# STRICT GEOMETRIC MODEL BASED ON AFFINE TRANSFORMATION FOR REMOTE SENSING IMAGE WITH HIGH RESOLUTION

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#### **ABSTRACT:**

The successful lunch of IKONOS and QUICKBIRD, and a serious of plans for acquiring the remote sensing images with high resolution let it become possible that the basic spatial information is obtained from the remote sensing images in low costing and sort period. The first processing step is calculating the parameters of image position and orientation. Because of the very strong relativity of the traditional parameters of remote image with high resolution, the parameter calculation is not solved perfectly up to now. A strict geometric model, proposed in this paper, adopts a new method with three steps of transformations based on parallel ray projection. The first step is reducing the three dimensional space to the image space by the similar transformation. Then, the small space is projected to the level plane, which passes the center of the image plane, by parallel rays (Affine transformation). Finally, the level image is transformed to the original declining image. Every step of the model is strict, and the map function of each transformation is the first order polynomials and other simple function. The final calculation of the parameters is for the linear equations with good status. As a result, the problem of the relativity of image parameter calculation is solved completely. Some experiments are carried on for a lot of images with 10, 3 and 1-meters resolution. All of the results are quite perfect. Thus, the validity of the strict geometric model has been verified.

## 1. INTRODUCTION

The quick development of the economy and the society requires the suitable 3D representation for districts, states and the earth. The digital elevation model (DEM) and georeferenced remote sensing images with 1-meter resolution are the base. Recently, the most effect means, by which acquisition of the spatial information of the earth is the fastest, is remote sensing technology, which development is the quickest. That is, the spatial information of the earth is captured from the images taken by variance sensors installed in the space vehicles. The IKONOS-2, which can acquire the remote sensing image with 1-meter resolution for commercial application, had been lunched successfully in September 1999. The QUICKBIRD, which can acquire the image with 0.62-meter resolution, had been lunched successfully in 2001 too. Furthermore, a serious of lunch plans for acquiring the remote sensing images with high resolution are being executed. Thus, it is becoming possible that this basic spatial information could be obtained using the remote sensing images with high resolution in low costing and sort period.

In order to process the captured remote sensing image as georeferenced image, the first step is calculating the parameters of image position and orientation. After the parameters are computed, the original image can be rectified precisely based on the corresponding DEM, so that the image becomes georeferenced. However, as a result of that the photographic station of the remote sensing images with high resolution is very high, and the photographic viewing angle is very small, there is very strong relativity between their traditional position and orientation parameters. Because of the very strong relativity of the image parameters, the calculation of the image parameters is not solved perfectly all the time. For example, for the SPOT image with 10-meters resolution, the reasonable values of the image parameters can be computed hardly by the traditional calculation method, which includes 12 parameters, the six position and orientation parameters an their linear variance rates. Although many algorithms for overcoming the strong relativity, such as grouping iteration, combining relative items, etc., had been proposed, the reasonable solution could not be obtained in some cases. Up to now, the algorithm, which is used relatively frequently, is the fitting based on reasonable polynomials, proposed by Kratky (V. Kratky, 1989a and V. Kratky, 1989b). It is an approximate method. If the better result is desired, the polynomials with higher order should be applied, therefore, the more control points should be needed. Even though, the reasonable solution could not be obtained sometimes.

Okamoto (Okamoto, 1981, 1988; Okamoto & Akamatsu, 1992a; b) had proposed a model based on affine projection. Susumu Hattori (Susumu Hattori etc. 2000) and Tetsu Ono (Tetsu Ono etc. 2000) had further investigated and used the model. Under the hypothesis that the central projection is approximately as the same of the parallel projection in the case of the small viewing angle, the calculation of position and orientation parameters using the affine model could overcome the strong relativity between their position and orientation parameters of SPOT images. It is effective in the case of the maps with smaller scale from SPOT images, which demands the lower precision. But it is an approximate method nonetheless. The image with higher resolution is the same as the SPOT incompletely. Its viewing angle is quite smaller, and the relativity of the parameters must be quite stronger. It is needed to investigate whether the method would be suitable for the case of the maps with larger scale from the images with about 1-meter resolution, which demands the higher precision. Therefore, searching a strict mathematical model of the calculation for the position and orientation parameters of the remote sensing image with higher resolution,

which can overcome the relativity, and solving the problem completely, is quite important for its application.

The strict geometric model of the calculation for the position and orientation parameters of the remote-sensing image with higher resolution, proposed in this paper, adopts the method with three steps of transformations. Recently, the remote sensing image with higher resolution is similar as SPOT image, which is imaging by pushbrooming ahead with the linear array CCD. That is, it is central projection in the scanning direction, and parallel projection in the flight direction. The first step of the strict geometric model is reducing the three dimensional space to the image space by the similar transformation. Then, the small space is projected to the level plane, which passes the center of the image plane, by parallel rays (Affine transformation). Finally, the level image is transformed to the original declining image. Every step of the new method is the strict, and the map function of each transformation is the first order polynomials and other simple function. The final calculation of the parameters is for the linear equations with good status. As a result, the problem of the relativity of image parameter calculation is solved completely.

By the strict geometric model of remote sensing image with high resolution mentioned above, some experiments are carried on for more than ten pairs of SPOT images, some of which could not be processed and generate right result by old algorithms. Some pairs of the images with 3 and 1-meters resolution are used in the experiments too. All of the experimental results are quite perfect. Thus, the validity of the new method has been verified.

#### 2、 IMAGE GEOMETRY OF PARALLEL RAY PROJECTION

As shown in Fig.1, the intersection line of the ground level plane and the plane consisted of principle ray  $Sx_0$  and straight-line  $x_0x'$  passed by push-broom linear array, is  $X_0X'$ . The perpendicular of  $X_0X'$ , which passes S, crosses  $X_0X'$  at O. In coordinate system O-XYZ, OX' is X-axes, and OS is Z-axes.

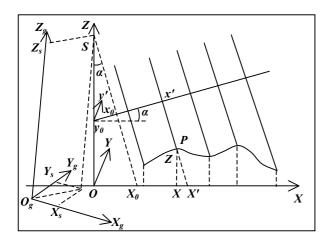


Fig.1 Imaging Geometry of Parallel Ray Projection

The straight line, which passes point **O** and parallels the motion direction of CCD linear array, is **Y**-axes (It is possible that the **Y**-axes is not perpendicular to plane **O**-**XY**). The coordinates of the point **O** in ground coordinate system **O**<sub>g</sub>-**X**<sub>g</sub>**Y**<sub>g</sub>**X**<sub>g</sub> are  $(X_O Y_O Z_O)$ . Then, the relationship between the coordinates of a ground point P in two coordinate systems (X, Y, Z) and  $(X_g, Y_g, X_g)$ , is a 3-D affine transformation:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} X_g - X_o \\ Y_g - Y_o \\ Z_g - Z_o \end{pmatrix}$$
(1)

If let Px' parallel the principle ray  $Sx_0$ ,  $x_0SO=\alpha$ , and the surface profile of the ground is projected to the image plane by the ray, which parallels  $Sx_0$ , then

$$\begin{array}{ll} \mathbf{x}^{*} - \mathbf{x}_{\theta} = (X_{\theta} - X_{1}) \cos \alpha &= (X + Z \sin \alpha - X_{1}) \cos \alpha & (2) \\ \mathbf{y}^{*} - \mathbf{y}_{\theta} &= \mathbf{Y} & (3) \end{array}$$

From Eq. (1), (2) and (3):

$$\begin{aligned} x' - x_{\theta} &= (r_{11}(X_g - X_O) + r_{12}(Y_g - Y_O) + r_{13}(Z_g - Z_O) + \\ & (r_{31}(X_g - X_O) + r_{32}(Y_g - Y_O) + r_{33}(Z_g - Z_O))sina - X_1)cosa \quad (4) \\ y' - y_{\theta} &= r_{21}(X_e - X_O) + r_{22}(Y_e - Y_O) + r_{23}(Z_e - Z_O) \quad (5) \end{aligned}$$

namely:

$$\begin{aligned} \mathbf{x}' = a_0' + a_1' X_g + a_2' Y_g + a_3' Z_g \tag{6} \\ \mathbf{y}' = b_0' + b_1' X_g + b_2' Y_g + b_3' Z_g \tag{7} \end{aligned}$$

Eq.(6) and (7) show that imaging by parallel ray projection is the affine transformation from 3-D to 2-D.

### **3. STRIC GEOMETRIC MODEL**

# **3.1** Relationship of Image Coordinates (x , y) and Space Coordinates (Xg, Yg, Zg).

Within plane *XOZ*, perpendicular from  $x_{\theta}$  to *OZ* crosses *OZ* at *O*'. Let m = SO / SO'. Centering at *S*, the real surface model is reduced with m times by similar transformation (Fig. 2), and (*x*'', *y*'') are the coordinates of the image by parallel projection. Thus

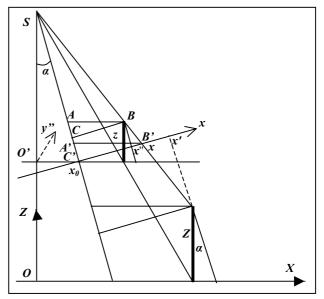


Fig. 2 Imaging Profile

$$x'' - x_0 = (x' - x_0) / m$$
  

$$y'' - y_0 = (y' - y_0) / m = y - y_0$$
  

$$z = Z / m$$

If f is the principle distance,  $\alpha$  is side watch angle, x and y are the coordinates of image, from Fig.2 the equations are acquired:

$$\frac{BC}{B'C'} = \frac{AB}{A'B'} = \frac{SA}{SA'}$$

That is:

$$\frac{x''-x_0}{x-x_0} = \frac{f - \frac{z}{\cos \alpha}}{f - (x - x_0)tg \,\alpha} = \frac{f - \frac{Z}{m \cos \alpha}}{f - (x - x_0)tg \,\alpha}$$
$$x'-x_0 = m \,\frac{f - \frac{Z}{m \cos \alpha}}{f - (x - x_0)tg \,\alpha} (x - x_0) \tag{8}$$

$$y''-y_0 = (y'-y_0)/m = y - y_0$$
  
$$y'-y_0 = m(y - y_0)$$
 (9)

From Eq. (4), (5), (8) and (9):

$$\frac{f - \frac{Z}{m\cos\alpha}}{f - xtg\alpha} (x - x_0) = a_0 + a_1 X + a_2 Y + a_3 Z$$
(10)

$$(y - y_0) = b_0 + b_1 X + b_2 Y + b_3 Z$$
(11)

Eq. (10) and (11) show the strict mathematical relationship of the image coordinates (x, y) and the space coordinates  $(X_g, Y_g,$  $Z_g$ ).

#### 3.2 Relationship of Space Coordinates $(X_g, Y_g, Z_g)$ and Left and Right Image Coordinates $(x_l, y_l)$ and $(x_r, y_r)$ :

Let subscript l denote elements of left image, subscript r denote elements of right image, four linear equations can be acquired from Eq.(10), (11) and the coordinates  $(x_l, y_l)$  and  $(x_r, y_r)$  of left and right images:

$$a_{l1}X_{g} + a_{l2}Y_{g} + \left(a_{l3} + \frac{x_{l} - x_{l0}}{m\cos\alpha_{l}(f - (x_{l} - x_{l0})tg\alpha_{l})}\right)Z_{g} = \frac{f(x_{l} - x_{l0})}{f - (x_{l} - x_{l0})tg\alpha_{l}} - a_{l0}$$
(12)

$$b_{l1}X_g + b_{l2}Y_g + b_{l3}Z_g = y_l - y_{l0} - b_{l0}$$
(13)

$$a_{r1}X_{g} + a_{r2}Y_{g} + \left(a_{r3} + \frac{x_{r} - x_{r0}}{m\cos\alpha_{r}\left(f - (x_{r} - x_{r0})tg\alpha_{r}\right)}\right)Z_{g} = \frac{f(x_{r} - x_{r0})}{f - (x_{r} - x_{r0})tg\alpha_{r}} - a_{r0}$$
(14)

$$b_{r1}X_g + b_{r2}Y_g + b_{r3}Z_g = y_r - y_{r0} - b_{r0}$$
(15)

Eq. (12) to (15) are the strict mathematical relationship of the space Coordinates (Xg, Yg, Zg) and left and right image coordinates (xl, yl) and (xr, yr).

#### 4. CALCULATION STRATEGY

#### 4.1 Calculation of Parameters

Because  $\alpha$  in the left of Eq.(10) is unknown, the equation is not linear. The calculation procedure is iterative based on the linearization. For simplifying, let x denote  $x - x_{\theta}$ , y denote  $y - y_{\theta}$ , X denote  $X_g$ , Y denote  $Y_g$  and Z denote  $Z_g$  in the next part of this paper. The Eq. (10) is linearized as following error equation:

$$da_{0} + X_{i}da_{1} + Y_{i}da_{2} + Z_{i}da_{3} +$$

$$x_{i}\left[\frac{Z_{i}\sin\alpha}{m(f - x_{i}tg\alpha)\cos^{2}\alpha} - \frac{x_{i}(f - Z_{i}/(m\cos\alpha))}{(f - x_{i}tg\alpha)^{2}\cos^{2}\alpha}\right]d\alpha +$$

$$a_{0} + X_{i}a_{1} + Y_{i}a_{2} + Z_{i}a_{3} - \frac{f - \frac{Z_{i}}{m\cos\alpha}}{f - xtg\alpha} = 0$$
(16)

Using error equation (16) and more than 5 control points,  $\alpha$ ,  $a_{\theta}$ ,  $a_1$ ,  $a_2$  and  $a_3$  can be solved iteratively. From Eq. (11), the linear equation can be acquired:

$$b_0 + X_i b_0 + Y_i b_1 + Z_i b_2 - y_i = 0$$
 (17)

Using Eq.(17),  $b_0$ ,  $b_1$ ,  $b_2$  and  $b_3$  can be solved directly without the iteration.

#### 4.2 Calculation of Image Coordinates (x, y) from Space Coordinates (X, Y, Z).

After  $a_1$ ,  $a_2$ ,  $a_3$ ,  $b_4$ ,  $b_1$ ,  $b_2$  and  $b_3$  are computed using control points, following equations can be adopted for calculating the image coordinates (x, y) from space coordinates (X, Y, Z).

$$x = (a_0 + a_1 X + a_2 Y + a_3 Z) \frac{f - xtg\alpha}{f - \frac{Z}{m\cos\alpha}}$$
(18)  

$$y = b_0 + b_1 X + b_2 Y + b_3 Z$$
(19)

(19)

**4.3** Calculation of Space Coordinates 
$$(X, Y, Z)$$
 from Left and Right Image Coordinates  $(x_l, y_l)$  and  $(x_r, y_r)$ :

Algorithm 1: From Eq. (12) to (15), the linear equations can be acquired:

Or denote as AX=L, and then the resolution is

$$X = (A^T A)^{-1} A^T L.$$

Algorithm 2: Combining Eq. (13) and (12), (15) and (14), expunging the Y, the 1-order equations with two unknowns X and Z are determined, from which the X and Z can be solved. Then  $Y_1$  and  $Y_2$  can be computed by and Z from Eq. (13) and (15). Therefore

$$Y = (Y_1 + Y_2)/2$$

In this way,  $Y_{I}$ -  $Y_{2}$  can be used for evaluating the quality of the solution.

### 5. EXPERIMENTS

By the strict geometric model of remote sensing image with high resolution mentioned above, some experiments are carried on for more than ten pairs of SPOT images, some of which could not be processed and generate right result by old algorithms. Some pairs of the remote sensing images with 3-meter and 1-meter resolution, including IKONOS images, are used in the experiment also. Tab.1 shows the RMSE of ground coordinates from the control points after parameter computation of stereo image pair. Tab.2 shows the RMSE of ground coordinates from the control and check points after parameter computation of SPOT stereo image pair with 78 known points. The last line of Tab.2 is the results with arbitrarily selected 10 points as control points and the other 68 points as check points, which indicates the solution is very stable. All of the other experimental results are quite perfect too. Thus, the validity of the new method has been verified.

Pixel	Point	X-rmse	Y-rmse	Z-rmse
Ground	Number	(m)	(m)	(m)
Resolution				
10 m	21	14.091	13.980	4.760
10 m	12	7.340	5.084	5.843
10 m	78	9.081	12.212	5.247
10 m	9	13.904	14.742	8.676
10 m	9	10.283	14.632	3.519
10 m	5	0.999	3.624	0.436
10 m	5	0.032	0.171	0.086
3 m	15	4.912	7.138	0.798
1 m	12	0.742	0.804	0.985
1 m	6	0.317	0.367	0.808

Table 1. Experiment Results Of Stereo Image Pairs

#### 6. CONCLUSION

Presented the geometric model above is the strict in theory, which needs no prior parameters of sensor trajectory except more than four control points. The experimental results shows that the parameter computation of remote sensing image with high resolution based on it is very stable. The problem of the relativity of image parameter calculation is solved completely by the strict geometric model.

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Control Point Number	X-rmse (m)	Y-rmse (m)	Z-rmse (m)	Check Point Number	X-rmse (m)	Y-rmse (m)	Z-rmse (m)
67	9.401	11.745	5.311	11	7.745	15.282	4.753
49	9.537	10.898	5.378	29	11.242	15.397	6.374
30	9.456	10.148	4.172	48	10.553	13.853	7.111
10	3.487	5.856	3.989	68	11.749	15.553	6.072

Table 2. Experiment Results Of SPOT Stereo Image Pairs with Check Points