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Competition in Modular Clusters

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Abstract

The last twenty years have witnessed the rise of disaggregated "clusters," "networks," or "ecosystems" of firms. In these clusters the activities of R&D, product design, production, distribution, and system integration may be split up among hundreds or even thousands of firms. Different firms will design and produce the different components of a complex artifact (like the processor, peripherals, and software of a computer system), and different firms will specialize in different stages of a complex production process. This paper considers the pricing behavior and profitability of these so-called *modular clusters*. In particular, we investigate a possibility hinted at in prior work: that for composite goods, a vertical pricing externality operating across complements can offset horizontal competition between substitutes. In this paper, we isolate the offsetting price effects and show how they operate in large (as well as small) clusters. We argue that it is possible in principle for a modular cluster of firms to mimic the pricing behavior and profitability of a vertically integrated monopoly. We then use our model to compare open and closed standards regimes, to understand how commoditization affects a cluster, to determine the relative profits of platform firms and firms that depend on the platform, and to assess the impact of horizontal and vertical mergers. Our model highlights a collective action problem: what is good for an individual firm is often not good for the cluster. We speculate that this conflict may be a source of strategic tension in platform firms.

Key words: oligopoly pricing, vertical integration, modularity, cluster

JEL classification: D21, D40, L13, L22, L23, M11, O31

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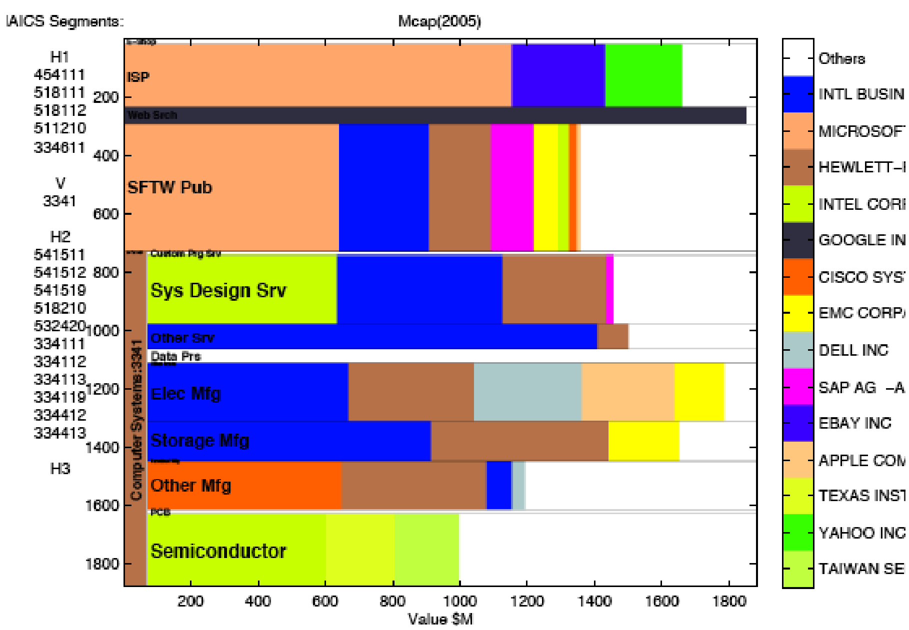
1 Introduction

The last twenty years have witnessed the rise of disaggregated "clusters," "networks," or "ecosystems" of firms in a number of industries including computers, telecommunications and pharmaceuticals. In these clusters, different firms design and produce the various components of a complex artifact (like the processor, peripherals, and software of a computer system), and different firms specialize in the various stages of a complex production process. Such clusters or networks of firms have been described by many scholars, including Powell (1987), Langlois and Robertson (1992), Saxenian (1994), Baldwin and Clark (2000), Sturgeon (2002), Bresnahan and Gambardella (2003), Iansiti and Levien (2004), Staudenmayer et al. (2005), Fallick et al. (2006), Gawer and Henderson (2007), Boudreau (2006), and Eisenmann et al. (2007).

Industry clusters can be envisioned using layer maps (Grove, 1996; Fransman, 2002; Fixson and Park, 2007). For example, Figure 1 is a map of the computer industry in 2005. Each horizontal band, or layer, represents the market for a component or a stage of production that contributes to a whole computer system. Layers are defined by NAICS industry classification codes (on the left), and include semiconductor design and manufacturing, storage device manufacturing, system design services, software publishing, and Internet service provision. The colored rectangles are sized in proportion to the market capitalization of the largest firms in each layer. The white areas at the far right represent the total market capitalization of the other Compustat-listed companies in each layer. In 2005, there were over 1,500 publicly traded firms in this cluster.

A computer system is a composite good, that is, a collection of components that are assembled into a working whole. To be functional, a modern computer system requires products and services from essentially all of the layers shown in Figure 1. Although consumers do not typically buy every component in a separate transaction (some components may be bundled), each component is an economic good that is subject to the pricing decisions of the firm that supplies it. In setting the price of a particular component, each firm competes against other suppliers of the same type of component, but also influences the demand for systems, hence the demand for complementary components. This is the industrial structure that Brandenburger and Nalebuff (1996) labeled "co-opetition." In addition to computers, other composite goods include vacations (travel, lodging, food, activities); movies (writing, acting, filming, production, distribution, exhibition, concessions); and telecommunications (handsets, transmission equipment, network infrastructure, software, services).

Figure 1
Layer Map for the Greater Computer Industry in 2005
(Area equals market capitalization)



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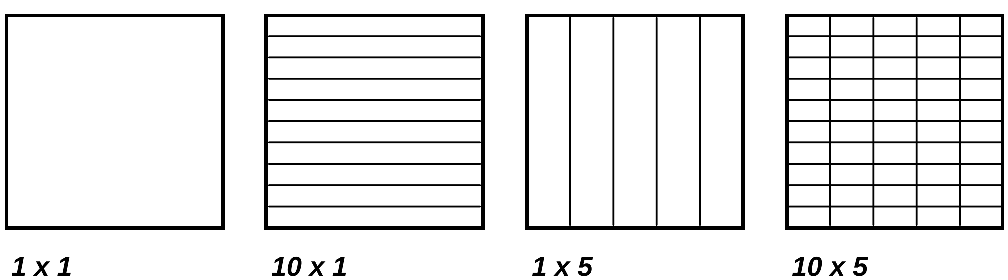
A fundamental tenet of economic theory is that an increase in the number of sellers of a good reduces equilibrium prices relative to a monopoly. An equally fundamental but less celebrated result, first derived by Cournot (1838), is that splitting the supply of complementary goods across two monopolies increases prices relative to a vertically integrated monopoly. If the complementary goods constitute the successive stages of a supply chain, the latter effect is known as "double marginalization" or the "chain of monopolies" effect (Spengler, 1950; Tirole, 1988). But, more generally, this *vertical pricing externality* applies to *any set* of complementary products or services that are combined into a composite good (Nalebuff, 2000). Formally, the vertical pricing externality is the dual of Cournot competition in quantities (Sonnenschein, 1968).

The purpose of this paper is to study competitive equilibria in *modular clusters* characterized by horizontal competition and the vertical pricing externality. By our definition, a modular cluster has two properties. First, the firms in the cluster make products that are both substitutes for other products in the same layer and complements to products in other layers. Second, the components are *modular*, in the sense of being mutually compatible and interoperable (Baldwin and Clark, 2000; Farrell and Weiser, 2003). In other words, a component from one layer will work in systems containing any combination of components from other layers. This "mix and match" assumption (cf. Matutes and Regibeau, 1988) means that the pricing of each component is influenced by the prices of all other components.

Modular clusters are based on technical architectures in which the standards of interoperability—often defined as part of a "platform"—are open and public (Farrell and Weiser, 2003; Henderson, 2005). In Section 5, we will discuss what happens when the standards are closed and/or proprietary.

To make our analysis tractable, we will lean heavily on symmetry. We define a *symmetric* modular cluster as one with J vertical layers and N firms competing in each layer. Layer maps of four symmetric clusters are shown in Figure 2. For clusters such as these, we will combine a Hotelling-style model of horizontally differentiated products with a Cournot-style model of complementary components. The model yields a pricing equilibrium for each $J \times N$ cluster configuration. Then, holding demand fixed, we will compare equilibria across configurations.

Figure 2
Four Symmetric Modular Clusters
(*J* layers x *N* firms per layer)



The comparison yields a somewhat surprising result. As the number of layers and the number of firms per layer increase, price competition within layers drives prices down, while the vertical pricing externality across layers drives prices up. The two effects offset one another, and thus a large symmetric modular cluster of firms can in principle arrive at the same system price and aggregate profitability as a single, profit-maximizing monopoly that controls 100% of every layer. We first show this for an illustrative case with a simple demand function, and then generalize the results. Our main results hold for a potentially large class of symmetric modular clusters.

Our model can be used to study open vs. closed standards regimes, the commoditization of layers, platform monopolies, and horizontal and vertical mergers. Counter-intuitively, we find that clusters based on open, public standards may have higher prices and be more profitable than clusters based on closed, proprietary standards. Also, by increasing the number of layers in cluster, component makers may be able recapture rents from the platform proprietor. Finally, consistent with prior theoretical and empirical work (cf. Lafontaine and Slade, 2007), a series of horizontal mergers will raise prices and lower profits, while a series of vertical mergers will lower both prices and profits.

The rest of the paper is organized as follows. Section 2 discusses prior work. In Section 3, to build intuition, we set up a simple model and show the results of a numerical example. Section 4 generalizes the model and results. In Section 5, we extend the model to look at open vs. closed standards, the commoditization of layers, platform monopoly, and horizontal and vertical mergers. Section 6 concludes.

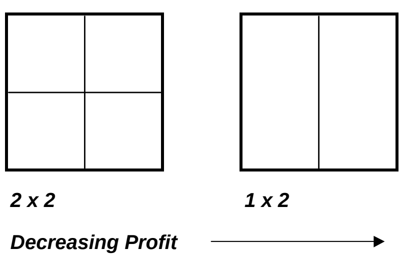
2 Related Work

Most of the literature on oligopoly pricing has focused on single-stage production processes (Tirole, 1988; Vives, 1999), while the recent literature on vertical integration has focused primarily on the design of optimal bilateral contracts or tradeoffs between efficiency and vertical foreclosure (Choi and Yi, 2000; Linnemer, 2003; Chemla, 2003; Rey and Tirole, 2007). Thus with the few exceptions described below, prior work does not consider the potentially offsetting effects of horizontal competition and the vertical pricing externality.

An important exception to this rule is a seminal paper by Economides and Salop (1992). They showed that a 2×2 configuration (two layers with two firms in each layer) might be able to mimic the pricing and profitability of a single firm (the 1×1 configuration). But the precise circumstances that gave rise to that outcome were difficult to discern from their model. We discuss their modeling approach in relation to ours in Section 4 below.

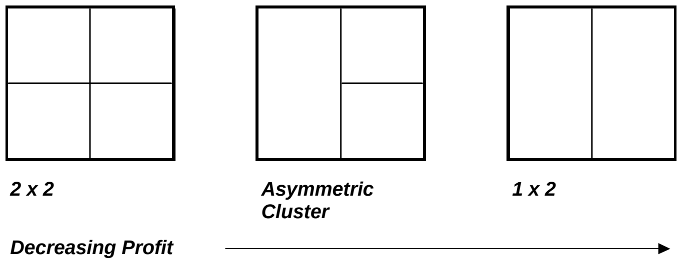
Rey and Stiglitz (1995), Nalebuff (2000) and Casadesus-Masanell et al. (2007) also consider cases in which both horizontal competition and the vertical pricing externality influence prices and profitability. First, Rey and Stiglitz model a duopoly in which two upstream firms may sell through perfectly competitive or differentiated retailers. (Differentiation among retailers is achieved by giving them exclusive territories.) They find that retail prices and aggregate profits are higher when the retailers are differentiated. The Rey and Stiglitz case with perfectly competitive retailers corresponds to a "one-layer duopoly" or, in our notation, a 1×2 configuration. Differentiated retailers correspond to a 2×2 configuration (Figure 3). Although our assumptions are different, we also find that system prices and aggregate profits are higher in a 2×2 than a 1×2 cluster configuration.

Figure 3
Cluster Configurations Analyzed by Rey and Stiglitz (1995)



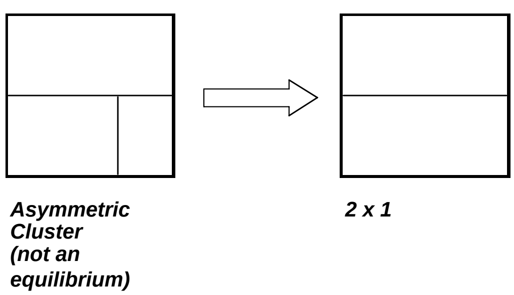
Nalebuff looks at a configuration in which a vertically integrated firm competes against vertically separated complementors. He constructs a product-location model in the tradition of Hotelling (1929) and shows that, when a vertical merger takes place, system prices decline. In our terminology, he compares a 2×2 configuration to an asymmetric configuration with one vertically integrated firm and a 1×2 configuration (Figure 4). Like Rey and Stiglitz, he finds that prices and aggregate profits are higher in the 2×2 than the 1 x 2 configuration. The results we derive below are consistent with these.

Figure 4
Cluster Configurations Analyzed by Nalebuff (2000)



Casadesus-Masanell et al. consider another asymmetric configuration, in which a single firm dominates one layer and two firms compete in a complementary layer. In the layer with competition, the products are differentiated in terms of quality. (If the two firms' products were priced the same, one would dominate the other.) If there are positive marginal costs in each layer, then there is no equilibrium in which the firm with the lower product quality survives. Thus an asymmetric cluster devolves in equilibrium to a 2×1 configuration (Figure 5).

Figure 5
Cluster Configurations Analyzed by Casadesus-Masanell, Nalebuff and Yoffie (2007)



These papers all hint at the fact that a $J \times N$ cluster of firms can be more profitable than a vertically integrated *N*-opoly ($1 \times N$) or a set of *J* complementary monopolies ($J \times 1$). In this paper, we seek to establish when and why this is true.

In addition to these theoretical papers, in the field of strategy, there is a large, growing literature on "platform competition." Gawer and Henderson (2007) define a *platform* as a "core element of the technological system" that is "strongly functionally interdependent with most of the other components of [the] system, … [and] there is no demand for components when they are isolated from the overall system" (p. 4). In other words, a platform is a set of unique components that drive the demand for all other components.[1] From the perspective of those who build on it, the platform is a natural monopoly, although it may be supplied as a public good.

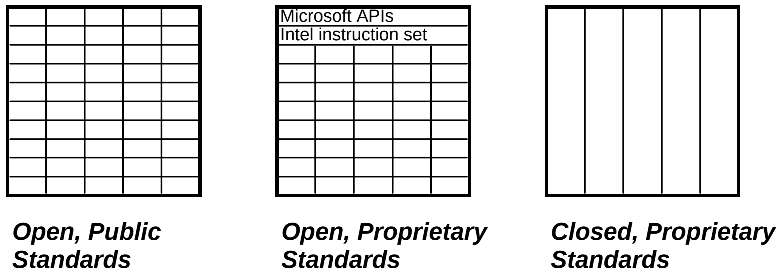
In a modular system, the design rules or architectural standards that serve to make the components compatible are a set of unique components on which all others depend (Baldwin and Clark, 2000; Farrell and Weiser, 2003; West, 2003). All other components are mix-and-matchable, but the standards of compatibility are (by definition) an input to every system. Thus standards are a key part of every platform, although platforms can contain more than just standards.[2] We follow Henderson (2005) in classifying standards according to their degree of *access* to component designers (open or closed), and the mode of *control* over their use and evolution (public or proprietary).

Closed, public standards (e.g., interface specifications for military systems) fall outside the scope of our model. *Open, public standards* (e.g., the Internet protocols) are freely available to all, hence give rise to modular clusters. *Open, proprietary standards* (e.g., the instruction set of an Intel chip or the application program interfaces of Windows) are available to all, but the proprietor may charge for their use. These give rise to asymmetric clusters in which one (or more) layers are monopolies and other layers are modular. Finally, *closed, proprietary standards* (e.g., parts specifications for an automobile) are accessible only via a special contract with the proprietor. With closed, proprietary standards, components that work one system will not function in another. The result is a set of vertical "silos," a $1 \times N$ cluster configuration, where N is the number of proprietors promulgating different standards. Figure 6 summarizes the correspondence between types of standards and cluster configurations.

[1] As Gawer and Henderson (2007) point out, this definition differs from that used in the multi-sided market literature (cf. Parker and Van Alstyne, 2005; Armstrong, 2006; Rochet and Tirole, 2006).

[2] Proprietary platforms usually comprise both standards and other components of the system. Public platforms are often only design rules, i.e., pure standards, possibly supplemented by an example of their implementation.

Figure 6
The Correspondence between Standards Regimes and Cluster Configurations



A strength of our model is that we can derive pricing equilibria for each of these standards regimes under consistent assumptions. We can then compare outcomes across regimes in terms of aggregate profits and the distribution of profits across layers and firms. We are not aware of any theoretical model of platform competition that supports such comparisons. We are also not aware of any model involving both horizontal competition and the vertical pricing externality that yields results for large clusters of firms. For these reasons, we believe that despite our model's necessarily stylized assumptions, it can serve as a bridge between theoretical models of the strategic interaction of small numbers of firms and the rich and growing body of empirical work focused on large industry clusters.

In the next section, we introduce our model by looking at a special case with a very simple demand structure. We can derive closed-form expressions for prices and aggregate profit for this case, holding costs and demand functions fixed but varying cluster configurations. In the section after, we generalize these results.

3 A Simple Model with Bilinear Demand

In this section we present a model of pricing in a $J \times N$ modular cluster with a simple demand structure that yields closed-form results. We derive unique symmetric equilibria for any combination of J and N. We then compute prices and profits for an array of cluster configurations that vary by J and N. We show that clusters exist which, in equilibrium, arrive at the same prices and profits as the 1×1 configuration—a monopoly that supplies all of the components of the system.

3.1 Basic Assumptions

Consider a symmetric cluster with J layers and N firms per layer, as illustrated in Figure 2. We assume that a functioning system requires exactly one component from each layer. We index the layers by j and firms within each layer by i, and focus on the ith firm in the jth layer, identified by the subscript ij.

Each firm in the cluster produces and sells a product in a given layer. A firm sets the price of its product to maximize its profit, independently of the other firms. We assume that the demand faced by a firm is a function of that firm's price, p_{ij}, the average price of its *substitutes*, $\overline{p}_{vj}$, and the average price of its *complements*, $\overline{p}_k$. These averages are known to the firm and taken to be fixed. For simplicity, we set all firms' marginal costs to zero. (For our demand structure, the optimal price is the cost plus a markup; the prices we derive can thus be interpreted as markups over marginal cost.)

Hence firm ij seeks to maximize

$$\pi_{ij} = p_{ij}\, q_{ij}(p_{ij}; \overline{p}_{vj}, \overline{p}_k) . \tag{1}$$

Let the quantity demanded of firm ij be given by

$$q_{ij} = a(p_{ij}; \overline{p}_{vj})\, b(p_{ij}; \overline{p}_k)\, Q_0 , \tag{2}$$

where

$$a = \begin{cases} 1 & \text{if } N = 1 \\ \dfrac{1}{N} + s\left(\overline{p}_{vj} - p_{ij}\right) & \text{if } N > 1 \end{cases}$$

$$b = 1 - r\left[(J-1)\overline{p}_k + p_{ij}\right].$$

Both a and b are linear in prices, so we say that the demand function is bilinear.

The parameter Q_0 is the maximum demand for systems, i.e., the number that would be sold if the price of all components were zero. The parameters s and r determine the elasticity of firm ij's demand to price changes by its competitors and complementors respectively. For a high value of s, a small difference in the price of the firm's product compared to its substitutes will yield a large change in the quantity sold, either negative or positive depending whether firm ij's price is higher or lower than the average. The r

parameter relates the demand for the focal firm's product to the average price of systems containing this product. Since a system requires one component from each layer, the total price of a system containing a component produced by firm *ij* is p_{ij} plus the prices of $J-1$ other components, whose average is defined as $\overline{p}_k$.

The model combines Hotelling-style competition among firms producing horizontally differentiated substitutes with Cournot-style vertical pricing externalities among producers of complementary components. This can be seen by looking at equation (2). If b is held constant and $N>1$, then total demand is fixed and the firm faces the simplest form of differentiated-goods competition: its market share goes down as it raises its own price, and up as the average price of substitutes rises. If, on the other hand, a is held constant (as is the case when $N=1$), firm *ij*'s demand is a linear function of the system price. This is the simplest demand function that gives rise to the vertical pricing externality (cf. Tirole, 1988, pp. 174–175).

3.2 Solving for Equilibrium Prices and Profits

Under these assumptions, firm *ij*'s first-order condition for profit maximization is

$$\begin{aligned} &1-r\left[(J-1)\overline{p}_k+2p_{ij}\right]=0 && \text{if } N=1 \\ &3rsp_{ij}^2-2\left(\frac{r}{N}+s+rs\left[\overline{p}_{vj}-(J-1)\overline{p}_k\right]\right)p_{ij}+\left(s\overline{p}_{vj}+\frac{1}{N}\right)\left[1-r(J-1)\overline{p}_k\right]=0 && \text{if } N>1. \end{aligned} \tag{3}$$

(We have divided out the common factor of Q_0.) To keep the model tractable, we restrict our attention to symmetric pure-strategy Nash equilibria in which the prices of all firms are equal, i.e., $\overline{p}_{vj}=\overline{p}_k=p_{ij}$. Substituting into (3) yields

$$\begin{aligned} &1-r(J+1)p_{ij}=0 && \text{if } N=1 \\ &Jrsp_{ij}^2-\left(\frac{J+1}{N}r+s\right)p_{ij}+\frac{1}{N}=0 && \text{if } N>1. \end{aligned} \tag{4}$$

Note that we derived the first-order condition for each firm before imposing the assumption of symmetry. This implies that firms in the cluster select their optimal prices taking the average prices of their substitutes and complements as given. In other words, when setting their prices, they do not expect other firms to behave as they do.

Solving these equations yields the unique symmetric equilibrium price, p_{ij}^*, for firms in the cluster:

$$p_{ij}^* = \begin{cases} \dfrac{1}{(J+1)r} & \text{if } N = 1 \\ \dfrac{1}{2Jr} + \dfrac{J+1}{2JNs} - \dfrac{1}{2JNrs}\sqrt{\left((J+1)r + Ns\right)^2 - 4JNrs} & \text{if } N > 1. \end{cases} \tag{5}$$

We have taken the negative root of the quadratic equation for $N > 1$ since it always yields economically meaningful prices ($0 < p_{ij}^* < 1/Jr$, so that $q_{ij} > 0$), whereas the positive root always leads to negative demand. It is straightforward to verify the second-order condition by taking the derivative of (3) with respect to p_{ij} and testing the sign. For $N = 1$ the profit function is everywhere concave in p_{ij}, and for $N > 1$ it is concave at all symmetric solutions to (3), so the stationary point identified by (5) is indeed a local profit maximum for all firms.

Of interest for comparing equilibria across cluster configurations is the equilibrium *system price*, $P_{J\times N}^*$, which is simply *J* times the component price given in (5):

$$P_{J\times N}^* = \begin{cases} \dfrac{J}{(J+1)r} & \text{if } N = 1 \\ \dfrac{1}{2r} + \dfrac{J+1}{2Ns} - \dfrac{1}{2Nrs}\sqrt{\left((J+1)r + Ns\right)^2 - 4JNrs} & \text{if } N > 1. \end{cases} \tag{6}$$

Finally, the *aggregate profit* for the cluster is *NJ* times the individual firm profit from (1) when all firms charge the equilibrium price:

$$\Pi_{J\times N}^* = \begin{cases} \dfrac{JQ_0}{(J+1)^2 r} & \text{if } N = 1 \\ \left[\dfrac{J-1}{2Ns} + \dfrac{J+1}{2(Ns)^2}\left(\sqrt{\left((J+1)r + Ns\right)^2 - 4JNrs} - (J+1)r\right)\right]Q_0 & \text{if } N > 1. \end{cases} \tag{7}$$

3.3 Comparison of Equilibria

Next we use a numerical example to compare prices and aggregate profits across the symmetric equilibria of different cluster configurations. Figure 7 shows an array of symmetric configurations in which each cell represents a different industry structure, comprising some number of layers and some number of firms per layer.

Figure 7
An Array of Symmetric Cluster Configurations

More Firms in Each Layer →

More Layers ↓

1x1	1x2	1x3	1x4	…	1xN
2x1	2x2	2x3	2x4	…	2xN
3x1	3x2	3x2	3x4	…	3xN
4x1	4x2	4x3	4x4	…	4xN
…				…	
Jx1	Jx2	Jx3	Jx4	…	JxN

For given parameter values, we can solve for the symmetric equilibrium system price and aggregate profit for each cluster configuration (each cell in the array). We can then compare outcomes across the configurations. Let $Q_0 = 2\times10^6$, $r = 0.0005$, and $s = 0.001$. These assumptions imply that total demand can range from zero to two million units; system prices may range from zero to \$2,000; and, *ceteris paribus*, for every dollar increase in the price of a substitute component, the *ij* firm's share of revenue in its layer declines by 0.1%.

Table 1 shows system prices for configurations in which *J* and *N* range from 1 to 20. Consistent with the idea that more competition in layers should cause prices to fall, equilibrium system prices decrease as *N* increases across each row. And consistent with the known effects of the vertical pricing externality, as the number of layers increases and each component accounts for less of the total system, system prices rise as *J* rises down each column. The combination of these two effects leads to an interesting outcome, which can be seen by looking at the bold numbers down the diagonal: *There are cluster configurations in which the equilibrium system price of a cluster of firms setting prices independently is the same as the system price charged by a vertically integrated monopoly.*

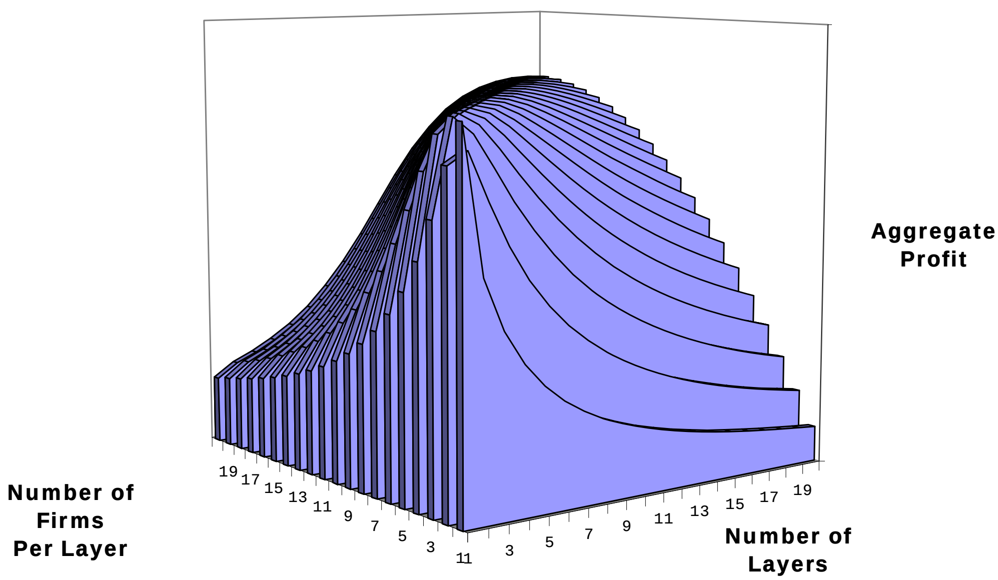
Table 1
System Price for Various Configurations of Symmetric Modular Clusters: $J, N \in \{1,\ldots,20\}$

System Price		Firms in Each Layer (N)																			
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	1	**1000**	382	279	219	180	153	133	117	105	95	87	80	74	69	64	61	57	54	51	49
	2	1333	719	543	431	357	304	264	233	209	190	173	159	148	138	129	121	114	108	102	97
	3	1500	**1000**	785	634	528	451	394	349	313	283	259	239	221	206	193	181	171	162	154	146
	4	1600	1219	**1000**	825	694	596	521	462	415	377	345	318	294	274	257	241	228	216	205	195
	5	1667	1382	1184	**1000**	852	736	646	575	517	469	430	396	367	342	321	301	284	269	255	243
Layers (J)	**6**	1714	1500	1333	1157	**1000**	871	768	685	617	561	514	474	440	410	384	361	341	323	306	291
	7	1750	1586	1451	1293	1137	**1000**	886	793	716	652	598	552	512	478	447	421	397	376	357	340
	8	1778	1649	1543	1407	1260	1121	**1000**	898	813	741	680	628	584	545	510	480	453	429	408	388
	9	1800	1697	1613	1500	1368	1232	1108	**1000**	908	829	762	705	655	611	573	539	509	482	458	436
	10	1818	1734	1667	1575	1460	1333	1210	1098	**1000**	916	843	780	725	678	636	598	565	535	508	484
	11	1833	1764	1709	1634	1537	1423	1304	1191	1089	**1000**	922	855	795	743	698	657	621	588	559	532
	12	1846	1788	1743	1681	1600	1500	1389	1278	1175	1082	**1000**	928	865	809	759	715	676	640	609	580
	13	1857	1807	1770	1719	1652	1566	1465	1360	1257	1161	1076	**1000**	933	873	820	773	731	693	658	627
	14	1867	1824	1792	1750	1694	1621	1532	1434	1333	1238	1150	1071	**1000**	937	881	831	785	745	708	675
	15	1875	1838	1811	1775	1728	1667	1590	1500	1405	1310	1221	1140	1066	**1000**	941	888	840	797	757	722
	16	1882	1850	1826	1796	1757	1705	1638	1559	1470	1378	1290	1207	1131	1062	**1000**	944	894	848	807	769
	17	1889	1860	1840	1814	1780	1736	1680	1610	1529	1442	1355	1271	1194	1123	1058	**1000**	947	899	855	816
	18	1895	1869	1851	1829	1800	1763	1714	1653	1581	1500	1416	1333	1255	1182	1116	1055	**1000**	950	904	862
	19	1900	1877	1861	1842	1817	1785	1743	1691	1627	1553	1473	1392	1314	1240	1172	1110	1052	**1000**	952	908
	20	1905	1884	1870	1853	1831	1804	1768	1723	1667	1600	1526	1448	1371	1297	1227	1163	1104	1050	**1000**	954

A feature of this model, which we will carry over to the more general case, is that if system prices are the same in two different equilibria, total revenue and profit will be the same as well. Hence those cluster configurations that arrive at the same system price as a vertically integrated monopoly will be as profitable as the monopoly.

Figure 7 graphs aggregate profit for the array of cluster configurations whose system prices are given in Table 1. The 1×1 configuration (the vertically integrated monopoly) is located in the corner that appears closest to the reader. Proceeding outward along the axis to the right leads to configurations with an increasing number of layers; along the axis to the left lie configurations with an increasing number of firms per layer. Looking along those axes, it is clear that for a single layer or a single firm per layer, aggregate profit falls if one adds firms or layers. This is consistent with the idea that horizontally competitive or vertically separated firms, which cannot coordinate their prices, are not as profitable as a single integrated monopoly. However, as is well known, horizontally competitive firms set prices that are too low relative to the monopoly price, while vertically separated firms set prices that are too high. This is borne out by looking across the top row and down the first column of Table 1.

Figure 8
Aggregate Profit for Various Configurations of Symmetric Modular Clusters
$J, N \in \{1, \ldots, 20\}$



Just as system prices vary systematically along the rows and columns of Table 1, aggregate profit also exhibits a regular pattern. Along both the rows and columns, it starts low at the axis, rises to a maximum and then falls.[3] The maximum in any row or column may be as high as the profit of the vertically integrated monopoly. Thus, under the assumptions of the model, a symmetric modular cluster with the "right number" of layers and firms per layer can be as profitable as one big firm that controls the whole cluster.

However, the results in Table 1 and Figure 8 were derived under specific numerical assumptions. We do not, on the strength of a single case, know how general these effects are likely to be. In the next section, we provide conditions under which these equilibrium price and profit patterns hold for more general demand conditions.

[3] In contrast, Nalebuff (2000) finds that aggregate profit rises monotonically with the number of layers. The difference in our results is due to the fact that Nalebuff assumes the total number of purchasers is fixed, and thus as prices rise, the total demand for systems remains the same. In our model, as system prices rise, demand falls, and once the system price exceeds the price charged by a vertically integrated monopoly, further price increases cause aggregate profit to fall.

4 Prices and Profits in a General Model

We now present a more general version of the model, which allows asymmetric cluster configurations and a larger class of demand functions. We then return to the symmetric case to show that under appropriate conditions, symmetric pricing equilibria are unique and exhibit the same pattern of prices and profits we saw in the previous section. All proofs are in the Appendix.

4.1 The General Model

As in the previous section, consumers purchase a system divided into $J \geq 1$ layers, where each layer represents a component that is essential to the proper functioning of the system. Within each layer *j*, there are $N_j \geq 1$ firms indexed by *i*. Each firm sells a horizontally differentiated variant of the component indicated by its layer. These products are equivalent in function and interact with products in the other layers in the same way, so they can be assembled in any combination.

Each firm chooses a price, p_{ij}, for its product.[4] The variable cost of each unit is c_{ij}. The demand faced by a typical firm is given by

$$q_{ij} = a_{ij}(\cdot)\, b_{ij}(\cdot)\, Q_0 \tag{8}$$

where $a_{ij} \in [0,1]$ and $b_{ij} > 0$ are functions of the product prices chosen by all firms in the cluster. As in Section 3, the multiplicative form of this expression will allow us to separate the impact of price changes by the firm's horizontal competitors and vertical complementors. To simplify the analysis, let a_{ij} and b_{ij} be twice continuously differentiable and concave in p_{ij}, i.e., $\partial^2 a_{ij} / \partial p_{ij}^2 \leq 0$ and $\partial^2 b_{ij} / \partial p_{ij}^2 \leq 0$.

In the absence of any additional restrictions on a_{ij} and b_{ij}, each could be a function of the prices set by every firm in the cluster. Indeed, every unique system could have its own demand function, giving rise to $\prod_{j=1}^{J} N_j$ distinct system demands, as well as a *derived demand* function for each of the $\sum_{j=1}^{J} N_j$

[4] A firm may sell multiple products in a layer at different prices, in which case p_{ij} can be interpreted as a weighted average across the firm's product line. Similar reasoning applies to costs. In this case we assume that firms' revenues and profits are affected only through these averages, so we still only need to consider one price and cost per firm.

firms, obtained by adding up the demands for the $\prod_{k\in[1,J]\setminus j} N_k$ unique systems in which firm *ij*'s product appears. This was the approach taken by Economides and Salop in their seminal 1992 paper. A limitation of this approach is that for industries with a large number of layers or firms, modeling the demand for unique systems becomes analytically and even computationally intractable.

After examining this approach in detail, we elected to take a different route. We assume that a_{ij} is affected only by the firm's own price *and those of other firms in the same layer*, while b_{ij} is affected by the firm's price *and those of firms in other layers*. This decomposition, formalized below, allows us to obtain analytical results for both symmetric and simple asymmetric modular clusters.

4.1.1 Competition Within Layers

Under any cluster configuration, for all $i \in [1, N_j]$ and $j \in [1, J]$, let

$$\frac{\partial a_{ij}}{\partial p_{ij}} < 0 \text{ if } N_j > 1, \text{ or } = 0 \text{ if } N_j = 1 \tag{9a}$$

$$\frac{\partial a_{ij}}{\partial p_{vj}} > 0 \text{ for all } v \in [1, N_j] \setminus i \tag{9b}$$

$$\frac{\partial a_{ij}}{\partial p_{vk}} = 0 \text{ for all } v \in [1, N_k] \text{ and } k \in [1, J] \setminus j\,. \tag{9c}$$

Conditions (9a) and (9b) require that a_{ij} responds to prices just as market shares would in a typical model of oligopolistic price competition: as a firm raises its own price, a_{ij} decreases (unless it is a monopoly). Conversely, as its competitors raise their prices, a_{ij} increases.

Condition (9c) reflects our simplified approach to cluster pricing by requiring a_{ij} to be unaffected by changes in the price of any complementary product. This crucial assumption rules out "special complementarities" between products in different layers, i.e., situations in which the attractiveness of a firm's product relative to others in its layer depends on the price charged by a specific complementor. We rule out such situations not because they do not exist in reality, but because we think it makes sense to treat them as second-order phenomena in the context of this model.

We further assume that the effects of (9a) and (9b) balance each other out, so that the sum of the a_{ij} terms equals one in each layer:

$$\sum_{i=1}^{N_j} a_{ij}(\cdot) = 1 \text{ for all } j \in [1, J]. \tag{10}$$

This condition means that as a firm raises its price and thereby loses the patronage of some fraction of the preference space, other firms in the layer gain the customers in the same fraction of the space. The number of such customers may decrease, however, due to the orthogonal effect we describe next.

4.1.2 Complements Across Layers

The conditions we impose on b_{ij} closely parallel those for a_{ij}. Under any cluster configuration, for all $i \in [1, N_j]$ and $j \in [1, J]$,

$$\frac{\partial b_{ij}}{\partial p_{ij}} < 0 \tag{11a}$$

$$\frac{\partial b_{ij}}{\partial p_{vk}} < 0 \text{ for all } v \in [1, N_k] \text{ and } k \in [1, J] \setminus j \tag{11b}$$

$$\frac{\partial b_{ij}}{\partial p_{vj}} = 0 \text{ for all } v \in [1, N_j] \setminus i. \tag{11c}$$

Condition (11a) means that as a firm raises its price, some consumers will exit the market entirely rather than switch to a product of another firm. (11b) captures the vertical pricing externality, stipulating that a price increase by any complement reduces demand for ij's component. In parallel with (9c), condition (11c) reflects our simplifying assumption that b_{ij} terms are not affected by the pricing actions of firms in the same layer (their influence comes through a_{ij}).

We assume that a functioning system must contain exactly one product from each layer, thus the total number of units sold is the same in each layer:

$$\sum_{i=1}^{N_j} q_{ij}(\cdot) = \sum_{i=1}^{N_k} q_{ik}(\cdot) \equiv Q(\cdot) \text{ for all } j, k \in [1, J]. \tag{12}$$

4.1.3 Profit Maximization

Under these assumptions, the profit of a typical firm is

$$\begin{aligned} \pi_{ij} &= \left(p_{ij} - c_{ij}\right) q_{ij}(\cdot) \\ &= \left(p_{ij} - c_{ij}\right) a_{ij}(\cdot)\, b_{ij}(\cdot)\, Q_0 . \end{aligned} \tag{13}$$

The first-order condition for profit maximization is

$$\frac{\partial \pi_{ij}}{\partial p_{ij}} = a_{ij}(\cdot)\, b_{ij}(\cdot)\, Q_0 + \left(p_{ij} - c_{ij}\right)\left[a_{ij}(\cdot)\, b'_{ij}(\cdot) + b_{ij}(\cdot)\, a'_{ij}(\cdot)\right] Q_0 = 0\,, \tag{14}$$

where $a'_{ij} \equiv \partial a_{ij} / \partial p_{ij}$ and $b'_{ij} \equiv \partial b_{ij} / \partial p_{ij}$.

Taken together, the set of first-order conditions for all firms in the cluster is a system of $\sum_{j=1}^{J} N_j$ equations in the same number of unknowns—one price variable for each firm. A solution to this system that satisfies the second-order condition for each firm is a Nash equilibrium in prices. Our first proposition establishes the existence of such an equilibrium.

Proposition 1. Under the assumptions of the general model, there is at least one pure-strategy Nash equilibrium in prices for every cluster configuration.

Stronger conditions are needed to ensure a unique equilibrium for each configuration (see, e.g., Rosen, 1965). Since our main results concern the symmetric model presented in the next section, we defer further consideration of uniqueness for now.

4.2 The Symmetric Model

To derive our main results, we restrict the general model in three ways. First, let there be the same number of firms in each layer: $N_j \equiv N$ for all $j \in [1, J]$. Second, let $c_{ij} \equiv C / J$ for all firms *ij*, where *C* denotes the total cost of a system, which we assume to be invariant to both *J* and *N*, as well as to the particular product choices of individual consumers. Third, let a_{ij} and b_{ij} be identical for all firms within a given configuration, so that, in a $J \times N$ cluster, $a_{ij} \equiv a_{J\times N}$ and $b_{ij} \equiv b_{J\times N}$ for all $i \in [1, N]$ and $j \in [1, J]$.

4.2.1 Profit Maximization Under Symmetry

Our strategy to render the model analytically tractable even for large *J* and *N* is to solve the profit maximization problem for a single representative firm under the assumption that all firms choose the same product price, as we know must be true at a symmetric equilibrium. To indicate the situation in which all firms choose a particular price *p*, we can write each of the functions defined in the previous section using a single argument. We thus denote firm-level demand under symmetric pricing by

$$q_{J\times N} = a_{J\times N}(p)\, b_{J\times N}(p) Q_0 \,. \tag{15}$$

Applying condition (10), it follows immediately that

$$a_{J\times N} = 1/N \,. \tag{16}$$

Note that we did not assume that p is an equilibrium price; this identity follows from symmetry alone.

When we write down a single-variable equation for the equilibrium price in a $J \times N$ configuration, we need to ensure that its solution yields the same Nash outcome as solving the corresponding system of JN equations explicitly. To do this, we must be careful to maintain the assumption that each firm acts independently—in other words, that symmetry does not imply coordinated behavior. In particular, every firm must maintain the expectation that if it unilaterally raises or lowers its price, its competitors and complementors will not necessarily follow suit.

From a modeling perspective this means we need to take derivatives of the profit functions and their components *before* imposing symmetry. Therefore let

$$\left(\partial a_{ij} / \partial p_{ij}\right)\Big|_{p_{ij}=p_{vk}=p} \equiv a'_{J\times N}(p)$$
$$\left(\partial b_{ij} / \partial p_{ij}\right)\Big|_{p_{ij}=p_{vk}=p} \equiv b'_{J\times N}(p)$$

for all $i, v \in [1, N]$ and $j, k \in [1, J]$.[5]

These definitions and restrictions reduce the system of first-order conditions for Nash equilibrium defined in (14) to a single equation in one variable:

$$\begin{aligned} \pi'_{J\times N}(p) \equiv \left.\frac{\partial \pi_{ij}}{\partial p_{ij}}\right|_{p_{ij}=p_{vk}=p} &= a_{J\times N}(p)\, b_{J\times N}(p) Q_0 \\ &+ \left(p - \frac{C}{J}\right)\left[a_{J\times N}(p)\, b'_{J\times N}(p) + b_{J\times N}(p)\, a'_{J\times N}(p)\right] Q_0 = 0 \end{aligned} \tag{17}$$

with the symmetric equilibrium price given by

[5] To see that $a'_{J\times N} \neq \partial a_{J\times N} / \partial p$, note from (16) that $\partial a_{J\times N} / \partial p = 0$ for all configurations, whereas condition (9a) requires $a'_{J\times N} < 0$ unless $N = 1$. For $b'_{J\times N}$ and $\partial b_{J\times N} / \partial p$, the sign of both should be negative but the magnitude of the latter will typically be larger because it represents the decrease in demand associated with an increase in price by a firm *and* its complementors rather than the firm on its own.

$$p^*_{J\times N} = \frac{-a_{J\times N}(p^*_{J\times N})\, b_{J\times N}(p^*_{J\times N})}{a_{J\times N}(p^*_{J\times N})\, b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N})\, a'_{J\times N}(p^*_{J\times N})} + \frac{C}{J}\,. \tag{18}$$

The following proposition establishes the existence and uniqueness of this equilibrium. While it does not rule out the existence of asymmetric price equilibria, it motivates our focus on the comparative statics of symmetric cluster configurations.

Proposition 2. Under the assumptions of the symmetric model, equation (18) has exactly one solution, which is the unique symmetric Nash equilibrium in prices for a $J\times N$ cluster.

4.2.2 System Price, Total Demand, and Aggregate Profit

We now define the system price, total demand, and aggregate profit for a $J\times N$ modular cluster.

The *system price* associated with a given product price for a $J\times N$ cluster, denoted $P_{J\times N}$, is the sum of the prices of the *J* components that make up a whole system. By our symmetry assumptions, all of these component prices are the same, so we can simply multiply a representative product price, *p*, by the number of layers to obtain the system price. Conversely, we can "back out" a symmetric product price given a system price *P*:

$$\begin{aligned} P_{J\times N}(p) &= Jp \\ p_{J\times N}(P) &= P/J\,. \end{aligned} \tag{19}$$

Total demand, denoted $Q_{J\times N}$, is the sum of the demands for the products in each layer. Total demand is the same for every layer by (12), and by our symmetry assumptions is simply the product of *N* and $q_{J\times N}$. By convention, we will always write $Q_{J\times N}$ as a function of the system price and $q_{J\times N}$ as a function of the product price, yielding the following relationship:

$$Q_{J\times N}(P) = \sum_{i=1}^{N} q_{J\times N}(P/J) = N\, q_{J\times N}(P/J) \tag{20}$$

where $q_{J\times N}$ is given by (15). Through (19) and (20), $Q_{J\times N}$ is well defined for all system prices between zero and the price that leads both component and system demands to be zero.

Aggregate profit for the $J\times N$ cluster, denoted $\Pi_{J\times N}$, is also defined in terms of the system price:

$$\Pi_{J\times N}(P) = (P-C)\,Q_{J\times N}(P)\,. \tag{21}$$

This function is zero if $P_{J\times N}$ or $Q_{J\times N}$ is zero, positive and continuous between those two bounds, and strictly quasiconcave.[6]

4.2.3 The Conservation of Total Demand and Cross-Cluster Regularity Conditions

The main purpose of the model is to compare prices and aggregate profits across different cluster configurations. In this section, we make assumptions that allow us to do this.

First, in order to make meaningful comparisons, we need to hold external demand conditions constant while varying configurations. To do this, we assume that for any pair of cluster configurations, if system prices are the same then total demand will also be the same:

If $P_{J\times N} = P_{K\times M}$, then $Q_{J\times N} = Q_{K\times M}$, for all J, N, K, M.

This assumption in turn implies

$$Q_{J\times N}(P) = Q_{K\times M}(P) \equiv Q(P) \tag{22a}$$

and

$$\Pi_{J\times N}(P) = \Pi_{K\times M}(P) \equiv \Pi(P)\,. \tag{22b}$$

In other words, across all configurations, total demand and aggregate profit depend only on the system price, which we can denote simply by P.

This assumption is restrictive, but it is fundamentally what allows us to compare equilibria across configurations. It also has a natural interpretation: it says that *end users care only about system prices and not how the cluster is organized*. Specifically, end users do not care how the components of their systems were divided up and assembled (the determinants of *J*), nor do they care about the levels of concentration in the component markets (the determinants of *N*). Regardless of the internal industrial organization of the cluster, for a given system price, the same total number of customers will show up and purchase the systems that most closely match their preferences. This condition relaxes Nalebuff's (2000) assumption that the total number of customers remains the same regardless of system price. In our model, a higher

[6] Strict quasiconcavity holds because firm profits are strictly quasiconcave (as shown in Appendix A.1), aggregate profit is *JN* times firm profit by symmetry, and quasiconcavity is preserved under monotonic transformations.

system price causes total demand to fall *by the same amount* in every configuration.

When two clusters have the same system price, how are their component prices related? If the clusters have the same number of layers, then by (19), if $P_{J\times M} = P_{J\times N}$, then $p_{J\times M} = p_{J\times N}$. However, if the clusters have a *different* number of layers, then, the same system price must be split over a different number of components. In that case, it follows from (19) that

$$\text{If } P_{K\times M} = P_{J\times N}\text{, then } p_{K\times M} = \frac{J}{K} p_{J\times N}\,. \tag{23}$$

Thus when comparing *component* prices across clusters with different layer structures, we must re-apportion the system price to the layers using equation (23).

Finally, in addition to the critical assumption about the conservation of total demand, we impose four regularity conditions to ensure that demand is well-behaved under cluster reconfigurations. First, consistent with our strategy of decomposing the demand functions into separate terms reflecting horizontal competition (a_{ij}) and the vertical pricing externality (b_{ij}), we assume that the marginal impact of horizontal price competition is not affected by the number of vertical layers, and symmetrically, the marginal impact of the vertical pricing externality is not affected by the number of firms in each layer:

$$a'_{J\times N}(p) = a'_{K\times N}(p) \quad \text{for all } J, K \geq 1 \tag{24a}$$

$$b'_{J\times M}(p) = b'_{J\times N}(p) \quad \text{for all } M, N \geq 1\,. \tag{24b}$$

Second, we assume that as the *number of firms per layer* goes up, the marginal impact of horizontal price competition stays the same or gets stronger. And as the *number of layers* goes up, the marginal impact of the vertical pricing externality stays the same or gets stronger. In terms of the demand functions, this means that a' becomes (weakly) more negative as the number of firms per layer increases, while b' becomes (weakly) less negative as the number of layers increases:

$$\left|a'_{J\times M}(p)\right| \leq \left|a'_{J\times N}(p)\right| \quad \text{if } N \geq M\text{, for all } J \geq 1 \tag{24c}$$

$$\left|b'_{J\times N}(p)\right| \geq \left|b'_{K\times N}(p)\right| \quad \text{if } K \geq J\text{, for all } N \geq 1\,. \tag{24d}$$

With the conservation of total demand and these regularity conditions providing the necessary linkages across configurations, we can now state and prove our main results.

4.3 Pricing and Profitability in Symmetric Clusters

Returning to the thought experiment of Section 3.3, where we compared system prices and profits across different configurations, we now show that the pattern of prices in Table 1 and profits in Figure 8 is characteristic of the symmetric model as defined above. Our results establish that large clusters of firms can mimic the pricing behavior, and hence the profitability, of a monopoly.

Proposition 3 shows that the pattern observed in Table 1—equilibrium system prices that decrease across each row and increase down each column—holds for all demand functions consistent with our model, and for clusters of any size.

Proposition 3. (a) Consider two cluster configurations, $J \times N$ and $J \times M$, where $1 \le M < N$. Under the assumptions of the symmetric model, $P^*_{J\times M} > P^*_{J\times N}$. In other words, for any number of layers, equilibrium system prices are strictly *decreasing* in the number of firms competing in each layer.

(b) Consider two cluster configurations, $J \times N$ and $K \times N$, where $1 \le J < K$. Under the assumptions of the symmetric model, $P^*_{J\times N} < P^*_{K\times N}$. In other words, for any number of firms in each layer, equilibrium system prices are strictly *increasing* in the number of layers.

Next we find that for any number of layers, *J*, there is a number of firms per layer, N^*, that yields an equilibrium system price close to the monopoly price, $P^*_{1\times 1}$ (in the sense that the monopoly price lies between $P^*_{J\times N^*}$ and $P^*_{J\times(N^*+1)}$), and $P^*_{J\times N^*}$ converges to the monopoly price as *J* grows large. An analogous result holds for every *N* and associated J^*.

Proposition 4. For every *J*, there is an N^* such that $P^*_{J\times N^*} \ge P^*_{1\times 1} > P^*_{J\times(N^*+1)}$. Likewise, for every *N*, there is a J^* such that $P^*_{J^*\times N} \le P^*_{1\times 1} < P^*_{(J^*+1)\times N}$. As $J \to \infty$, $P^*_{J\times N^*} \to P^*_{1\times 1}$, and as $N \to \infty$, $P^*_{J^*\times N} \to P^*_{1\times 1}$.

Large clusters can thus mimic the pricing behavior of a monopoly. Because, under our assumptions, aggregate profit depends on the system price alone, it follows that large clusters can be just as profitable, in the aggregate, as a monopoly. This finding, which we formalize in our final proposition, explains and generalizes the existence of the striking "ridge" of monopoly-level profits visible in Figure 8.

Proposition 5. For every *J*, aggregate profits increase as *N* approaches N^* (as defined in Proposition 4), then decrease from N^*+1 onward. Similarly, for every *N*, aggregate profits increase as *J* approaches J^*, then decrease from J^*+1 onward. As $J \to \infty$, $\Pi^*_{J\times N^*} \to \Pi^*_{1\times 1}$, and as $N \to \infty$, $\Pi^*_{J^*\times N} \to \Pi^*_{1\times 1}$.

To recap, we have shown that the highest aggregate profit accrues to the cluster configurations that most closely approximate the pricing behavior of one big firm (Proposition 5). Large symmetric clusters can approximate monopoly pricing arbitrarily closely (Proposition 4), thus large clusters can in principle be just as profitable, in the aggregate, as a vertically integrated monopoly. An immediate but significant implication of this finding is that large clusters can generate more profit than either chains of monopolies ($J\times 1$ clusters) or vertically integrated oligopolies ($1\times N$).

5 Applications and Extensions

In this section we apply and extend our model to various real-world phenomena. First, as indicated in Section 2, our model permits us to compare prices and profits across different standards regimes. We adopt Henderson's (2005) typology and classify standards as (1) open, public; (2) closed, proprietary; or (3) open, proprietary. (Closed, public standards are typically developed and controlled by governments, and lie outside the scope of our model.) We compare open, public to closed, proprietary standards, and then consider the phenomenon of commoditization. Following that, we look at open but proprietary standards and platform monopolies, and the impact of horizontal and vertical mergers. We close this section by discussing how conflicts between the good of the firm and the good of the cluster may create strategic tension within the largest firms in a cluster.

5.1 Open, Public Standards vs. Closed, Proprietary Standards

Our model assumes full compatibility among components in different layers, so that a working system can be assembled using any combination of products from each layer. This kind of compatibility occurs when the underlying technology is characterized by what Henderson (2005) calls *open, public standards*. In such cases, the design rules or architectural standards that support compatibility are supplied as a public good, freely accessible to all. There is, by definition, no platform monopoly in such systems—no essential component controlled by a single firm. Most computer hardware works approximately this way: an end user can buy cases, power supplies, motherboards, memories, hard drives, video cards, keyboards, monitors and mice from any of dozens of manufacturers and rely on them to work together thanks to standards like ATX, DDR, IDE, USB and VGA. And if end users do not want to assemble all the necessary components themselves, there are many system integrators willing to do it for them. The system integrators form another layer of the cluster: they compete with each other for customers, and their services are fully mix-and-matchable with the other components.

The opposite case is a system based on *closed, proprietary standards*. In the starkest version of this case, a lead firm (often called an original equipment manufacturer or OEM) enters into contracts with a set of suppliers who agree to make components to its specifications. The lead firm then integrates the components and sells the resulting systems to end users, setting prices subject to horizontal competition from other OEMs. Profits are distributed via negotiations between the lead firm and its suppliers. The automotive industry has historically operated like this, with many key parts (e.g., dashboards, seat assemblies and braking systems) designed in close collaboration with the major automakers and manufactured by captive subsidiaries or independent suppliers under long-term contracts.

Products designed to different proprietary standards are not technologically substitutable, and do not compete with one another. Thus closed, proprietary standards do not give rise to cross-price effects in the same way that open, public standards do. Under a regime in which each OEM sets its own standards, the final integrated products are effectively fixed bundles. For pricing purposes, the situation is equivalent to a modular cluster with one layer (the OEMs), hence product prices and aggregate profits will be the same as in a vertically integrated $1 \times N$ cluster.

In contrast, under open, public standards, OEMs and their closed systems disappear and are replaced by system integrators, who are just one layer among many. The system integrators do not control the component makers' access to the market, and their pricing decisions apply only to their own layer. Consequently system integrators' incentives to cut prices are diminished relative to the OEMs'. Prices and profits will be that of a $J \times N$ cluster.

Our model thus predicts that moving from closed, proprietary standards to open, public standards will *increase* system prices and aggregate profits of the cluster as a whole. At first glance, this seems counter-intuitive, but it is perfectly logical given the assumptions of our model. In a cluster with full mix-and-match compatibility, as we have seen, the vertical pricing externality serves to offset the effects of horizontal competition. Open, public standards thus mitigate what Nalebuff (2000) calls "bundle against bundle" competition, which occurs when oligopolists compete head-to-head by selling whole systems. In our model, as in his, "bundle against bundle is ferocious competition."

However, under open, public standards the dominant firm in each "silo" will no longer be able extract all of the available rents from its suppliers and complementors. Because complementary products are no longer tied to the technology of a single lead firm, profits will shift away from the dominant layer to other parts of the industry. Thus, although the totality of firms is better off operating under open, public standards, firms in the erstwhile dominant layer may be worse off.

5.2 Commoditization of Layers

Although different on the surface, the phenomenon known as "commoditization" leads to the same overall outcome as closed, proprietary standards. Practitioners speak of a layer becoming "commoditized," when competition causes prices to fall to the point where no significant profits can be earned in that layer (Christensen and Raynor, 2003; Schrage, 2007). In our model, commoditization is equivalent to letting the number of firms in a layer grow large. With their prices driven to marginal costs, the pricing decisions for products in the commoditized layer no longer affect the demand for their complements. In terms of our model, the commoditized layer disappears from the cluster. Thus when a layer is commoditized, a $J \times N$ cluster devolves into a $(J-1) \times N$ cluster.

Under the assumptions of our model, commoditization of a layer will cause system prices to fall. Aggregate profits in the rest of the cluster may go up or down, depending on where the pre- and post-commoditization system prices lie relative to the "one big firm" system price (see Proposition 4). However, as multiple layers become commoditized, the cluster comes closer to the $1 \times N$ configuration, which has both low prices and low aggregate profits. In effect, the commoditization of layers brings the remaining firms into more intense competition with one another. In a cluster where most layers have been commoditized, the firms in the non-commoditized layers will mimic the effects of fierce "bundle against bundle" competition, even though they are selling components rather than actual bundles.

Again, we must distinguish between the impact of commoditization on the cluster as a whole and the impact on individual firms. It is quite possible for firms in the surviving, non-commoditized layers to be better off, even though aggregate profits have dropped precipitously. Table 2 makes this point. It computes *profit per firm* for the array of configurations whose system prices and aggregate profits were shown above in Table 1 and Figure 8 respectively. Profit per firm declines strictly down the columns, while aggregate profits are highest in the cells indicated by bold, italic numbers along the diagonal. The latter cells correspond to the ridge in Figure 8, where the aggregate profit of the cluster equals the profit of one big firm.

Table 2
Profit per Firm for Various Configurations of Symmetric Modular Clusters: $J, N \in \{1, \ldots, 20\}$

Profit		Firms in Each Layer (N)																			
per Firm		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	1	***1000***	309	160	98	66	47	35	28	22	18	15	13	11	10	8	7	7	6	5	5
	2	444	230	132	85	59	43	33	26	21	17	14	12	11	9	8	7	6	6	5	5
	3	250	***167***	106	72	52	39	30	24	20	16	14	12	10	9	8	7	6	6	5	5
	4	160	119	***83***	61	45	35	28	22	18	15	13	11	10	8	7	7	6	5	5	4
	5	111	85	64	***50***	39	31	25	20	17	14	12	11	9	8	7	6	6	5	5	4
Layers (J)	**6**	82	63	49	41	***33***	27	23	19	16	13	12	10	9	8	7	6	6	5	5	4
	7	63	47	38	33	28	***24***	20	17	15	13	11	10	8	7	7	6	5	5	4	4
	8	49	36	29	26	23	21	***18***	15	13	12	10	9	8	7	6	6	5	5	4	4
	9	40	29	23	21	19	18	16	***14***	12	11	10	8	8	7	6	5	5	5	4	4
	10	33	23	19	17	16	15	14	12	***11***	10	9	8	7	6	6	5	5	4	4	4
	11	28	19	15	14	13	12	12	11	10	***9***	8	7	7	6	6	5	5	4	4	4
	12	24	16	12	11	11	10	10	10	9	8	***8***	7	6	6	5	5	4	4	4	3
	13	20	13	10	9	9	9	9	8	8	7	7	***6***	6	5	5	5	4	4	4	3
	14	18	11	9	8	7	7	7	7	7	7	6	6	***5***	5	5	4	4	4	3	3
	15	16	10	8	7	6	6	6	6	6	6	6	5	5	***5***	4	4	4	4	3	3
	16	14	9	7	6	5	5	5	5	5	5	5	5	5	4	***4***	4	4	3	3	3
	17	12	8	6	5	5	4	5	5	5	5	5	5	4	4	4	***4***	3	3	3	3
	18	11	7	5	4	4	4	4	4	4	4	4	4	4	4	4	3	***3***	3	3	3
	19	10	6	5	4	4	3	3	3	4	4	4	4	4	4	3	3	3	***3***	3	3
	20	9	5	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	***3***	2

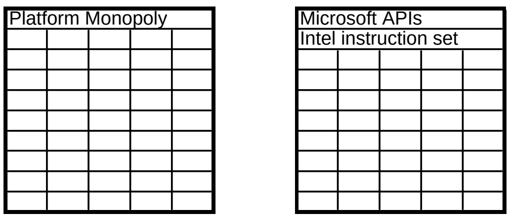
The table shows that under the assumptions of our numerical example, individual firms benefit from the commoditization of their complementors. From the perspective of an individual firm or a particular layer of firms, the cluster's aggregate profitability is an externality, and thus, in general, we would not expect individual firms to eschew commoditization in order to keep their complementors in business. But the difference between what is best for the cluster and what is best for individual firms may give rise to strategic tension between firms and their investors. We will return to this issue in Section 5.5.

5.3 Open, Proprietary Standards and Platform Monopoly

The opposite of commoditization is the domination of a layer by a single firm. This occurs when the standards of compatibility are open, i.e., available to all, but proprietary, i.e., owned by a single firm. The proprietor of this essential component is typically said to control "the platform." If it succeeds in dominating the platform layer, it has a "platform monopoly."

We can represent a cluster with a platform monopoly as a $(J-1) \times N$ cluster "attached to" a layer with a single firm (see Figure 9a). In some cases, however, the platform may be split into two or more complementary layers. This is approximately the situation in the personal computer market where Microsoft and Intel control different complementary standards, hence there are two platform monopolies (see Figure 9b). In this section, we examine the case of a single platform monopoly, using the bilinear model of Section 3, and describe how the analysis can be extended to multiple platform layers.

Figure 9
Two Types of Proprietary Platform



(a) Platform Monopoly

(b) Split Proprietary Platform

We begin by looking back at equations (1) and (2). In the case of a single proprietary platform (a platform monopoly), there will be one firm (subscripted "*11*") for whom $N = 1$, while for the rest of the cluster (subscripted "*ij*"), $N > 1$. The profit functions for these two types of firm are, for the platform monopoly:

$$\pi_{11} = p_{11}\left(1 - r\left[p_{11} + (J-1)\overline{p}_k\right]\right)Q_0 \tag{25a}$$

and for the other firms in the cluster:

$$\pi_{ij} = p_{ij}\left[\frac{1}{N} + s\left(\overline{p}_{vj} - p_{ij}\right)\right]\left(1 - r\left[p_{11} + (J-2)\overline{p}_k + p_{ij}\right]\right)Q_0 \tag{25b}$$

where p_{11} and p_{ij} are the prices of the platform and a representative component.

Deriving the first-order conditions and imposing symmetry, we obtain a system of two equations in the two unknown prices:

$$1 - 2rp_{11} - r(J-1)p_{ij} = 0 \tag{26a}$$

$$\left(\frac{1}{N} - sp_{ij}\right)\left(1 - r\left[p_{11} + (J-1)p_{ij}\right]\right) - \frac{r}{N}p_{ij} = 0\,. \tag{26b}$$

We can use (26a) to solve for p_{11} in terms of p_{ij}:

$$p_{11} = \frac{1}{2}\left[\frac{1}{r} - (J-1)p_{ij}\right]. \tag{27}$$

Substituting this expression into (26b) obtains:

$$(J-1)rsp_{ij}^2 - \left(\frac{J+1}{N}r + s\right)p_{ij} + \frac{1}{N} = 0 \tag{28}$$

which is identical to (4) except for the replacement of J by $J-1$ in the quadratic term. We can use (28) to obtain a closed-form solution for equilibrium p_{ij}^*, and substitute that expression into (27) to obtain p_{11}^*:

$$p_{11}^* = \frac{1}{4r} - \frac{J+1}{4Ns} + \frac{1}{4Nrs}\sqrt{\left((J+1)r + Ns\right)^2 - 4(J-1)Nrs} \tag{29a}$$

$$p_{ij}^* = \frac{1}{2(J-1)r} + \frac{J+1}{2(J-1)Ns} - \frac{1}{2(J-1)Nrs}\sqrt{\left((J+1)r + Ns\right)^2 - 4(J-1)Nrs}. \tag{29b}$$

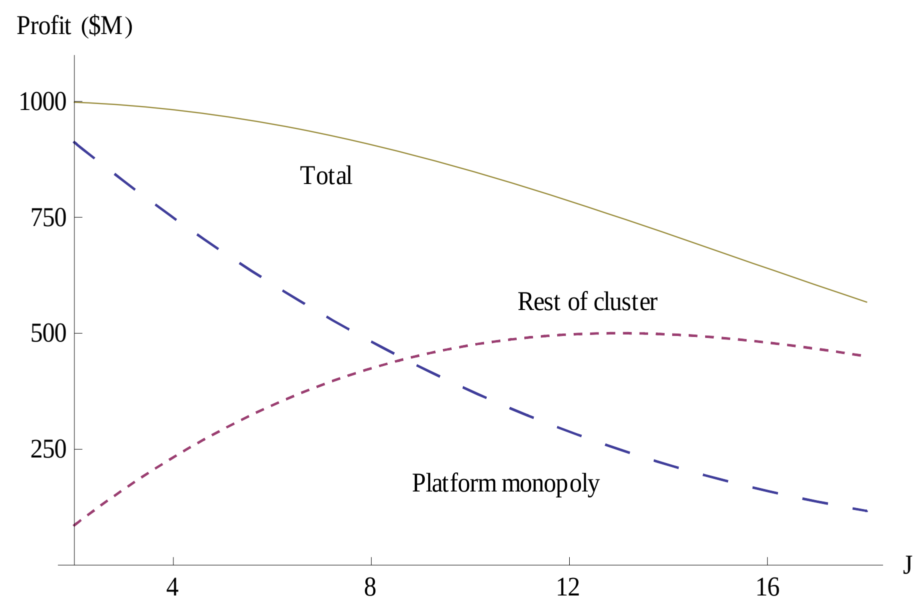
Using the same numerical parameter values as before, we can then obtain, for any combination of *J* and *N*, (1) the platform price and profit; (2) the prices and profits of the rest of the system; and (3) the system price and aggregate profit of the cluster. Moreover, this procedure generalizes to platforms with any number of layers.

Exhaustive analysis of the full array of platform configurations lies beyond the scope of the paper. The results are by and large consistent with those derived for symmetric clusters. For example, as we would expect, the platform monopoly's price and profit increase as the number of firms per layer (N) in the other layers goes up, converging on the 1×1 system price and profit as N grows large. In other words, not surprisingly, as the other layers become more competitive, an ever-larger share of the cluster's value is appropriated by the platform owner.

Interestingly, fixing the number of firms per layer reveals a possible "counterattack" by the rest of the cluster.[7] As the number of layers in the rest of the cluster increases, the vertical pricing externality they impose creates pricing pressure on the platform monopoly. As long as the number of layers does not become too high, profits will shift from the platform monopoly to the component makers. For example, Figure 10 shows that increasing *J* from 4 to 11, while holding *N* fixed at 10, maximizes the profit of the component firms while driving down the profit of the platform owner by about 50%.

[7] The word "counterattack" should not be taken literally, however. The firms in the rest of the cluster are, by assumption, decentralized and uncoordinated.

Figure 10
Increasing the Number of Layers Benefits Component Makers and Harms the Platform Monopoly



5.3.1 Platform Envelopment

In the simple bilinear case, there is no difficulty in splitting a proprietary platform into several complementary layers. This in turn allows us to model the impact on prices and profit of a strategic option that Eisenmann et al. (2007) call "platform envelopment." As they explain, *platform envelopment* is "entry by one platform provider into another's market, combining its own functionality with the target's in a multi-platform bundle." They go on to observe that "[d]ominant firms that otherwise are sheltered from entry … due to strong network effects and high switching costs may be vulnerable to an adjacent platform provider's envelopment attack." Indeed, in 30 examples of platform competition, Eisenmann et al. found that 13 involved platform envelopment. It was the most common strategic move by a wide margin.

In our scheme, platform envelopment involves a reduction in the number of layers, for example a transition from the configuration in Figure 9b to the one in Figure 9a. Because it mitigates the vertical pricing externality (by reducing the number of layers), platform envelopment causes system prices to fall. To the extent that the effect shown in Figure 10 generalizes to the real world, platform envelopment will always tend to benefit the enveloper though it may harm the rest of the cluster. Even in the stark theoretical form of our model, this observation provides a plausible explanation for why so many firms with proprietary platforms have found envelopment to be an attractive strategic option.

5.4 Horizontal and Vertical Mergers

We now consider the impact of horizontal and vertical mergers on cluster pricing and profits. Our basic results are intuitive and consistent with prior work (see, e.g., Lafontaine and Slade, 2007 and references therein). Basically, in our model, a series of horizontal mergers (one in each layer to preserve symmetry) raises prices and may raise or lower aggregate profit depending on where the pre- and post-merger prices fall in relation to the price charged by a vertically integrated monopoly. Conversely, a series of vertical mergers reduces prices and again may raise or lower aggregate profit.

From a public policy perspective, therefore, horizontal mergers unambiguously reduce consumer welfare. In contrast, consistent with the analysis of the Chicago School (cf. Bork, 1978), vertical mergers increase welfare by lowering prices. (In our model, costs are fixed by assumption, and thus mergers have no impact on efficiency.) These basic concepts are well known, and have been extensively discussed in the literature (Farrell and Weiser, 2003).

5.5 Strategic Tensions: The Firm vs. the Cluster

Our model brings to the fore the contrast between what is good for the cluster vs. what is good for an individual firm. As discussed above in the section on commoditization, there is a pecuniary externality embedded in the cluster form of industrial organization. In general, individual firms are better off (1) merging; (2) commoditizing their complementors; and/or (3) promulgating closed, proprietary standards. But the most profitable cluster configuration (other than one big firm) has many differentiated firms in many complementary layers operating under open, public standards. From the perspective of aggregate profitability, the configurations to avoid lie toward the edges of the configuration array: the $1 \times N$ form of system-to-system competition and the $J \times 1$ chain of monopolies.

The conflict between the good of the cluster and the good of individual firms in turn means that the incentives of firms and their investors are not always well aligned. Investors, who can hold portfolios of firms, tend to prefer large, multi-layered clusters organized around open standards because such configurations, according to our model, can closely approximate the pricing and profits of one big firm.

This conflict, we think, is real and generates strategic tension within the larger firms in a cluster, particularly the platform firms. Specifically, the larger firms have the greatest potential to harm the cluster through mergers, commoditization, and the imposition of closed, proprietary standards. But these firms are also responsible to and often governed by portfolio investors (e.g., venture capitalists) who are interested in the success of the cluster as a whole. Thus senior managers of these firms will feel pressure to be stewards—not only of their own companies, but of the cluster as a whole.[8]

Recent empirical analysis of platform competition provides some evidence of strategic tension within large, platform firms. We offer two examples. First, Iansiti and Levien (2004), who studied Microsoft in detail, argue that large groups of firms should be viewed as "ecosystems." Large firms and especially platform providers have a long-term interest in the diversity of their ecosystems. Therefore, they argue, enlightened self-interest dictates that such firms should support and channel resources to smaller, dependent firms in complementary layers.

Second, Gawer and Henderson (2007) describe conflicts within Intel over entering new markets based on the Intel processor platform. To address the concerns of developers that Intel might leverage its proprietary intellectual property to gain advantage over its complementors in adjacent markets, the company created a separate organizational unit, the Intel Architecture Lab (IAL), which was "explicitly structured as a cost center and rewarded for its success in 'promoting the health of the ecosystem' as a whole" (p. 3). At the same time, the company committed not to commoditize these markets entirely by frequently entering them through product groups that were publicly expected to earn profits. On balance, Intel managed to support its ecosystem "by making money *but not too much*" (p. 18, emphasis in original).

In summary, with respect to major strategic moves such as the promulgation of standards, the commoditization of complements, and especially mergers and acquisitions, the value of individual firms and the aggregate profit and value of the cluster as a whole may be diametrically opposed. Managers of large firms, especially platform providers, may feel this conflict keenly. When managers are aware of this

[8] Even managers that do not feel this pressure from external stakeholders may believe that their own firm's long-term interests lie in being part of a large, profitable cluster.

conflict—even if they do not fully understand its root causes—they will feel compelled to balance the welfare of their own firm with the general welfare of the cluster. We can see evidence of these cross-pressures in the organizational structure and practices of firms like Microsoft and Intel.

6 Conclusion

In this paper we investigated the possibility that pressures to reduce prices within horizontally differentiated product markets can be offset by pressures to increase prices in markets for complementary goods. The recognition of this possibility is not new—in fact, it underlies the models and results of Economides and Salop (1992), Rey and Stiglitz (1995), Nalebuff (2000), and Casadesus-Masanell et al. (2007), among others. This work has hinted at the fact that a disaggregated cluster of firms might be able get closer to the prices and profitability of a single firm than a chain of monopolies or a set of vertically integrated oligopolies. We have sought first to clarify and generalize these results by constructing a model that is both simpler at its core and more "scalable" than those of the existing literature, in the sense that it can be extended to symmetric clusters of arbitrary size. Our main contribution has been to isolate the offsetting price effects in a model, and show how they might operate in large as well as small clusters of firms.

We then used our model to gain insight into a set of real-world phenomena: open vs. closed standards regimes, the commoditization of layers, platform monopolies, and horizontal and vertical mergers. In some cases, e.g., mergers, our results are consistent with prior work. In other cases, our results are somewhat surprising: for example, our model shows that clusters operating under open, public standards may have higher prices and profits than those operating under closed, proprietary standards.

Our model has many limitations. It is entirely static; in particular, the dynamics of entry and exit lie outside the scope of the model. For the sake of tractability, it assumes a higher degree of symmetry across firms and layers than is characteristic of the real world. (As we showed in Section 5, the model can be extended to deal with simple forms of asymmetry, such as platform monopoly, but even the extensions rely heavily on symmetry.) We do not consider potential cost differences across cluster configurations; by assumption, all configurations are equally efficient. We also model product variety in a very simplistic way, and do not consider how different configurations may affect the range of systems available to customers. Moreover, we neglect the specific nature of contracts between firms in a cluster, which might affect prices, demand and profit.

Despite its limitations, the model opens up several avenues for future research. First, from a theoretical perspective, it would be interesting to relax the symmetry assumptions further and to consider the dynamics of price formation in conjunction with changing cluster structures. In particular, the Nash equilibrium condition places high demands on common knowledge and the speed of price adjustment, and it would be interesting to explore whether adaptive firms in large clusters with changing structures can find (or come close) to an equilibrium. Future work in this direction may need to rely on computational experiments.

Empirically, our model makes one provocative prediction: that clusters based on open standards (either public or proprietary) in general have higher system prices and aggregate profits than clusters based on closed, proprietary standards. This prediction, while crisp, is difficult to test because of the many confounding factors left out of our model. But in principle, the hypothesis might be tested on industry sector-level data, by looking for natural experiments, or through case studies.

In terms of organizational theory, our model motivates and explains a type of strategic tension that has already been observed by Iansiti and Levien (2004), Gawer and Henderson (2007), and others. The tension made stark by the model is between the good of an individual firm and the good of the cluster. Strategic moves such as mergers, commoditization of complements, and the promulgation of closed, proprietary standards are profitable for individual firms, but if widely adopted, can destroy a cluster and its profits. Thus there is a potential collective action problem (Olson, 1971) in maintaining a cluster's size and diversity. In the absence of antitrust regulation, clusters would devolve into one big firm—the 1×1 configuration. However, if horizontal mergers are disallowed but vertical mergers allowed, then there is a risk that a cluster might devolve into a single-layer oligopoly—the $1 \times N$ configuration.

This problem in turn causes the interests of firms to be in conflict with the interests of their investors. Large firms and platform firms must decide how to deal with this tension. And it is possible that, in a dynamic setting where firms can choose between different platforms, the most successful large firms will be those who place significant weight on the welfare of the cluster in forming their strategies.

Finally, our model opens up interesting avenues of research in the field of innovation. Beginning with Schumpeter (1942), many economists have argued that high levels of innovation are consonant with relatively large firms with market power, who can earn enough quasi-rents to pay for their investments in new products and processes. (See Baumol, 2002 for a recent version of this argument.) But, at the same time, some management scholars claim that very high levels of innovation are characteristic of clusters of firms operating under modular product architectures. (See Langlois and Robertson, 1992; Sanchez and Mahoney, 1996; Teece, 1996; and Langlois, 2002 for versions of this argument.)

Our model gives theoretical support to the latter claim: we show that because of the way they offset horizontal competition and the vertical pricing externality, large, decentralized clusters can be as profitable in aggregate as a vertically integrated monopoly, and more profitable than a one-layer oligopoly or a chain of monopolies. Moreover, in a large cluster, the system is broken up into many small components, hence clusters make it relatively easy for capital-constrained firms and entrepreneurs to gain a foothold with a modular innovation that is limited in scope.

Cluster forms of industrial organization may not be conducive to all kinds of innovation, however. In particular, innovations that add new layers of functionality to the system, and thus increase *total demand*, will not be adequately rewarded relative to the value they create. The reason is akin to the vertical pricing externality: the benefits of increased demand will be spread over all layers and thus only part of the value created will be captured by the innovator, even if it is a monopoly in its own layer. Gawer and Henderson (2007) observe that Intel actively channels resources to innovations at other firms that it believes will increase the total demand for its platform. Similar behavior has been documented in some two-sided markets, as for example, when platform firms subsidize the creation of new forms of content (Hagiu, 2006). Understanding how cluster configurations affect incentives to supply different forms of innovation, and how firms respond to these cross-layer dependencies in formulating their long-term strategies, seems to be a promising avenue for future research.

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Appendix

A.1 Proof of Proposition 1

The standard theorems on the existence of a pure-strategy Nash equilibrium in a normal-form game require the set of each player's feasible actions to be a compact and convex subset of a Euclidean space, and the payoffs to be continuous in the vector of actions and quasiconcave in the action of each player (see, e.g., Fudenberg and Tirole, 1991, p. 34). Here the actions are price choices p_{ij}. Although we have not set a finite ceiling on prices, the compactness of the action sets can be ensured by mapping the real line onto a closed and bounded interval (Debreu, 1952). The payoffs $\pi_{ij} \in \mathbb{R}$ are continuous by the continuity of a_{ij} and b_{ij}. We now show that π_{ij} is quasiconcave in p_{ij}.

Since π_{ij} is differentiable, we can establish quasiconcavity by showing that for any pair of distinct prices u and v in the domain of π_{ij}, $\pi_{ij}(v) \geq \pi_{ij}(u)$ implies $\pi_{ij}'(u)(v-u) \geq 0$. (The prices of the other firms are taken as fixed, so we omit them from the notation to reduce clutter.) Consider two cases: $v > u$, where u lies on the upward slope of the profit function, and $u > v$, where u lies on the downward slope.

In the first case, let $x = v - u$. Then rewrite $\pi_{ij}(v) \geq \pi_{ij}(u)$ as $\left(u + x - c_{ij}\right) q_{ij}(v) - \left(u - c_{ij}\right) q_{ij}(u) \geq 0$. Rearranging terms and dividing by $v - u$ yields

$$q_{ij}(v) + \left(u - c_{ij}\right) \frac{q_{ij}(v) - q_{ij}(u)}{v - u} \geq 0 .$$

By the mean value theorem, the fraction in this expression is the value of the derivative of the demand function for some $z \in (u, v)$. Letting $z = u + y$ and expanding q_{ij} according to (8), we have

$$a_{ij}(u+x)\,b_{ij}(u+x)Q_0 + \left(u - c_{ij}\right)\left[a_{ij}(u+y)\,b'_{ij}(u+y) + b_{ij}(u+y)\,a'_{ij}(u+y)\right]Q_0 \geq 0\,. \tag{A1}$$

To show that this expression implies $\pi'_{ij}(u)(v-u) \geq 0$, we divide the latter by $v - u$ and expand π'_{ij} according to (14):

$$a_{ij}(u)\,b_{ij}(u)Q_0 + \left(u - c_{ij}\right)\left[a_{ij}(u)\,b'_{ij}(u) + b_{ij}(u)\,a'_{ij}(u)\right]Q_0 \geq 0\,. \tag{A2}$$

Noting that $0 < y < x$, we have $a_{ij}(u) > a_{ij}(u+y) > a_{ij}(u+x)$ and $b_{ij}(u) > b_{ij}(u+y) > b_{ij}(u+x)$**,** since a_{ij} and b_{ij} are monotonically decreasing in firm *ij*'s price by assumptions (9a) and (11a); $a'_{ij}(u) \geq a'_{ij}(u+y)$ and $b'_{ij}(u) \geq b'_{ij}(u+y)$ by the assumed concavity of a_{ij} and b_{ij}. It follows that (A1) implies (A2) for all $u > v$.

The second case proceeds analogously, but instead we let $x = u - v$ and $z = u - y$. This yields

$$a_{ij}(u-x)\,b_{ij}(u-x)Q_0 + \left(u - c_{ij}\right)\left[a_{ij}(u-y)\,b'_{ij}(u-y) + b_{ij}(u-y)\,a'_{ij}(u-y)\right]Q_0 \leq 0\,. \tag{A3}$$

Again, we need to show that this implies $\pi'_{ij}(u)(v-u) \geq 0$. Diving by $v - u$ and expanding π'_{ij} as before:

$$a_{ij}(u)\,b_{ij}(u)Q_0 + \left(u - c_{ij}\right)\left[a_{ij}(u)\,b'_{ij}(u) + b_{ij}(u)\,a'_{ij}(u)\right]Q_0 \leq 0\,. \tag{A4}$$

Now $0 < y < x$, so we have $a_{ij}(u) < a_{ij}(u-y) < a_{ij}(u-x)$ and $b_{ij}(u) < b_{ij}(u+y) < b_{ij}(u+x)$ by assumptions (9a) and (11a), as well as $a'_{ij}(u) \leq a'_{ij}(u-y)$ and $b'_{ij}(u) \leq b'_{ij}(u-y)$ by the concavity of a_{ij} and b_{ij}. It follows that (A3) implies (A4) for all $v > u$. Taking this result together with the first case, we conclude that π_{ij} is quasiconcave in p_{ij} over its entire domain, and thus a Nash equilibrium exists. □

In fact, π_{ij} is strictly quasiconcave in p_{ij} since (A2) and (A4) hold with strict inequalities. We use this fact in the proof of Proposition 3 below.

A.2 Proof of Proposition 2

Recall that (18) expresses the first-order condition for a symmetric equilibrium. To show that it has a unique solution, we define a new function *h* with the property that $h(p) = 0$ if and only if $p = p^*_{J\times N}$:

$$h(p) = p - p^*_{J\times N} = p + \frac{a_{J\times N}(p)\,b_{J\times N}(p)}{a_{J\times N}(p)\,b'_{J\times N}(p) + b_{J\times N}(p)\,a'_{J\times N}(p)} - \frac{C}{J}\,.$$

Since $p^*_{J\times N} \geq 0$ exists by Proposition 1, $h(0) \leq 0$. We now show that *h* is monotonically increasing in *p*.

Taking the derivative of h with respect to p and rearranging terms yields

$$h'(p) = 1 + \frac{a_{J\times N}(p)^2\left[b'_{J\times N}(p)^2 - b_{J\times N}(p)\,b''_{J\times N}(p)\right] + b_{J\times N}(p)^2\left[a'_{J\times N}(p)^2 - a_{J\times N}(p)\,a''_{J\times N}(p)\right]}{\left(a_{J\times N}(p)\,b'_{J\times N}(p) + b_{J\times N}(p)\,a'_{J\times N}(p)\right)^2}.$$

Noting that $a_{J\times N}(p) \geq 0$ and $b_{J\times N}(p) > 0$ by definition, $a'_{J\times N}(p) < 0$ and $b'_{J\times N}(p) < 0$ by assumptions (9a) and (11a), and $a''_{J\times N}(p) \leq 0$ and $b''_{J\times N}(p) \leq 0$ by the concavity of $a_{J\times N}$ and $b_{J\times N}$, we find that $h'(p) > 0$ for all p.

Since h is continuous and monotonic, by the intermediate value theorem it must pass through zero exactly once, yielding a unique solution to (18). This solution is, by definition, a symmetric Nash equilibrium, and since (18) is a necessary condition, it is the only one. □

A.3 Proof of Proposition 3

Our strategy is to "test" the equilibrium component price of one configuration against the equilibrium first-order condition of the other. In part (a), we show that the $J \times M$ marginal profit function evaluated at the $J \times N$ equilibrium component price is positive, therefore the $J \times N$ equilibrium price is on the rising slope of the $J \times M$ profit function. It follows that the $J \times M$ equilibrium system price is higher than the $J \times N$ equilibrium system price. Thus equilibrium system prices go *down* as the number of firms per layer increases.

Similarly, in part (b), we show that the $K \times N$ marginal profit function is positive when evaluated at the $J \times N$ equilibrium system price re-apportioned over K layers. It follows that the $K \times N$ equilibrium system price is higher than the $J \times N$ equilibrium system price. Thus equilibrium system prices go *up* as the number of layers increases.

The proof requires the following lemma:

Lemma. Under the conservation of total demand, $b_{K\times M}(P/K) = b_{J\times N}(P/J)$.

Proof. Expanding (22a), using (19) and (20), we have

$$Q_{K\times M}(P) = Mq_{K\times M}(p_{K\times M}) = Nq_{J\times N}(p_{J\times N}) = Q_{J\times N}(P)$$

where $p_{K\times M}(P) = P/K$ and $p_{J\times N}(P) = P/J$. Substituting for the q terms from (15):

$$M\,a_{K\times M}(p_{K\times M})\,b_{K\times M}(p_{K\times M})\,Q_0 = N\,a_{J\times N}(p_{J\times N})\,b_{J\times N}(p_{J\times N})\,Q_0\,.$$

But in a symmetric cluster, $a_{J\times M} = 1/M$ and $a_{J\times N} = 1/N$. Substituting again and canceling terms obtains the result. □

We now proceed to the proof itself.

Part (a). From (17), the marginal profit function of a firm in a $J\times M$ symmetric cluster is

$$\pi'_{J\times M}(p) = a_{J\times M}(p)\, b_{J\times M}(p) Q_0 + \left(p - \frac{C}{J}\right)\left[a_{J\times M}(p)\, b'_{J\times M}(p) + b_{J\times M}(p)\, a'_{J\times M}(p)\right] Q_0 \qquad \text{(A5)}$$

with $\pi'_{J\times M}(p^*_{J\times M}) = 0$ by the first-order condition.

We wish to determine the sign of (A5) when *p* is the product price in a $J\times M$ cluster corresponding to the equilibrium system price in a $J\times N$ cluster. To do this, we need to convert the $J\times M$-subscripted terms in (A5) to their $J\times N$ equivalents. We have five cross-cluster conditions that together permit this conversion.

First, from equation (23), since the two clusters have the same number of layers:

$$p = p^*_{J\times N}\,.$$

Second, from equation (16):

$$a_{J\times M}(p) = \frac{1}{M} = \frac{N}{M}\, a_{J\times N}(p^*_{J\times N})\,.$$

Third, from the lemma:

$$b_{J\times M}(p) = b_{J\times N}(p^*_{J\times N})\,.$$

Fourth, from equation (24c):

$$a'_{J\times M}(p) = \gamma a'_{J\times N}(p^*_{J\times N})$$

where γ is a scaling parameter such that $0 < \gamma \le 1$. (Recall from Section 4.2.3 that adding firms to a layer weakly strengthens the cross-price elasticity of demand.)

Fifth, from equation (24b):

$$b'_{J\times M}(p) = b'_{J\times N}(p^*_{J\times N})\,.$$

Substituting these five conditions into equation (A5) obtains

$$\begin{aligned}\pi'_{J\times M}(p) &= \frac{N}{M} a_{J\times N}(p^*_{J\times N})\, b_{J\times N}(p^*_{J\times N}) Q_0 + \\ &\left(p^*_{J\times N} - \frac{C}{J}\right)\left[\frac{N}{M} a_{J\times N}(p^*_{J\times N})\, b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N}) \cdot \gamma a'_{J\times N}(p^*_{J\times N})\right] Q_0 .\end{aligned} \tag{A6}$$

This expression is the marginal profit that a firm in a $J \times M$ configuration would obtain at the $J \times N$ equilibrium system price. Through the cross-cluster conditions, all the elements are now expressed in terms of $J \times N$ -subscripted functions.

We can now substitute into (A6) an explicit expression for the $J \times N$ cluster's equilibrium product price, which is given by equation (18):

$$p^*_{J\times N} = \frac{-a_{J\times N}(p^*_{J\times N})\, b_{J\times N}(p^*_{J\times N})}{a_{J\times N}(p^*_{J\times N})\, b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N})\, a'_{J\times N}(p^*_{J\times N})} + \frac{C}{J} . \tag{A7}$$

Making the substitution and collecting terms yields

$$\begin{aligned}\pi'_{J\times M}(p^*_{J\times N}) &= \left(\frac{N}{M} - \frac{U}{D}\right) a_{J\times N}(p^*_{J\times N})\, b_{J\times N}(p^*_{J\times N}) Q_0 \\ &= \left(1 - \frac{M}{N}\frac{U}{D}\right)\frac{N}{M} a_{J\times N}(p^*_{J\times N})\, b_{J\times N}(p^*_{J\times N}) Q_0\end{aligned} \tag{A8}$$

where

$$U \equiv \frac{N}{M} a_{J\times N}(p^*_{J\times N})\, b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N}) \cdot \gamma a'_{J\times N}(p^*_{J\times N})$$

$$D \equiv a_{J\times N}(p^*_{J\times N})\, b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N})\, a'_{J\times N}(p^*_{J\times N}) .$$

Because $\gamma \le 1$ and $M < N$, $MU / ND < 1$ and the leading term of (A8) is positive. Since the terms $a_{J\times N}(p^*_{J\times N})$ and $b_{J\times N}(p^*_{J\times N})$ are positive and Q_0 is positive, the entire expression is positive.

Because the profit function is strictly quasiconcave in prices (see Appendix A.1), this means that $p^*_{J\times M} > p^*_{J\times N}$, i.e., the equilibrium component price in the $J \times M$ configuration is greater than the equilibrium component price in the $J \times N$ configuration. Multiplying both sides by *J* yields $Jp^*_{J\times M} > Jp^*_{J\times N}$. It follows immediately from the definition of the system price that $P^*_{J\times M} > P^*_{J\times N}$ for all *M* and *N* such that $1 \le M < N$. □

Part (b). We now test the $J \times N$ equilibrium system price in the marginal profit function of a $K \times N$ cluster, where $1 \le J < K$. Using (17) again, the marginal profit function is

$$\pi'_{K\times N}(p) = a_{K\times N}(p)\,b_{K\times N}(p)Q_0 + \left(p - \frac{C}{K}\right)\left[a_{K\times N}(p)\,b'_{K\times N}(p) + b_{K\times N}(p)\,a'_{K\times N}(p)\right]Q_0 . \tag{A9}$$

As before, we wish to determine the sign of (A9) when p is the product price in a $K\times N$ cluster corresponding to the equilibrium system price in a $J\times N$ cluster. Again, to do this, we need to convert the $K\times N$ -subscripted terms in (A5) to their $J\times N$ equivalents, which a slightly different set of cross-cluster conditions will enable us to do.

First, from equation (23), because the two clusters have different numbers of layers:

$$p = \frac{J}{K}p^*_{J\times N} .$$

Second, from equation (16):

$$a_{K\times N}(p) = \frac{1}{N} = a_{J\times N}(p^*_{J\times N}) .$$

Third, from the lemma:

$$b_{K\times N}(p) = b_{J\times N}(p^*_{J\times N}) .$$

Fourth, from equation (24a) and the concavity of $a_{J\times N}$:

$$a'_{K\times N}(p) = \gamma a'_{J\times N}(p^*_{J\times N})$$

where $0 < \gamma \le 1$. (Note that $a'_{K\times N}(p) = a'_{J\times N}(p)$ by (24a), but $p \le p^*_{J\times N}$ since $J < K$, so by the concavity of $a_{J\times N}$ we have $a'_{J\times N}(p) \ge a'_{J\times N}(p^*_{J\times N})$ and thus $a'_{K\times N}(p) \ge a'_{J\times N}(p^*_{J\times N})$.)

Fifth, from equation (24d) and the concavity of $b_{J\times N}$:

$$b'_{K\times N}(p) = \lambda b'_{J\times N}(p^*_{J\times N})$$

where $0 < \lambda \le 1$. (Recall from Section 4.2.3 that adding layers to a cluster weakly moderates the impact of a price increase by the focal firm. Here we need the concavity of $b_{J\times N}$ for the same reason as above, but we have $b'_{K\times N}(p) \ge b'_{J\times N}(p) \ge b'_{J\times N}(p^*_{J\times N})$ so we only need one parameter λ .)

Substituting these five conditions into equation (A9) obtains

$$\begin{aligned}\pi'_{K\times N}(p) &= a_{J\times N}(p^*_{J\times N})\,b_{J\times N}(p^*_{J\times N})Q_0 + \\ &\left(\frac{J}{K}p^*_{J\times N} - \frac{C}{K}\right)\left[a_{J\times N}(p^*_{J\times N})\cdot\lambda b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N})\cdot\gamma a'_{J\times N}(p^*_{J\times N})\right]Q_0 .\end{aligned} \tag{A10}$$

This expression is the marginal profit that a firm in a $K\times N$ configuration would obtain at the $J\times N$

system price. Through the cross-cluster conditions, all the elements are again expressed in terms of $J \times N$ -subscripted functions.

We can now substitute into (A10) an explicit expression for the $J \times N$ cluster's equilibrium product price, using equation (18) and $p = (J/K) p^*_{J\times N}$. Making the substitution and collecting terms yields

$$\pi'_{K\times N}(\frac{J}{K} p^*_{J\times N}) = \left(1 - \frac{J}{K}\frac{V}{D}\right) a_{J\times N}(p^*_{J\times N}) b_{J\times N}(p^*_{J\times N}) Q_0 \tag{A11}$$

where

$$V \equiv a_{J\times N}(p^*_{J\times N}) \cdot \lambda b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N}) \cdot \gamma a'_{J\times N}(p^*_{J\times N})$$

$$D \equiv a_{J\times N}(p^*_{J\times N}) b'_{J\times N}(p^*_{J\times N}) + b_{J\times N}(p^*_{J\times N}) a'_{J\times N}(p^*_{J\times N}) .$$

Because $\gamma, \lambda \le 1$ and $J < K$, $JV / KD < 1$ and the leading term of (A11) is positive. Since the terms $a_{J\times N}(p^*_{J\times N})$ and $b_{J\times N}(p^*_{J\times N})$ are positive and Q_0 is positive, the entire expression is positive.

Because the profit function is strictly quasiconcave in prices (see Appendix A.1), this means that $p^*_{K\times N} > (J/K) p^*_{J\times N}$, i.e., the equilibrium component price in the $K \times N$ configuration is greater than the reapportioned equilibrium component price in the $J \times N$ configuration. Multiplying both sides by K yields $Kp^*_{K\times N} > Jp^*_{J\times N}$, from which it follows immediately that $P^*_{K\times N} > P^*_{J\times N}$ for all J and K such that $1 \le J < K$. □

A.4 Proof of Proposition 4

The two sets of inequalities follow from Proposition 3, which establishes the monotonicity of equilibrium system prices with respect to *J* and *N*.

To show that $P^*_{J\times N^*} \to P^*_{1\times 1}$ as $J \to \infty$, we first show that $N^* \to \infty$ as $J \to \infty$. Relaxing the constraint that the number of firms per layer be an integer, let $\hat{N}$ be the real number for which $P^*_{J\times \hat{N}} = P^*_{1\times 1}$. ($N^*$ is thus the closest integer less than or equal to $\hat{N}$.) Using the fact that $a_{J\times \hat{N}} = 1/\hat{N}$ by (16) and $b_{J\times \hat{N}}(P^*_{1\times 1}/J) = b_{1\times 1}(P^*_{1\times 1})$ by the lemma of Appendix A.3, we obtain after some algebra:

$$\hat{N} = J\left[\frac{b'_{1\times 1}(P^*_{1\times 1})}{a_{J\times \hat{N}}(P^*_{1\times 1}/J)\, b'_{J\times \hat{N}}(P^*_{1\times 1}/J) + b_{J\times \hat{N}}(P^*_{1\times 1}/J)\, a'_{J\times \hat{N}}(P^*_{1\times 1}/J)}\right]$$

where the term in brackets is strictly positive by the assumptions of the symmetric model, and does not converge to zero as $J \to \infty$. Therefore, as J grows large, $\hat{N}$ and N^* do also.

Next, note that $\lim_{N\to\infty} P^*_{J\times N} - P^*_{J\times(N+1)} = 0$ because $P^*_{J\times N}$ is a strictly decreasing function of N (by Proposition 3) and is bounded below by zero. Since $P^*_{J\times N^*} \geq P^*_{1\times 1} > P^*_{J\times(N^*+1)}$ by construction, we conclude that the monopoly price is "squeezed" between $P^*_{J\times N^*}$ and $P^*_{J\times(N^*+1)}$ as $J \to \infty$. The proof that $P^*_{J^*\times N} \to P^*_{1\times 1}$ as $N \to \infty$ is directly analogous. □

A.5 Proof of Proposition 5

The proof makes use of the following lemma, which establishes the monopoly profit as a benchmark for the highest aggregate profit attainable by a cluster, and the fact that it is attainable only by replicating the system price of a monopoly.

Lemma. Consider two configurations with equilibrium system prices P' and P'', and cluster profits Π' and Π'', respectively. If $P'' < P' < P^*_{1\times 1}$ or $P'' > P' > P^*_{1\times 1}$, then $\Pi'' < \Pi' < \Pi^*_{1\times 1}$. Likewise, if $P' = P^*_{1\times 1}$ then $\Pi' = \Pi^*_{1\times 1}$.

Proof. Given the conservation of total demand and the other assumptions of the symmetric model, all configurations share a common cluster profit function, $\Pi = (P - C)Q(P)$. Since Q is continuous, downward-sloping and concave, Π is strictly concave in its argument, the system price. (We need only consider prices for which for $P \geq C$, since by (18) equilibrium prices always exceed unit costs.) By definition, $P^*_{1\times 1}$ is the price that maximizes the cluster profit in the monopoly case, and thus for all configurations. Therefore, as the system price approaches $P^*_{1\times 1}$ from above or below, the cluster profit increases and approaches the maximum, $\Pi^*_{1\times 1}$. If the system price equals $P^*_{1\times 1}$, then aggregate cluster profit equals the maximum profit, $\Pi^*_{1\times 1}$. □

The existence of profit peaks near $J \times N^*$ and $J^* \times N$ configurations follows from Proposition 4 and the lemma. For $1 \leq N' < N^*$, $P^*_{J\times N'} > P^*_{J\times N^*} \geq P^*_{1\times 1}$ and thus $\Pi^*_{J\times N'} < \Pi^*_{J\times N^*} \leq \Pi^*_{1\times 1}$. For $N'' > N^* + 1$, $P^*_{J\times N''} < P^*_{J\times(N^*+1)} < P^*_{1\times 1}$ and thus $\Pi^*_{J\times N''} < \Pi^*_{J\times(N^*+1)} < \Pi^*_{1\times 1}$. Similar reasoning holds for $1 \leq J' < J^*$ and $J'' > J^* + 1$. The convergence result follows directly from Proposition 4 and equations (21) and (22). □

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