Capital Cash Flows:
A Simple Approach to Valuing Risky Cash Flows

Richard S. Ruback
Graduate School of Business Administration
Harvard University
Boston, MA 02163

email: rruback@hbs.edu

ABSTRACT

This paper presents the Capital Cash Flow method for valuing risky cash flows. I show that the Capital Cash Flow method is equivalent to discounting Free Cash Flows by theweighted average cost of capital. Because the interest tax shields are included in the cash flows, the Capital Cash Flow approach is easier to apply when the level of debt changes or when a specific amount of debt is projected. The paper also compares the Capital Cash Flow method to the Adjusted Present Value method and provides consistent leverage adjustment formulas for both methods.

Draft
May 8, 2000

Comments Welcome

Capital Cash Flows:
A Simple Approach to Valuing Risky Cash Flows

1. Introduction

The most common technique for valuing risky cash flows is the Free Cash Flow method. In that method, the tax deductibility of interest is treated as a decrease in the cost of capital using the after-tax weighted average cost of capital (WACC). Interest tax shields are therefore excluded from the Free Cash Flows. Because the weighted average cost of capital is affected by changes in capital structure, the Free Cash Flow method poses several implementation problems in highly leveraged transactions, restructurings, project financings and other instances in which capital structure changes over time. In these situations, the capital structure has to be estimated and those estimates have to be used to compute the appropriate weighted average cost of capital in each period. Under these circumstances, the Free Cash Flow method can be used to correctly value the cash flows, but it is not straightforward.[1]

This paper presents an alternative method for valuing risky cash flows. I call this method the Capital Cash Flow (CCF) method because the cash flows include all of the cash available to capital providers, including the interest tax shields. In a capital structure with only ordinary debt and common equity, Capital Cash Flows equal the flows available to equity—net income plus depreciation less capital expenditure and the increase in working capital—plus the interest paid to debtholders. The interest tax shields decrease taxable

---

[1] See Loffler (1998) and Miles and Ezzell (1980).

income, decrease taxes and thereby increase after-tax cash flows. In other words, Capital Cash Flows equal Free Cash Flows plus the interest tax shields. Because the interest tax shields are included in the cash flows, the appropriate discount rate is before-tax and corresponds to the riskiness of the assets.

Although the Free Cash Flow and Capital Cash Flow methods treat interest tax shields differently, the two methods are algebraically equivalent. In other words, the Capital Cash Flow method is a different way of valuing cash flows using the same assumptions and approach as the Free Cash Flow method. The advantage of the Capital Cash Flow method is its simplicity. When debt is forecasted in dollar amounts or when capital structure changes over time, the Capital Cash Flow method is much easier to use because the interest tax shields are included in the cash flows. Also, the expected asset return depends on the riskiness of the asset and therefore does not change when capital structure changes. As a result, the discount rate for the Capital Cash Flows does not have to be re-estimated every period. In contrast, when using the FCF method, the after-tax weighted average cost of capital (WACC) has to be re-estimated every period. Because the WACC depends on value-weights, the value of the firm has to be estimated simultaneously. The CCF method avoids this complexity so that it is especially useful in valuing highly levered firms whose forecasted debt is usually expressed in dollars and whose capital structure changes substantially over time.

The Capital Cash Flow method is closely related to my work on valuing riskless cash flows (Ruback (1986)) and to Stewart Myers' work on the Adjusted Present Value (APV) method (Myers (1974)). In my paper on riskless cash flows, I showed that the interest tax shields associated with riskless cash flows can be equivalently treated as increasing cash flows by the interest tax shield or as decreasing the discount rate to the after-tax riskless rate.

The analysis in this paper presents similar results for risky cash flows; namely, risky cash flows can be equivalently valued by using the Capital Cash Flow method with the interest tax shields in the cash flows or by using the Free Cash Flow method with the interest shields in the discount rate.

The Adjusted Present Value method is generally calculated as the sum of Free Cash Flows discounted by the cost of assets plus interest tax shields discounted at the cost of debt. It results in a higher value than the Capital Cash Flow method because it assigns a higher value to interest tax shields. The interest tax shields that are discounted by the cost of debt in the APV method are discounted by the cost of assets explicitly in the CCF method and implicitly in the Free Cash Flow method. Stewart Myers suggested the term "Compressed APV" to describe the CCF method because the APV method is equivalent to CCF when the interest tax shields are discounted at the cost of assets. However, most descriptions of APV suggest discounting the interest tax shields at the cost of debt (Luehrman (1997)).

The Adjusted Present Value method treats the interest tax shields as being less risky than the assets because the level of debt is implicitly assumed to be a fixed dollar amount. The intuition is that interest tax shields are realized roughly when interest is paid so that the risk of the shields matches the risk of the payment. This matching of the risk of the tax shields and the interest payment only occurs when the level of debt is fixed. Otherwise, the risk of the shields depends on both the risk of the payment *and* systematic changes in the amount of debt. Because the risk of a levered firm is a weighted average of the risk of an unlevered firm and the risk of the interest tax shields, the presence of less risky interest tax shields reduces the risk of the levered firm. As a result, a tax adjustment has to be made when unlevering an equity beta to calculate an asset beta.

The Capital Cash Flow method, like the Free Cash Flow method, assumes that debt is proportional to value. The higher the value of the firm, the more debt the firm uses in its financial structure. The more debt used, the higher the interest tax shields. The risk of the interest tax shields therefore depends on the risk of the debt as well as the changes in the level of the debt. When debt is a fixed proportion of value, the interest tax shields will have the same risk as the firm, even when the debt is riskless. Because the interest tax shields have the same risk as the firm, leverage does not alter the beta of the firm. As a result, no tax adjustment has to be made when calculating asset betas.

The primary contributions of this paper are to introduce the Capital Cash Flow method of valuation, to demonstrate its equivalency to the Free Cash Flow method, and to show its relation to the Adjusted Present Value method. The Capital Cash Flow method has been used in teaching materials to value cash flow forecasts, in Kaplan and Ruback (1995) to value highly levered transactions, and in Hotchkiss, Gilson and Ruback (1998) to value firms emerging from Chapter 11 reorganizations.[2] Also, finance textbooks contain some of the ideas about the relation between the discount rate for interest tax shields, unlevering formulas, and financial policy. This paper provides the basis for the applications of Capital Cash Flows and highlights the linkages between the three methods of cash flow valuation.

Section 2 of this paper describes the mechanics of the Capital Cash Flow method, including the calculation of the cash flows and the discount rate. Section 3 shows that the Capital Cash Flow method is equivalent to the Free Cash Flow method through an example and then with a more general proof. Section 4 relates the Capital Cash Flow and the Adjusted Present Value methods and shows that the difference between the two methods

---

[2] Teaching materials include Ruback (1989, 1995a, 1995b).

depends on the implicit assumption about the financial policy of the firm. I also show that the assumption about financial policy has implications regarding the impact of taxes on risk and thereby determines the approach used to transform equity betas into asset betas.

2. Mechanics of Capital Cash Flow Valuation

The present value of Capital Cash Flows is calculated by discounting the CCFs by the expected asset return, K_A. This section details the calculation of the CCFs in Section 2.1 and explains the calculations of K_A in Section 2.2. An example is presented in Section 2.3.

2.1 Calculating Capital Cash Flows   Capital Cash Flows include all of the cash flows that are paid or could be paid to any capital provider. By including cash flows to all security holders, CCFs measure all of the after-tax cash generated by the assets. Since CCFs measure the after-tax cash flows from the enterprise, the present value of these cash flows equals the value of the enterprise.

Figure A summarizes the calculation of Capital Cash Flows. The calculations depend on whether the cash flow forecasts begin with net income (NI) or earnings before interest and taxes (EBIT).

The Net Income Path   Net income includes any tax benefit from debt financing because interest is deducted before computing taxes. Net income is therefore increased by the interest tax shields. Cash flow adjustments and noncash interest are added to net income to determine the available cash flow. Cash flow adjustments include those adjustments required to transform the accounting data into cash flow data. Typical adjustments include adding depreciation and amortization because these are noncash subtractions from net income.

Figure A: Calculating Capital Cash Flows
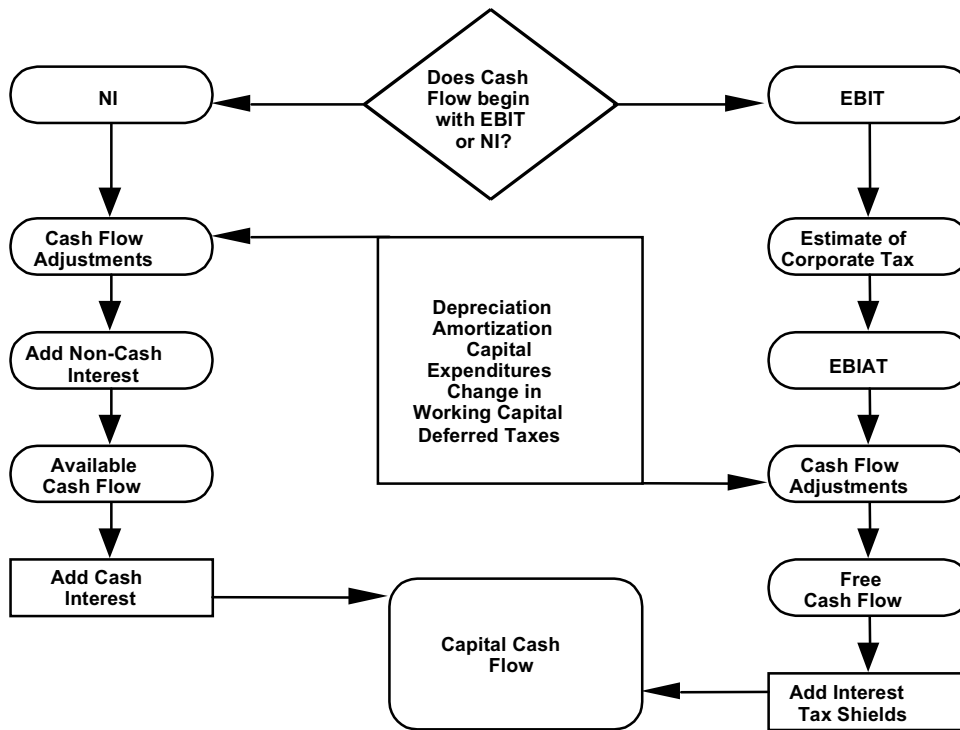
Capital expenditures are subtracted from net income because these cash outflows do not appear on the income statement and thus are not deducted from net income. Subtracting the increases in working capital transforms the recognized accounting revenues and costs into cash revenues and costs. Net income is net of noncash interest, if any. Because noncash interest is not a cash outflow, it is added to net income to compute the available cash flow. The label 'available cash flow' often appears in projections and measures the funds available for debt repayments or other corporate uses. Capital Cash Flow is computed by adding cash interest to available cash flow so that cash flows represent the after-tax cash available to all cash providers.

The EBIT Path When cash flow forecasts present EBIT instead of net income, corporate taxes have to be estimated to calculate earnings before interest and after taxes (EBIAT). Typically the taxes are estimated by multiplying EBIT by a historical marginal tax

rate. EBIAT is then adjusted using the cash flow adjustments that transform the accounting data into cash flow data. EBIAT plus cash flow adjustments equals Free Cash Flow, which is used to compute value using the after-tax weighted average cost of capital (WACC). Free Cash Flows equal Capital Cash Flows less the interest tax shields. Interest tax shields, therefore, have to be added to the Free Cash Flows to arrive at the Capital Cash Flows. The interest tax shields on both cash and noncash debt are added because both types of interest tax shields reduce taxes and thereby increase after-tax cash flow.

The EBIT path should yield the same Capital Cash Flows as the net income path. In practice, however, the net income path is usually easier and more accurate than the EBIT path. The primary advantage of the net income path is that it uses the corporate forecast of taxes, which should include any special circumstances of the firm. Taxes are rarely equal to the marginal tax rate times taxable income. The EBIT path involves estimating taxes, usually by assuming a constant average tax rate. This ignores the special circumstances of the firm and adds a likely source of error.

2.2 Calculating the Expected Asset Return The appropriate discount rate to value Capital Cash Flows (CCFs) is a before-tax rate because the tax benefits of debt financing are included in the CCFs. The pre-tax rate should correspond to the riskiness of the CCFs. One such discount rate is the pre-tax weighted average cost of capital:

$$Pre-tax\ WACC = \frac{D}{V}K_D + \frac{E}{V}K_E \qquad (2.1)$$

where D/V is the debt-to-value ratio; E/V is the equity-to-value ratio, and K_D and K_E are the respective expected debt and equity returns. Using the pre-tax WACC as a discount rate is correct, but there is a much simpler approach. Note that the expected returns in (2.1) are

determined by the Capital Asset Pricing Model (CAPM):

$$K_D = R_F + \beta_D R_P \tag{2.2}$$

$$K_E = R_F + \beta_E R_P \tag{2.3}$$

where R_F is the risk-free rate, R_P is the risk premium, and β_D and β_E are the debt and equity betas, respectively. Substituting (2.2) and (2.3) into (2.1) yields:

$$Pre-tax\ WACC = \frac{D}{V}(R_F + \beta_D R_P) + \frac{E}{V}(R_F + \beta_E R_P). \tag{2.4}$$

Simplifying:

$$Pre-tax\ WACC = R_F + \left(\frac{D}{V}\beta_D + \frac{E}{V}\beta_E\right)R_P. \tag{2.5}$$

The beta of the assets, β_U, is a weighted average of the debt and equity beta:

$$\beta_U = \frac{D}{V}\beta_D + \frac{E}{V}\beta_E. \tag{2.6}$$

Substituting (2.6) into (2.5) provides a simple formula for the pre-tax WACC which is also labeled as the Expected Asset Return, K_A:

$$K_A = Pre-tax\ WACC = R_F + \beta_U R_P \tag{2.7}$$

Note that the pre-tax expected asset return depends only on the market-wide parameters for the risk-free rate, R_F, and the risk premium, R_P, and on the unlevered asset beta, β_U. The debt-to-value and equity-to-value ratios are *not* in (2.7). K_A, therefore, does *not* depend on capital structure and does *not* have to be recomputed as capital structure changes. This means that the debt-to-value and equity-to-value ratios do *not* have to be estimated to use the Capital Cash Flow valuation method. This eliminates much of the complexity encountered when applying the FCF method.

The discount rate for the Capital Cash Flows is simple to calculate regardless of the

capital structure. It takes two steps. First, estimate the asset beta, β_U. Second, use β_U, together with the risk-free rate, R_F, and the risk premium, R_P, to compute the expected asset return, K_A. For example, if the asset beta is assumed to be 1.0, the risk free rate is assumed to equal 10% and the risk premium assumed to be 8%, the expected asset return is 18%.

2.3 Numerical Example   **Table 1** contains a numerical example that demonstrates the Capital Cash Flow method. The example assumes an initial investment of $100,000 to be depreciated equally over three years. Panel A details the assumptions. The asset beta is 1.0 and the forecasted expected pre-tax operating profits are $50,000 in year one, $60,000 in year two, and $70,000 in year three. The risk-free rate is assumed to be 10%, the risk premium is assumed to be 8%, and the tax rate is assumed to be 33%. The debt is assumed to be risky, with a debt beta of 0.3. The project is financed with debt so that the initial debt is $100,000 at the beginning of year one, $65,000 at the beginning of year two, and $20,000 at the beginning of year three.

The Capital Cash Flow is calculated by the following the net income path. The cash flow available is equal to net income plus noncash adjustments. CCF is calculated by adding the expected interest to the cash flow available.

The value of the Capital Cash Flows is calculated using the expected asset return. The easiest way to calculate the asset return is to use the asset beta in the CAPM. Using a risk-free rate of 10%, an asset beta of 1.0 and a risk premium of 8% yields an expected asset return of 18%. The asset return does not depend on leverage because it is a pre-tax cost of capital. It remains constant even though the leverage changes through time. As Panel B of **Table 1** shows, discounting the CCFs at the expected asset return results in a value of $117,773.

Table 1: An Example of Capital Cash Flow Valuation

Panel A: Assumptions

Market parameters

Asset beta, β_A =	1
Debt beta, β_D =	0.3
Risk-free debt rate, R_F =	10%
Risk premium, R_P =	8%
Tax rate, τ =	33%

Expected Cash Flows ($)	year 1	year 2	year 3
Operating Profit	50,000	60,000	70,000
Less: Depreciation	33,333	33,333	33,333
EBIT	16,667	26,667	36,667
Less: Expected Interest (#1)	12,400	8,060	2,480
Pre-tax income	4,267	18,607	34,187
Less: Taxes	1,408	6,140	11,282
Net Income	2,859	12,466	22,905
Non-cash Adjustments (#2)	34,333	34,333	34,333
Cash Flow Available	37,192	46,800	57,238
Beginning Debt	100,000	65,000	20,000

Panel B: Capital Cash Flow Valuation

	year 1	year 2	year 3
Cash Flow Available	37,192	46,800	57,238
Plus: Expected Interest (#1)	12,400	8,060	2,480
Capital Cash Flow	49,592	54,860	59,718
Cost of Assets, K_A (#3)	18.0%	18.0%	18.0%
Discount factor	0.8475	0.7182	0.6086
Present Value of CCFs	42,027	39,399	36,346
Total Enterprise Value	117,773		

Note #1: Expected Interest is calculated using the Expected Cost of Debt from the CAPM (riskfree rate plus the debt beta times the risk premiu
Note #2: Noncash adjustments include depreciation plus $10,000 of other adjustments.
Note #3: Expected asset return is calculated using the assumed asset beta in the CAPM with the assumed riskless debt rate and risk premium

3. The Relation Between Capital Cash Flow and Free Cash Flow Valuation

3.1 Numerical Example   **Table 2** presents a Free Cash Flow valuation of the same cash flows valued using Capital Cash Flows in Panel A of Table 1. The Free Cash Flows are calculated from EBIT, which is reduced by the hypothetical taxes on EBIT to determine EBIAT. Adding the non-cash adjustments to EBIAT results in Free Cash Flows.

The Free Cash Flows are valued using the after-tax weighted average cost of capital (WACC). The WACC has two components: the after-tax cost of debt and the levered cost of equity. The after-tax cost of debt depends on the assumed riskiness of the debt with the cost of debt calculated as its CAPM expected return using (2.2). The levered cost of equity is calculated by levering the asset beta to determine the levered equity beta. Because the fraction of debt is not the same each year, the WACC and its components need to be recomputed each year.

The formula for levering the asset or unlevered beta is:

$$\beta_E = \left(\beta_U - \frac{D}{V}\beta_D \right) / \frac{E}{V} \tag{3.1}$$

which requires information on the value of the firm to compute the percentage of debt and equity in the capital structure.[3]   Generally, an iterative or dynamic programming approach is used to solve for a consistent estimate of enterprise value. However, because the value is already computed in Table 1, that value can be used to compute the debt and equity proportions. Based on the implied equity-to-value ratio of 15.1% in the first year, the asset beta of 1.0 and the debt beta of 0.3, the implied equity beta is 4.94.   Using the CAPM and the assumed market parameters, the expected cost of equity is 49.5% in the first year. Weighting the

---

[3] This formula is derived in Section 4.2 of this paper.

Table 2: An Example of Free Cash Flow Valuation

Panel A: Assumptions

<u>Market parameters</u>

Asset beta, β_A =	1
Debt beta, β_D =	0.3
Risk-free debt rate, R_F =	10%
Risk premium, R_P =	8%
Tax rate, τ =	33%

Expected Cash Flows ($)	year 1	year 2	year 3
Operating Profit	50,000	60,000	70,000
Less: Depreciation	33,333	33,333	33,333
EBIT	16,667	26,667	36,667
Less: Expected Interest (#1)	12,400	8,060	2,480
Pre-tax income	4,267	18,607	34,187
Less: Taxes	1,408	6,140	11,282
Net Income	2,859	12,466	22,905
Non-cash Adjustments (#2)	34,333	34,333	34,333
Cash Flow Available	37,192	46,800	57,238
Beginning Debt	100,000	65,000	20,000

Panel B: Free Cash Flow Valuation

	year 1	year 2	year 3
EBIT	16,667	26,667	36,667
Less: Tax on EBIT	5,500	8,800	12,100
EBIAT	11,167	17,867	24,567
Non-cash Adjustments (#2)	34,333	34,333	34,333
Free Cash Flows	45,500	52,200	58,900
<u>Capitalization</u>			
Total Enterprise Value (#3)	117,773	89,380	50,609
Debt	100,000	65,000	20,000
<u>WACC Calculations</u>			
Debt			
Percent	84.9%	72.7%	39.5%
After-tax cost (#4)	8.3%	8.3%	8.3%
Contribution (#5)	7.1%	6.0%	3.3%
Equity			
Percent	15.1%	27.3%	60.5%
Equity beta (#6)	4.94	2.87	1.46
Cost (#7)	49.5%	32.9%	21.7%
Conribution (#8)	7.5%	9.0%	13.1%
After-tax WACC	14.5%	15.0%	16.4%
Discount factor	0.8732	0.7591	0.6523
Present Value of FCFs	39,729	39,626	38,418
Total Enterprise Value	117,773		

Note #1: Expected Interest equals the beginning debt times Expected Cost of Debt (=riskfree rate+debt beta*risk premium).

Note #2: Noncash adjustments include depreciation plus $10,000 of other adjustments.

Note #3: Total Enterprise Value is the present value of the remaining cash flows.

Note #4: After-tax cost of debt is the Expected Cost of Debt times (1-tax rate).

Note #5: Debt contribution is the After-tax Expected Cost of Debt times the percent debt.

Note #6: Equity beta is determined by levering the asset beta ((asset beta - debt beta contribution)/percent equity).

Note #7: Cost of equity is calculated using the CAPM as the riskfree rate plus the equity beta times the risk premium.

Note #8: Equity contribution is the cost of equity times the percent equity.

expected after-tax cost of debt and the expected cost of equity by their proportions in the capital structure results in a WACC of 14.5% for the first year.

The capital structure changes in each period because the ratio of the value of the remaining cash flows and the amount of debt outstanding does not remain constant throughout the life of the project. Repeating the process of valuing the enterprise, determining the debt and equity proportions, unlevering the asset beta, and estimating the equity cost of capital according to the CAPM, results in a weighted average cost of capital of 15.0% for the second year and 16.4% for the third year. These after-tax WACCs rise as the percentage of debt in the capital structure, and the corresponding amount of the interest tax shields, falls.

Total Enterprise Value is calculated by discounting the FCFs by the after-tax WACCs. Since the after-tax WACCs change, the discount rate for each period is the compounded rate that uses the preceding after-tax WACCs. The resulting value of the Free Cash Flows is $117,773, exactly the same value as obtained in the Capital Cash Flow calculations in Panel B of Table 1.

3.2 Proof of Equivalency This section shows that the Capital Cash Flow method is equivalent to the Free Cash Flow method. To keep the analysis simple, assume the asset being valued generates a constant pre-tax operating cash flow of X. This cash flow is before tax but after cash adjustments such as depreciation, capital expenditures, and changes in working capital. The after-tax operating cash flow, $X(1-\tau)$ equals earnings before interest and after-tax plus the cash flow adjustments where τ is the tax rate. This after-tax operating cash flow measures the cash flow of the firm if it were all equity financed. Therefore, $X(1-\tau)$ equals Free Cash Flow.

The value, V_{FCF}, is calculated using the Free Cash Flow method by discounting the free cash flows by the after-tax WACC:

$$V_{FCF} = \frac{X(1-\tau)}{WACC} \qquad (3.2)$$

where V is the value of the project being valued. WACC, the after-tax weighted average cost of capital, is defined as:

$$WACC = \frac{D}{V}K_D(1-\tau) + \frac{E}{V}K_E \qquad (3.3)$$

with D and E equal to the market value of debt and equity, respectively; $K_D(1-\tau)$ is the after-tax expected cost of debt; and K_E is the expected cost of equity.

The Capital Cash Flow is the expected cash flow to all capital providers with its projected financing policy, including any benefits of interest tax shields from its financial structure. Since Free Cash Flow measures the cash flow assuming a hypothetical all equity capital structure, Capital Cash Flow is equal to Free Cash Flow plus interest tax shields:

$$CCF = FCF + Interest\,Tax\,Shield = X(1-\tau) + \tau K_D D \qquad (3.4)$$

where $\tau K_D D$ is the interest tax shield calculated as the tax rate $[\tau]$ times the interest rate on the debt $[K_D]$ times the amount of debt outstanding, D.

Value is calculated using the Capital Cash Flow method, V_{CCF}, by discounting the Capital Cash Flows by the expected return on assets. The expected asset return is measured using the Capital Asset Pricing Model and the asset beta (β_U) of the project being valued:

$$V_{CCF} = \frac{X(1-\tau) + \tau K_D D}{R_F + \beta_U R_P} \qquad (3.5)$$

where R_F is the risk-free rate and R_P is the risk premium.

The goal is to show that the value obtained using FCFs and WACC is the *same* as the value obtained using CCFs and K_A. In other words, the goal is to show that (3.2) is identical to (3.5). Combining (3.2) and (3.3):

$$V_{FCF} = \frac{X(1-\tau)}{K_D \frac{D}{V}(1-\tau) + \frac{E}{V}K_E} \qquad (3.6)$$

In (3.6) K_E and K_D are measured using the Capital Asset Pricing Model according to (2.2) and (2.3). By substituting the equality between the pre-tax WACC and the cost of assets from (2.7):

$$V_{FCF} = \frac{X(1-\tau)}{(R_F + \beta_U R_P) - \tau K_D \frac{D}{V}} = \frac{X(1-\tau)}{K_A - \tau K_D \frac{D}{V}} \qquad (3.7)$$

Multiplying both sides by the denominator on the right-hand-side of (3.7) yields:

$$V_{FCF}(K_A) - \tau K_D D = X(1-\tau) \qquad (3.8)$$

Rearranging terms by adding $\tau R_F D$ to both sides and dividing by the cost of assets shows that:

$$V_{FCF} = \frac{X(1-\tau) + \tau D K_D}{K_A} = V_{CCF} \qquad (3.9)$$

which is identical to (3.5). Thus, this proof shows that the FCF approach in (3.2) and the CCF approach of (3.5) will, when correctly applied, result in identical present values for risky cash flows.

3.3 Choosing Between Capital Cash Flows and Free Cash Flow Methods

The proof in Section 3.2 shows that the Capital Cash Flow method and the Free Cash Flow method are equivalent because they make the same assumptions about cash flows, capital structure, and taxes. When applied correctly using the same information and

assumptions, the two methods provide identical answers. The choice between the two methods, therefore, is governed by ease of use. The ease of use, of course, is determined by the complexity of applying the method and the likelihood of error.

The form of the cash flow projections generally dictates the choice of method. In the simplest valuation exercise, when the cash flows do not include the interest tax shields and the financing strategy is specified as broad ratios, the Free Cash Flow method is easier than the Capital Cash Flow method. To apply the FCF method, the discount rate can be calculated in a straightforward manner using prevailing capital market data and information on the target capital structure. Because that target structure does not (by assumption) change over time, a single weighted average cost of capital can be used to value the cash flows. This type of valuation often occurs in the early stages of a project valuation before the detailed financial plan is developed. When the goal is to get a simplified 'back-of-the-envelope' value, the FCF method is usually the best approach.

When the cash flow projections include detailed information about the financing plan, the Capital Cash Flow method is generally the more direct valuation approach. Because such plans typically include the forecasted interest payments and net income, the CCFs are simply computed by adding the interest payments to the net income and making the appropriate non-cash adjustments. These cash flows are valued by discounting them at the expected cost of assets. This process is simple and straightforward even if the capital structure changes through time. In contrast, applying the Free Cash Flow method is more complex and more prone to error because, as illustrated in Section 3.1 and Table 2, firm and the equity value have to be inferred to apply the FCF method. Also, the CCF method can easily incorporate

complex tax situations. Therefore, in most transactions, restructurings, leverage buyouts and bankruptcies, the CCF method will be the easier to apply.

4.0 Capital Cash Flows and Adjusted Present Value.

Both Capital Cash Flows and Adjusted Present Value can be expressed as:

$$\text{Value} = \text{Free Cash Flows Discounted at } K_A + \text{Interest Tax Shields Discounted at } K_{ITS}$$

where K_{ITS} is the discount rate for interest tax shields. For both methods, the discount rate for the Free Cash Flows is the cost of assets, (K_A) which is generally computed using the CAPM with the beta of an unlevered firm. The methods differ in K_{ITS}, the discount rate for interest tax shields: the APV method generally uses the debt rate; the CCF method uses the cost of assets, K_A. APV assigns a higher value to the interest tax shields so that values calculated with APV will be higher than CCF valuations.[4]

To gauge how much higher APV valuations are relative to CCF valuations, **Table 3** calculates the difference in values assuming perpetual cash flows and interest tax shields. I define the value of the interest tax shields in the CCF valuation as a proportion, γ, of the all equity value. The ratio of V_{APV} to V_{CCF} becomes:

$$\frac{V_{APV}}{V_{CCF}} = \frac{1 + \gamma\left(\dfrac{K_A}{K_D}\right)}{1 + \gamma} \tag{4.1}$$

Table 3 presents the percentage differences between the APV and CCF valuations. For example, if $K_D = 10\%$ and $K_A = 15\%$, the ratio of the expected asset return to the expected

---

[4] Inselbag and Kaufold (1997) present examples of Free Cash Flow and APV valuations that result in identical values for debt policies with both fixed debt and proportional debt. This occurs because they infer the equity costs that result equivalence in their FCF valuations instead of obtaining discount rates from the CAPM.

debt return is 1.5, locating it in the middle column of Table 3. If the tax rate is 36% and the

debt is 42% of the all equity value, the value of the interest tax shield is about 15% of the all

equity value, locating it in the middle row of Table 3. In this example, therefore, the APV

approach would provide a discounted cash flow value that is 7% higher than the CCF value.

Table 3: Percentage Differences Between APV Values and CCF Values (V_{APV}/V_{CCF})

Tax Shield/ All Equity Value [2]	Ratio of Expected Asset Return to Debt Rate (K_A/K_D) [1]		
	1.25	1.50	1.75
10%	2%	5%	7%
15%	3%	7%	10%
20%	4%	8%	13%

Note #1: Calculations assume perpetual cash flows and interest tax shields.
Note #2: All Equity Value is the Free Cash Flows discounted at the cost of assets.

In the CAPM framework, the discount rate for the interest tax shields should depend

on the beta of the interest tax shields:

$$K_{ITS} = \text{Risk free rate} + \beta_{ITS} * \text{Risk Premium} \tag{4.2}$$

When debt is assumed fixed, Section 4.1 shows that the beta of the interest tax shidds

equals the beta of the debt. This implies that the appropriate discount rate for the interest tax

shields is the debt rate, which is the rate used in the APV method. It also implies that the

interest tax shields reduce risk so that a tax effect should appear when unlevering equity

betas. When debt is assumed proportional to value, Section 4.2 shows that the beta of the

interest tax shields is equal to the unlevered or asset beta. This implies that the appropriate

discount rate is the cost of assets, which is the rate used in the CCF method. It also implies

that taxes have no effect on the transformation of equity betas into asset betas.

4.1 Fixed Debt

When debt is fixed as a dollar amount, D, that does not change as the value of the firm changes, the value of the interest tax shields is:

$$V_{ITS,t} = \frac{\tau \bar{K}_D D}{K_{D,t}}$$
(4.3)

where $\bar{K}_D$ is the fixed yield on the debt, $K_{D,t}$ is the cost of debt in period t, and τ is the tax rate. The value of the debt can change through time if K_D is fixed and the cost of debt changes. Assuming $\bar{K}_D$ is the fixed yield,

$$V_{D,t} = \frac{D\bar{K}_D}{K_{D,t}}$$
(4.4)

Substituting (4.4) into (4.3), the value of the interest tax shield at time t therefore can be expressed as:

$$V_{ITS,t} = \tau V_{D,t}$$
(4.5)

The beta of the interest tax shields, β_{ITS}, equals:

$$\beta_{ITS} = \frac{Cov(V_{ITS,t}, R_M)}{V_{ITS,t-1} Var(R_M)}$$
(4.6)

Substituting (4.5) into (4.6) and simplifying,

$$\beta_{ITS} = \frac{Cov(V_{D,t}, R_M)}{V_{D,t-1} Var(R_M)} = \beta_D$$
(4.7)

The beta of the interest tax shields is therefore equal to the beta of the debt when the debt is assumed to be a fixed dollar amount.[5] If the debt is assumed to be riskless, the interest tax shields will also be riskless. If the debt is risky, the interest tax shields will have the same amount of systematic risk as the debt. This result shows that the practice of discounting interest tax shields by the expected return on the debt is appropriate when the debt is assumed to be a fixed dollar amount.

The assumption of fixed debt and the result that the beta of interest tax shields equals the debt beta implies that leverage reduces the systematic risk of the levered assets. The value of a levered firm (V_L) exceeds the value of an unlevered or all equity firm (V_U) by value of the interest tax shields from the debt of the levered firm (V_{ITS}):

$$V_L = V_U + V_{ITS} \qquad (4.8)$$

Equation (4.8) holds in each time period and abstracts from differences between levered and unlevered firms other than taxes. Also, the analysis assumes strictly proportional taxes. I assume that interest is deductible and that interest tax shields are realized when interest is paid. The beta of the levered firm, β_L, is a value-weighted average of the unlevered beta, β_U, and the beta of the interest tax shields, β_{ITS}, is the beta of the interest tax shields:

$$\beta_L = \frac{V_U}{V_L}\beta_U + \frac{V_{ITS}}{V_L}\beta_{ITS} \qquad (4.9)$$

When the beta of the interest tax shields equals the debt beta, equation (4.9) simplifies to:

---

[5] When debt is assumed to be fixed in value instead of a fixed dollar amount, the beta of the interest tax shields is zero regardless of the debt beta.

$$\beta_L = \beta_U - \tau \frac{D}{V_L}(\beta_U - \beta_D) \tag{4.10}$$

The beta of a levered firm, β_L, can also be expressed as a value weighted average of the debt and equity of the levered firm:

$$\beta_L = \frac{E}{V_L}\beta_E + \frac{D}{V_L}\beta_D \tag{4.11}$$

Where E is the equity of the levered firm, β_E is the equity beta and β_D is the debt beta. Setting (4.10) equal to (4.11):

$$\frac{E}{V_L}\beta_E + \frac{D}{V_L}\beta_D = \beta_L = \beta_U - \tau \frac{D}{V_L}(\beta_U - \beta_D) \tag{4.12}$$

which can be simplified as:

$$\beta_E = \left(\beta_U - \frac{D}{V_L}(\beta_D + \tau(\beta_U - \beta_D))\right) / \frac{E}{V_L} \tag{4.13}$$

Thus the equity beta is equal to the asset beta less the proportion of debt borne by the debt holder and the reduction due to the tax effect and scaled by leverage. The equity beta is reduced by the tax effect because the government absorbs some of the risk of the cash flows. With fixed debt, the interest tax shields portion of the cash flows are insulated from fluctuations in the market value of the firm.

When the debt is riskless, the beta of the debt is zero. Therefore, (4.13) simplifies to:

$$\beta_E = \frac{E + D(1-\tau)}{E}\beta_U \tag{4.14}$$

Equation (4.14) is the standard unlevering formula that correctly includes tax effects when the debt is assumed to be fixed and assumes a zero debt beta. In the next sub-section I show

that when debt is assumed to be proportional to firm value, taxes do not appear in the unlevering formula.

4.2 Proportional Debt

When the value of debt is assumed to be proportional to total enterprise value, the firm varies the amount of debt outstanding in each period so that:

$$V_D = \delta V_U \tag{4.15}$$

where δ is the proportionality coefficient and V_U is the value of the unlevered firm. The value of the interest tax shields is the tax rate times the value of the debt so that

$$V_{ITS} = \tau V_D = \tau \delta V_U \tag{4.16}$$

Substituting (4.16) into the definition of the beta of the interest tax shields from (4.4):

$$\beta_{ITS} = \frac{Cov(V_{ITS,t}, R_M)}{V_{ITS,t-1} Var(R_M)} V_{ITS,t-1} \tag{4.17}$$

$$= \frac{Cov(\tau \delta V_{Ut}, R_M)}{\tau \delta V_{U,t-1} Var(R_M)}$$

$$= \frac{Cov(V_{U,t}, R_{M,t})}{V_{U,t-1} Var(R_M)}$$

$$= \beta_U$$

The equality between the beta of the interest tax shields and the beta of the unlevered firm implies that the rate used to discount the interest tax shields is equal to K_A, the unlevered or asset cost of capital.

The equality between the betas for the interest tax shields and the assets also implies that there is no levering/unlevering tax effect. From (4.9) the beta of a levered firm is

a weighted average of the beta of the unlevered firm and the beta of the interest tax shields. Since the asset beta equals the interest tax shield beta, the beta of the levered firm equals the beta of the unlevered firm. To calculate the beta of levered equity, (4.11) can be restated as:

$$\beta_E = \left(\beta_U - \frac{D}{V}\beta_D \right) / \frac{E}{V} \tag{4.18}$$

This result means that tax terms should not be include when applying the Capital Cash Flow or Free Cash Flow methods.[6]

4.3 Choosing Between Capital Cash Flows and Adjusted Present Value Methods

Section 4.2 shows that the difference between the Capital Cash Flow and the Adjusted Present Value methods is the implicit assumption about the determinates of leverage. CCF (and equivalently FCF) assumes that debt is proportional to value; APV assumes that debt is fixed and independent of value. Debt cannot literally be strictly proportional to value at all levels of firm value. For example, when a firm is in financial distress, the option component of risky debt increases, thereby distorting the proportionality. Nevertheless, Graham and Harvey (1999) report that about 80% of firms have some form of target debt-to-value ratio, and that the target is tighter for larger firms. That suggests that the CCF approach is more appropriate than the APV approach when valuing corporations.

In practice, valuations are often performed on forecasts that make assumptions about debt policy. When that policy is characterized as a target debt-to-value ratio, the proportional policy seems more accurate. In project finance or leveraged buyout situations, however, the forecasts typically are characterized as a changing dollar amount of debt in each year. These

---

[6] Kaplan and Ruback (1995) incorrectly used tax adjustments to unlever observed equity betas to obtain asset betas when applying the Capital Cash Flow method. Correcting this error does not meaningfully change the results of Kaplan and Ruback (1995).

amounts can, of course, be characterized as a changing percentage of value or as a changing dollar amount through time. It isn't obvious from the forecasts themselves whether the assumption of proportional debt or fixed debt is the better description of debt policy. The answer in these circumstances depends on the likely dynamic behavior. If debt policy adheres to the forecasts regardless of the evolution of value through time, the fixed assumption is probably better. Alternatively, if debt is likely to increase as the firm expands and value increases, then the proportional assumption is probably better.

There are circumstances when the fixed debt assumption is more accurate. These cases typically involve some tax or regulatory restriction on debt, such as industrial revenue bonds that are fixed in dollar amounts. Luehrman (1997) presents an example of APV valuation in which debt is assumed to be a constant fraction of *book* value. To the extent that book value does not respond to market forces, a fraction of book value is a fixed dollar amount. The CCF and APV methods can, of course, be combined. In Hochkiss, Gilson, and Ruback (2000), for example, the value of firms emerging from bankruptcy are valued as the capital cash flow value of their continuing operations plus the value of their fixed net operating losses discounted at a debt rate.

In most corporate circumstances, however, debt levels ought to change as market values change. Theories of debt policy generally suggest that debt changes as value changes.[7] Thus, for most applications, the proportional debt assumption appears to be a more accurate description of corporate behavior. That means that the Capital Cash Flow or the equivalent FCF method of valuation will generally be preferred to APV and that asset beta calculations should not include tax adjustments.

5.0 Conclusions

This paper presents the Capital Cash Flow (CCF) method of valuing risky cash flows. The CCF method is simple and intuitive. The after-tax capital cash flows are just the before-tax cash flows to both debt and equity, reduced by taxes including interest tax shields. The discount rate is the same expected return on assets that is used in the before-tax valuation. Because the benefit of tax deductible is included in the cash flows, the discount rate does not change when leverage changes.

The CCF method is algebraically equivalent to the popular method of discounting Free Cash Flows by the after-tax weighted average cost of capital. But in many instances, the Capital Cash Flow method is substantially easier to apply and, as a result, is less prone to error. The ease of use occurs because the Capital Cash Flow method puts the interest tax shields in the cash flows and discounts by a before-tax cost of assets. The cash flow calculations can generally rely on the projected taxes, and the cost of assets does not generally change through time even when the amount of debt changes. In contrast, when applying the Free Cash Flow method, taxes need to be inferred, and the cost of capital changes as the amount of debt changes.

The Capital Cash Flow method is closely related to the Adjusted Present Value method. Adjusted Present Value is generally calculated as the sum of operating cash flows discounted by the cost of assets plus interest tax shields discounted at the cost of debt. The interest tax shields that are discounted by the cost of debt in the APV method are discounted by the cost of assets in the Capital Cash Flow method. The Adjusted Present Value method

---

[7] Arzac (1996) suggests a "recursive APV approach" that recognizes that excess available cash flow is typically used to repay senior debt after a leveraged buyout.

results in a higher value than the Capital Cash Flow method because it treats the interest tax shields as being less risky than the firm as a whole because the level of debt is implicitly assumed to be a fixed dollar amount. As a result, a tax adjustment is made when unlevering an equity beta to calculate an asset beta. In contrast, the Capital Cash Flow method, like the FCF method, makes the more economically plausible assumption that debt is proportional to value. The risk of the interest tax shields therefore matches the risk of the assets.

References

Arzac, Enrique R., 1996, "Valuation of highly leveraged firms," *Financial Analysts Journal* (July/August), 42-49.

Gilson, Stuart C., Edith S Hotchkiss, and Richard S. Ruback, 2000, "Valuation of bankrupt firms," *Review of Financial Studies*, forthcoming.

Graham, John R. and Campbell R. Harvey, 1999, "The theory and practice of corporate finance: Evidence from the field," unpublished paper.

Inselbag, Isik and Howard Kaufold, 1997, "Two DCF approaches for valuing companies under alternative financing strategies and how to choose between them," *Journal of Applied Corporate Finance*, 10, 1 (Spring), 114-122

Kaplan, Steven N. and Richard S. Ruback, 1995, "The valuation of cash flow forecasts: an empirical analysis," *The Journal of Finance*, Volume L, No. 4, September 1995, pp. 1059-1093.

Kaplan, Steven N. and Richard S. Ruback, 1996, "The market pricing of cash flow forecasts: discounted cash flow vs. the method of comparables," *Journal of Applied Corporate Finance*, Volume 8, No. 4, Winter 1996, pp. 45-60.

Luehrman, Timothy A., 1997, "Using APV: A Better Tool for Valuing Operations," *Harvard Business Review*, May-June, 145-154.

Loffler, Andreas, 1998, "WACC approach and nonconstant leverage," unpublished paper.

Miles, James A. and John R. Ezzell, 1980, "The weighted average cost of capital, perfect capital markets, and project life: a clarification," *Journal of Financial and Quantitative Analysis*, 15, 3 (September), 719-730.

Myers, Stewart C, 1974, "Interactions of corporate financing and investment decisions—implications for capital budgeting," *The Journal of Finance*, 29, 1-25.

Ruback, Richard S., 1986, "Calculating the market value of riskless cash flows," *Journal of Financial Economics*, Volume 15, No. 3 (March 1986), pp. 323-339.

Ruback, Richard S., 1989, "Teaching Note for RJR Nabisco," Harvard Business School Case No. 5-289-057.

Ruback, Richard S., 1995a, "Technical Note for Capital Cash Flow Valuation," Harvard Business School Case No. 295-069.

Ruback, Richard S., 1995b, "Technical Note for Introduction to Cash Flow Valuation Methods: Harvard Business School Case No. 295-155.