第四章

电磁波的传播

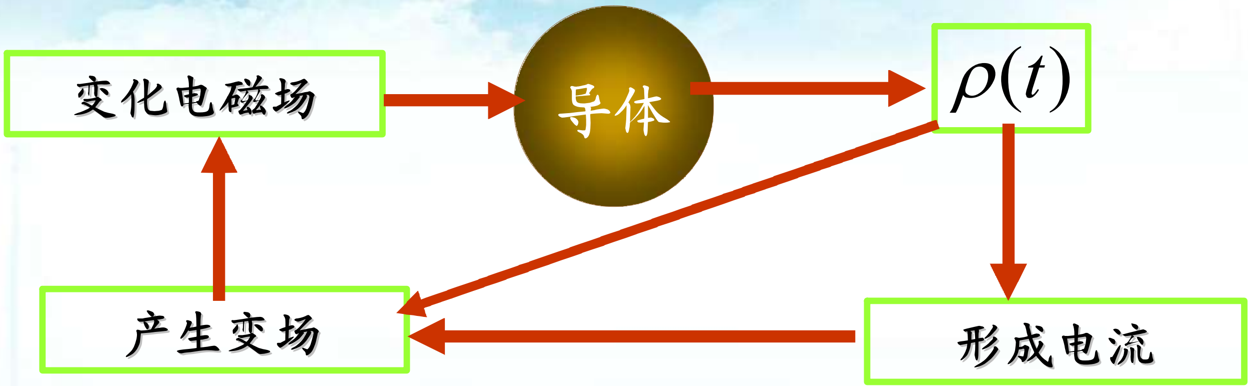
4.3导体中的电磁波

导体中电磁波传播特分析

- 导体中的自由电子与电磁场相互作用，形成传导电流产生热损耗.
- 导体内部的电磁波是一种衰减的电磁波。
- 导体中电磁波传播不同于真空或介质中电磁波的传播形式。

4.3.1. 导体内的自由电荷分布

1.导体内电磁场与自由电子的作用方式



讨论的条件设定:

- 均匀导体
- 周期时间作为特征时间间隔

$$\nabla\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0 \quad \xrightarrow{\vec{J}=\sigma\vec{E}} \quad \sigma\nabla\cdot\vec{E}+\frac{\partial\rho}{\partial t}=0$$

均匀导体

$$\nabla\cdot\vec{D}=\rho \quad \xrightarrow{\vec{D}=\varepsilon\vec{E}} \quad \varepsilon\nabla\cdot\vec{E}=\rho$$

$$\frac{\partial\rho(t)}{\partial t}=-\frac{\sigma}{\varepsilon}\rho(t)$$

$$\rho(t)=\rho_0 e^{-\frac{\sigma}{\varepsilon}t}=\rho_0 e^{-\frac{t}{\tau}}$$

导体内部的电荷按指数规律衰减

特征时间

$$\tau=\frac{\varepsilon}{\sigma}$$

2. 良导体条件

$$\rho(t)=\rho_0 e^{-\frac{\sigma}{\varepsilon}t}=\rho_0 e^{-\frac{t}{\tau}}$$

经过τ时间后，ρ已较小，若电磁波的周期$T>>\tau$，则有

$$T=\frac{1}{f}>>\tau \qquad f\sim\omega$$

$$\omega<<\tau^{-1}=\frac{\sigma}{\varepsilon}$$

$$\frac{\sigma}{\varepsilon\omega}>>1$$

良导体的条件

结论:

良导体内 $\rho(t)=0$ **电荷分布在导体表面薄层内**

对于常用的电磁频率，**金属导体**便可视为**良导体**

4.3.2导体内的电磁波

1．复电容率

$$\begin{cases} \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} & \nabla\cdot\vec{D}=\rho\approx 0 \\ \nabla\times\vec{H}=\vec{J}+\dfrac{\partial\vec{D}}{\partial t} & \nabla\cdot\vec{B}=0 \end{cases}$$

良导体内

$$\rho\approx 0,\qquad \boldsymbol{J}=\sigma\boldsymbol{E}$$

$$\vec{D}=\varepsilon\vec{E}\qquad \vec{B}=\mu\vec{H}$$

时谐（定态）→ $\dfrac{\partial}{\partial t}\Leftrightarrow -i\omega t$

$$\vec{E}(\vec{x},t)=\vec{E}(\vec{x})e^{-i\omega t}$$

$$\vec{H}(\vec{x},t)=\vec{H}(\vec{x})e^{-i\omega t}$$

$$\begin{cases} \nabla\times\vec{E}=i\omega\mu\vec{H} \\ \nabla\times\vec{H}=(-i\omega\varepsilon+\sigma)\vec{E} \\ \nabla\cdot\vec{E}\approx 0 \\ \nabla\cdot\vec{H}=0 \end{cases}$$

$$\nabla\times\vec{H}=(-i\omega\varepsilon+\sigma)\vec{E}=-i\omega[\varepsilon+i\frac{\sigma}{\omega}]\vec{E}=-i\omega\varepsilon'\vec{E}$$

$$\varepsilon'=\varepsilon+i\frac{\sigma}{\omega}$$

引入复介电场数

$$\nabla\times\vec{\boldsymbol{H}}=\frac{\partial\vec{\boldsymbol{D}}}{\partial t}+\vec{\boldsymbol{J}}=\vec{\boldsymbol{J}}_D+\vec{\boldsymbol{J}}=\vec{\boldsymbol{J}}_{总}$$

$$\overline{\boldsymbol{P}}=\frac{1}{2}Re\vec{\boldsymbol{J}}^*_{总}\cdot\vec{\boldsymbol{E}}$$

平均热功率密度

$$=\frac{1}{2}Re\left(\frac{\partial\vec{\boldsymbol{D}}^*}{\partial t}\cdot\vec{\boldsymbol{E}}\right)+\frac{1}{2}Re(\vec{\boldsymbol{J}}^*\cdot\vec{\boldsymbol{E}})$$

$$=\frac{1}{2}Re\left(-i\omega\varepsilon\vec{\boldsymbol{E}}^*\cdot\vec{\boldsymbol{E}}\right)+\frac{1}{2}Re(\sigma\vec{\boldsymbol{E}}^*\cdot\vec{\boldsymbol{E}})$$

$$=\frac{1}{2}Re\left(-i\omega\varepsilon E_0^2\right)+\frac{1}{2}Re(\sigma E_0^2)$$

$$=0+\frac{1}{2}\sigma E_0^2$$

$$\nabla\times\vec{H}=-i\omega\varepsilon'\vec{E}=-i\omega(\varepsilon+i\frac{\sigma}{\omega})\vec{E}$$

实部对应位移电流的贡献，无耗散

虚部对应传导电流的贡引起能耗，$\frac{1}{2}\sigma E_0^{\ 2}$

2．导体中的亥姆霍兹方程及平面波解

$$\begin{cases} \nabla\times\vec{E}=i\omega\mu\vec{H} \\ \nabla\times\vec{H}=-i\omega\varepsilon'\vec{E} \\ \nabla\cdot\vec{E}\approx 0 \\ \nabla\cdot\vec{H}=0 \end{cases}$$

微分消元 → 亥姆霍兹方程

$$\begin{cases} \nabla^2\vec{E}+k''^2\vec{E}=0 \\ \vec{H}=-\dfrac{i}{\omega\mu}\nabla\times\vec{E} \end{cases}$$

$$\varepsilon'=\varepsilon+i\frac{\sigma}{\omega}$$

复波数 $k''=\omega\sqrt{\mu\varepsilon'}$

平面波解

$$\vec{E}(\vec{x},t)=\vec{E}_0 e^{i(\vec{k}''\cdot\vec{x}-\omega t)}$$

复波矢可表示为： $\vec{k}''=\vec{\beta}+i\vec{\alpha}$

3.衰减常数与相位常数

$$k'' = \omega\sqrt{\mu\varepsilon'}$$

平面电磁波解改写为: $\vec{E}(\vec{x}) = \vec{E}_0 e^{-\vec{\alpha}\cdot\vec{x}} e^{i(\vec{\beta}\cdot\vec{x})}$

衰减常数 $\vec{\alpha}$ ----- 描述波振幅在导体内的衰减程度

相位常数 $\vec{\beta}$ -----描述波空间传播的位相关系 $v = \dfrac{\omega}{\beta}$

可由

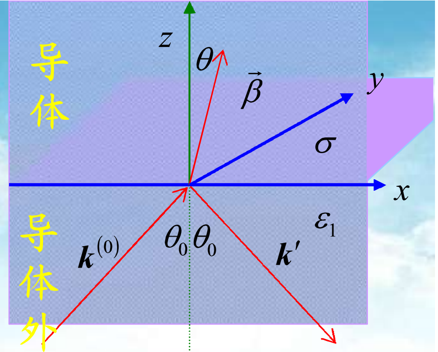
$$\begin{cases} \vec{k}'' = \vec{\beta} + i\vec{\alpha} \\ \varepsilon' = \varepsilon + i\dfrac{\sigma}{\omega} \\ k''^2 = \omega^2\mu\varepsilon' \end{cases} \Rightarrow \begin{cases} \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \\ \vec{\alpha}\cdot\vec{\beta} = \dfrac{1}{2}\omega\mu\sigma \end{cases}$$

解出了 $\vec{\alpha}$ $\vec{\beta}$就可以确定导体中的电磁波

$$\vec{\alpha} = \alpha_x\vec{e}_x + \alpha_y\vec{e}_{xy} + \alpha_z\vec{e}_z$$

$$\vec{\beta} = \beta_x\vec{e}_x + \beta_y\vec{e}_{xy} + \beta_z\vec{e}_z$$

$k_x = k_x^{(0)}$ ， $k_x = k_x''$ $k_y = k_y'' = 0$ $\vec{k}'' = \vec{\beta} + i\vec{\alpha}$



$k_x'' = \beta_x + i\alpha_x$ $k_y'' = \beta_y + i\alpha_y$ $k_z'' = \beta_z + i\alpha_z$

$\beta_x + i\alpha_x = k_x^{(0)}$ $\alpha_x = 0,$ $\beta_x = k_x^{(0)}$

$\beta_y + i\alpha_y = 0$ $\alpha_y = 0,$ $\beta_y = 0$

$\vec{\alpha} = \alpha_x \vec{e}_x + \alpha_y \vec{e}_{xy} + \alpha_z \vec{e}_z$

$\vec{\beta} = \beta_x \vec{e}_x + \beta_y \vec{e}_{xy} + \beta_z \vec{e}_z$

$\beta_x = k_x^{(0)}$ $\beta_y = 0$ $\beta_z = \sqrt{\beta^2 - \beta_x^2}$

$\alpha_x = 0$ $\alpha_y = 0$ $\alpha_z = \alpha$

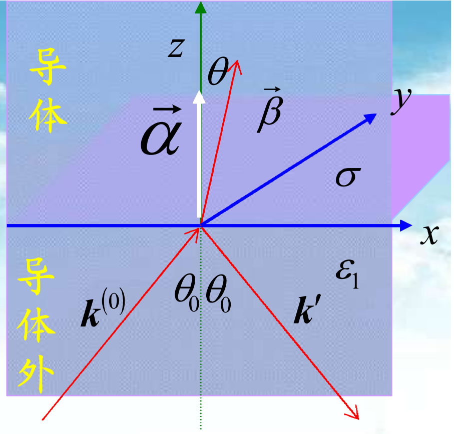
$$\begin{cases} \beta^2 - \alpha^2 = \omega^2 \mu\varepsilon \longrightarrow \beta_x^2 + \beta_z^2 - \alpha^2 = \omega^2 \mu\varepsilon \\ \vec{\alpha} \cdot \vec{\beta} = \frac{1}{2}\omega\mu\sigma \longrightarrow \alpha\beta_z = \frac{1}{2}\omega\mu\sigma \end{cases}$$

可求出 α 和 β 的x和y分量就可以求出其z分量

$$\begin{cases} \beta^2-\alpha^2=\omega^2\mu\varepsilon \\ \vec{\alpha}\cdot\vec{\beta}=\frac{1}{2}\omega\mu\sigma \end{cases}$$

$\Rightarrow \beta_x^{\ 2}+\beta_z^{\ 2}-\alpha^2=\omega^2\mu\varepsilon$

$\Rightarrow \alpha\beta_z=\frac{1}{2}\omega\mu\sigma$



$\beta_x=k_x^{(0)}\quad \beta_y=0$

$\alpha_x=0\quad \alpha_y=0$

$\beta_x=k_x^{(0)}=\frac{\omega}{v_0}\sin\theta_0$　　已知量

$$\beta_z^{\ 2}=\frac{1}{2}(\omega^2\mu\varepsilon-\beta_x^{\ 2})+\frac{1}{2}[(\omega^2\mu\varepsilon-\beta_x^{\ 2})^2+\omega^2\mu^2\varepsilon^2]^{1/2}$$

$$\alpha^2=-\frac{1}{2}(\omega^2\mu\varepsilon-\beta_x^{\ 2})+\frac{1}{2}[(\omega^2\mu\varepsilon-\beta_x^{\ 2})^2+\omega^2\mu^2\varepsilon^2]^{1/2}$$

$\Rightarrow \vec{\alpha}=\alpha_z\vec{e}_z=\alpha\vec{e}_z$（即$\vec{\alpha}\perp$分界面指向导体内部，波沿$\vec{z}$方向衰减）

4.3.3. 趋肤效应和穿透深度

$k_x = k_x^{(0)}$， $k_x = k''_x$

$k_y = k''_y = 0$

$\vec{k}'' = \vec{\beta} + i\vec{\alpha}$

$\vec{k}'' = \vec{\beta} + i\vec{\alpha}$

1.求解 α 和 β

$$\vec{E}(\vec{x}) = \vec{E}_0 e^{-\vec{\alpha}\cdot\vec{x}} e^{i(\vec{\beta}\cdot\vec{x})}$$

$$\begin{cases} \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \\ \vec{\alpha}\cdot\vec{\beta} = \frac{1}{2}\omega\mu\sigma \end{cases}$$

电磁波在良导体中传播时，只能沿着导体表面进行，其内部很快衰减为零。

讨论垂直入射情况 $k_x = k_y = 0$

$$\alpha_x = \beta_x = \alpha_y = \beta_y = 0$$

$$\boldsymbol{\alpha}\cdot\boldsymbol{\beta} = \alpha_z\cdot\beta_z = \alpha\cdot\beta$$

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_0 e^{-\alpha z} e^{i(\beta z - \omega t)}$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon$$

$$\alpha\beta = \frac{1}{2}\omega\mu\sigma$$

$$\alpha^4 + \omega^2\mu\varepsilon\alpha^2 - \frac{1}{4}\omega^2\mu^2\sigma^2 = 0$$

$$\alpha^2 = \omega^2\mu\varepsilon\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}} - 1\right)\right]$$

$$\alpha = \omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}} - 1\right)\right]^{1/2}$$

$$\beta^2-\alpha^2=\omega^2\mu\varepsilon$$

$$\alpha\beta=\frac{1}{2}\omega\mu\sigma$$

$$\alpha=\omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}}-1\right)\right]^{1/2}$$

良导体

$$\frac{\sigma}{\omega\varepsilon}>>1$$

$$\alpha=\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\beta=\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\beta^2-\alpha^2=\omega^2\mu\varepsilon$$

$$\alpha\beta=\frac{1}{2}\omega\mu\sigma$$

$$\beta=\omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}}+1\right)\right]^{1/2}$$

结论:在良导体中(垂直入射)

$$\alpha\approx\beta=\sqrt{\frac{\omega\mu\sigma}{2}}$$

2. 穿透深度

$$\vec{E}(\vec{x},t)=\vec{E}_0 e^{-\alpha z} e^{i(\beta_x x+\beta_z z-\omega t)}$$

穿透深度 δ：波幅降至原值 $1/e$ 的传播距离

$$\vec{E}_0 e^{-\alpha z} = \vec{E}_0 e^{-1}$$

波幅

$$z=\frac{1}{\alpha} \Rightarrow \frac{|\vec{E}_0|}{e} \Rightarrow \delta=\frac{1}{\alpha}$$

良导体 →

$$\delta \approx \sqrt{\frac{2}{\omega\mu\sigma}}$$

3.趋肤效应

对于良导体，当电磁波频率为交变频率时，电磁场及交频电流集中在导体表面薄层

例如，当 $\omega \approx 100$ 兆赫，铜 $\delta \sim 0.7\times10^{-3} cm \approx 0.007mm$

4. 导体内磁场与电场的关系（垂直入射为例）

$$\vec{E}=\vec{E}_0 e^{i\vec{k}''\cdot\vec{x}}$$

$$\vec{E}(\vec{x},t)=\vec{E}_0 e^{i(\vec{k}''\cdot\vec{x}-\omega t)}$$

$$\nabla\times\vec{E}=i\omega\mu\vec{H}\Rightarrow \vec{H}=-\frac{i}{\omega\mu}\nabla\times\vec{E}=\frac{\vec{k}''\times\vec{E}}{\omega\mu}=\frac{\vec{\beta}+i\vec{\alpha}}{\omega\mu}\times\vec{E}$$

对良导体 $\vec{H}=\frac{(\beta+i\alpha)\vec{n}\times\vec{E}}{\omega\mu}\approx\sqrt{\frac{\sigma}{\omega\mu}}(\frac{1+i}{\sqrt{2}})\vec{n}\times\vec{E}=\sqrt{\frac{\sigma}{\omega\mu}}e^{i\frac{\pi}{4}}\vec{n}\times\vec{E}$

结论1：磁场的相位滞后电场 $\pi/4$

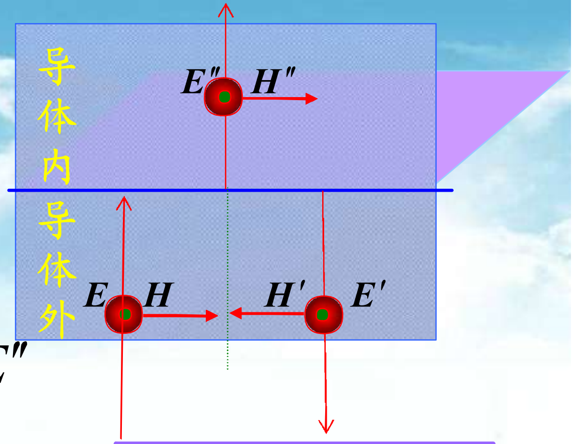
$$\alpha\approx\beta=\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\sqrt{\frac{\mu}{\varepsilon}}\frac{|\boldsymbol{H}|}{|\boldsymbol{E}|}=\sqrt{\frac{\mu}{\varepsilon}}\sqrt{\frac{\sigma}{\omega\mu}}=\sqrt{\frac{\sigma}{\varepsilon\omega}}>>1$$

$$\frac{\text{磁能}}{\text{电能}}=\frac{\mu H^2}{\varepsilon E^2}=\frac{\sigma}{\omega\varepsilon}>>1$$

结论**2:**在金属导体中，磁能远远大于电能**.**

4.3.4. 导体表面上的反射



以真空中的正入射为例

导体外 $H=\sqrt{\dfrac{\varepsilon_0}{\mu_0}}E$

导体中 $H''=\sqrt{\dfrac{\sigma}{\mu\omega}}e^{i\frac{\pi}{4}}E''=\sqrt{\dfrac{\sigma}{2\mu\omega}}(1+i)E''$

$H_{1t}=H_{2t}$ $\quad$ $E_{1t}=E_{2t}$

$H-H'=H''$ $\quad$ $\sqrt{\dfrac{\varepsilon_0}{\mu_0}}(E-E')=\sqrt{\dfrac{\sigma}{\mu_0 2\omega}}(1+i)E''$

$$E-E'=\sqrt{\frac{\sigma}{2\omega\varepsilon_0}}E''(1+i)$$

$$E+E'=E''$$

可解出振幅比

$$E - E' = \sqrt{\frac{\sigma}{2\omega\varepsilon_0}} E''(1+i)$$

$$E + E' = E''$$

真空正入射振幅比

$$\frac{E'}{E} = -\frac{1+i-\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}}{1+i+\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}} = \frac{a+i}{b+i}$$

反射系数：反射能流与入射能流之比

$$\frac{|\vec{\boldsymbol{E}}|}{|\vec{\boldsymbol{B}}|} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \to \frac{|\vec{\boldsymbol{E}}|}{|\mu_0\vec{H}|} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \longrightarrow |\vec{H}| = \sqrt{\frac{\mu_0}{\varepsilon_0}}|\vec{\boldsymbol{E}}|$$

$$R = \frac{|\vec{E}'\times\vec{H}'|}{|\vec{E}\times\vec{H}|} = \frac{|E'|^2}{|E|^2} = \frac{\left(1-\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}\right)^2+1}{\left(1+\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}\right)^2+1}$$

（真空正入射）

$$R=\frac{\left(1-\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}\right)^2+1}{\left(1+\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}\right)^2+1}$$

在良导体条件下化简此式

舍去二阶无穷小

$$\frac{\sigma}{\omega\varepsilon_0}>>1 \rightarrow x=\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}<<1$$

$$R=\frac{2-2x+x^2}{2+2x+x^2}\doteq\frac{1-x}{1+x}\doteq(1-x)(1-x)$$

$$R\approx 1-2x=1-2\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}$$

$$\sigma\uparrow \qquad R\rightarrow 1$$

在无线电波段垂直入射情况下，金属导体的反射系数接近于**1**，电磁能量几乎全部被反射。