Topology Partitioning with Fault-Tolerant Mapping

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Abstract: While partitioning a network for scalability and manageability purposes, we identify the significance of survivability of individual partitions. We formulate and solve "2-SRLG-connected" topology partitioning using a flexible mapping onto the physical topology.

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1. Introduction

As the size of optical networks grows, networks are often organized into hierarchical partitions for scalable routing operations and manageable services. Such a partitioned network also provides faster protection by decreasing recovery time. A practical partitioning example is the Private Network-Network Interface (P-NNI) protocol that splits a topology into P-NNI peer groups.

In optical networks, multiple logical links (or lightpaths) are mapped on physical links, which frequently share resources such as cable, conduit, and right of way (ROW). Therefore, a failure of shared resources potentially disrupts a group of logical links, which is called a shared risk link group (SRLG). More often than not, optical networks experience SRLG failures, rather than single logical link or node failures. Failures are critical because of huge data loss. Failures are even more critical if it results in the disconnection of a partition. We call a partition connected if there exists a path between any two nodes. A partition is k-SRLG-connected if it remains connected even if any k-1 SRLGs fail, where $k \geq 1$. In this paper, we consider k = 2, i.e., we look for 2-SRLG-connected (2SC) partitions. The significance of 2SC partitions compared to 1-SRLG-connected partitions (which is the traditional case) is threefold:

1. In a 1-SRLG-connected partition, we may not find protected (SRLG-disjoint) paths using the partition alone;
2. Upon an SRLG failure, a 1-SRLG-connected partition may become disconnected, disabling the restoration;
3. A 1-SRLG-connected partition being disconnected at single failures forfeits regular system operations and requires frequent system re-partitioning at extra overhead and performance degradation.

Therefore, in this study, we pursue the 2SC property within each partition while we find a mapping from the logical topology to the physical topology. Obviously, it is not always the case that for any 2SC mapping there exists a 2SC partitioning. Thus far, the only proven case is that a k-connected network (link or node connectivity) has k 1-connected partitions [1]. Therefore, we need to add extra logical links (i.e., link augmentation) to make the logical topology 2SC so that restoration and protection within a partition are possible and that the partition itself remains functional/survivable upon single SRLG failures. Our partitioning problem is novel since existing studies mainly focus either on 1-connected (link or node connectivity) graph decomposition or on 2SC mapping without partitioning. We provide and solve a mathematical formulation of the problem. To achieve survivability, we use an approach that finds two SRLG-disjoint flows, rather than an approach that finds a fault-tolerant cutset. As described in [2], the number of constraints in the cutset based approach is greater (exponential) than that in the flow based approach (polynomial). Thus, our mathematical formulation is more efficient compared to the formulations in [3, 4] in finding survivability.

The rest of this paper is organized as follows. Section 2 defines the problem and analyzes the complexity. Section 3 describes a mathematical formulation. Section 4 shows numerical results, and Section 5 concludes the paper.

2. Problem Statement and Hardness

Let us consider a physical topology (V_p, E_p), where V_p is the set of nodes and E_p is the set of bidirectional fibers. We are given a logical topology (V_l, E_l), where V_l is the set of source/destination nodes and E_l is the set of lightpaths to be established. We are also given the underlying SRLG set R. We need to find Q partitions of the logical topology and an associated lightpath mapping such that every partition is 2SC and bounded to the maximum size S_{max} and the minimum size S_{min}.

As described earlier, we consider adding new lightpaths to enforce the 2SC property for each partition. Even with such flexibility, the problem has the non-deterministic polynomial time complexity (i.e., in the class of NP-hard). To prove this, we can set Q = 1 and let each SRLG consists of a single physical link. The problem then reduces to the 2-link-connected mapping problem, which is described to be NP-hard in [4].

3. Solution using Mathematical Formulation

In this section, a solution based on an Integer Linear Programming (ILP) formulation of the problem is given. As expected, we want to minimize the cost of resource usage (e.g., wavelength-links) in the mapping. Hence, the objective is to minimize $\sum_{\substack{(s,t)\in E_k \\ (i,j)\in E_p}} f_{ij}^{st}$, where binary variable f_{ij}^{st} is 1 if the logical link (s,t) is routed on physical link (i,j) and E_k denotes the set of links of the complete graph on logical nodes. The mapping of logical links is modeled by multi-commodity flows. One unit of flow from node s to t is routed only when x^{st} = 1, i.e., the link (s,t) is included in the logical topology. As given below, the constraint set *i*) generates such a mapping. The constraint set *ii*) ensures the inclusion of given logical links. The binary variable y_r^{st} is 1 if the logical link (s,t) is routed through a physical link that shares the SRLG $r \in R$. In this case, (s,t) is subject to the same risk, which is captured by the constraint set *iii*). The constraint sets *iv*) and *v*) produce Q partitions of the logical topology within permissible sizes, where z_q^s is 1 if the logical node s is included in the partition q. To ensure the 2-SRLG-connected property among intra-partition links, we find two SRLG-disjoint flows for each node-pair within a partition. At first, in the constraint set *vi*), the variable g^{st} sorts out $\{s,t\}$ node-pair, where both nodes are within the same partition. Then, we find two flows on the logical topology as given by the constraint set *vii*), where a_{mn}^{st} and b_{mn}^{st} are such flows. The constraint set *viii*) guarantees that each flow must be entirely within the corresponding partition. The SRLG-disjointness of the two flows is ensured by the constraint set *ix*). Similar to the 2SC property among intra-partition links, we use additional constraints to impose the 2SC property among inter-partition links, which are omitted in this paper due to the space limitation.

Constraint set *i*)

$$\sum_{\substack{j\ s.t.\\ (i,j)\in E_p}} f_{ij}^{st} - \sum_{\substack{j\ s.t.\\ (i,j)\in E_p}} f_{ji}^{st} = \begin{cases} x^{st}, & \text{if s = i} \\ -x^{st}, & \text{if t = i} \\ 0, & \text{otherwise.} \end{cases}$$

Constraint set *ii*) $x^{st} = 1,\ \forall (s,t) \in E_l.$

Constraint set *iii*)

$$y_r^{st} \le \sum_{(i,j)\in r} f_{ij}^{st},$$

$$|r| \times y_r^{st} \ge \sum_{(i,j)\in r} f_{ij}^{st}.$$

Constraint set *iv*) $\sum_{1\le q\le Q} z_q^s = 1.$

Constraint set *v*)

$$S_{min} \le \sum_{s\in V_l} z_q^s \le S_{max}.$$

Constraint set *vi*)

$$g^{st} \le z_q^s - z_q^t + 1,$$

$$g^{st} \le z_q^t - z_q^s + 1,$$

$$1 + g^{st} \ge z_q^s + z_q^t.$$

Constraint set *vii*)

$$\sum_{n\in V_l} a_{mn}^{st} - \sum_{n\in V_l} a_{nm}^{st} = \begin{cases} g^{st}, & \text{if s = m} \\ -g^{st}, & \text{if t = m} \\ 0, & \text{otherwise,} \end{cases}$$

$$\sum_{n\in V_l} b_{mn}^{st} - \sum_{n\in V_l} b_{nm}^{st} = \begin{cases} g^{st}, & \text{if s = m} \\ -g^{st}, & \text{if t = m} \\ 0, & \text{otherwise.} \end{cases}$$

Constraint set *viii*)

$$2 \times a_{mn}^{st} \le x^{st} + g^{mn},$$

$$2 \times b_{mn}^{st} \le x^{st} + g^{mn}.$$

Constraint set *ix*)

$$a_{mn}^{st} + y_r^{mn} \le \alpha_r^{st} + 1,$$

$$b_{mn}^{st} + y_r^{mn} \le \beta_r^{st} + 1,$$

$$\alpha_r^{st} + \beta_r^{st} \le 1.$$

4. Numerical Results

We have solved the problem using ILOG CPLEX. Since the solution become intractable for bigger networks, we use smaller network topologies to study the relationship among the cost of a solution, the number of risks, and the number

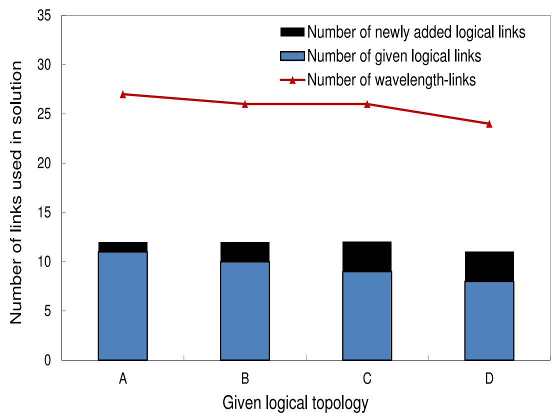

Fig. 1. Relationship between cost and logical topology.

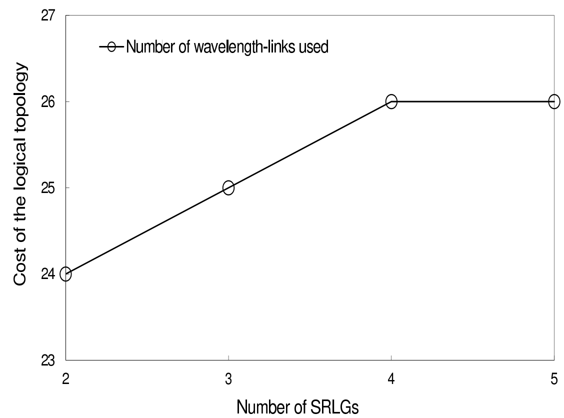

Fig. 2. Increased cost at higher number of risks.

of new lightpaths added to the logical topology to generate survivable individual partitions. We use the total number of wavelength-links required to map a logical topology as the cost metric of a solution. We assume that the given physical topology is 2SC, otherwise a survivable partitioning solution can not exist. However, it is not guaranteed that a solution of the problem is available even if the physical topology is 2SC. All of the given logical links (or lightpaths) remain in the final solution. More logical links are added when necessary to achieve 2SC partitions. For a physical topology of 7 nodes, 12 links, and up to 5 SRLGs, we test with four different logical topologies: topology A has 11 lightpaths, topology B has 10 lightpaths, topology C has 9 lightpaths, and topology D has 8 lightpaths. As depicted in Fig. 1, for $Q = 2$, the fewer links are given in the logical topology, the more new links are needed to ensure survivable partitions. For example, an optimal solution for topology A needs 1 new logical link, whereas an optimal solution for topology D needs 3 new logical links. However, a fewer number of given logical links allows us to reduce the cost of 2SC partitioning, since we can add logical links in places where it requires fewer wavelength-links to map on the physical topology. As shown in Fig. 2, the cost of an optimal solution is increased when we increase the number of SRLGs in the physical topology. The reason is that, with the increase of risks, both the number of lightpaths and the number of wavelength-links used for each lightpath are generally increased to enforce 2SC partitions.

5. Conclusion

In this paper, we argue that survivable mapping of a logical topology on a physical topology must be associated with individual partitions to allow protection and restoration within each partition and to avoid frequent re-partitioning of a large network upon single SRLG failures. Since a given logical topology does not always guarantee a 2-SRLG-connected solution, we consider the augmentation of the logical topology (i.e., inclusion of additional lightpaths) to ensure survivability. We find that the fewer links are given in the initial logical topology, the lower cost is needed to achieve 2-SRLG-connected partitions. The cost is increased at higher number of SRLGs. The work in this paper can be extended to provide even higher degrees of fault-tolerance in network partitioning, e.g., 3-SRLG-connectivity and so on.

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