B_3 - \frac{v}{c^2}E_2)$$

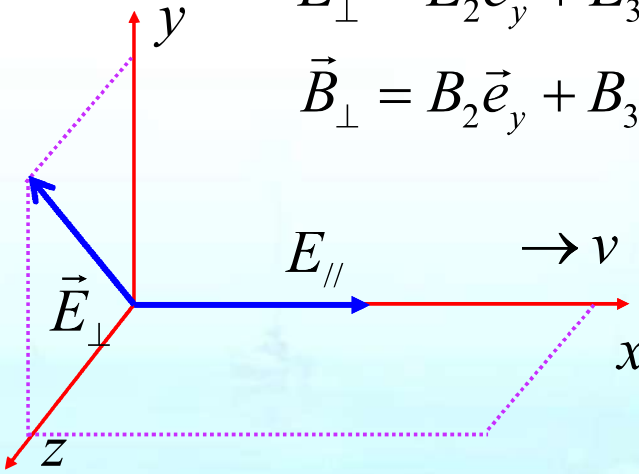
电磁场按平行和垂直于坐标系运动方向分解，则电磁场的变换式可写成

$$\begin{cases} \vec{E}'_{//} = \vec{E}_{//} \\ \vec{E}'_{\perp} = \gamma(\vec{E} + \vec{v} \times \vec{B})_{\perp} \end{cases}$$

$$\begin{cases} \vec{B}'_{//} = \vec{B}'_{//} \\ \vec{B}'_{\perp} = \gamma(\vec{B} - \frac{1}{c^2}\vec{v} \times \vec{E})_{\perp} \end{cases}$$

$$\vec{E}_{\perp} = E_2\vec{e}_y + E_3\vec{e}_z$$

$$\vec{B}_{\perp} = B_2\vec{e}_y + B_3\vec{e}_z$$



平行与垂直相对于速度方向而言

$$\vec{E}'_{\perp} = \gamma(\vec{E} + \vec{v} \times \vec{B})_{\perp}$$

$$= \gamma \vec{E}_{\perp} + \gamma [v\vec{e}_x \times (B_1\vec{e}_x + B_2\vec{e}_y + B_3\vec{e}_z)]_{\perp}$$

$$= \gamma \left(E_2\vec{e}_y + E_3\vec{e}_z \right) + \gamma [vB_2\vec{e}_z - vB_3\vec{e}_y]_{\perp}$$

$$= \gamma \left(E_2 - vB_3 \right)\vec{e}_y + \gamma (vB_2 + E_3)\vec{e}_z$$

$$\vec{E}'_{\perp} = E'_2\vec{e}_y + E'_3\vec{e}_z$$

$$E'_2 = \gamma(E_2 - vB_3) \qquad E'_3 = \gamma(E_3 + vB_2)$$

$$\begin{cases} \vec{E}'_{//} = \vec{E}_{//} & \vec{B}'_{//} = \vec{B}'_{//} \\ \vec{E}'_{\perp} = \gamma(\vec{E} + \vec{v} \times \vec{B})_{\perp} & \vec{B}'_{\perp} = \gamma(\vec{B} - \frac{1}{c^2}\vec{v} \times \vec{E})_{\perp} \end{cases}$$

当v<<c时，上式过渡到非相对论电磁场变换式

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B} \qquad \vec{B}' = \vec{B} - \frac{\vec{v}}{c^2} \times \vec{E}$$

至此，关于电磁现象的参考系问题完全得到解决，电动力学基本方程式对任意惯性参考系成立。在坐标变换下，势按四维矢量变换，电磁场按四维张得变换。

运用矩阵运算可得电磁场量的变换关系

$$F'_{\mu\gamma} = a_{\mu\lambda} a_{\nu\tau} F_{\lambda\tau} = a_{\mu\lambda} F_{\lambda\tau} \tilde{a}_{\tau\nu} \qquad \therefore F' = aF\tilde{a}$$

$$F' = \begin{bmatrix} 0 & B_3' & -B_2' & -\frac{i}{c}E_1' \\ -B_3' & 0 & B_1' & -\frac{i}{c}E_2' \\ B_2' & -B_1' & 0 & -\frac{i}{c}E_3' \\ \frac{i}{c}E_1' & \frac{i}{c}E_2' & \frac{i}{c}E_3' & 0 \end{bmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{bmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{bmatrix} \begin{pmatrix} \gamma & 0 & 0 & -i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$F' = \begin{bmatrix} 0 & B_3' & -B_2' & -\frac{i}{c}E_1' \\ -B_3' & 0 & B_1' & -\frac{i}{c}E_2' \\ B_2' & -B_1' & 0 & -\frac{i}{c}E_3' \\ \frac{i}{c}E_1' & \frac{i}{c}E_2' & \frac{i}{c}E_3' & 0 \end{bmatrix} = \begin{bmatrix} 0 & \gamma(B_3 - \frac{v}{c^2}E_2) & -\gamma(B_2 + \frac{v}{c^2}B_3) & -\frac{i}{c}E_1 \\ -B_3' & 0 & B_1 & -\frac{i}{c}\gamma(E_2 - vB_3) \\ \gamma(B_2 + \frac{v}{c^2}B_3) & -\gamma(B_3 - \frac{v}{c^2}E_2) & 0 & -\frac{i}{c}\gamma(E_3 + vB_2) \\ \frac{i}{c}E_1 & \frac{i}{c}\gamma(E_2 - vB_3) & \frac{i}{c}\gamma(E_3 + vB_2) & 0 \end{bmatrix}$$

四维矢量变换与时空变换的对应

$x_\mu = (\boldsymbol{x}, ict)$

空间分量对应：$x_i \leftrightarrow \{u_i\gamma_u,\ A_i,\ k_i,\ J_i\}$

时间分量对应：$\{\gamma_u,\ \varphi/c^2,\ \omega/c^2,\ \rho\} \leftrightarrow t$

$u_i\gamma_u \leftrightarrow U_\mu = (\boldsymbol{u}, ic)\gamma_u \leftrightarrow \gamma_u$

$A_i \leftrightarrow A_\mu = (\boldsymbol{A}, i\varphi/c) \leftrightarrow \varphi/c^2$

$k_i \leftrightarrow k_\mu = (\boldsymbol{k}, iw/c) \leftrightarrow \omega/c^2$

$J_i \leftrightarrow J_u = (\boldsymbol{J}, ic\rho) \leftrightarrow \rho$

$x_1' = (x_1 - vt)\gamma$

$x_2' = x_2$

$x_3' = x_3$

$t' = (t - \frac{v}{c^2}x_1)\gamma$

对时空变换进行对应置换，可得相应的相对论变换式。

变换举例:

$x_\mu = (\boldsymbol{x}, ict)$

$x_i \Leftrightarrow A_i$

$A_\mu = (\boldsymbol{A}, i\varphi / c)$

$t \Leftrightarrow \varphi / c^2$

$x_1' = (x_1 - vt)\gamma$ $x_2' = x_2$ $x_3' = x_3$ $t' = (t - \frac{v}{c^2} x_1)\gamma$

$A_1' = (A_1 - v\frac{\varphi}{c^2})\gamma$ $A_2' = A_2$ $A_3' = A_3$ $\varphi' / c^2 = (\varphi / c^2 - \frac{v}{c^2} A_1)\gamma$

$\varphi' = (\varphi - vA_1)\gamma$

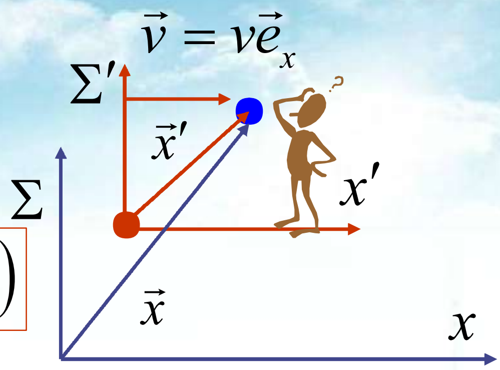
与教材**P196-197**一致

[例1] 求匀速 $\vec{v}$ 运动的带电荷为e的粒子的电磁场。

1.建立两个参照系

设 Σ'系的原点固定在粒子上，

粒子相对 Σ 以匀速v沿x轴正向运动.



$$\vec{x}=\left(x\vec{e}_x + y\vec{e}_y + z\vec{e}_z\right)$$

$$\vec{x}'=\left(x'\vec{e}_x + y'\vec{e}_y + z'\vec{e}_z\right)$$

$$r'^2 = \left(x'^2 + y'^2 + z'^2\right)$$

$$r^2 = \left(x^2 + y^2 + z^2\right)$$

$$r' = \left|\vec{x}'\right| \qquad r = \left|\vec{x}\right|$$

2.求在 Σ'系上测得的电场与磁场

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{e\vec{x}'}{r'^3} = \frac{e}{4\pi\varepsilon_0 r'^3}\left(x'\vec{e}_x + y'\vec{e}_y + z'\vec{e}_z\right)$$

$$\vec{B}' = 0$$

$$E_i' = \frac{ex_i'}{4\pi\varepsilon_0 r'^3}$$

$$B_i' = 0$$

3.求在 Σ 系上测得的电场与磁场

$$E_1 = E_1' \qquad E_2 = \gamma(E_2' + vB_3') \qquad E_3 = \gamma(E_3' - vB_2')$$

$$B_1 = B_1' \qquad B_2 = \gamma(B_2' - \frac{v}{c^2}E_3') \qquad B_3 = \gamma(B_3' + \frac{v}{c^2}E_2')$$

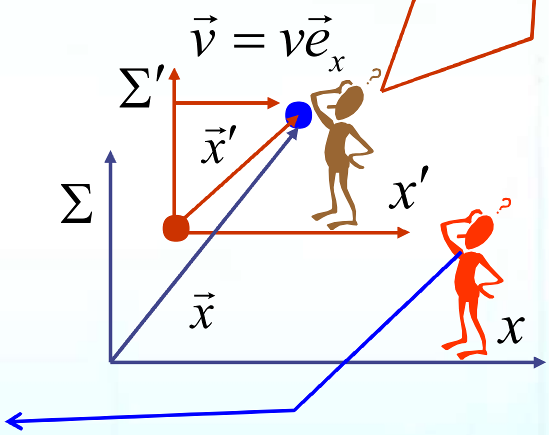
$$E_i' = \frac{ex_i'}{4\pi\varepsilon_0 r'^3}$$

$$B_i' = 0$$

$$E_x = \frac{ex'}{4\pi\varepsilon_0 r'^3}, \qquad B_x = 0$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\gamma\frac{ey'}{r'^3}, \qquad B_y = -\frac{1}{4\pi\varepsilon_0}\gamma\frac{vez'}{c^2 r'^3}$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\gamma\frac{ez'}{r'^3}, \qquad B_z = \frac{1}{4\pi\varepsilon_0}\gamma\frac{vey'}{c^2 r'^3}$$



设 $t=0$ 时粒子运动正好与 Σ 系的原点重合，并且我们就在这一时刻在 Σ 系中测量空间的场。用 Σ 系中的坐标表示电场与磁场。

$$x' = \gamma x, \qquad y' = y, \qquad z' = z$$

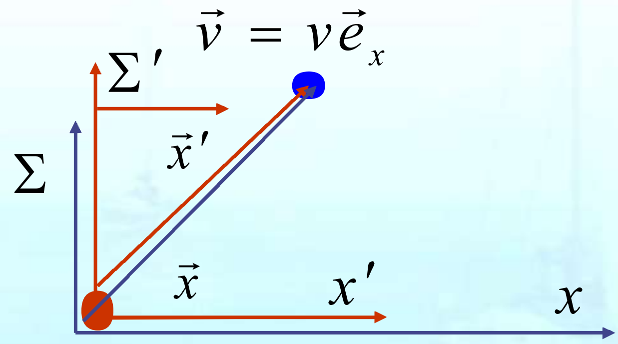
$$r'^2 = x'^2 + y'^2 + z'^2 = \gamma^2 x^2 + y^2 + z^2 = \gamma^2 \left(x^2 + (\frac{y}{\gamma})^2 + (\frac{z}{\gamma})^2 \right)$$

$$r'^2 = \gamma^2 \left(x^2 + (1 - \frac{v^2}{c^2})(y^2 + z^2) \right) = \gamma^2 \left[(1 - \frac{v^2}{c^2})(x^2 + y^2 + z^2) + \frac{v^2}{c^2} x^2 \right]$$

$$r' = \gamma \left[(1 - \frac{v^2}{c^2}) r^2 + \frac{(\vec{v} \cdot \vec{x})^2}{c^2}) \right]^{1/2}$$

***t*=0时两系原点重合**

$$\vec{v} = v\vec{e}_x \qquad \vec{v} \cdot \vec{x} = vx$$



$$\vec{E}=\left(E_x\vec{e}_x+E_y\vec{e}_y+E_z\vec{e}_z\right)$$

$$=\frac{e}{4\pi\varepsilon_0 r'^3}\left(x'\vec{e}_x+y'\gamma\vec{e}_y+z'\gamma\vec{e}_z\right)$$

$$E_x=\frac{ex'}{4\pi\varepsilon_0 r'^3} \qquad x'=\gamma x\ ,$$

$$E_y=\frac{1}{4\pi\varepsilon_0}\gamma\frac{ey'}{r'^3} \qquad y'=y$$

$$E_z=\frac{1}{4\pi\varepsilon_0}\gamma\frac{ez'}{r'^3} \qquad z'=z$$

$$\vec{E}==\frac{e}{4\pi\varepsilon_0 r'^3}\left(x\gamma\vec{e}_x+y\gamma\vec{e}_y+z\gamma\vec{e}_z\right)$$

$$r'=\gamma\left[(1-\frac{v^2}{c^2})r^2+\frac{(\vec{v}\cdot\vec{x})^2}{c^2})\right]^{1/2}$$

$$\vec{E}==\frac{\gamma e}{4\pi\varepsilon_0 r'^3}\vec{x}$$

$$\vec{x}=\left(x\vec{e}_x+y\vec{e}_y+z\vec{e}_z\right)$$

$$\vec{E}=\left(1-\frac{v^2}{c^2}\right)\frac{e\vec{x}}{4\pi\varepsilon_0\left[(1-\frac{v^2}{c^2})r^2+\frac{(\vec{v}\cdot\vec{x})^2}{c^2})\right]^{3/2}}$$

Σ系上测得的电场

Σ系中测得的磁感应强度

$$E_x = \frac{ex'}{4\pi\varepsilon_0 r'^3} \qquad x' = \gamma x\ ,$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\gamma\frac{ey'}{r'^3} \qquad y' = y$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\gamma\frac{ez'}{r'^3} \qquad z' = z$$

$$B_x = 0$$

$$B_y = -\frac{1}{4\pi\varepsilon_0}\gamma\frac{vez'}{c^2 r'^3} = -\frac{v}{c^2}E_z$$

$$B_z = \frac{1}{4\pi\varepsilon_0}\gamma\frac{vey'}{c^2 r'^3} = \frac{v}{c^2}E_y$$

$$\vec{v} = v\vec{e}_x$$

$$\vec{B} = \frac{\vec{v}}{c^2}\times\vec{E} = \frac{1}{c^2}\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ v & 0 & 0 \\ E_x & E_y & E_z \end{vmatrix}$$

4. 讨论电磁场的物理特征:

$$\vec{E}=\left(1-\frac{v^2}{c^2}\right)\frac{e\vec{x}}{4\pi\varepsilon_0\left[(1-\frac{v^2}{c^2})r^2+\frac{(\vec{v}\cdot\vec{x})^2}{c^2})\right]^{3/2}}$$

$$\vec{B}=\frac{\vec{v}}{c^2}\times\vec{E}$$

（1） 当$v<<c$时，略去$(\frac{v}{c})^2$ 的二级小量，得到

$$\vec{E}=\frac{e\vec{x}}{4\pi\varepsilon_0 r^3}=\vec{E}_{\text{静电场}} \qquad \vec{B}=\frac{\vec{v}}{c^2}\times\vec{E}=\frac{\mu_0 e\vec{v}\times\vec{x}}{4\pi r^3}$$

运动电粒子产生的电场等于其静止时产生的电场。

运动电粒子产生的磁场相当于一电流元$e\,v$产生的磁场

（2）当 $v \sim c$ 时

$$\vec{E} = \left(1-\frac{v^2}{c^2}\right)\frac{e\vec{x}}{4\pi\varepsilon_0\left[(1-\frac{v^2}{c^2})r^2+\frac{(\vec{v}\cdot\vec{x})^2}{c^2})\right]^{3/2}}$$

$$\vec{B} = \frac{\vec{v}}{c^2}\times\vec{E}$$

在与 $\vec{v}_\perp$ 方向上（即 $\vec{v}\cdot\vec{x}=0$ ）

$$\vec{E} = \gamma\frac{e\vec{x}}{4\pi\varepsilon_0 r^3} >> \vec{E}_{静电场} \quad (\theta=\frac{\pi}{2})$$

垂直方向远大于静电场，强度最大。

在与 $\vec{v}$ //的方向上，（即$\vec{v}\cdot\vec{x}=vr$ ）

$$\vec{E} = (1-\frac{v^2}{c^2})\frac{e\vec{x}}{4\pi\varepsilon_0 r^3} << \vec{E}_{静电场} \quad (\theta=0)$$

平行方向远小于静电场，强度最弱。

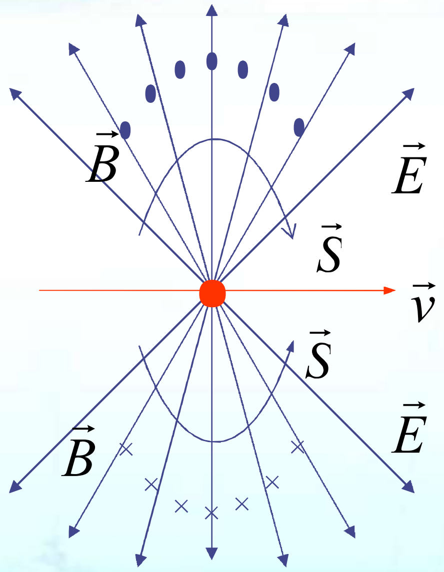
高速运动的点电荷产生的电场不再具有球对称性

(a)场趋向于垂直速度方向的平面近域集中，集中的程度与粒子运动速度有关。当 $v \to c$ 时，场基本上集中分布在垂直于运动方向且过粒子的平面附近.

$$\vec{E} = \gamma \frac{e\vec{x}}{4\pi\varepsilon_0 r^3} >> \vec{E}_{静电场} \quad (\theta = \frac{\pi}{2})$$

$$\vec{E} = (1-\frac{v^2}{c^2})\frac{e\vec{x}}{4\pi\varepsilon_0 r^3} << \vec{E}_{静电场} \quad (\theta = 0)$$

(b)场线沿着运动方向压缩，形成近垂直区域密集，近平行方向区域稀疏的图景。这与空间收缩效应相联系。



(c)能流分布　$\vec{S} = \vec{E} \times \vec{H}$　$\vec{S} \perp \vec{x}$

没有能流沿着径向方面辐射出去，能流是在以粒子为中心的球面上流动。能流伴随着粒子一起运动.

$$\vec{S} = \vec{E} \times \vec{H} = \vec{E} \times \vec{B} / \mu_0 = \frac{1}{\mu_0 c^2} \vec{E} \times (\vec{v} \times \vec{E})$$

$$= \varepsilon_0 \left[\vec{v} E^2 - \vec{E}(\vec{E} \cdot \vec{v}) \right]$$

$$\boxed{\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}}$$