第四章

电磁波的传播

4.5 波导

4.5.1高频电磁能量的传输

问题: *a*)高频幅射产生损耗问题。
b)趋肤效应，电阻增大，损耗加大。

1. 有界空间中的电磁波

TEM（transverse electromagnetic wave）波

电场和磁场在垂直传播方向上振动的电磁波。平面电磁波在无界空间中传播时就是典型的TEM波。

采用空心金属管（波导）来传播电磁波。

高频时电压的概念失效，用场的方法来研究电磁信号传输问题。

电磁波在良导体中及外表的电磁场特点

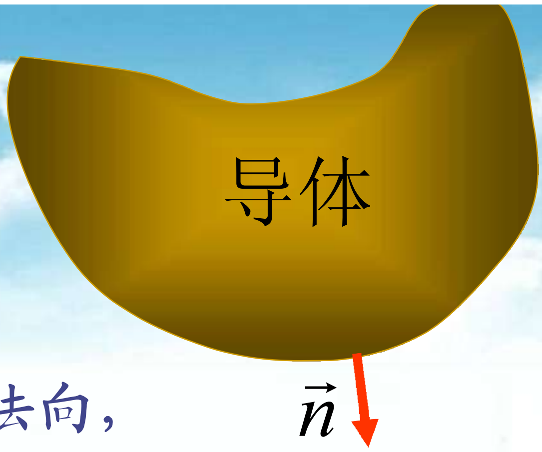
$$R \approx 1-2x = 1-2\sqrt{\frac{2\omega\varepsilon_0}{\sigma}} \to 1$$ 几乎全部反射

$$\boxed{\rho(t) = \rho_0 e^{-\frac{t}{\tau}}}$$ 体内几乎无剩余电荷

讨论$\sigma \to \infty$ 的理想导体(一般金属接近理想导体)。假定它的穿透深度 $\delta \to 0$（$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$）。

对于理想导体 $R = 1,\ \delta \to 0 \quad \rho = 0$

2. 理想导体的边界条件



$$\vec{n}\times(\vec{E}_2-\vec{E}_1)=0 \qquad \begin{cases}\vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\sigma\\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0\end{cases}$$

$$\vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{\alpha}$$

且令导体为介质1，则n为导体外法向，

体内电磁场： E_1=B_1=0

$$\boxed{\vec{n}\times\vec{H}=\vec{\alpha} \qquad \vec{n}\cdot\vec{D}=\sigma}$$

确定电流和电荷分布

$$\boxed{\vec{n}\times\vec{E}=0 \qquad \vec{n}\cdot\vec{B}=0}$$

用来定解，仅一个独立

定解条件可选： $\vec{n}\times\vec{E}=0$

结论：理想导体表面上的切向电场为0，法向磁场为0

定态电磁波的基本方程

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 0$$

边界附近的体现，可作为定解条件

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

设导体表面法向为z轴

x,y为切向分量

$$\frac{\partial E_n}{\partial n} = \frac{\partial E_z}{\partial z} = 0$$

$\vec{n}$

导体内

导体表面的电场边界条件

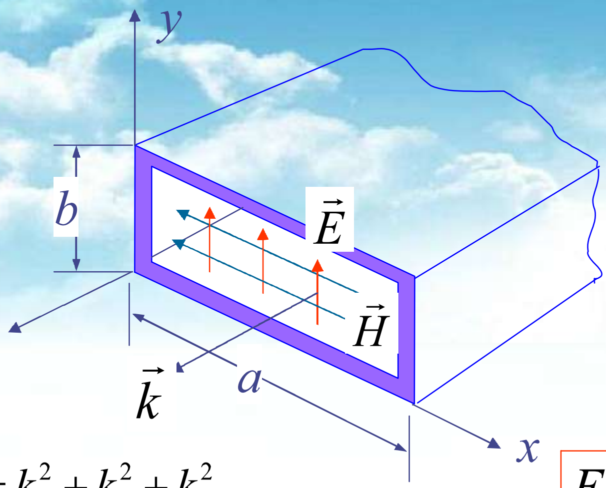
电场切向分量为0 → $E_t = 0$

电场法向分量的法向导数为0 → $\dfrac{\partial E_n}{\partial n} = 0$

4.5.2矩形波导（Waveguide）中的电磁波

1.沿z方向传播的时谐电磁波

$$\vec{E}(x,y,z)=\vec{E}(x,y)e^{ik_z z}$$



2..方程与边界条件

$$\nabla^2\vec{E}+k^2\vec{E}=0$$

$$\nabla^2 E_i+k^2 E_i=0 \qquad i=1,2,3 \qquad k^2=k_x^2+k_y^2+k_z^2$$

$$E_t=0$$

$$\frac{\partial E_n}{\partial n}=0$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial y^2}\right)E_i(x,y)+(k^2-k_z^2)E_i(x,y)=0$$

$$E_i=(C_1\cos k_x x+D_1\sin k_x x)(C_2\cos k_y y+D_2\sin k_y y)$$

边界条件

$$x=0,\ x=a \qquad E_y=E_z=0 \qquad \frac{\partial E_x}{\partial x}=0$$

$$y=0,\ y=b \qquad E_z=E_x=0 \qquad \frac{\partial E_y}{\partial y}=0$$

在边界上的电场:

电场函数值为0,取正弦项

电场法向导数为0取余弦项

$$E_i = (C_1 \cos k_x x + D_1 \sin k_x x)(C_2 \cos k_y y + D_2 \sin k_y y)$$

$$x=0,\ x=a \quad E_y = E_z = 0 \quad \frac{\partial E_x}{\partial x} = 0$$

$$y=0,\ y=b \quad E_z = E_x = 0 \quad \frac{\partial E_y}{\partial y} = 0$$

电场函数值为0,取正弦项

电场法向导数为0取余弦项

3.由边界条件定解

$x=0,$

$E_y = E_z = 0 \quad \dfrac{\partial E_x}{\partial x} = 0$

$y=0$

$E_z = E_x = 0 \quad \dfrac{\partial E_y}{\partial y} = 0$

$$E_x(x,y,z) = A_1 \cos k_x x \sin k_y y e^{ik_z z}$$

$$E_y(x,y,z) = A_2 \sin k_x x \cos k_y y e^{ik_z z}$$

$$E_z(x,y,z) = A_3 \sin k_x x \sin k_y y e^{ik_z z}$$

$$\nabla \cdot \vec{\boldsymbol{E}} = 0$$

$$A_1 k_x + A_2 k_y - ik_z A_3 = 0$$

*A*的分量只有两个独立，每一个确定的*k*，对应有两个独立的偏振波型

$$E_x(x,y,z) = A_1 \cos k_x x \sin k_y y e^{ik_z z}$$
$$E_y(x,y,z) = A_2 \sin k_x x \cos k_y y e^{ik_z z}$$
$$E_z(x,y,z) = A_3 \sin k_x x \sin k_y y e^{ik_z z}$$

4.确定波矢量的分量

$x = a, \quad E_y = 0 \qquad k_x = \dfrac{m\pi}{a} \qquad m = 0, 1, 2\cdots$

$y = b, \quad E_x = 0 \qquad k_y = \dfrac{n\pi}{b} \qquad n = 0, 1, 2\cdots$

$$\vec{\boldsymbol{E}}(x,y,z) = \vec{\boldsymbol{E}}(x,y)e^{ik_z z}$$
$$\vec{\boldsymbol{H}} = -\frac{i}{\omega\mu}\nabla\times\vec{\boldsymbol{E}}$$
$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b}$$
$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$
$$A_1 k_x + A_2 k_y - ik_z A_3 = 0$$

$\vec{H}$ 的解由 $\vec{H} = -\dfrac{i}{\omega\mu}\nabla\times\vec{E}$ 确定

5.两种独立波型的选择

(1). E_z=0，此种波型称为TE波,横电型。

(2).另一种波型选H_z=0，为TM波，横磁型 。

每一组m n决定了一个k_{mn},
对应着两个独立波型TM和TE波

$$TE_{mn}, \qquad TM_{mn}$$

$$A_1 k_x + A_2 k_y - i k_z A_3 = 0$$

其它mn对应的波型不独立，

可分解成这两个独立波型的迭加 .

4.5.3 截止频率

$$k=\omega\sqrt{\mu\varepsilon}\qquad k_z=\sqrt{k^2-(k_x^2+k_y^2)}\qquad \boxed{\vec{E}(x,y,z)=\vec{E}(x,y)e^{ik_z z}}$$

波导导通电磁波的要求 $k_z>0,\rightarrow k^2>k_x^2+k_y^2$

对应的频率为

$$\omega\sqrt{\mu\varepsilon}=\sqrt{\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}}\qquad \boxed{\omega_{c,mn}=\frac{\pi}{\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}}$$

设$a>b$，则最低截止频率为 $\omega_{c,10}=\dfrac{\pi}{a\sqrt{\mu\varepsilon}}$

对于某一频率 $\boxed{\omega}$ 电磁波，选择适当的波导尺寸，让TE_{10}波通过，其它高次波型截止，保证单模传输。

$$\boxed{\omega_{c,10}<\omega<\omega_{c,mn}}\qquad (m>1,n>0)$$

4.5.4 TE_{10}的电磁场与管壁电流

1. TE_{10}的电场表示

$$E_x(x,y,z) = A_1 \cos k_x x \sin k_y y e^{ik_z z} \quad (1)$$

$$E_y(x,y,z) = A_2 \sin k_x x \cos k_y y e^{ik_z z} \quad (2)$$

$$E_z(x,y,z) = A_3 \sin k_x x \sin k_y y e^{ik_z z} \quad (3)$$

$$\vec{E}(x,y,z) = \vec{E}(x,y)e^{ik_z z}$$

$$\vec{H} = -\frac{i}{\omega\mu}\nabla\times\vec{E}$$

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b}$$

$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$

$$A_1 k_x + A_2 k_y - ik_z A_3 = 0$$

$$k_x = \frac{m\pi}{a},\ k_y = \frac{n\pi}{b}, \qquad TE_{10}:\ k_x = \frac{\pi}{a},\ k_y = 0,\ E_x = 0$$

$$E_x = 0$$

$$E_y(x,y,z) = \frac{i\omega\mu a}{\pi} H_0 \sin\frac{\pi x}{a} e^{ik_z z}$$

$$E_z = 0$$

其中$A_2 = \dfrac{i\omega\mu a}{\pi} H_0$

2. TE_{10}的磁场表示

$$\vec{\boldsymbol{H}} = -\frac{i}{\omega\mu}\nabla\times\vec{\boldsymbol{E}} = -\frac{i}{\omega\mu}\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

$$\boxed{\begin{aligned} E_y &= \frac{i\omega\mu a}{\pi}H_0\sin\frac{\pi x}{a}e^{ik_z z} \\ E_z &= E_x = 0 \end{aligned}}$$

$$H_x = \frac{i}{\omega\mu}\frac{\partial E_y}{\partial z} = -\frac{ik_z a}{\pi}H_0\sin\frac{\pi x}{a}e^{ik_z z}$$

$$H_y = 0$$

$$H_z = -\frac{i}{\omega\mu}\frac{\partial E_y}{\partial x} = H_0\cos\frac{\pi x}{a}e^{ik_z z}$$

常数H_0由波的振幅来确定（波在激发时已定）

3.TE_{10}为两列平面波的迭加

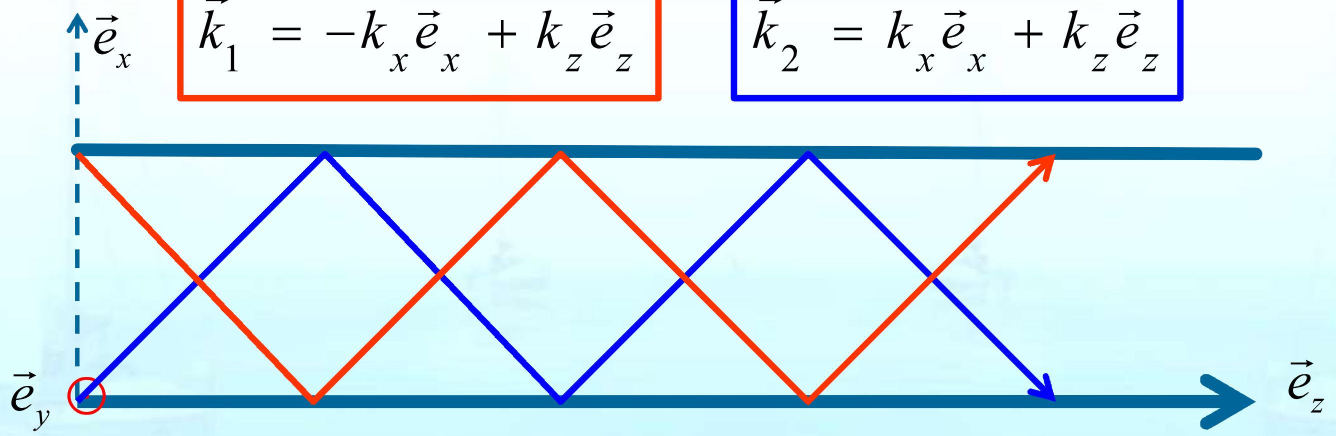
$$E = E_y(x, y, z) = \frac{i\omega\mu a}{\pi} H_0 \sin\frac{\pi x}{a} e^{ik_z z}$$

$$E(x, y, z, t) = \frac{i\omega\mu a}{\pi} H_0 \frac{1}{2i}\left(e^{ik_x x} - e^{-ik_x x}\right) e^{i(k_z z - \omega t)}$$

$$\vec{E}(x, y, z, t) = E_0 e^{i(k_x x + k_z z - \omega t)} \vec{e}_y - E_0 e^{i(-k_x x + k_z z - \omega t)x} \vec{e}_y$$

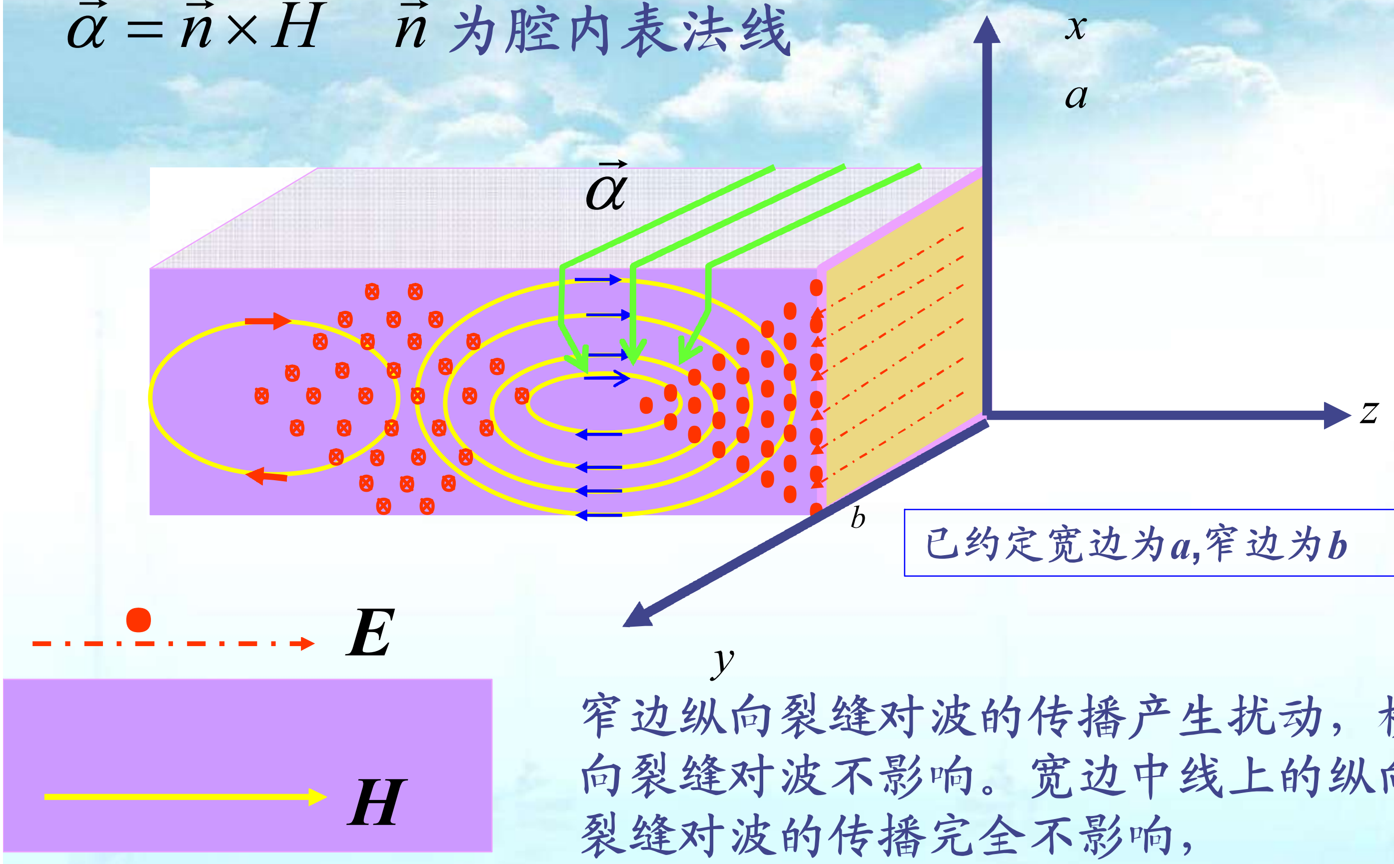
$$\boxed{\vec{k}_1 = -k_x \vec{e}_x + k_z \vec{e}_z}$$

$$\boxed{\vec{k}_2 = k_x \vec{e}_x + k_z \vec{e}_z}$$



4.TE_{10}波的电磁场和管壁电流

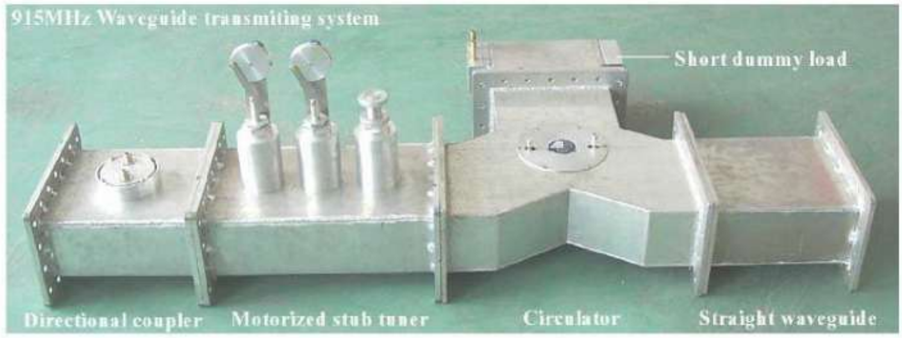
$\vec{\alpha}=\vec{n}\times\vec{H}$　$\vec{n}$ 为腔内表法线



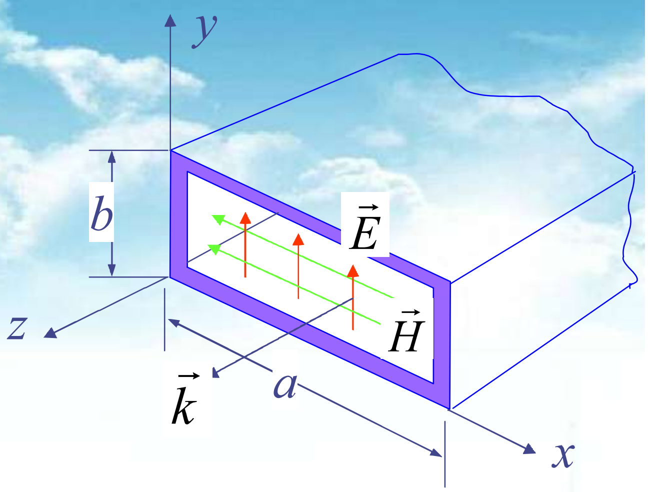
已约定宽边为*a*,窄边为*b*

窄边纵向裂缝对波的传播产生扰动，横向裂缝对波不影响。宽边中线上的纵向裂缝对波的传播完全不影响，


2450MHz Waveguide Transmiting System
Magnetron
Circulator
Launcher
ggeco.en.alibaba.com
Short dummy load
Motorized stub tuner


915MHz Waveguide transmiting system
Short dummy load
Directional coupler
Motorized stub tuner
Circulator
Straight waveguide

波导小结



1.方程: $\nabla^2 \vec{E} + k^2 \vec{E} = 0$

2.Z向传播: $\vec{E}(\vec{x},t) = \vec{E}(x,y)e^{i(k_z z - \omega t)}$

3.边界条件: $E_t = 0 \qquad \dfrac{\partial E_n}{\partial n} = 0$

4.解:

$$E_x(x,y,z) = A_1 \cos k_x x \sin k_y y e^{ik_z z}$$
$$E_y(x,y,z) = A_2 \sin k_x x \cos k_y y e^{ik_z z}$$
$$E_z(x,y,z) = A_3 \sin k_x x \sin k_y y e^{ik_z z}$$

5.最低截止频率

$$\omega_{c,10} = \frac{\pi}{a\sqrt{\mu\varepsilon}}$$

6.两种独立波型: TE_{mn} TM_{mn}

7.常用单模传输波型: TE_{10}波

8.管壁电流横向流动，宽边中线上无电流。