

# Second-Order PMD Monitoring from Adaptive FIR-Filter Tap Coefficients in a Digital Coherent Receiver

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**Abstract:** We propose a straightforward algorithm to estimate second-order PMD from adaptive FIR-filter tap coefficients in a digital coherent optical receiver. The novel algorithm is verified with 100-Gbit/s dual-polarization QPSK experiments.

**OCIS codes:** (060.2330) Fiber optic communication; (060.1660) Coherent communications; (060.2920) Homodyning.

## 1. Introduction

A digital coherent optical receiver enables high spectral efficiency by the use of multi-level modulation formats and polarization multiplexing [1]. Such a receiver allows polarization demultiplexing and equalization of all deterministic linear impairments, namely, chromatic dispersion (CD), polarization-mode dispersion (PMD) and polarization-dependent loss (PDL) by using four finite-impulse-response (FIR) filters structured in a two-by-two butterfly configuration. After the filters are adapted by a suitable algorithm, we can construct a frequency dependent two-by-two matrix, with four elements equal to the transfer functions of the adapted four FIR filters. This matrix is nothing but the inverse transfer matrix of the channel, and contains combined effects of CD, PMD and PDL. Monitoring of CD, first-order PMD and PDL from this matrix has been attracted significant attention, and several efforts have been demonstrated so far [2-4].

On the other hand, separation of second-order PMD (SOPMD) from this matrix has not been investigated yet. In this paper, we propose an algorithm to estimate SOPMD and its components, namely, polarization-dependent chromatic dispersion (PCD) and depolarization of the principal states of polarization (PSP). The proposed algorithm is verified with 100-Gbit/s dual-polarization QPSK experiments. To the best of our knowledge, this is the first demonstration of SOPMD monitoring from the adaptive FIR filters in a digital coherent optical receiver.

Throughout the remainder, vectors and matrices are in boldface letters; superscripts  $(\bullet)^*$ ,  $(\bullet)^T$ ,  $(\bullet)^{-1}$ ,  $(\bullet)^H$  denote complex conjugate, transpose, inverse, and conjugate transpose, respectively; subscript  $(\bullet)_\omega$  is the differentiation with respect to the angular frequency; and functions  $DFT(\bullet)$  and  $arg(\bullet)$  denote discrete Fourier transform and argument, respectively.

## 2. Proposed algorithm

Considering a modest channel power to operate in the linear region, the transfer function of a fiber can be modeled as a concatenation of CD, PMD and PDL elements, given as  $D(\omega)$ ,  $\mathbf{U}(\omega)$  and  $\mathbf{K}$ , respectively, and the Jones matrix  $\mathbf{T}$  represents the birefringence of the fiber as

$$\mathbf{H}_{fiber}(\omega) = D(\omega)\mathbf{U}(\omega)\mathbf{K}\mathbf{T}. \quad (1)$$

The polarization demultiplexing and equalization of all these linear impairments can be done in the digital domain by using four complex-valued multi-tap FIR filters arranged in a butterfly configuration. Provided that the filter tap length is sufficient enough compared to the impulse response of the channel, we can obtain the channel transfer function given by a single monitoring matrix  $\mathbf{M}(\omega)$  as

$$\mathbf{M}(\omega) \approx \mathbf{H}_{fiber}(\omega) = \left\{ DFT \begin{bmatrix} \mathbf{h}_{xx}(n) & \mathbf{h}_{xy}(n) \\ \mathbf{h}_{yx}(n) & \mathbf{h}_{yy}(n) \end{bmatrix} \right\}^{-1}. \quad (2)$$

The tap coefficient vector is given as  $\mathbf{h}_{ij}(n) = [h_{ij,0}(n) \ h_{ij,1}(n) \ \dots \ h_{ij,L}(n)]^T$ , where  $i$  and  $j$  are any one of  $x$  or  $y$ .

Eqs. (1) and (2) yield

$$\begin{aligned} \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega) &= D(\omega + \Delta\omega)\mathbf{U}(\omega + \Delta\omega)\mathbf{K}\mathbf{T}\{D(\omega)\mathbf{U}(\omega)\mathbf{K}\mathbf{T}\}^{-1} \\ &= D(\omega + \Delta\omega)D(\omega)^* \mathbf{U}(\omega + \Delta\omega)\mathbf{K}\mathbf{T}(\mathbf{K}\mathbf{T})^{-1}\mathbf{U}(\omega)^{-1} \end{aligned}$$

## OWN3.pdf

$$= c\mathbf{U}(\omega + \Delta\omega)\mathbf{U}(\omega)^H, \quad (3)$$

where  $\Delta\omega$  denotes the DFT angular-frequency resolution and the scalar term  $D(\omega + \Delta\omega)D(\omega)^*$  is expressed as  $c$ . By using the explicit form of  $\mathbf{U}(\omega)$ , Eq. (3) can be written as

$$\begin{aligned} \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega) &= c\mathbf{R}_1^{-1} \begin{bmatrix} e^{j(\omega + \Delta\omega)\Delta\tau/2} & 0 \\ 0 & e^{-j(\omega + \Delta\omega)\Delta\tau/2} \end{bmatrix} \mathbf{R}_1 \mathbf{R}_1^{-1} \begin{bmatrix} e^{-j\omega\Delta\tau/2} & 0 \\ 0 & e^{j\omega\Delta\tau/2} \end{bmatrix} \mathbf{R}_1 \\ &= c\mathbf{R}_1^{-1} \begin{bmatrix} e^{j\Delta\omega\Delta\tau/2} & 0 \\ 0 & e^{-j\Delta\omega\Delta\tau/2} \end{bmatrix} \mathbf{R}_1, \end{aligned} \quad (4)$$

where  $\Delta\tau$  is the differential group delay (DGD) between two orthogonal PSP, and  $\mathbf{R}_1$  is a unitary matrix converting two PSP into the  $x$ - and  $y$ -polarization. Eq. (4) clearly shows that eigen values of the matrix  $\mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)$ ,  $\rho_{1,2}$ , are associated with group delays of the two PSP. Thus, the DGD  $\Delta\tau$  can be estimated as

$$\Delta\tau(\omega + \Delta\omega/2) = \left| \frac{\arg(\rho_1 / \rho_2)}{\Delta\omega} \right|. \quad (5)$$

The necessary condition to avoid ambiguities arising from the multi-valued argument function is  $\Delta\tau \Delta\omega < \pi$ .

The SOPMD vector is described by the derivative of the fiber PMD vector and can be written as

$$\vec{\tau}_\omega = \hat{p}\Delta\tau_\omega + \Delta\tau\hat{p}_\omega, \quad (6)$$

where  $\hat{p}$  is the unit vector pointing the direction of the slow PSP. The magnitude of the first component  $\Delta\tau_\omega$  is the change of DGD with the angular frequency, causing PCD. Since frequency dependent DGD can be estimated from Eq. (5), PCD can be calculated as

$$PCD = |\Delta\tau_\omega| = \left| \frac{\Delta\tau(\omega_1) - \Delta\tau(\omega_2)}{\omega_2 - \omega_1} \right|. \quad (7)$$

The second-term  $\Delta\tau\hat{p}_\omega$  relates to PSP depolarization, that is, a rotation of the PSP with the angular frequency. The eigen vectors of the matrix  $\mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)$  locate the PSP as a function of the angular frequency. Let the eigen vectors are  $|t_{1,2}\rangle$  and  $\mathcal{S}$  is the Stokes vector corresponding to the slow PSP  $|t_1\rangle$ ; then the unit vector  $\hat{p}$  can be found as  $\hat{p} = \mathcal{S} / |\mathcal{S}|$ . The angular rate of PSP rotation  $|\hat{p}_\omega|$  from  $\omega_1$  to  $\omega_2$  can be found as

$$|\hat{p}_\omega| = \frac{\cos^{-1}\{\hat{p}(\omega_2) \cdot \hat{p}(\omega_1)\}}{\omega_2 - \omega_1}. \quad (8)$$

The magnitude of total SOPMD can be estimated as

$$SOPMD = \sqrt{PCD^2 + DEP^2}. \quad (9)$$

### 3. Experimental results and discussions

To verify the proposed algorithm, we conduct SOPMD monitoring experiments. We used a commercial all-order PMD emulator (PMDE) between the 100-Gbit/s dual-polarization transmitters and a phase and polarization diverse coherent optical receiver. The PMDE uses three programmable DGD sections separated by polarization controller to generate all-order PMD with tunable statistics. The transmitter and local oscillator lasers were distributed-feedback laser diodes (DFB-LD) having a 3-dB linewidth of 150 kHz and a center wavelength of 1552 nm. A 50-Gbit/s NRZ QPSK signal was generated using a LiNbO<sub>3</sub> optical IQ modulator (IQM) from two streams of pre-coded data from a pulse-pattern generator (PPG) with 2<sup>9</sup>-1 pseudo-random binary sequences (PRBS). A 100-Gbit/s dual-polarization signal was then produced in the split-delay-combine manner. Then the signal passed through the PMDE. The transmitted signal was pre-amplified by an erbium-doped fiber amplifier (EDFA) before it was incident on a coherent optical receiver employing phase and polarization diversities. The received power was fixed so that BER was around 3×10<sup>-4</sup>. The outputs were sampled and digitized at 50 GSample/s with analog-to-digital converters (ADCs), and stored for offline digital signal processing (DSP).

In the DSP circuit, we performed two samples per symbol-based equalization: the signal was polarization demultiplexed and equalized by 33-tap butterfly-structured FIR filters. For filter adaptation we used the constant-modulus algorithm (CMA). After being converged, the filter tap coefficients were used for monitoring.

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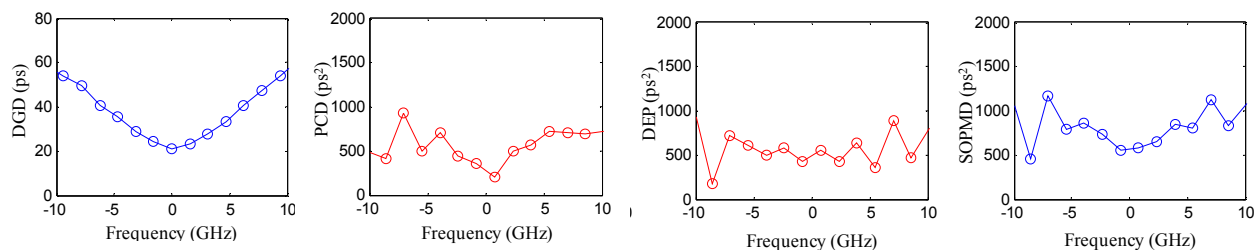


Fig. 1: Estimation example of DGD, SOPMD, PCD and DEP from a random sample.

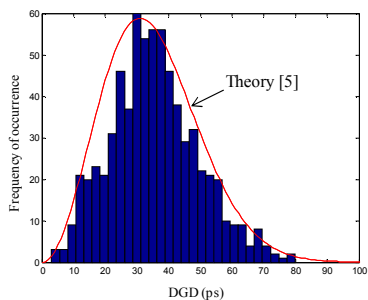


Fig. 2: Histogram of estimated DGD.

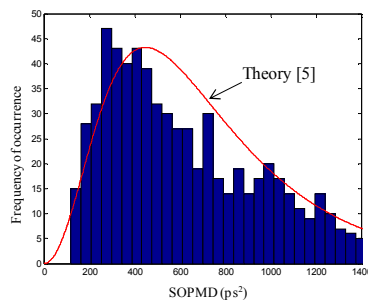


Fig. 3: Histogram of estimated SOPMD.

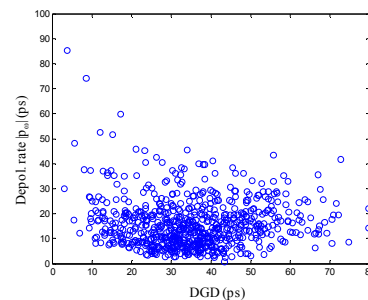


Fig. 4: Scatter plot of estimated depolarization rate.

We set the PMDE to generate a Maxwellian-distributed DGD with the mean value of 35 ps at the refreshing rate of 10 ms/sample. Fig. 1 shows an example of the estimation of DGD, SOPM and its components for a random sample. We restrict our estimation to several taps around the center tap which correspond to frequency range from -10 GHz to +10 GHz, due to the bandwidth limitation in the transmitter and receiver. Fig. 2 shows the histogram of estimated DGD from 700 measurements. The result fits well to the theoretical Maxwellian distribution by the red curve. Its mean value is 35.45 ps, which is very close to the set value. Fig. 3 shows the estimation of the corresponding SOPMD histogram. The distribution matches with the theoretical shape [5].

Fig. 4 is the scatter plot of the depolarization rate as a function of corresponding DGD, which shows that the depolarization rate tends to be high when DGD is small. This correlation is in a good agreement with the previously published results [6].

#### 4. Conclusion

We have demonstrated SOPMD estimation from the adaptive FIR-filter tap coefficients in a digital coherent optical receiver. The proposed monitoring algorithm is also validated by experiments, and the monitoring results match well with predicted statistics.

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