A General Minimum Lower-Order Confounding Criterion for Two-Level Regular Designs

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Abstract

Based on the hierarchical ordering principle of factorial effects in experimental design, we propose an aliased effect-number pattern (AENP) as a criterion to judge a two-level regular design; such a pattern contains the basic information of non-aliased effects as well as effects aliased at varying degrees in a design. A design that sequentially maximizes the numbers in the AENP is called a general minimum lower-order confounding (GMLOC) design. The new criterion is thus called a GMLOC criterion and several results follow. First, the word-length pattern, as the core of the minimum aberration (MA) criterion, is a function of the AENP, thus the MA criterion can be considered as a special case of the new criterion. The same also holds for the clear effects criterion under the hierarchical ordering principle. Furthermore, since the estimation capacity of a design can be calculated as a function of the new pattern, its corresponding criterion can be treated as a special case of the GMLOC criterion as well. From the new pattern, certain ties between the MA and clear effects criteria are revealed. In addition, we introduce in this paper a concept of estimation ability for regular designs, and infer that a GMLOC design is simply a design with the best estimation ability. At last, a simple algorithm for computing the AENP is provided. All the GMLOC designs for 16 and 32 runs and some comparisons with MA designs are tabulated in the Appendix.

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1 Introduction

One of the main tasks in experimental design is to find good designs and to analyze experimental data more effectively, so that more effects and more possible models related to the effects in experiments can be estimated. Regular designs have been the most commonly considered designs in practice due to their simple confounding structure.

The effect hierarchy principle is one of most important principles having been used in experiments (Wu and Hamada (2000)). The principle reveals that a lower-order effect is likely more important than a higher-order one and effects of the same order are equally important. Therefore, to estimate more important parameters and models, a good design should minimize the confounding between the lower-order effects. Aimed at such a purpose, many optimality criteria have been proposed and discussed in the literature.

In this paper, we restrict ourselves to the discussion of the case of two-level regular designs. A regular 2^{n-m} design is determined by m independent defining relations. A defining relation is given by a word of letters which are labels of factors denoted by $1, 2, \ldots, n$. All possible products of the m independent defining words constitute its defining contrast subgroup, denoted by $G = \{I, w_1, \ldots, w_{2^m-1}\}$. Starting from the subgroup, there are quite a few optimality criteria for choosing good designs; the following four we describe appear to be the most popular.

The first one is the maximum resolution criterion proposed by Box and Hunter (1961). The number of letters in a word in G of a design d is called the length of the word and the length of the shortest word in G is called the resolution of d. The goal is then to choose the designs with maximum resolution. For a given pair of n and m, there may exist many designs with the same maximum resolution among which only some of the designs are considered good ones; this criterion is unable to compare them to find the best ones.

To extract good designs from the set returned by using the maximum resolution criterion, Fries and Hunter (1980) proposed the minimum aberration (MA) criterion. It is based on the word-length pattern (WLP):

$$W = (A_1, A_2, A_3, A_4, \ldots, A_n) \tag{1}$$

where A_i denotes the number of words with length i in G. According to this criterion, for any two designs, one compares the first unequal A_r's in their WLPs and the one with the smaller A_r is said to have less aberration. The design with the least aberration is called an MA design. In the past two and half decades, the MA criterion has been the most popular choice for finding good designs, and consequently much attention has been paid to its theory and constructions. Many related papers have been published since 1980, including Franklin (1984), Chen and Wu (1991), Chen (1992), Tang and Wu (1996), Chen and Hedayat (1996), Tang and Wu (1996), Suen, Chen and Wu (1997), Cheng, Steinberg and Sun (1999), Mukerjee and Wu (2001). Zhang and Park (2000), Zhang and Shao (2001), Ai and Zhang (2004), Zhu and Zeng (2005), Cheng and Tang (2005) and so on. However, sometimes the use of the MA criterion can fail to detect good designs under the effect hierarchy principle (Wu and Hamada (2000)).

The third one is the clear effects criterion proposed first by Wu and Chen (1992). To remedy the above problem of the MA criterion, they introduced the notion of clear effects. A main effect or two-factor interaction is said to be clear if none of its aliases are main effects or two-factor interactions (2fi's). The clear effects criterion selects designs which sequentially maximizes the numbers of clear main effects and clear 2fi's. Thus a design with more clear main effects and 2fi's is better. Recent results on the clear effects criterion include Chen and Hedayat (1998), Tang, Ma, Ingram and Wang (2002), Wu and Wu (2002), Ai and Zhang (2004), Yang, Liu and Zhang (2005), Chen, Li, Liu and Zhang (2005), Yang, Li, Liu and Zhang (2005). But the clear effects criterion can only be used when there exist designs having clear effects. In addition, like the maximum resolution criterion, there may be many equally good designs under the clear effects criterion, but this criterion can not judge which one is better.

Furthermore, although for given parameters n and m, both the MA criterion and clear effects criterion usually lead to the same set of optimal designs, especially when $n-m$ is small, in certain cases the optimal designs obtained by the two criteria may conflict each other. For example, consider the following two 2^{9-4} designs.

Example 1.

$$d_1: I = 1236 = 1247 = 1258 = 13459;$$
$$d_2: I = 1236 = 1247 = 1348 = 23459,$$

where I is the column with all entries zeros. The word-length patterns of d_1 and d_2 are (0,0,0,6,8,0,0,1,0) and (0,0,0,7,7,0,0,0,1). Both of the designs have 9 clear main effects since they are of resolution IV, but d_1 only has 8 clear 2fi's and d_2 has 15 clear 2fi's.

Under the MA criterion, d_1 is an MA 2^{9-4} design, implying that d_1 has less aberration than d_2, and hence d_1 is better than d_2. But under the clear effects criterion, d_2 contains maximum number of clear main effects and 2fi's in all 2^{9-4} designs and so it is optimal and better than d_1.

The fourth one is the criterion of estimation capacity. Sun (1993) first introduced the notion of estimation capacity, where the idea is to estimate as many as possible models, involving all the main effects and some 2fi's. Cheng and Mukerjee (1998), Cheng, Steinberg and Sun (1999), and Ai and Zhang (2004) studied it in detail, and obtained some "good" designs which have maximum estimation capacity (MEC). These designs can be used to estimate as many as possible models involving all the main effects and some special 2fi's, but they need a strong assumption that all the other 2fi's, those that are not involved in the models but are aliasing the 2fi's in the models, are absent or negligible.

In the face of so many criteria, one may ask the following questions: What relationships are there between the criteria? Why do the criteria originating from the same ideas, especially the MA and clear effects criteria, lead to the same optimal designs in most cases, but can still give conflicting results? Why do the existing good criteria have themselves defect? What is the basic information being contained in the defining contrast group G indeed? Is there a criterion which more reasonably reflect the effect hierarchy principle? In this paper we try to answer these questions.

In Section 2 we introduce a new aliasing pattern for judging two-level regular designs, called the aliased effect-number pattern (AENP), and based on the AENP we propose a general minimum lower-order confounding criterion for rank-ordering regular 2^{n-m} designs and choosing optimal designs under the effect hierarchy principle. Relations of the new criterion with the MA, clear effects and MEC criteria are studied in Sections 3, 4 and 5, respectively. Especially, the ties between the MA and clear criteria is addressed in Section 4. A novel criterion, maximum estimation ability criterion, is proposed in Section 6. A simple algorithm for computing the AENP is given in Section 7. In Section 8 we make a simplification to the AENP and give more usages of the AENP via examples. The optimal designs of 16- and 32-run and 64-run with some parameters under the new criterion and some comparisons with the MA and clear criteria are tabulated in the Appendix.

2 A New Aliasing Pattern and Minimum Lower-Order Confounding Criterion

In this section, we first introduce a new aliasing pattern for characterizing 2^{n-m} designs and then based on the new pattern suggest a new criterion for rank-ordering the designs.

In order to give a good aliasing pattern and criterion, we need to explore further the basic information hidden in the subgroup G. We note that the confounding between effects in a design contains information on how the effects of all different orders are confounded by other effects. First, we consider a description of an i-order effect being aliased by j-order effects, for all i and j. To characterize the confoundedness of i-order effects by j-order effects, two basic elements should be considered. The first one is that for a given i-order effect, how severe it is aliased by j-order effects, and we call the severity an aliased degree. If the i-order effect is aliased by k j-order effects simultaneously, it is said that the i-order effect is aliased by j-order effects at degree k. In particular, if $k = 0$, then the i-order effect is not aliased by j-order effects. The second consideration is how many i-order effects are aliased by j-order effects at a given degree k. We use the notation ${}_i^{\#}C_j^{(k)}$ to denote the number of i-order effects aliased by j-order effects at degree k. Thus, for a design, we have a set

$$\{{}_i^{\#}C_j^{(k)}, i, j = 0, 1, \ldots, n, k = 0, 1, \ldots, K_j\}, \tag{2}$$

where $K_j = \binom{n}{j}$ and use this set to reflect the whole confounding between effects in the design.

Obviously, the numbers in (2) are not equally important, we need to arrange them in a particular order. Clearly, for an i-order effect, the lesser the degree at which it is aliased by other effects, the easier it can be estimated. In particular, if it is aliased at degree 0 by lower-order effects and higher-order effects are negligible, then it can be estimated without confounding. In addition, since the total number of i-order effects in a 2^{n-m} design is a constant $\binom{n}{i}$, the larger the number ${}_i^{\#}C_j^{(0)}$ is, the less severe the i-order effects are confounded by j-order effects. Subsequently, under the condition of maximizing the number ${}_i^{\#}C_j^{(0)}$, the larger the number ${}_i^{\#}C_j^{(1)}$ is, the less severe the i-order effects are confounded by j-order effects, and so on. Consider $\{{}_i^{\#}C_j^{(k)},\ k = 0, 1, \ldots, K_j\}$. Since the larger the degree k is, the more severe the effect is aliased, we should rank the confounded numbers of i-order effects by j-order effects from degree 0 to most severe degree in the following order, denoted by the vector

$$ {}_i^{\#}C_j = ({}_i^{\#}C_j^{(0)}, {}_i^{\#}C_j^{(1)}, \ldots, {}_i^{\#}C_j^{(K_j)}). \tag{3} $$

It turns out that this vector indicates a distribution of the total number of i-order effects aliased by j-order effects on the degrees $k = 0, 1, \ldots, K_j$. When $i = 0$ or $j = 0$, 0-order effect appears and is just the total mean effect, which is most important in a design. It is easy to see that ${}_i^{\#}C_0 = ({}_i^{\#}C_0^{(0)}, {}_i^{\#}C_0^{(1)})$, ${}_0^{\#}C_j = (1, 0, \ldots)$ for $A_j = 0$ and ${}_0^{\#}C_j = (0, \ldots, {}_0^{\#}C_j^{(A_j-1)} = 0, 1)$ for $A_j \neq 0$, where the A_j's are the components in the word-length pattern of the design.

Now we consider the ranking of the different vectors ${}_i^{\#}C_j$'s. First we ignore ${}_0^{\#}C_0$, ${}_0^{\#}C_1$ and ${}_1^{\#}C_0$ since we always have ${}_0^{\#}C_0 = (1, 0, \ldots, 0)$, ${}_0^{\#}C_1 = (1, 0, \ldots, 0)$ and ${}_1^{\#}C_0 = (n, 0, \ldots, 0)$ for the 2^{n-m} designs in consideration. Let us consider the remaining ones. According to the effect hierarchy principle, we should rank ${}_1^{\#}C_1$ first and then consider the vectors related to two-factor interactions. If two-factor interactions are not negligible, then we should rank the vectors ${}_2^{\#}C_0$, ${}_1^{\#}C_2$, ${}_2^{\#}C_1$ and ${}_2^{\#}C_2$ in order as $({}_2^{\#}C_0,\ {}_1^{\#}C_2,\ {}_2^{\#}C_1,\ {}_2^{\#}C_2)$. The reason for placing ${}_2^{\#}C_0$ at the first place here is that the number involves if the total mean effect, the most important effect, can be estimated under the assumption that 2-factor effects can not be neglected. ${}_1^{\#}C_2$ being put before ${}_2^{\#}C_1$ is due to the fact that main effects are more important than 2-factor interactions. ${}_2^{\#}C_2$ should be placed last. If the three-order effects are not negligible, following the similar arguments above we should rank the vectors ${}^{\#}_{3}C_0,\ {}^{\#}_{1}C_3,\ {}^{\#}_{3}C_1, {}^{\#}_{2}C_3,\ {}^{\#}_{3}C_2$ and ${}^{\#}_{3}C_3$ in order as $({}^{\#}_{3}C_0,\ {}^{\#}_{1}C_3,\ {}^{\#}_{3}C_1, {}^{\#}_{2}C_3,\ {}^{\#}_{3}C_2,\ {}^{\#}_{3}C_3)$, and so on. The general rule can be described as follows: (i) if $\max(i,j) < \max(s,t)$ then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$, (ii) if $i+j < s+t$ then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$, and (iii) if $i+j = s+t$ and $i < s$ then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$. Therefore, according to the principle that lower-order effects are more important than higher-order effects, we shall rank the numbers in set (2) as follows:

$$\begin{aligned} {}^{\#}C = (&{}^{\#}_{1}C_1,\ {}^{\#}_{2}C_0,\ {}^{\#}_{1}C_2,\ {}^{\#}_{2}C_1,\ {}^{\#}_{2}C_2,\ {}^{\#}_{3}C_0,\ {}^{\#}_{1}C_3,\ {}^{\#}_{3}C_1,\ {}^{\#}_{2}C_3,\ {}^{\#}_{3}C_2,\ {}^{\#}_{3}C_3, \\ &{}^{\#}_{4}C_0,\ {}^{\#}_{1}C_4,\ {}^{\#}_{4}C_1, {}^{\#}_{2}C_4,\ {}^{\#}_{4}C_2,\ {}^{\#}_{3}C_4,\ {}^{\#}_{4}C_3,\ {}^{\#}_{4}C_4, \ldots). \end{aligned} \tag{4}$$

We call the ordering (4) an aliased effect-number pattern (AENP). Such a pattern, as well as set (2), contains the basic information of non-aliased effects as well as effects aliased at varying degrees in a design. In the Appendix we give the complete AENPs of three designs in Table 14.

One of the main purposes of experimental design is to estimate as many as possible the effects of factors, especially the lower-order effects, e.g., the main effects and 2fi's. Hence, a "good" design should minimize the confounding between the lower-order effects, i.e., it should maximize the entries of ${}^{\#}C$ sequentially.

Based on ${}^{\#}C$, we define a general minimum lower-order confounding (GMLOC) criterion as follows. The GMLOC criterion selects designs having

the least GMLOC as the optimal designs.

Definition 1. *Let ${}^{\#}C_l$ be the l-th component of ${}^{\#}C$, and ${}^{\#}C(d_1)$ and ${}^{\#}C(d_2)$ the aliased effect-number patterns of two designs d_1 and d_2. Suppose ${}^{\#}C_l$ is the first component such that ${}^{\#}C_l(d_1)$ and ${}^{\#}C_l(d_2)$ are different from each other. If ${}^{\#}C_l(d_1) > {}^{\#}C_l(d_2)$, then d_1 is said to have less general lower-order confounding than d_2. A design d is said to have general minimum lower-order confounding if no other design has less general lower-order confounding than d.*

Definition 1 shows us that a GMLOC design is simply one which sequentially maximizes the components ${}^{\#}_{i}C^{(k)}_{j}$'s of ${}^{\#}C$ in (4).

Example 2. *Let us consider the following two 2^{8-3} designs determined by:*

$$\begin{aligned} d_3 &: I = 1236 = 1247 = 1358, \\ d_4 &: I = 1236 = 1247 = 1348. \end{aligned}$$

Since they are of resolution IV, there does not exist any alias between the main effects and 2fi's in both d_3 and d_4, and obviously we have ${}_1^{\#}C_1(d_3) = {}_1^{\#}C_1(d_4) = (8, 0, \ldots, 0)$, ${}_2^{\#}C_0(d_3) = {}_2^{\#}C_0(d_4) = (8)$, ${}_1^{\#}C_2(d_3) = {}_1^{\#}C_2(d_4) = (8, 0, \ldots, 0)$ and ${}_2^{\#}C_1(d_3) = {}_2^{\#}C_1(d_4) = (28, 0, \ldots, 0)$. Also, via an easy calculation (Refer to our simple algorithm given in Section 7), we can obtain: ${}_2^{\#}C_2(d_3) = (4, 18, 6, 0, \ldots, 0)$ and ${}_2^{\#}C_2(d_4) = (7, 0, 21, 0, \ldots, 0)$. Therefore, the confounding between the 2fi's of d_3 is more severe than that of d_4, as ${}_2^{\#}C_2^{(0)}(d_3) = 4 < {}_2^{\#}C_2^{(0)}(d_4) = 7$. Hence the design d_4 has less general lower-order confounding than design d_3.

Remark 1. ${}_i^{\#}C_j$ and ${}_i^{\#}C_j$ do not have symmetric property with respect to i and j. This can be observed for design d_3 in Example 2, where we note that ${}_1^{\#}C_2^{(0)}(d_3) = 8 \neq 28 = {}_2^{\#}C_1^{(0)}(d_3)$ and ${}_1^{\#}C_2(d_3) \neq {}_2^{\#}C_1(d_3)$.

We have directly the following theorem from Definition 1:

Theorem 1. *A GMLOC 2^{n-m} design must be one with maximum resolution in all 2^{n-m} designs.*

Proof. For any given n and m, suppose that the maximum resolution of the 2^{n-m} designs is R. Then there exits at least one 2^{n-m} design, say d, which has resolution R and for the design d, by the definition of resolution and the meaning of ${}_i^{\#}C_j^{(0)}$ we have ${}_i^{\#}C_j(d) = (\binom{n}{i}, 0, \ldots, 0)$ for any i and j satisfying $i + j < R$. On the other hand, the fact that ${}_i^{\#}C_j(d) = (\binom{n}{i}, 0, \ldots, 0)$ for any i and j with $i + j < R$ implies that the design d has sequentially maximized all the components of ${}_i^{\#}C_j$'s with $i + j < R$ in (4), for all 2^{n-m} designs. Therefore, according to Definition 1, a GMLOC 2^{n-m} design at least satisfies that ${}_i^{\#}C_j(d) = (\binom{n}{i}, 0, \ldots, 0)$ for any i and j with $i + j < R$. It follows that any GMLOC 2^{n-m} design must have resolution no less than R. But we have supposed that the maximum resolution of 2^{n-m} designs is R, thus the resolution of a GMLOC 2^{n-m} design must be R. The proof of the theorem is completed.

3 Relations with Minimum Aberration Criteria

To study the relations of the GMLOC criterion with the MA criterion, we need to study the relations of the world-length pattern, as the core of MA, with the AENP, as the core of GMLOC. First, We have the following theorem.

Theorem 2. *For any 2^{n-m} design, its word-length pattern W, given in (1), is a function of $\{{}_i^{\#}C_j^{(k)}, i, j = 0, 1, \ldots, n, k = 1, \ldots, K_j\}$ in the following three forms:*

1. $A_i = {}_i^{\#}C_0^{(1)}, i = 1, \ldots, n$;

2. A_j is a function of ${}_0^{\#}C_j$, $j = 1, \ldots, n$;

3. For any i, A_i is a function of ${}_sC_t, s, t = 1, \ldots, n$ in (6), where ${}_sC_t$ is a function of $\{{}_s^{\#}C_t^{(k)}, k = 1, \ldots, K_j\}$ as in (7), and sequentially minimizing A_i's of W is equivalent to sequentially minimizing ${}_sC_t$'s of C in (6).

Proof. By definition of the AENP, results 1 and 2 in the theorem are trivially implied. Let us focus on result 3 only.

Considering the 2^{n-m} designs with resolution at least III, Zhang and Park (2000) defined ${}_iC_j$ as the number of alias relations of i- and j-order effects in a design and obtained a general formula for calculating ${}_iC_j$ with $i \leq j$ as:

$$ {}_iC_j = \sum_{l=0}^{i} \binom{n-(j-i+2l)}{i-l} \binom{j-i+2l}{l} A_{j-i+2l}, \quad i, j = 1, 2, \ldots, n, \tag{5} $$

where $\binom{x}{0} = 1$, $\binom{x}{y} = 0$ for $x < y$ or $x < 0$, and $A_i = 0$ for $i \leq 2$ or $i > n$.

Furthermore, they proposed to use the following sequence

$$ C = ({}_1C_1,\ {}_1C_2,\ {}_2C_2,\ {}_1C_3,\ {}_2C_3,\ {}_3C_3,\ {}_1C_4,\ {}_2C_4,\ {}_3C_4,\ {}_4C_4,\ \ldots) \tag{6} $$

as a characterization to choose optimal designs. Based on equation (5), they showed that sequences (1) and (6) can be determined from each other. Also they proved that sequentially minimizing sequence (6) is equivalent to sequentially minimizing sequence (1).

By the definition of ${}_iC_j$ and comparing with the definition of alias sets for a regular design, it is easy to see that

$$ {}_iC_j = \begin{cases} \sum_{k=1}^{K_i} k \cdot {}_i^{\#}C_i^{(k)}/2, & \text{if } i = j, \\ \sum_{k=1}^{K_j} k \cdot {}_i^{\#}C_j^{(k)}, & \text{if } i \neq j. \end{cases} \tag{7} $$

Thus result 3 is proved.

From Theorem 2, immediately we have the following corollary:

Corollary 1. *The designs with different word-length patterns must have different AENPs.*

But the inverse of the corollary does not hold and the AENP is not a function of word-length pattern W. It follows that designs with different AENPs may have the same word-length pattern. This is illustrated by the following example.

Example 3. *Consider the two* 2^{12-7} *designs:*

$$d_5 : I = 126 = 137 = 238 = 12349 = 1235t_0 = 45t_1 = 12345t_2,$$
$$d_6 : I = 126 = 137 = 248 = 349 = 125t_0 = 135t_1 = 145t_2,$$

where t_0, t_1, t_2 *denote the factors 10, 11, 12. The word-length patterns of* d_5 *and* d_6 *are both* $W = (0, 0, 8, 15, 24, 32, 24, 15, 8, 0, 0, 1)$*. But the sequence of the aliased effect-number pattern of* d_5 *and* d_6 *are different and the first different components of them are* ${}_2^{\#}C_2^1(d_5) = 60$ *and* ${}_2^{\#}C_2^1(d_6) = 54$*. Incidentally we can infer that* d_5 *has less general lower-order confounding than* d_6 *by Definition 1.*

Consequently, the AENP is a more refined pattern than the word-length pattern, when they are used to judge designs.

An interesting observation is that the two sequences (4) and (6) have the following relation. After dropping #, sequence (4) becomes

$$C = ({}_1C_1,\ {}_2C_0,\ {}_1C_2,\ {}_2C_1,\ {}_2C_2,\ {}_3C_0,\ {}_1C_3,\ {}_3C_1,\ {}_2C_3,\ {}_3C_2,\ {}_3C_3,\ {}_4C_0,\ {}_1C_4,\ {}_4C_1, \ldots). \tag{8}$$

Note that ${}_iC_j = {}_jC_i$ and ${}_iC_0 = A_i$ is a function of $({}_1C_1,\ {}_1C_2,\ {}_2C_2, \ldots,\ {}_{\lfloor i/2\rfloor}C_{(i-\lfloor i/2\rfloor)})$ by (5). Sequence (8) is turned into (6) after dropping ${}_iC_j$ for $i > j$ and the ${}_iC_0$'s.

From Theorem 2, we can see that the MA criterion only uses the information from $\{{}_i^{\#}C_j^{(k)}, i, j = 0, 1, \ldots, n, k = 1, \ldots, K_j\}$, but not $\{{}_i^{\#}C_j^{(0)}, i, j = 0, 1, \ldots, n, \}$. We note that although ${}_i^{\#}C_j^{(0)}$ can determine the sum $\sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)}$, it cannot determine the vector $({}_i^{\#}C_j^{(1)}, \ldots, {}^{\#}C_j^{(K_j)})$ and ${}_iC_j = \sum_{k=1}^{K_j} k \cdot {}_i^{\#}C_j^{(k)}$. Therefore, it is possible for two designs d and d' with ${}_i^{\#}C_j^{(0)}(d) > {}_i^{\#}C_j^{(0)}(d')$ to have $\sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)}(d) < \sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)}(d')$, but at the same time, ${}_iC_j = \sum_{k=1}^{K_j} k \cdot {}_i^{\#}C_j^{(k)} > {}_iC_j = \sum_{k=1}^{K_j} k \cdot {}_i^{\#}C_j^{(k)}$.

Let us now recall the two designs d_1 and d_2 in Example 1. We have ${}_1^{\#}C_2(d_1) = {}_1^{\#}C_2(d_2) = (9, 0, \ldots)$, ${}_2^{\#}C_1(d_1) = {}_2^{\#}C_1(d_2) = (36, 0, \ldots)$ and ${}_2^{\#}C_2(d_1) = (8, 24, 0, 4)$ and ${}_2^{\#}C_2(d_2) = (15, 0, 21)$. Although ${}_2^{\#}C_2^{(0)}(d_1) = 8 < {}_2^{\#}C_2^{(0)}(d_2) = 15$, we still have

${}_2C_2(d_1) = 1 \times 24 + 3 \times 4 = 36 < {}_2C_2(d_2) = 2 \times 21 = 41$. Thus, by sequentially minimizing (6) the MA criterion infers that d_1 is better than d_2 and d_1 is a MA design. However, under the effect hierarchical principle, d_2 is better than d_1, since d_2 has 15 clear 2fis while d_1 has only 8 (both have 9 clear main effects). Perhaps using only partial information in the AENP is a reason why sometimes the MA criterion fails to detect optimal designs under the effect hierarchy principle.

From equation (7), one can find that ${}_iC_j$ is a linear function of the components of ${}_i^{\#}C_j$ with k as the weight of ${}_i^{\#}C_j^{(k)}$. In addition, a design which sequentially maximizes the components of ${}_i^{\#}C_j$ tends to minimize ${}_iC_j$. Hence, the optimal designs under MA and GMLOC criteria are consistent in many cases, especially for designs with small runs (for more examples, see the tables in Appendix). However, there are quite a few optimal designs under the two criteria that differ from each other, since they are established on different bases. One more example is shown below.

Example 4. *Consider the three 2^{13-7} designs with 64 runs (designs 13-7.7, 13-7.2, and 13-7.1 in Table 13):*

$$\begin{aligned}
d_7 &: I = 12347 = 34568 = 2459 = 1456t_0 = 256t_1 = 136t_2 = 235t_3,\\
d_8 &: I = 12347 = 3458 = 2459 = 356t_0 = 256t_1 = 456t_2 = 346t_3,\\
d_9 &: I = 12347 = 34568 = 2459 = 1456t_0 = 246t_1 = 12356t_2 = 256t_3,
\end{aligned}$$

The word-length patterns of d_7, d_8 and d_9 are respectively

$$\begin{aligned}
d_7 &: (0, 14, 28, 24, 24, 17, 12, 8, 0, 0, 0),\\
d_8 &: (0, 26, 12, 24, 28, 13, 20, 0, 4, 0, 0),\\
d_9 &: (0, 14, 33, 16, 16, 33, 14, 0, 0, 0, 1)
\end{aligned}$$

and the most important part of their AENP are shown in the following table:

	d_7		d_8		d_9	
${}_i^{\#}C_j$	$j=1$	$j=2$	$j=1$	$j=2$	$j=1$	$j=2$
$i=1$	13	13	13	13	13	13
$i=2$	78	20, 36, 18, 4	78	23, 0, 24, 16, 15	78	36, 0, 42

According to the MA criterion, d_7 is the best and it is an MA design, the second best one is d_9 and the worst is d_8. However, from the above table of their AENPs, it is easy to see that they all have 13 clear main effects, d_7 only has 20 clear 2fi's, d_8 has 23 clear 2fi's, and d_9 has 36 clear 2fi's. Therefore, according the GMLOC and clear effects criteria their order of optimality is d_9, d_8 and d_7. The best design

d_7 in MA criterion becomes the worst one under the other two criteria. The MA criterion fails to detect the best design in this case as well.

Zhu and Zeng (2005) proposed the minimum M-aberration (MMA) criterion for selecting "good" designs and proved that the MMA criterion is more detailed than MA. To show the relation between the GMLOC and MMA criteria, the definition of MMA is reviewed below.

Let G be the defining contrast subgroup of d. Zhu and Zeng (2005) defined an order $\lhd$ between the effects of d. Suppose that $i_1 \cdots i_k$ and $j_1 \cdots j_l$ are two different effects, $i_1 \cdots i_k$ is said to be smaller than $j_1 \cdots j_l$ if $k < l$ or if $k = l$ and $i_1 \cdots i_k$ should be listed ahead of $j_1 \cdots j_l$ lexicographically. If $i_1 \cdots i_k$ is "smaller" than $j_1 \cdots j_l$, it will be written as $i_1 \cdots i_k \lhd j_1 \cdots j_l$. Applying $\lhd$ to the effects of a given alias set, the "smallest" effect in the alias set is referred to as the alias set leader. If an alias set has $i_1 \cdots i_k$ as its alias set leader, it is said to be a k-order alias set and denoted by $i_1 \cdots i_k G$. If two effects e_1 and e_2 of order i and j are aliased with each other, and they are included in a k-order alias set, then the aliasing between e_1 and e_2 is of type $(i,j)_k$. Let $M_{(i,j)_k}$ be the number of pairs of aliased effects which are of the type $(i,j)_k$. Define

$$M = (M_{(1,2)_1}, M_{(2,2)_2}, M_{(2,2)_1}, M_{(1,3)_1}, M_{(2,3)_2}, M_{(2,3)_1}, \\ M_{(1,4)_1}, M_{(3,3)_3}, M_{(3,3)_2}, M_{(3,3)_1}, \ldots)$$

and call M the aliasing type pattern of a design. The MMA criterion chooses the designs which sequentially minimize the components of M as the optimal ones.

Clearly, M and ${}^{\#}C$ have the following relation:

$$\sum_{k \geq 0} M_{(i,j)_k} = \begin{cases} \sum_{k \geq 1} k \cdot {}_i^{\#}C_j^{(k)}/2, & \text{if } i = j, \\ \sum_{k \geq 1} k \cdot {}_i^{\#}C_j^{(k)}, & \text{if } i \neq j. \end{cases}$$

Similar to the word-length pattern (1), M still depends directly on the number of pairs of i- and j-order effects aliased with each other. Hence, the MMA criterion, though more elaborate than the MA, still can not lead to the optimal in certain cases, as illustrated below.

Example 1 (continued). *Note that $M(d_1) = (0, 18, 0, \ldots)$ and $M(d_2) = (0, 21, 0, \ldots)$.*

Thus d_1 is better than d_2 under the MMA criterion, which still conflicts with result from the GMLOC and clear effects criteria.

4 Relations with Clear Effects Criterion

To make clear the relations of the new criterion with the clear effects criterion, we first note some results related to the clear effects criterion.

For regular 2^{n-m} designs, we have the following lemmas.

Lemma 1. *When* $2^{n-m-1} < n < 2^{n-m}-1$*, there exist only the designs with resolution* $R \leq \text{III}$*, and for any* 2^{n-m} *design with resolution* III*, it has neither any clear main effect nor any clear two-factor interaction.*

Lemma 2. *When* $2^{n-m-2} + 1 < n \leq 2^{n-m-1}$*, there exist resolution* IV *designs, but any such resolution* IV 2^{n-m} *design does not contain any clear two-factor interaction. If a* $2^{n-(n-k)}$ *design contains clear two-factor interaction for* $2^{n-m-2} < n \leq 2^{n-m-1}$*, then its resolution must be less than* IV*.*

Lemma 3. *When* $M(n-m) < n \leq 2^{n-m-2} + 1$*, there exist* 2^{n-m} *designs with resolution* IV *which contain clear two-factor interactions, where* $M(n-m)$ *is the maximum number of factors that can be accommodated in a* 2^{n-m} *design with the maximum resolution at least* V*.*

Lemma 4. *Consider the* 2^{n-m} *designs which have resolution at least* III*. Then* ${}_1^{\#}C_2^{(0)}$ *is simply the number of clear main effects in a design, and* ${}_2^{\#}C_2^{(0)} - {}_1^{\#}C_2^{(1)}$ *is simply the number of clear 2fi's in a design.*

Lemmas 1, 2 and 3 are the results of Chen and Hedayat (1998), which also gave a complete classification of the existence of clear two-factor interactions (2fi's) in regular 2^{n-m} designs with resolution III or IV. The proofs of these lemmas can be found from their paper. We only give a proof of Lemma 4 below.

By the definition of clear main effect, for a 2^{n-m} design with resolution at least III, we always have ${}_1^{\#}C_1^{(0)} = n$. This means that no main effect is aliased by any other main effect, thus the number ${}_1^{\#}C_2^{(0)}$ is simply the number of clear main effects in the design. By the definition of clear two-factor interaction, if a two-factor interaction is clear, it must be in an alias set which only contains itself as the unique 2fi and does not contain any main effects. The number of such alias sets is simply the number of clear 2fi's in the design. By the definitions of ${}_2^{\#}C_2^{(0)}$ and ${}_1^{\#}C_2^{(1)}$, the number

${}^{\#}_{2}C_2^{(0)} - {}^{\#}_{1}C_2^{(1)}$ is simply the number of the alias sets satisfying the two conditions above. Lemma 4 is therefore proved.

Since a main effect is more important than a 2fi under the hierarchical assumption, designs with most clear main effects are always preferred.

In addition, for a given number of clear main effects, designs with the most clear 2fi's should be selected as the optimal ones under the clear effects criterion. The following Theorem 3 reveals the relation between the new criterion and clear effects criterion and shows that the clear effects criterion is a special case of the GMLOC criterion.

Theorem 3. *The clear effects criterion selects the 2^{n-m} designs which sequentially maximize ${}^{\#}_{1}C_2^{(0)}$ and ${}^{\#}_{2}C_2^{(0)}$ as the optimal ones when $n \leq 2^{n-m-1}$. For given n and m, if an optimal design under the clear effects criterion exists, then the GMLOC criterion must be at its best for the optimal clear effects criterion designs, where the meaning of "best" is under the GMLOC criterion, as given in Definition 1.*

Proof. Without lost of generality, we only consider the designs with resolution at least III. For the first part, by Lemma 1, when $2^{n-m-1} < n \leq 2^{n-m} - 1$, there are no designs with resolution higher than III and any 2^{n-m} design in this case does not contain any clear main effects and 2fi's, hence the clear effects criterion can not be used to select the optimal designs. Consider the case where $n \leq 2^{n-m-1}$. By Lemma 4, the clear effects criterion maximizes ${}^{\#}_{1}C_2^{(0)}$ and ${}^{\#}_{2}C_2^{(0)} - {}^{\#}_{1}C_2^{(1)}$ sequentially. And in this case, by Lemmas 2 and 3, there exist resolution IV designs, and the clear effects criterion must choose the designs with resolution IV to obtain all clear main effects. Note that the number ${}^{\#}_{1}C_2^{(1)}$ for designs with resolution IV is equal to 0, thus the proof of the first part is finished.

For the second part, consider the four cases for n. For the case $2^{n-m-1} < n < 2^{n-m} - 1$, by Lemma 1, all the considered designs must be of resolution III and have no any clear main effects and clear 2fi's, thus the GMLOC design also does not contain any clear main effects and clear 2fi's, finishing the proof in this case. For the case $2^{n-m-2} + 1 < n \leq 2^{n-m-1}$, since there exist designs with resolution IV (no higher than IV) by Lemma 2, by Theorem 1 the GMLOC design, say d', must be of resolution IV. Note that for any resolution IV design d, ${}^{\#}_{1}C_1(d) = (n, 0, \ldots, 0)$, ${}^{\#}_{2}C_0(d) = (\binom{n}{2}, 0)$, ${}^{\#}_{1}C_2(d) = (n, 0, \ldots, 0)$, ${}^{\#}_{2}C_1(d) = (\binom{n}{2}, 0, \ldots, 0)$ and ${}^{\#}_{2}C_2^{0}(d)$ is simply the number of clear 2fi's of d. Again by Lemma 2, for this case we have ${}^{\#}_{2}C_2^{(0)}(d') = 0$ and hence the GMLOC design d' must be one and the best one among the optimal designs under the clear criterion, which contain n clear main effects and 0 clear 2fi. For the case $M(n-m) < n \leq 2^{n-m-2}+1$, by Lemma 3, there exist resolution-IV designs, then the GMLOC design, say d'', also must be a resolution IV design and sequentially maximizes the components of sequence (4). Thus ${}^{\#}_{1}C_2^{(1)}(d'') = 0$ and d'' also maximizes the number ${}^{\#}_{2}C_2^{(0)}$. Therefore, d'' has n clear main effects and maximum number of clear 2fi's, and it is the best one among all optimal designs under the clear effects criterion. For the case $n \leq M(n-m)$, by noting that there exist designs with resolutions at least V, the GMLOC design is obviously optimal under the clear effects criterion; this completes the proof.

As a popular choice for selecting good designs, the clear effects criterion can not be used in many situations. For example, when $n > 2^{n-m-1}$, Lemma 1 tells us that the resolution III designs existed do not contain any clear main effects or 2fi's. However, the GMLOC criterion can be used for all the range of parameters. When $2^{n-m-2}+1 < n \leq 2^{n-m-1}$, all of the resolution-IV 2^{n-m} designs existed make no difference under the clear effects criterion. The GMLOC criterion can discriminate them further, as illustrated below.

Example 5. *Consider the designs 10-5.1,10-5.2, 10-5.3 and 10-5.4 in Table 6 in the Appendix. The clear effects criterion does not distinguish between the four designs, while the GMLOC criterion can. We can see that design 10-5.1 is the best, since it has 40 2fi's being aliased by only one 2fi, among them about 20 2fi's may be de-aliased by some follow-up designs if the experimenter wishes to estimate them. Design 10-5.4 is the worst one, since all 45 2fi's are aliased by 3 2fi's, thus any one of them would be difficult to de-alias with follow-up designs due to the severity of the aliases. From the perspective of an experimenter, the best choice would obviously be design 10-5.1.*

Now we need to analyze designs 8-4.3 and 8-4.4 (denoted as d and d') in Table 5 in the Appendix, which is an exceptional example. Design d has one clear main effect and one clear 2fi and design d' has one clear main effect and 7 clear 2fi's, but why the former is judged to be better than the latter? Let us look at their AENPs. We have ${}^{\#}_{2}C_2(d) = {}^{\#}_{2}C_2(d') = (7,0,21)$, but ${}^{\#}_{1}C_2(d) = (1,6,0,1)$ and ${}^{\#}_{1}C_2(d') = (1,0,0,7)$.

It follows that design d has 6 main effects confounded by only one 2fi; these main effects may be de-aliased easily by some follow-up experiments so that they can be estimated. On the other hand, design d' has 7 main effects which are confounded by 3 2fi's and hence it can hardly be de-aliased by follow-up experiments. Therefore, although design d' has 6 more clear 2fi's than d, it needs to sacrifice 6 main effects which may be estimated in d. Hence according to the effect hierarchy principle, design d should be preferred.

Based on the analysis above, we can conclude that the GMLOC criterion is more refined and a more reasonable choice than the clear effects criterion when judging designs.

Now let us discuss the ties between the MA and clear effects criteria, which are revealed by our new development. From our analysis in Sections 3 and 4, we have found that the MA criterion only uses the information from $\{{}^{\#}_{i}C^{(k)}_{j}, i, j = 0, 1, \ldots, n, k = 1, \ldots, K_j\}$ in the set (2) and the clear effects criterion only uses the information from $\{{}^{\#}_{i}C^{(k)}_{j}, i, j = 0, 1, \ldots, n, k = 0\}$. In other words, they separately use different parts of the information contained in the same set. As mentioned above, the two parts have the relation ${}^{\#}_{i}C^{(0)}_{j} + \sum_{k=1}^{K_j} {}^{\#}_{i}C^{(k)}_{j} = \binom{n}{j}$ for any i and j. Thus the larger the number ${}^{\#}_{i}C^{(0)}_{j}$ we choose, the less the number $\sum_{k=1}^{K_j} {}^{\#}_{i}C^{(k)}_{j}$ we obtain. In most cases, when ${}^{\#}_{i}C^{(0)}_{j}$ is large, the weighed sum $iC_j = \sum_{k=1}^{K_j} k \cdot {}^{\#}_{i}C^{(k)}_{j}$ tends to be small. Thus sequentially maximizing the sequence $({}^{\#}_{1}C^{(0)}_{2}, {}^{\#}_{2}C^{(0)}_{2}, \ldots)$ tends to sequentially minimize the sequence (6). Perhaps this is the reason why in most cases, the two criteria would give the same optimal designs. However, although the relationship between the number ${}^{\#}_{i}C^{(0)}_{j}$ and the sum $\sum_{k=1}^{K_j} {}^{\#}_{i}C^{(k)}_{j}$ is rather clear, the same cannot be said between ${}^{\#}_{i}C^{(0)}_{j}$ and the weighted sum $iC_j = \sum_{k=1}^{K_j} k \cdot {}^{\#}_{i}C^{(k)}_{j}$. Therefore, conflicting results from the two criteria may appear as shown in examples given so far.

5 Relations with Maximum Estimation Capacity Criterion

Cheng and Mukerjee (1998) and Cheng, Steinberg and Sun (1999) discussed the estimation capacity of a design d. Let $E_r(d)$ denote the number of models containing all the main effects and r 2fi's, $1 \le r \le n(n-1)/2$, which can be estimated by the design d. The design d is said to dominate a design d' if $E_r(d) \ge E_r(d')$ for all r and with strict inequality for at least one r. Furthermore, a design which maximizes $E_r(d)$ for all r is said to have maximum estimation capacity (MEC). The MEC criterion selects the designs with MEC as the optimal ones. We only consider designs with resolution at least III in this section since there are at least two main effects aliased with each other in any resolution-II design and these two main effects can not be estimated at the same time.

Clearly, there are ${}_2^{\#}C_2^{(k)}/(k+1)$ alias sets containing $k+1$ 2fi's and ${}_1^{\#}C_2^{(k+1)}/(k+1)$ alias sets containing $k+1$ 2fi's and one main effects. An alias set contains at most $l = \min\{\lfloor n/2 \rfloor, 2^m\}$ 2fi's, where $\lfloor x \rfloor$ is the integer part of x. Then all of the alias sets containing 2fi's but none of the main effects can be partitioned into l classes and the i-th class includes the alias sets containing $i+1$ 2fi's for $i = 0, 1, \ldots, l-1$. Let $\mathcal{C}_i$ be the i-th class. Then $|\mathcal{C}_i| = ({}_2^{\#}C_2^{(i)} - {}_1^{\#}C_2^{(i+1)})/(i+1)$, where $|\cdot|$ denote the cardinality of a set. Note that there may exist $|\mathcal{C}_i| = 0$ for some i. By the definition of $E_r(d)$, it is easy to obtain the following theorem:

Theorem 4. *$E_r(d)$ can be expressed as a function of ${}_2^{\#}C_2$ and ${}_1^{\#}C_2$ as follows:*

$$E_r(d) = \begin{cases} \sum \cdots \sum_{r_0+\cdots+r_{l-1}=r} \prod_{i=0}^{l-1} \binom{|\mathcal{C}_i|}{r_i} (i+1)^{r_i}, & \text{if } r \le f, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 \le r_i \le |\mathcal{C}_i|$, $f = 2^{n-m} - 1 - n$.

This theorem reveals the relation between the MEC and GMLOC criteria. Namely, the MEC criterion can be seen as one that optimizes a function of the AENP. The following provides further analysis to illuminate this point.

It is clear that $\lhd$ defined in Section 3 can be applied to the alias set leaders, so the alias sets can be rank-ordered from the "smallest" to the "largest" with the "smallest" alias set G receiving rank 0 and the "largest" receiving rank $2^{n-m}-1$. For a 2^{n-m} design of resolution III or more, let m_i denote the number of 2fi's in the i-th alias set and $m = (m_{n+1}, \ldots, m_{n+f})$, where $f = 2^{n-m} - 1 - n$. Recall that a vector $u = (u_1, \ldots, u_s)$ is said to be upper weakly majorized by $v = (v_1, \ldots, v_s)$ if $\sum_{i=1}^t u_{[i]} \ge \sum_{i=1}^t v_{[i]}$ for $1 \le t \le s$, where $u_{[1]} \le u_{[2]} \le \ldots \le u_{[s]}$ and $v_{[1]} \le v_{[2]} \le \ldots \le v_{[s]}$ are the ordered components of u and v, respectively. A sufficient condition for d_1 to dominate d_2 is given in the following Lemma 5 by Cheng, Steinberg and Sun (1999).

Lemma 5. *If $m(d_1)$ is upper weakly majorized by $m(d_2)$ and $m(d_1)$ cannot be obtained from $m(d_2)$ by permuting its components, then d_1 dominates d_2 with respect to the criterion of estimation capacity.*

Lemma 5 shows that a design d will behave well under the MEC criterion if $\sum_{i=n+1}^{n+f} m_i(d)$ is large and $m_{n+1}(d), \ldots, m_{n+f}(d)$ are close to one another, i.e., a design d tends to behave well if $\sum_{i=n+1}^{n+f} m_i(d)$ is large and $\sum_{i=n+1}^{n+f} m_i^2(d)$ is small under such a criterion. Note that $\sum_{i=n+1}^{n+f} m_i(d) = \sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)$ and $\sum_{i=n+1}^{n+f} m_i^2(d) = \sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)^2$. Then a design d which maximizes $\sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)$ and minimizes $\sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)^2$ tends to behave well under the MEC criterion.

6 Maximum Estimation Ability

The optimal designs under the MEC criterion can estimate as many as possible models involving all the main effects and some 2fi's with the assumption that all other 2fi's not involved in the model are negligible. Such an assumption is too strong to justify if some 2fi's not included in the model are active. In such cases people would prefer to choose designs with smaller degree aliasing between the 2fi's.

To avoid the strong assumption, we introduce the notion of estimation ability and propose an maximum estimation ability criterion.

Now let us consider the classes $\mathcal{C}_i$ for $i = 0, 1, \ldots, l-1$. Note that there are $i+1$ 2fi's in each alias set in the class $\mathcal{C}_i$. Hence, a smaller i implies aliasing between the 2fi's in the alias sets of $\mathcal{C}_i$ to a lesser degree. Any model involving all the main effects and $r \leq |\mathcal{C}_0|$ 2fi's can be estimated without bias under the weaker assumption of absence of interactions involving at least three factors. And any model involving all the main effects and $|\mathcal{C}_0| < r \leq |\mathcal{C}_0| + |\mathcal{C}_1|$ 2fi's can be estimated under the assumption of absence of $|\mathcal{C}_1|$ 2fi's in the alias sets of $\mathcal{C}_1$ and interactions involving at least three factors. Similarly, any model involving all the main effects and $\sum_{i=0}^{j} |\mathcal{C}_i| < r \leq \sum_{i=0}^{j+1} |\mathcal{C}_i|$ ($j = 0, 1, \ldots, l-1$) 2fi's can be estimated under the assumption of absence of $i|\mathcal{C}_i|$ 2fi's in the alias sets of $\mathcal{C}_i$ for $i = 0, \ldots, j$ and interactions involving at least three factors. For convenience, we call a model involving only the 2fi's in the alias sets of $\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_i$ an i-class model in the following. A good design should sequentially maximize $|\mathcal{C}_i|$ for $i = 0, 1, \ldots, l-1$ since such a design can be used to estimate all the main effects and 2fi's with aliasing between the 2fi's to the least degree. A design which sequentially maximizes $|\mathcal{C}_i|$ for $i = 0, 1, \ldots, l-1$ is said to be a design with maximum estimation ability. The criterion selecting such designs as the optimal ones is called the maximum estimation ability (MEA) criterion.

The optimal designs under the MEA criterion can estimate the model involving all the main effects and some 2fi's with confounding between the 2fi's to the least degree, especially for models containing a few 2fi's. If the experimenter wishes to de-alias the confounding between the 2fi's, he/she needs only to perform a few follow-up experiments.

Note that $|\mathcal{C}_i| = ({}_2^{\#}C_2^{(i)} - {}_1^{\#}C_2^{(i+1)})/(i+1)$. For given ${}_1^{\#}C_2$ and ${}_2^{\#}C_1$, sequentially maximizing the components of ${}_2^{\#}C_2$ is equivalent to sequentially maximizing $|\mathcal{C}_i|$ for $i = 0, 1, \ldots, l-1$. Hence a GMLOC design sequentially maximizes the estimation ability of i-class models for $i = 0, 1, \ldots, l-1$. Under the effect hierarchy principle, the estimatability of the main effects is our first concern, thus a good design must sequentially maximize ${}_1^{\#}C_2$ and ${}_2^{\#}C_1$. Therefore, in any case, a GMLOC design can sequentially maximize the estimation ability of i-class models for $i = 0, 1, \ldots, l-1$.

7 Algorithm for AENP and GMLOC Designs with 16- and 32-run

In this section, we give an algorithm for computing AENP through an example. Although the series (4) appear complicated, the algorithm is in fact quite simple.

Consider a 2^{n-m} regular design d. Let us use a $2^m \times n$ matrix D to express the defining contrast subgroup G of the design d, where the entry (i, j) of D equals 1 if the i-th word in G contains letter j and 0 otherwise. We call D the defining structure matrix (or defining pencil matrix) of design d. In matrix D, the first row is a vector of 0's, which corresponds to the element I in the subgroup G, and every other row indicates a word in G.

Let S denote the set of all effects of n factors in d, where a k-order effect $i_1 \cdots i_k$

of d is expressed as an n-dimensional row vector with the i_1-th, ..., i_k-th entries ones and zeros otherwise. The effects in G or the defining structure matrix D are those which are aliased with I, the total mean effect. Let the column sum (ordinary addition) of D be as its marginal column.

The algorithm of computing ${}^{\#}_{i}C_j(d)$ can be described as follows:

Step 1. Set S_0 to be the empty set. And set ${}^{\#}_{i}C_j^{(k)} = 0$ for all $i, j = 0, 1, \ldots, n, k = 0, 1, \ldots, K_j$.

Step 2. Let $S = S \setminus S_0$. Selecting one vector (i.e. a effect) from S (can be from lower order to higher order). Adding (in module 2) it to the every row of D, we obtain the aliased-effect matrix D' of the selected effect. From the matrix D', we can get the alias set T to which the selected effect belongs (also the element of T is expressed as an n-dimensional vector). And then set $S_0 = S_0 \cup T$ and $i = j = 0$.

Step 3. Let p_i and q_j be the numbers of i- and j-order effects in T respectively (just count the numbers of i's and j's at the marginal column by the D' in the table respectively). We set ${}^{\#}_{i}C_j^{(q_j)} = {}^{\#}_{i}C_j^{(q_j)} + p_i$ if $i \neq j$ or ${}^{\#}_{i}C_j^{(q_j-1)} = {}^{\#}_{i}C_j^{(q_j-1)} + p_i$ if $i = j$. Then repeat this step for all cases: $1 \leq i + j \leq n, i, j = 1, \ldots, n$.

Step 4. Stop if $|S_0| = 2^n$ and go to Step 2 otherwise, where $|\cdot|$ is the cardinality of a set.

Let us consider the design d_3 in Example 2 as an example. The defining contrast subgroup G of d_3 is

$$\{I, 1236, 1247, 1358, 2568, 3467, 145678, 234578\},$$

and its defining structure matrix D is given in Table 1.

Table 1. Defining structure matrix D of d_3

0	0	0	0	0	0	0	0	0
1	1	1	0	0	1	0	0	4
1	1	0	1	0	0	1	0	4
1	0	1	0	1	0	0	1	4
0	1	0	0	1	1	0	1	4
0	0	1	1	0	1	1	0	4
1	0	0	1	1	1	1	1	6
0	1	1	1	1	0	1	1	6

The marginal column in the table is simply the distribution of word-lengths in the defining contrast subgroup G.

For the example, in step 1 the S is the set of all effects of 5 factors. For simplicity, we only consider to calculate ${}_2^{\#}C_j^{(k)}$'s of d_3.

At step 2, say, we select vector $(1,1,0,0,0,0,0,0)$ (2fi 12) from S. Adding it to every row of D in Table 1, we obtain the aliased-effect matrix D' and its marginal column of the 2fi 12 of d_3 which is shown in Table 2.

Table 2. Aliased-effect matrix D' of the 2fi 12 of d_3

1	1	0	0	0	0	0	0	2
0	0	1	0	0	1	0	0	2
0	0	0	1	0	0	1	0	2
0	1	1	0	1	0	0	1	4
1	0	0	0	1	1	0	1	4
1	1	1	1	0	1	1	0	6
0	1	0	1	1	1	1	1	6
1	0	1	1	1	0	1	1	6

The marginal column in the table is simply the distribution of effect orders in the alias set containing the 2fi 12.

From D' we obtain the alias set T of the 2fi 12:

$$T = \{12, 36, 47, 2358, 1568, 123467, 245678, 134578\}.$$

Set $S_0 = S_0 \cup T$ (i.e. add the row vectors of D' into S_0) and $i = j = 0$.

In step 3, in this case we take $i = j = 2$ and have $p_2 = q_2 = 3$ (the number of 2's at the marginal column in Table is 3), and then set ${}_2^{\#}C_2^{(2)} = {}_2^{\#}C_2^{(2)} + 3$. Considering $i = 2, j = 4$, have $p_2 = 3, q_4 = 2$, and set ${}_2^{\#}C_4^{(2)} = {}_2^{\#}C_4^{(2)} + 3$. Considering $i = 2$ and $j = 6$, have $p_2 = 3, q_6 = 3$, and set ${}_2^{\#}C_6^{(3)} = {}_2^{\#}C_6^{(3)} + 3$. No change for other ${}_2^{\#}C_j^{(k)}$'s.

In step 4, in this case only calculate the number of 2fi's in S_0, the number 3 is less than $\binom{5}{2} = 10$, then go to step 2. Set $S = S \setminus S_0$. Select one 2fi belonging to S, say 45, and add $(0,0,0,1,1,0,0,0)$ to the rows of Table 1. we obtain the aliased-effect matrix D'' of the 2fi 45 of d_3 which is shown in Table 3.

Table 3. Aliased-effect matrix D'' of the 2fi 45 of d_3

0	0	0	1	1	0	0	0	2
1	1	1	1	1	1	0	0	6
1	1	0	0	1	0	1	0	4
1	0	1	1	0	0	0	1	4
0	1	0	1	0	1	0	1	4
0	0	1	0	1	1	1	0	4
1	0	0	0	0	1	1	1	4
0	1	1	0	0	0	1	1	4

The marginal column in the table is simply the distribution of effect orders in the alias set containing the 2fi 45.

From Table 3, we can get the alias set containing 2fi 45:

$$T = \{45, 123456, 1257, 1348, 2468, 3567, 1678, 2378\}.$$

Let $S_0 = S_0 \cup T$. Since $p_2 = q_2 = 1$ (the number of 2's at the marginal column in Table 3 is 1), set ${}_{2}^{\#}C_2^{(0)} = {}_{2}^{\#}C_2^{(0)} + 1$. In same way to consider $i = 2, j = 4$ and $i = 2, j = 6$.

Repeat the procedure above, we can obtain ${}_{2}^{\#}C_2(d_3) = (4, 18, 6, 0\ldots, 0)$ and ${}_{2}^{\#}C_j^{(k)}, j = 1, 2, 3, \ldots, k = 0, 1, 2\ldots$ of d_3.

In the Appendix, in Table 14 we list the complete AENPs of d_1, d_2 and d_{10} in Examples 1 and 6. For convenience, we show the AENP in an array (or a matrix) of dimension 10×10, and the (i, j)-th entry of the array is ${}_{i}^{\#}C_j$. To save space, we omit the zeros after the last nonzero component of ${}_{i}^{\#}C_j$ and use 0^k to denote k successive zero entries. For example, the (1,3)-th entry $(1, 0^2, 8)$ of d_1 in Table 14 of the Appendix expands to ${}_{1}^{\#}C_3(d_1) = (1, 0, 0, 8, 0, \ldots, 0)$. Also, we tabulate the 16- and 32-run GMLOC designs and 64-run GMLOC designs with some parameters and some comparisons with MA and clear effects criteria in Table 6-13 of the Appendix. We make an illustration to the tables in the following.

Let a_1, a_2, a_3, a_4 and a_5 denote the five independent columns $(10000)'$, $(01000)'$, $(00100)'$, $(00010)'$ and $(00001)'$, respectively. Then any product of a_1, a_2, a_3, a_4 and a_5 also corresponds to a binary sequence, for example $a_1a_3a_5$ corresponds to $(10101)'$. Table 4 converts these binary sequences into decimal ones. A 2^{n-m} design can be obtained by selecting a subset of n columns of $C = \{1, \ldots, 2^{n-m} - 1\}$, consisting of $n - m$ independent columns and m additional columns.

For simplicity, in the tables we use n-m.i to denote the i-th good design according to the GMOLC criterion among 2^{n-m} designs with n factors and m independent defining words. The additional columns are listed in decimal in the second part of the tables. The third part of the tables is the AENP of the design, here we only list ${}_1^{\#}C_2$, ${}_2^{\#}C_1$, and ${}_2^{\#}C_2$ for short. We also list the WLP and the number of clear main effect and clear 2fi's for comparison in the fourth and fifth parts respectively. In the last part, the optimality order-numbers of the designs under GMLOC, MA and clear effects criterion respectively in all the isomorphism 2^{n-m} designs are listed. We only list the first ten good designs under GMLOC criterion if there are more than ten non-isomorphic designs, otherwise we list all of them in order. For parameter $n = 2^{n-m} - i, i = 1, 2, 3$, the design is unique up to isomorphism and hence is omitted.

8 Simplification of the AENP and Its More Usage

From the complete AENPs of three designs in Table 14 of the Appendix, one may say that the AENP of a design appears to be rather complicated. However, one should note that from an application point of view in design, the most important part of the AENP is only the small part at the top left corner. If we only consider designs in which three and higher order interactions are negligible, then we only need to be concerned with the 2×2 sub-matrix (or a sub-array) $({}_i^{\#}C_j)$ with $i, j = 1, 2$; this can usually discriminate different designs. If we consider designs in which only four and higher order interactions are negligible, then the 3×3 sub-matrix $({}_i^{\#}C_j)$, $i, j = 1, 2, 3$, suffices. However the former is more commonly used in practice. From these low-dimensional sub-matrices we can already obtain all the information concerning the numbers of clear main effects and two-factor interactions and the severity of confounding between the lower-order effects. As a consequence, the complete AENP can be simplified into a set of few numbers. If we only consider designs with resolution no less than III, we can also drop ${}_1^{\#}C_1$ from the matrices and leave only three entries for the 2×2 case. Thus in the tables related to 16-run, 32-run and 64-run designs in the Appendix, we list these three entries for every design.

Example 6. *Let us consider the 2^{9-4} designs d_1, d_2 in Example 1 and the following* *design* d_{10} *(they are 9-4.2, 9-4.1, and 9-4.3 in Table 6 respectively) defined by*

$$d_1 : I = 1236 = 1247 = 1258 = 13459;$$
$$d_2 : I = 1236 = 1247 = 1348 = 23459;$$
$$d_{10} : I = 1236 = 2347 = 1348 = 1249.$$

All the main effects of d_1*,* d_2 *and* d_{10} *are clear and the number of clear 2fi's of* d_1*,* d_2 *and* d_{10} *are 8, 15 and 8, respectively. The word-length patterns of the three designs are (0,0,0,6,8,0,0,1,0), (0,0,0,7,7,0,0,0,1) and (0,0,0,14,0,0,...), respectively.* d_1 *has MA in all* 2^{9-4} *designs, and* d_2 *has the most clear 2fi's in all* 2^{9-4} *designs. Note that*

$$\begin{array}{l}
{}^{\#}_{1}C_1(d_1) = {}^{\#}_{1}C_1(d_2) = {}^{\#}_{1}C_1(d_{10}) = (9, 0, \ldots, 0),\\
{}^{\#}_{2}C_0(d_1) = {}^{\#}_{2}C_0(d_2) = {}^{\#}_{1}C_1(d_{10}) = 36,\\
{}^{\#}_{1}C_2(d_1) = {}^{\#}_{1}C_2(d_2) = {}^{\#}_{1}C_1(d_{10}) = (9, 0, \ldots, 0),\\
{}^{\#}_{2}C_1(d_1) = {}^{\#}_{2}C_1(d_2) = {}^{\#}_{1}C_1(d_{10}) = (36, 0, \ldots, 0),\\
{}^{\#}_{2}C_2(d_1) = (8, 24, 0, 4, 0, \ldots, 0),\\
{}^{\#}_{2}C_2(d_2) = (15, 0, 21, 0, \ldots, 0),\\
{}^{\#}_{2}C_2(d_{10}) = (8, 0, 0, 28, 0, \ldots, 0).
\end{array}$$

Although according to the word-length pattern of MA criterion, d_1 *is better than* d_2 *and* d_{10}*, according to the AENP of GMLOC criterion,* d_2 *is obviously better than* d_1 *and* d_{10}*.* d_1 *and* d_{10} *make no difference under the clear effects criterion, but* d_1 *is better than* d_{10} *according to the GMLOC criterion, and the word-length pattern of* d_{10} *is much worse than that of* d_1*.*

In fact, the GMLOC criterion given by Definition 1 mainly adapts to the case where three or higher order interactions are negligible. If three or higher order interactions are active and need to be estimated, we probably have to make some necessary modifications. But we can still use the AENP or its set form (2).

Furthermore, the AENP has more usages. In what follows, we shall use it to perform more analysis using the above example.

As the core of the GMLOC, the AENP, in addition to being used to rank-order 2^{n-m} designs, also contains more useful information which one can find from it directly. For example, let us study the AENPs of designs d_1 and d_{10} in Example 6 further. Note that ${}^{\#}_{2}C_3^{(0)}(d_{10}) = 36$, which is simply the number of all the 2fi's of design d_{10}, i.e., there does not exist 2fi's aliased with any three-factor interactions (3fi's). Hence the eight clear 2fi's of d_{10} are all strongly clear. By studying ${}^{\#}_{2}C_2(d_1) = (8, 24, 0, 4)$, ${}^{\#}_{2}C_3(d_1) = (4, 0^2, 8, 24)$, ${}^{\#}_{3}C_1(d_1) = (60, 24)$, ${}^{\#}_{3}C_2(d_1) = (28, 32, 24)$ and

${}^{\#}_{3}C_3(d_1) = (0, 24, 24, 36)$, which are listed in Table 14, we can conclude that the aliasing cosets of d_1 containing 2fi's and 3fi's have five forms. They are: 1 coset containing four 2fi's but no 3fi's, 1 coset containing four 3fi's but no 2fi's, 12 cosets containing two 2fi's and two 3fi's, 8 cosets containing one 2fi and four 3fi's, and 8 cosets containing three 3fi's but no 2fi's. Then in ${}^{\#}_{2}C_1(d_1)$ the eight 2fi's non-aliased by main effects must be in the 8 cosets containing one 2fi and four 3fi's respectively. But each of such 8 cosets contains 3fi's. Hence, none of the eight clear 2fi's of d_1 is strongly clear. Although d_1 has less general lower-order confounding than d_{10} by the AENPs listed in Table 6 and 14, d_{10} is perhaps a good selection between d_1 and d_{10} for the experimenter if the interactions involving three factors can not be ignored, if he/she is willing to sacrifice 12 2fi's which may be de-aliased and estimated by some follow-up experiments. Note that the word-length of d_{10} is much worse than that of d_1 and the analysis above can not be arrived at from only the word-length patterns of d_1 and d_{10}. Hence the AENP contains more information of a design than the word-length pattern and we can easily retrieve such information from the AENP directly.

Finally, we have found that nearly all the existing criteria for choosing optimal designs can be expressed as a function of the AENP. Choosing different function of the AENP can lead to different criteria, such as the MA, the clear effects, the week MA, the MEC, the MMA, the MEA criteria and so on. For example, for the maximal designs of resolution IV proposed by Chen and Cheng (2006), we have that a 2^{n-m} design of resolution IV is maximal if and only if the design satisfies the two conditions: ${}^{\#}_{1}C_2^{(0)} = n$ and $\sum_{k\geq 1, j\geq 3} {}^{\#}_{j}C_2^{(k)} + \binom{n}{2} = 2^n - (n+1)2^m$.

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Appendix

Table 4. Design matrices for 16- and 32-run designs

1	**2**	3	**4**	5	6	7	**8**	9	10	11	12	13	14	15	**16**	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 5. 16-run GMLOC designs and comparisons with MA and Clear criteria

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
6-2.1	7 14	6; 15; 0,12,3	0, 3, 0, 0	6, 0	1, 1, 1
6-2.2	3 14	3,3; 12,3; 9,6	1, 1, 1, 0	3, 6	2, 2, 2
6-2.3	6 12	1,4,1; 9,6; 9,6	2, 1, 0, 0	1, 5	3, 4, 3
6-2.4	3 12	0,6;9,6;15	2, 0, 0, 1	0, 9	4, 3, 4
7-3.1	14 7 11	7; 21; 0^2,21	0, 7, 0, 0	7, 0	1, 1, 1
7-3.2	14 7 3	2,4,1; 15,6; 6,12,3	2, 3, 2, 0	2, 2	2, 2, 2
7-3.3	12 6 10	1,0,6; 9,12; 6,12,3	4, 3, 0, 0	1, 6	3, 5, 3
7-3.4	14 7 6	0,6,0,1; 12,9; 6,12,3	3, 3, 0, 0	0, 0	4, 4, 5
7-3.5	12 6 3	0,5,2; 12,9; 9,12	3, 2, 1, 1	0, 4	5, 3, 4
8-4.1	14 7 11 13	8; 28; 0^3,28	0, 14, 0, 0	8, 0	1, 1, 1
8-4.2	14 7 3 5	2,0,6; 16,12; 0,24,0,4	4, 6, 4, 0	2, 0	2, 4, 2
8-4.3	14 7 11 3	1,6,0,1; 19,9; 7,0,21	3, 7, 4, 0	1, 1	3, 2, 4
8-4.4	12 6 10 14	1,0^2,7; 7,21; 7,0,21	7, 7, 0, 0	1, 7	4, 6, 3
8-4.5	14 7 3 12	0,4,4; 16,12; 4,18,6	4, 5, 4, 2	0, 0	5, 3, 6
8-4.6	14 7 6 3	0,2,5,1; 13,15; 4,18,6	5, 5, 2, 2	0, 2	6, 5, 5
9-5.1	14 7 11 13 3	0,8,0^2,1; 24,12; 8,0^2,28	4, 14, 8, 0	0, 0	1, 1, 1
9-5.2	14 7 11 3 6	0,2,5,2; 18,18; 2,12,18,4	6, 10, 8, 4	0, 0	2, 3, 1
9-5.3	12 6 10 14 3	0,2,0,6,1; 12,24; 2,12,18,4	8, 10, 4, 4	0, 0	3, 5, 1
9-5.4	14 7 3 12 9	0^2,9; 18,18; 0,18,18	6, 9, 9, 6	0, 0	4, 2, 1
9-5.5	14 7 3 12 6	0^2,6,3; 15,21; 0,18,18	7, 9, 6, 6	0, 0	5, 4, 1
10-6.1	14 7 11 13 3 6	0^2,8,0,2; 21,24; 0,16,0,24,5	8, 18, 16, 8	0, 0	1, 1, 1
10-6.2	14 7 3 12 9 6	0^2,3,7; 18,27; 0,6,27,12	9, 16, 15, 12	0, 0	2, 2, 1
10-6.3	12 6 10 14 3 5	0^2,3,4,3; 15,30; 0,6,27,12	10, 16, 12, 12	0, 0	3, 4, 1
10-6.4	14 7 3 12 6 15	0^3,10; 15,30; 0^2,45	10, 15, 12, 15	0, 0	4, 3, 1
11-7.1	14 7 11 13 3 6 12	0^3,8,3; 19,36; 0^2,24,16,15	12, 26, 28, 24	0, 0	1, 1, 1
11-7.2	14 7 11 13 3 6 5	0^3,8,0,3; 16,39; 0^2,24,16,15	13, 26, 24, 24	0, 0	2, 3, 1
11-7.3	14 7 3 12 9 6 5	0^3,5,6; 16,39; 0^2,15,40	13, 25, 25, 27	0, 0	3, 2, 1
12-8.1	14 7 11 13 3 6 12 9	0^4,12; 18,48; 0^3,48,0,18	16, 39, 48, 48	0, 0	1, 1, 1
12-8.2	14 7 11 13 3 6 12 5	0^4,9,3; 15,51; 0^3,36,30	17, 38, 44, 52	0, 0	2, 2, 1

Table 6. 32-run GMLOC designs and comparisons with MA and Clear criteria

designs	add. columns	${}_1^{\#}C_2; {}_2^{\#}C_1; {}_2^{\#}C_2$	WLP	Cs	Orders G,M,C
7-2.1	30 7	7; 21; 15,6	0, 1, 2, 0	7, 15	1, 1, 1
7-2.2	28 7	7; 21; 9,12	0, 2, 0, 1	7, 9	2, 2, 2
7-2.3	28 14	7; 21; 6,12,3	0, 3, 0, 0	7, 6	3, 3, 3
7-2.4	30 3	4,3; 18,3; 21	1, 0, 1, 1	4, 18	4, 4, 4
7-2.5	28 3	4,3; 18,3; 15,6	1, 1, 0, 0	4, 12	5, 5, 5
7-2.6	28 6	4,3; 18,3; 15,6	1, 1, 1, 0	4, 12	6, 6, 5
7-2.7	24 12	2,4,1; 15,6; 15,6	2, 1, 0, 0	2, 11	7, 8, 7
7-2.8	24 3	1,6; 15,6; 21	2, 0, 0, 1	1, 15	8, 7, 8
8-3.1	30 7 11	8; 28; 13,12,3	0, 3, 4, 0	8, 13	1, 1, 1
8-3.2	28 14 22	8; 28; 7,0,21	0, 7, 0, 0	8, 7	2, 4, 2
8-3.3	28 14 7	8; 28; 4,18,6	0, 5, 0, 2	8, 4	3, 2, 3
8-3.4	28 14 3	8; 28; 0,24,0,4	0, 6, 0, 0	8, 0	4, 3, 4
8-3.5	30 7 12	5,3; 25,3; 16,12	1, 2, 3, 1	5, 13	5, 5, 5
8-3.6	28 14 13	5,3; 25,3; 13,12,3	1, 3, 2, 0	5, 10	6, 6, 6
8-3.7	30 7 3	3,4,1; 22,6; 22,6	2, 1, 2, 2	3, 18	7, 7, 7
8-3.8	28 6 3	3,4,1; 22,6; 16,12	2, 2, 1, 1	3, 12	8, 9, 8
8-3.9	30 7 6	3,4,1; 22,6; 16,12	2, 2, 2, 0	3, 12	9, 10, 8
8-3.10	28 14 6	3,4,1; 22,6; 13,12,3	2, 3, 2, 0	3, 9	10,11,10
9-4.1	30 7 11 13	9; 36; 15,0,21	0, 7, 7, 0	9, 15	1, 2, 1
9-4.2	30 7 11 19	9; 36; 8,24,0,4	0, 6, 8, 0	9, 8	2, 1, 2
9-4.3	28 14 22 26	9; 36; $8,0^2,28$	0, 14, 0, 0	9, 8	3, 5, 2
9-4.4	28 14 13 7	9; 36; 2,12,18,4	0, 10, 0, 4	9, 2	4, 4, 4
9-4.5	28 14 7 19	9; 36; 0,18,18	0, 9, 0, 6	9, 0	5, 3, 5
9-4.6	28 14 22 3	6,3; 33,3; 15,0,21	1, 7, 4, 0	6, 12	6, 7, 6
9-4.7	30 7 11 24	6,3; 33,3; 12,18,6	1, 5, 6, 2	6, 9	7, 6, 7
9-4.8	30 7 11 6	4,4,1; 30,6; 15,18,3	2, 4, 6, 2	4, 11	8, 9, 8
9-4.9	28 14 7 3	4,4,1; 30,6; 12,18,6	2, 5, 4, 2	4, 8	9, 10,9
9-4.10	28 14 7 10	4,4,1; 30,6; 12,18,6	2, 5, 5, 2	4, 8	10,11,9
10-5.1	30 7 11 19 29	10; 45; $0,40,0^2,5$	0, 10, 16, 0	10, 0	1, 1, 1
10-5.2	28 14 22 26 7	10; 45; 0,16,0,24,5	0, 18, 0, 8	10, 0	2, 4, 1
10-5.3	28 14 7 19 11	10; 45; 0,6,27,12	0, 16, 0, 12	10, 0	3, 3, 1
10-5.4	28 14 7 19 25	10; 45; $0^2,45$	0, 15, 0, 15	10, 0	4, 2, 1
10-5.5	28 14 22 26 3	7,3; 42,3; $17,0^2,28$	1, 14, 7, 0	7, 14	5, 6, 5
10-5.6	30 7 11 19 14	7,3; 42,3; 11,12,18,4	1, 10, 11, 4	7, 8	6, 5, 6
10-5.7	28 14 22 3 5	5,4,1; 39,6; 11,12,18,4	2, 10, 8, 4	5, 7	7, 10, 7
10-5.8	28 14 7 19 5	5,4,1; 39,6; 9,18,18	2, 9, 9, 6	5, 5	8, 9, 8
10-5.9	30 7 11 19 6	5,4,1; 39,6; 8,30,3,4	2, 8, 12, 4	5, 4	9, 8, 9
10-5.10	30 7 11 24 21	4,6; 39,6; 12,24,9	2, 7, 12, 7	4, 6	10, 7, 11
11-6. 1	28 14 22 26 7 11	11; 55; $0^2,24,16,15$	0, 26, 0, 24	11, 0	1, 2, 1
11-6. 2	28 14 7 19 25 11	11; 55; $0^2,15,40$	0, 25, 0, 27	11, 0	2, 1, 1
11-6. 3	28 14 22 26 7 3	6,4,1; 49,6; 10,16,0,24,5	2, 18, 14, 8	6, 6	3, 5, 3
11-6. 4	28 14 7 19 11 17	6,4,1; 49,6; 10,6,27,12	2, 16, 16, 12	6, 6	4, 4, 3
11-6. 5	30 7 11 19 29 6	6,4,1; 49,6; 4,28,18,0,5	2, 14, 22, 8	6, 0	5, 3, 5
11-6. 6	30 7 11 19 6 5	5,0,6; 43,12; 4,28,18,0,5	4, 14, 16, 8	5, 4	6, 15, 6
11-6. 7	28 14 7 19 11 18	4,6,0,1; 46,9; 10,6,27,12	3, 16, 12, 12	4, 4	7, 8, 7
11-6. 8	28 14 7 19 11 6	4,6,0,1; 46,9; 10,6,27,12	3, 16, 13, 12	4, 4	8, 9, 7
11-6. 9	28 14 7 19 25 3	4,6,0,1; 46,9; 10,0,45	3, 15, 13, 15	4, 4	9, 7, 7
11-6.10	30 7 11 24 21 14	4,5,2; 46,9; 8,24,15,8	3, 13, 19, 11	4, 3	10, 6, 10

Table 7. 32-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
12-7.1	28 14 22 26 7 11 13	12; 66; 0^3,48,0,18	0, 39, 0, 48	12, 0	1, 2, 1
12-7.2	28 14 7 19 25 11 13	12; 66; 0^3,36,30	0, 38, 0, 52	12, 0	2, 1, 1
12-7.3	28 14 22 26 7 3 5	6,0,6; 54,12; 0,36,0,24,0,6	4, 23, 28, 16	6, 0	3, 7, 3
12-7.4	28 14 22 26 7 11 3	5,6,0,1; 57,9; 11,0,24,16,15	3, 26, 22, 24	5, 5	4, 4, 4
12-7.5	28 14 7 19 25 11 6	5,6,0,1; 57,9; 11,0,15,40	3, 25, 23, 27	5, 5	5, 3, 4
12-7.6	28 14 22 26 7 3 9	4,4,4; 54,12; 4,32,0,20,10	4, 22, 28, 20	4, 0	6, 6, 7
12-7.7	30 7 11 19 29 6 12	4,4,4; 54,12; 4,22,27,8,5	4, 20, 32, 22	4, 0	7, 5, 7
12-7.8	28 14 7 19 11 17 18	4,2,5,1; 51,15; 4,22,27,8,5	5, 20, 28, 22	4, 2	8, 12, 6
12-7.9	28 14 7 19 11 6 13	3,8,0^2,1; 54,12; 11,0,24,16,15	4, 26, 20, 24	3, 3	9, 9, 9
12-7.10	28 14 7 19 25 11 3	3,8,0^2,1; 54,12; 11,0,15,40	4, 25, 19, 27	3, 3	10, 8, 9
13-8.1	28 14 22 26 7 11 13 19	13; 78 ;0^4,60,18	0, 55, 0, 96	13, 0	1, 1, 1
13-8.2	28 14 22 26 7 11 13 3	4,8,0^2,1; 66,12; 12,0^2,48,0,18	4, 39, 32, 48	4, 4	2, 4, 2
13-8.3	28 14 7 19 25 11 13 24	4,8,0^2,1; 66,12; 12,0^2,36,30	4, 38, 32, 52	4, 4	3, 2, 2
13-8.4	28 14 7 19 25 11 13 6	4,8,0^2,1; 66,12; 12,0^2,36,30	4, 38, 33, 52	4, 4	4, 3, 2
13-8.5	28 14 22 26 7 11 3 5	4,2,5,2; 60,18; 2,20,24,16,10,6	6, 31, 44, 40	4, 0	5, 9, 5
13-8.6	30 7 11 19 29 6 12 9	4,0,9; 60,18; 0,18,45,0,15	6, 28, 51, 42	4, 0	6, 6, 5
13-8.7	30 7 11 19 29 6 12 10	4,0,6,3; 57,21; 0,18,45,0,15	7, 28, 46, 42	4, 0	7, 14, 5
13-8.8	28 14 7 19 11 6 13 5	3,0,8,0,2; 54,24; 2,20,24,16,10,6	8, 31, 40, 40	3, 2	8, 29, 8
13-8.9	30 7 11 19 6 12 10 2	3,0,3,7; 51,27; 0,18,45,0,15	9, 28, 42, 42	3, 0	9, 46, 9
13-8.10	28 14 7 19 25 11 13 3	2,10,0^3,1; 63,15; 12,0^2,36,30	5, 38, 28, 52	2, 2	10, 5, 13
14-9.1	28 14 22 26 7 11 13 19 21	14; 91; 0^5,84,7	0, 77, 0, 168	14, 0	1, 1, 1
14-9.2	28 14 22 26 7 11 13 3 5	4,0,8,0,2; 67,24; 0,24,0,48,0,12,7	8, 45, 64, 72	4, 0	2, 8, 2
14-9.3	30 7 11 19 29 6 12 10 5	4,0,3,7; 64,27; 0,6,51,16,0,18	9, 41, 69, 72	4, 0	3, 11, 2
14-9.4	28 14 22 26 7 11 13 19 3	3,10,0^3,1; 76,15; 13,0^3,60,18	5, 55, 45, 96	3, 3	4, 2, 4
14-9.5	29 14 7 19 11 6 13 5 10	3,0^2,8,3; 55,36; 0,6,51,16,0,18	12, 41, 64, 72	3, 0	5, 66, 5
14-9.6	28 14 7 19 25 11 13 24 20	2,4,6,0,2; 67,24; 4,20,0,36,25,6	8, 43, 64, 80	2, 0	6, 6, 8
14-9.7	28 14 7 19 25 11 13 6 3	2,2,8,0,1,1; 64,27; 4,20,0,36,25,6	9, 43, 61, 80	2, 2	7, 13, 7
14-9.8	28 14 22 26 6 12 24 18 11	2,0^3,12; 43,48; 0,24,0,48,0,12,7	16, 45, 64, 72	2, 0	8, 124, 8
14-9.9	28 14 13 7 3 6 11 10 5	2,0^3,9,3; 40,51; 4,20,0,36,25,6	17, 43, 61, 80	2, 4	9, 125, 6
14-9.10	28 14 22 26 7 11 13 19 6	1,12,0^4,1; 73,18; 13,0^3,60,18	6, 55, 40, 96	1, 1	10, 3, 15
15-10.1	28 14 22 26 7 11 13 19 21 25	15; 105; 0^6,105	0, 105, 0, 280	15, 0	1, 1, 1
15-10.2	28 14 22 26 7 11 13 3 5 9	4,0^2,8,3; 69,36; 0^2,36,48,0^2,21	12, 57, 100, 120	4, 0	2, 13, 2
15-10.3	28 14 7 19 11 6 13 5 10 9	3,0^3,12; 57,48; 0^2,36,48,0^2,21	16, 57, 96, 120	3, 0	3, 98, 3
15-10.4	28 14 22 26 7 11 13 19 21 3	2,12,0^4,1; 87,18; 14,0^4,84,7	6, 77, 62, 168	2, 2	4, 2, 4
15-10.5	28 14 22 26 7 11 13 19 3 5	2,2,9,0^2,2; 75,30; 2,24,0^2,60,12,7	10, 61, 90, 136	2, 0	5, 5, 6
15-10.6	28 14 13 7 3 6 11 10 5 9	2,0^4,12,1; 39,66; 2,24,0^2,60,12,7	22, 61, 94, 136	2, 2	6, 144, 4
15-10.7	30 7 11 19 29 6 12 24 9 10	1,3,3,5,3; 69,36; 3,6,27,36,15,18	12, 53, 100, 136	1, 0	7, 11, 12
15-10.8	28 14 7 19 25 11 13 3 12 6	1,2,1,8,1,2; 63,42; 3,6,27,36,15,18	14, 53, 94, 136	1, 1	8, 39, 10
15-10.9	28 14 7 19 11 6 13 5 10 3	1,2,0^2,9,2,1; 51,54; 3,6,27,36,15,18	18, 53, 90, 136	1, 1	9, 127, 10
15-10.10	28 14 22 26 7 11 13 19 6 12	1,0,12,0^3,2; 69,36; 2,24,0^2,60,12,7	12, 61, 80, 136	1, 2	10, 14, 9

Table 8. 32-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
16-11.1	28 14 22 26 7 11 13 19 21 25 31	16; 120; 0^7,120	0,140, 0,448	16, 0	1, 1, 1
16-11.2	28 14 22 26 7 11 13 3 5 9 15	4,0^3,12; 72,48; 0^3,96,0^3,24	16, 76,144,192	4, 0	2, 20, 2
16-11.3	28 14 22 26 7 11 13 19 21 3 5	2,0,12,0^3,2; 84,36; 0,28,0^3,84,0,8	12, 84,124,224	2, 0	3, 4, 3
16-11.4	28 14 13 7 3 6 11 10 5 9 15	2,0^5,14; 36,84; 0,28,0^3,84,0,8	28, 84,140,224	2, 0	4, 144, 3
16-11.5	28 14 22 26 7 11 13 19 21 25 3	1,14,0^5,1; 99,21; 15,0^5,105	7,105, 84,280	1, 1	5, 2, 7
16-11.6	28 14 22 26 7 11 13 19 3 5 9	1,3,0,9,0,3; 75,45; 3,0,36,0,60,0,21	15, 73,140,216	1, 0	6, 9, 9
16-11.7	28 14 7 19 11 6 13 5 10 9 3	1,2,0^3,12,0,1; 51,69; 3,0,36,0,60,0,21	23, 73,132,216	1, 1	7, 140, 7
16-11.8	30 7 11 19 29 6 12 24 9 18 10	1,0,6,0,9; 72,48; 0,12,0,72,0,36	16, 68,144,224	1, 0	8, 14, 9
16-11.9	28 14 7 19 11 17 6 5 3 10 9	1,0,3,0,9,0,3; 60,60; 0,12,0,72,0,36	20, 68,132,224	1, 0	9, 106, 9
16-11.10	30 7 11 19 29 6 12 24 9 18 23	1,0^2,15; 75,45; 0^2,45,0,75	15, 65,156,232	1, 0	10, 6, 9
17-12.1	28 14 22 26 7 11 13 19 21 25 31 3	0,16,0^6,1; 112,24; 16,0^6,120	8,140,112,448	0, 0	1, 1, 1
17-12.2	30 7 11 19 29 6 12 24 9 10 5 15	0,4,0^2,9,4; 76,60; 4,0^2,48,60,0^2,24	20, 92,200,336	0, 0	2, 15, 1
17-12.3	28 14 22 26 7 11 13 3 5 9 15 6	0,4,0^3,12,0^2,1; 64,72; 4,0^2,48,60,0^2,24	24, 92,192,336	0, 0	3, 97, 1
17-12.4	28 14 22 26 7 11 13 19 21 25 3 6	0,2,13,0^4,2; 94,42; 2,28,0^4,98,8	14,112,168,364	0, 0	4, 2, 1
17-12.5	28 14 22 26 7 11 13 19 21 3 12 5	0,2,2,10,0^2,3; 82,54; 2,4,36,0^2,72,14,8	18, 96,192,348	0, 0	5, 5, 1
17-12.6	28 14 7 19 11 6 13 5 10 9 3 12	0,2,1,0^3,12,2; 46,90; 2,4,36,0^2,72,14,8	30, 96,184,348	0, 0	6,130, 1
17-12.7	28 14 22 26 7 11 13 19 21 3 5 6	0,2,0,12,0^3,2,1; 76,60; 2,4,36,0^2,72,14,8	20, 96,180,348	0, 0	7, 16, 1
17-12.8	28 14 22 26 6 12 24 18 10 20 30 3	0,2,0^5,14,1; 28,108; 2,28,0^4,98,8	36,112,196,364	0, 0	8,132, 1
17-12.9	30 7 11 19 29 6 12 24 9 18 10 5	0,1,3,3,6,4; 76,60; 1,6,9,36,45,18,21	20, 88,200,356	0, 0	9, 13, 1
17-12.10	28 14 22 26 7 11 13 19 3 5 9 6	0,1,3,1,8,1,2,1; 70,66; 1,6,9,36,45,18,21	22, 88,192,356	0, 0	10, 43, 1
18-13.1	28 14 22 26 7 11 13 19 21 25 31 3 6	0^2,16,0^5,2; 105,48; 0,32,0^5,112,9	16,148,224,560	0, 0	1, 1, 1
18-13.2	28 14 22 26 7 11 13 19 21 3 12 5 10	0^2,4,0,10,0,4; 81,72; 0,8,0,48,0,72,0,16,9	24,116,272,528	0, 0	2, 9, 1
18-13.3	30 7 11 19 29 6 12 24 17 10 5 15 9	0^2,4,0,3,11; 78,75; 0,8,0,12,85,24,0,24	25,112,275,536	0, 0	3, 13, 1
18-13.4	30 7 11 19 29 6 12 24 9 10 5 15 3	0^2,4,0,1,8,4,0,1; 69,84; 0,8,0,12,85,24,0,24	28,112,264,536	0, 0	4, 68, 1
18-13.5	28 14 22 26 7 11 13 3 5 9 15 6 12	0^2,4,0^3,12,0,2; 57,96; 0,8,0,48,0,72,0,16,9	32,116,256,528	0, 0	5,108, 1
18-13.6	28 14 22 26 7 11 13 19 21 25 3 6 12	0^2,3,12,0^3,3; 90,63; 0,6,39,0^3,84,24	21,126,259,532	0, 0	6, 2, 1
18-13.7	28 14 22 26 7 11 13 19 21 25 3 6 5	0^2,3,12,0^4,3; 87,66; 0,6,39,0^3,84,24	22,126,252,532	0, 0	7, 3, 1
18-13.8	28 14 22 26 6 12 24 18 10 20 30 3 5	0^2,3,0^4,12,3; 39,114; 0,6,39,0^3,84,24	38,126,252,532	0, 0	8,113, 1
18-13.9	28 14 22 26 7 11 13 19 21 3 12 24 5	0^2,2,4,8,0,4; 81,72; 0,4,12,40,0,54,35,8	24,114,272,540	0, 0	9, 7, 1
18-13.10	28 14 22 26 7 11 13 19 21 3 12 9 5	0^2,2,4,8,0,1,3; 78,75; 0,4,12,40,0,54,35,8	25,114,267,540	0, 0	10, 16, 1

Table 9. 32-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	$^{\#}_{1}C_2;^{\#}_{2}C_1;^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
19-14.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12	0^3,16,0^4,3; 99,72; 0^2,48,0^4,96,27	24,164,344,784	0, 0	1, 1, 1
19-14.2	28 14 22 26 7 11 13 19 21 25 31 3 6 5	0^3,16,0^5,3; 96,75; 0^2,48,0^4,96,27	25,164,336,784	0, 0	2, 2, 1
19-14.3	28 14 22 26 7 11 13 19 21 25 3 6 12 9	0^3,4,11,0^2,4; 87,84; 0^2,12,48,0^2,84,0,27	28,148,364,784	0, 0	3, 4, 1
19-14.4	28 14 22 26 7 11 13 19 21 25 3 6 12 24	0^3,4,11,0^2,4; 87,84; 0^2,12,48,0^2,63,48	28,147,364,791	0, 0	4, 3, 1
19-14.5	28 14 22 26 7 11 13 19 21 25 3 6 12 5	0^3,4,11,0^2,1,3; 84,87; 0^2,12,48,0^2,63,48	29,147,357,791	0, 0	5, 5, 1
19-14.6	28 14 22 26 7 11 13 19 21 3 12 24 5 10	0^3,4,2,8,5; 81,90; 0^2,12,8,50,48,28,16,9	30,140,370,800	0, 0	6,10, 1
19-14.7	28 14 22 26 7 11 13 19 21 3 12 9 5 10	0^3,4,2,8,0,4,1; 75,96; 0^2,12,8,50,48,28,16,9	32,140,360,800	0, 0	7,31, 1
19-14.8	30 7 11 19 29 6 12 24 9 10 5 15 3 14	0^3,4,1,0,8,4,2; 63,108; 0^2,12,8,50,48,28,16,9	36,140,348,800	0, 0	8,82, 1
19-14.9	28 14 22 26 7 11 13 19 21 3 12 5 10 6	0^3,4,0,10,0,4,0,1;72,99;0^2,12,8,50,48,28,16,9	33,140,352,800	0, 0	9,50, 1
19-14.10	30 7 11 19 29 6 12 24 9 18 23 3 14 13	0^3,4,0,9,6; 78,93; 0^2,12,0,45,90,0,24	31,137,369,811	0, 0	10, 7, 1
20-15. 1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9	0^4,16,0^3,4; 94,96; 0^3,64,0^3,96,0,30	32,189,480,1120	0, 0	1, 2, 1
20-15. 2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24	0^4,16,0^3,4; 94,96; 0^3,64,0^3,72,54	32,188,480,1128	0, 0	2, 1, 1
20-15. 3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 5	0^4,16,0^3,1,3; 91,99; 0^3,64,0^3,72,54	33,188,472,1128	0, 0	3, 3, 1
20-15. 4	28 14 22 26 7 11 3 17 13 5 23 24 30 12 10	0^4,6,0,14; 82,108; 0^3,24,0,108,0,48,0,10	36,173,492,1152	0, 0	4, 8, 1
20-15. 5	28 14 22 26 7 11 13 19 21 3 12 9 5 10 15	0^4,6,0,8,0,6; 70,120; 0^3,24,0,108,0,48,0,10	40,173,472,1152	0, 0	5, 50, 1
20-15. 6	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9	0^4,5,10,0,5; 85,105; 0^3,20,55,0,56,32,27	35,176,490,1148	0, 0	6, 5, 1
20-15. 7	28 14 22 26 7 11 13 19 21 25 3 6 12 24 17	0^4,5,10,0,5; 85,105; 0^3,20,55,0,35,80	35,175,491,1155	0, 0	7, 4, 1
20-15. 8	28 14 22 26 7 11 13 19 21 25 3 6 12 24 5	0^4,5,10,0,2,3; 82,108; 0^3,20,55,0,35,80	36,175,483,1155	0, 0	8, 9, 1
20-15. 9	28 14 22 26 7 11 13 19 21 25 3 6 12 9 5	0^4,5,10,0^2,4,1; 79,111; 0^3,20,55,0,56,32,27	37,176,476,1148	0, 0	9, 14, 1
20-15.10	28 14 22 26 6 12 24 18 10 20 30 3 5 7 9	0^4,5,0^2,8,4,3; 55,135; 0^3,20,55,0,56,32,27	45,176,452,1148	0, 0	10, 67, 1
21-16.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9	0^5,16,0^2,5; 90,120; 0^4,80,0^2,64,36,30	40,221,640,1600	0, 0	1, 2, 1
21-16.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 17	0^5,16,0^2,5; 90,120; 0^4,80,0^2,40,90	40,220,641,1608	0, 0	2, 1, 1
21-16.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 5	0^5,16,0^2,2,3; 87,123; 0^4,80,0^2,40,90	41,220,632,1608	0, 0	3, 3, 1
21-16.4	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 5	0^5,16,0^3,4,1; 84,126; 0^4,80,0^2,64,36,30	42,221,624,1600	0, 0	4, 6, 1
21-16.5	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 18	0^5,6,9,6; 84,126; 0^4,30,60,56,0,54,10	42,213,644,1624	0, 0	5, 5, 1
21-16.6	28 14 22 26 7 11 13 19 21 25 3 6 12 24 17 5	0^5,6,9,3,3; 81,129; 0^4,30,60,21,72,27	43,211,638,1638	0, 0	6, 8, 1
21-16.7	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 5	0^5,6,9,1,4,1; 78,132; 0^4,30,60,21,72,27	44,211,630,1638	0, 0	7, 14, 1
21-16.8	28 14 22 26 7 11 13 19 21 25 3 6 12 24 5 20	0^5,6,9,0,6; 78,132; 0^4,30,60,0,120	44,210,630,1646	0, 0	8, 8, 1
21-16.9	28 14 22 26 7 11 13 19 21 25 3 6 12 9 5 10	0^5,6,9,0^2,6; 72,138; 0^4,30,60,56,0,54,10	46,213,616,1624	0, 0	9, 35, 1
21-16.10	28 14 22 26 7 11 3 17 13 5 23 24 30 12 9 10	0^5,6,4,10,1; 78,132; 0^4,30,24,98,48,0,10	44,209,636,1644	0, 0	10, 7, 1

Table 10. 32-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
22-17.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 5	0^6,16,0,1,4,1; 81,150; 0^5,96,0,24,81,30	50,261,816,2240	0, 0	3, 5, 1
22-17.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 5 20	0^6,16,0^2,6; 81,150; 0^5,96,0^2,135	50,260,816,2249	0, 0	4, 4, 1
22-17.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 5 10	0^6,16,0^3,6; 75,156; 0^5,96,0,64,0,60,11	52,263,800,2224	0, 0	5, 16, 1
22-17.4	28 14 22 26 7 11 3 17 13 5 23 24 30 12 9 10 15	0^6,10,0,12; 75,156; 0^5,60,0,160,0^2,11	52,255,816,2264	0, 0	6, 15, 1
22-17.5	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18	0^6,16,0,6; 87,144; 0^5,96,0,64,0,60,11	48,263,832,2224	0, 0	1, 1, 1
22-17.6	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 17 5	0^6,16,0,3,3; 84,147; 0^5,96,0,24,81,30	49,261,825,2240	0, 0	2, 3, 1
22-17.7	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 5	0^6,7,15; 84,147; 0^5,42,119,0^2,70	49,259,833,2240	0, 0	7, 2, 1
22-17.8	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 18 5	0^6,7,10,4,1; 78,153; 0^5,42,77,48,54,10	51,255,821,2268	0, 0	8, 7, 1
22-17.9	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 5 10	0^6,7,9,0,6; 72,159; 0^5,42,77,48,54,10	53,255,805,2268	0, 0	9, 22, 1
22-17.10	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 18 10	0^6,7,8,6,0,1; 75,156; 0^5,42,77,48,54,10	52,255,812,2268	0, 0	10, 14, 1
23-18.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23	0^7,16,7; 85,168; 0^6,112,64,0^2,77	56,315,1064,3024	0, 0	1, 1, 1
23-18.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5	0^7,16,2,4,1; 79,174; 0^6,112,16,54,60,11	58,311,1050,3056	0, 0	2, 2, 1
23-18.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 5 10	0^7,16,1,0,6; 73,180; 0^6,112,16,54,60,11	60,311,1032,3056	0, 0	3, 9, 1
23-18.4	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 10	0^7,16,0,6,0,1; 76,177; 0^6,112,16,54,60,11	59,311,1040,3056	0, 0	4, 5, 1
23-18.5	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 5 20 10	0^7,16,0,5,2; 76,177; 0^6,112,0,81,60	59,310,1041,3065	0, 0	5, 4, 1
23-18.6	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 5 10 15	0^7,16,0^3,7; 64,189; 0^6,112,64,0^2,77	63,315,1008,3024	0, 0	6, 22, 1
23-18.7	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 18 5 15	0^7,10,7,6; 73,180; 0^6,70,64,108,0,11	60,307,1040,3080	0, 0	7, 8, 1
23-18.8	28 14 22 26 7 11 3 17 13 5 23 24 30 12 9 10 15 6	0^7,10,0,12,0,1; 64,189; 0^6,70,64,108,0,11	63,307,1016,3080	0, 0	8, 20, 1
23-18.9	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 5 3	0^7,9,13,0,1; 76,177; 0^6,63,120,0,70	59,308,1047,3073	0, 0	9, 3, 1
23-18.10	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 5 10 15	0^7,9,7,0,7; 64,189; 0^6,63,120,0,70	63,308,1015,3073	0, 0	10, 21, 1
24-19.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29	0^8,24; 84,192; 0^7,192,0^3,84	64,378,1344,4032	0, 0	1, 1, 1
24-19.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 15	0^8,18,0,6; 72,204; 0^7,144,0,120,0,12	68,370,1316,4096	0, 0	2, 4, 1
24-19.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 5	0^8,17,6,0,1; 75,201; 0^7,136,63,0,77	67,371,1324,4088	0, 0	3, 2, 1
24-19.4	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 5 10 15	0^8,17,0^2,7; 63,213; 0^7,136,63,0,77	71,371,1288,4088	0, 0	4, 14, 1
24-19.5	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20	0^8,16,4,4; 72,204; 0^7,128,36,90,22	68,369,1316,4106	0, 0	5, 3, 1
24-19.6	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 10 5	0^8,16,2,5,1; 69,207; 0^7,128,36,90,22	69,369,1306,4106	0, 0	6, 7, 1
24-19.7	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 3 10 5	0^8,11,11,2; 69,207; 0^7,88,117,60,11	69,367,1310,4120	0, 0	7, 6, 1
24-19.8	28 14 22 26 7 11 13 19 21 25 3 6 12 24 9 18 5 15 10	0^8,11,6,6,1; 63,213; 0^7,88,117,60,11	71,367,1292,4120	0, 0	8, 13, 1
24-19.9	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 3 10 20	0^8,9,15; 69,207; 0^7,72,144,60	69,366,1311,4129	0, 0	9, 5, 1
24-19.10	28 14 22 26 7 11 13 19 21 3 12 24 17 9 20 10 23 5 6	0^8,9,12,3; 66,210; 0^7,72,144,60	70,366,1301,4129	0, 0	10, 9, 1

Table 11. 32-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	$^{\#}_{1}C_2;^{\#}_{2}C_1;^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
25-20.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5	0^9,24,0^2,1; 72,228; 0^8,216,0^2,84	76,442,1656,5376	0, 0	1, 1, 1
25-20.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 5 10	0^9,18,5,2; 66,234; 0^8,162,60,66,12	78,438,1640,5412	0, 0	2, 3, 1
25-20.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 15 10	0^9,18,0,6,1; 60,240; 0^8,162,60,66,12	80,438,1620,5412	0, 0	3, 9, 1
25-20.4	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20 27	0^9,16,9; 66,234; 0^8,144,90,66	78,437,1641,5422	0, 0	4, 2, 1
25-20.5	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20 10	0^9,16,6,3; 63,237; 0^8,144,90,66	79,437,1630,5422	0, 0	5, 5, 1
25-20.6	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 3 10 20 5	0^9,13,12; 63,237; 0^8,117,150,33	79,436,1632,5430	0, 0	6, 4, 1
25-20.7	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 3 10 9 5	0^9,13,9,3; 60,240; 0^8,117,150,33	80,436,1622,5430	0, 0	7, 8, 1
25-20.8	28 14 22 26 7 11 13 19 21 3 12 24 17 9 20 6 27 5 10 18	0^9,10,15; 60,240; 0^8,90,210	80,435,1622,5440	0, 0	8, 6, 1
25-20.9	28 14 22 26 7 11 13 19 21 3 12 24 17 9 20 6 27 5 10 15	0^9,10,15; 60,240; 0^8,90,210	80,435,1623,5440	0, 0	9, 7, 1
26-21.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5 10	0^{10},24,0,2; 61,264; 0^9,240,0,72,13	88,518,2032,7032	0, 0	1, 1, 1
26-21.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20 27 10	0^{10},19,7; 58,267; 0^9,190,99,36	89,516,2023,7052	0, 0	2, 2, 1
26-21.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 5 10 15	0^{10},19,4,3; 55,270; 0^9,190,99,36	90,516,2012,7052	0, 0	3, 5, 1
26-21.4	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20 10 17	0^{10},16,10; 55,270; 0^9,160,165	90,515,2012,7063	0, 0	4, 3, 1
26-21.5	28 14 7 19 25 11 6 24 21 13 18 12 22 15 17 27 3 10 20 5 9	0^{10},16,10; 55,270; 0^9,160,165	90,515,2013,7062	0, 0	5, 4, 1
27-22.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5 10 20	0^{11},24,3; 51,300; 0^{10},264,48,39	100,606,2484,9064	0, 0	1, 1, 1
27-22.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5 10 15	0^{11},24,0,3; 48,303; 0^{10},264,48,39	101,606,2472,9064	0, 0	2, 3, 1
27-22.3	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 5 20 27 10 15	0^{11},21,6; 48,303; 0^{10},231,120	101,605,2473,9075	0, 0	3, 2, 1
28-23.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5 10 20 27	0^{12},28 ; 42,336 ; 0^{11},336,0,42	112,707,3024,11536	0, 0	1, 1, 1
28-23.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 9 18 23 29 5 10 20 15	0^{12},25,3 ; 39,339 ; 0^{11},300,78	113,706,3012,11548	0, 0	2, 2, 1

Table 12. 64-run GMLOC designs and comparisons with MA and Clear criteria

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
8-2.1	60 15	8; 28; 28	0, 0, 2, 1	8, 28	1, 1, 1
8-2.2	62 7	8; 28; 22,6	0, 1, 0, 2	8, 22	2, 2, 2
8-2.3	60 7	8; 28; 22,6	0, 1, 1, 0	8, 22	3, 3, 2
8-2.4	60 14	8; 28; 22,6	0, 1, 2, 0	8, 22	4, 4, 2
8-2.5	56 7	8; 28; 16,12	0, 2, 0, 0	8, 16	6, 5, 5
8-2.6	56 11	8; 28; 16,12	0, 2, 0, 1	8, 16	5, 6, 5
8-2.7	56 28	8; 28; 13,12,3	0, 3, 0, 0	8, 13	7, 7, 7
8-2.8	62 3	5,3; 25,3; 28	1, 0, 0, 1	5, 25	8, 8, 8
8-2.9	60 3	5,3; 25,3; 28	1, 0, 1, 0	5, 25	10, 9, 8
8-2.10	60 6	5,3; 25,3; 28	1, 0, 1, 1	5, 25	9, 10, 8
9-3.1	60 15 22	9; 36; 30,6	0, 1, 4, 2	9, 30	1, 1, 1
9-3.2	60 14 19	9; 36; 24,12	0, 2, 3, 1	9, 24	2, 2, 2
9-3.3	56 7 27	9; 36; 24,12	0, 2, 4, 0	9, 24	3, 3, 2
9-3.4	62 7 11	9; 36; 21,12,3	0, 3, 0, 4	9, 21	4, 4, 4
9-3.5	60 14 7	9; 36; 21,12,3	0, 3, 2, 0	9, 21	5, 6, 4
9-3.6	60 14 13	9; 36; 21,12,3	0, 3, 3, 0	9, 21	6, 7, 4
9-3.7	60 14 22	9; 36; 21,12,3	0, 3, 4, 0	9, 21	7, 8, 4
9-3.8	56 11 22	9; 36; 18,18	0, 3, 0, 4	9, 18	8, 4, 8
9-3.9	56 11 7	9; 36; 15,18,3	0, 4, 0, 2	9, 15	9, 9, 9
9-3.10	56 28 44	9; 36; 15,0,21	0, 7, 0, 0	9, 15	10, 12, 9
10-4.1	60 15 22 39	10; 45; 33,12	0, 2, 8, 4	10, 33	1, 1, 1
10-4.2	60 15 22 26	10; 45; 30,12,3	0, 3, 8, 3	10, 30	2, 4, 2
10-4.3	60 15 22 21	10; 45; 30,12,3	0, 3, 7, 4	10, 30	3, 3, 2
10-4.4	60 15 22 35	10; 45; 27,18	0, 3, 6, 4	10, 27	4, 2, 4
10-4.5	56 7 27 14	10; 45; 24,18,3	0, 4, 6, 2	10, 24	5, 5, 5
10-4.6	56 7 27 30	10; 45; 24,18,3	0, 4, 8, 0	10, 24	6, 6, 5
10-4.7	62 7 11 13	10; 45; 24,0,21	0, 7, 0, 7	10, 24	7, 15, 5
10-4.8	60 14 13 7	10; 45; 24,0,21	0, 7, 3, 0	10, 24	8, 17, 5
10-4.9	60 14 22 26	10; 45; 24,0,21	0, 7, 7, 0	10, 24	9, 18, 5
10-4.10	60 14 19 7	10; 45; 21,18,6	0, 5, 4, 2	10, 21	10, 7, 10
11-5.1	60 15 22 39 21	11; 55; 34,18,3	0, 4, 14, 8	11, 34	1, 1, 1
11-5.2	60 15 22 35 26	11; 55; 28,24,3	0, 5, 12, 7	11, 28	2, 3, 2
11-5.3	56 7 27 14 13	11; 55; 28,6,21	0, 8, 10, 4	11, 28	3, 11, 2
11-5.4	60 14 22 26 29	11; 55; 28,6,21	0, 8, 14, 0	11, 28	4, 13, 2
11-5.5	60 15 22 21 27	11; 55; 27,24,0,4	0, 6, 12, 8	11, 27	5, 6, 5
11-5.6	60 14 13 7 11	11; 55; $27,0^2,28$	0, 14, 4, 0	11, 27	6, 30, 5
11-5.7	60 15 22 35 57	11; 55; 25,30	0, 5, 10, 10	11, 25	7, 2, 7
11-5.8	60 15 22 35 21	11; 55; 25,24,6	0, 6, 10, 8	11, 25	8, 4, 7
11-5.9	60 15 22 39 19	11; 55; 25,24,6	0, 6, 12, 4	11, 25	9, 5, 7
11-5.10	60 14 19 7 37	11; 55; 22,24,9	0, 7, 8, 7	11, 22	10, 7, 10

Table 13. 64-run GMLOC designs and comparisons with MA and Clear criteria (continued)

designs	add. columns	${}^{\#}_{1}C_2; {}^{\#}_{2}C_1; {}^{\#}_{2}C_2$	WLP	Cs	Orders G,M,C
12-6.1	60 15 22 39 21 59	12; 66; 36,24,6	0, 6, 24, 16	12, 36	1, 1, 1
12-6.2	60 15 22 39 21 19	12; 66; 30,12,24	0, 10, 20, 8	12, 30	2, 5, 2
12-6.3	60 15 22 39 19 41	12; 66; 27,30,9	0, 8, 20, 14	12, 27	3, 2, 3
12-6.4	60 15 22 35 26 37	12; 66; 24,30,12	0, 9, 18, 13	12, 24	4, 3, 4
12-6.5	60 15 22 35 21 19	12; 66; 23,18,21,4	0, 12, 14, 12	12, 23	5, 12, 5
12-6.6	60 15 22 35 57 19	12; 66; 21,30,15	0, 10, 15, 16	12, 21	6, 4, 6
12-6.7	60 14 19 7 37 26	12; 66; 21,24,21	0, 11, 14, 15	12, 21	7, 6, 6
12-6.8	60 14 22 11 7 13	12; 66; 21,16,0,24,5	0, 18, 8, 8	12, 21	8, 35, 6
12-6.9	60 14 22 11 19 7	12; 66; 21,6,27,12	0, 16, 9, 12	12, 21	9, 28, 6
12-6.10	60 14 22 26 7 11	12; 66; 21,6,27,12	0, 16, 10, 12	12, 21	10, 29, 6
13-7.1	60 15 22 39 21 59 19	13; 78; 36,0,42	0, 14, 33, 16	13, 36	1, 2, 1
13-7.2	60 14 22 11 19 7 13	13; 78; 23,0,24,16,15	0, 26, 12, 24	13, 23	2, 37, 2
13-7.3	60 14 22 26 7 11 13	13; 78; 23,0,24,16,15	0, 26, 13, 24	13, 23	3, 38, 2
13-7.4	60 14 22 26 7 11 19	13; 78; 23,0,15,40	0, 25, 13, 27	13, 23	4, 34, 2
13-7.5	60 15 22 39 19 46 21	13; 78; 22,30,18,8	0, 15, 28, 20	13, 22	5, 5, 5
13-7.6	56 11 22 7 35 19 45	13; 78; 21,16,36,0,5	0, 18, 21, 24	13, 21	6, 14, 6
13-7.7	60 15 22 39 19 41 26	13; 78; 20,36,18,4	0, 14, 28, 24	13, 20	7, 1, 7
13-7.8	60 14 19 7 37 26 11	13; 78; 20,18,24,16	0, 19, 19, 25	13, 20	8, 16, 7
13-7.9	60 14 22 38 11 19 25	13; 78; 20,18,24,16	0, 19, 20, 24	13, 20	9, 17, 7
13-7.10	56 28 14 38 50 23 13	13; 78; 20,12,42,4	0, 18, 20, 28	13, 20	10, 13, 7
14-8.1	60 14 22 11 19 7 13 21	14; 91; 25,0^2,48,0,18	0, 39, 16, 48	14, 25	1, 42, 1
14-8.2	60 14 22 26 7 11 19 13	14; 91; 25,0^2,36,30	0, 38, 17, 52	14, 25	2, 40, 1
14-8.3	60 14 19 7 37 26 11 13	14; 91; 19,16,24,12,20	0, 30, 25, 44	14, 19	3, 17, 3
14-8.4	60 14 22 38 11 19 25 7	14; 91; 19,16,15,36,5	0, 29, 26, 46	14, 19	4, 15, 3
14-8.5	56 11 22 7 35 19 45 28	14; 91; 18,16,36,16,5	0, 26, 29, 48	14, 18	5, 11, 5
14-8.6	60 15 22 39 19 46 21 43	14; 91; 16,34,24,12,5	0, 23, 38, 38	14, 16	6, 5, 6
14-8.7	60 14 22 38 58 11 19 25	14; 91; 16,34,24,12,5	0, 23, 40, 36	14, 16	7, 6, 6
14-8.8	56 28 14 38 50 23 13 27	14; 91; 16,28,42,0,5	0, 22, 40, 41	14, 16	8, 2, 6
14-8.9	60 14 22 38 11 19 35 25	14; 91; 16,24,27,24	0, 25, 30, 50	14, 16	9, 9, 6
14-8.10	60 14 22 38 11 19 37 31	14; 91; 16,18,45,12	0, 24, 31, 54	14, 16	10, 7, 6

Table 14. AENPs of d_1, d_2 and d_{10} in Example 4.

${}^{\#}_{i}C_j^{(k)}(d_1)$	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
$i=0$	1	1	1	1	$0^6,1$	$0^8,1$	1	1	$0,1$	1
$i=1$	9	9	9	$1,0^2,8$	$0^4,8,0^3,1$	$0^3,8,0^2,1$	$1,0^3,8$	$1,8$	9	$8,1$
$i=2$	36	36	$8,24,0,4$	$4,0,24,0,8$	$4,0^2,8,24$	$0^4,32,0^3,4$	$0^2,24,8,4$	$12,0,24$	$28,8$	36
$i=3$	84	$60,24$	$28,32,24$	$0,24,24,36$	$0^3,32,48,0^3,4$	$4,0^2,24,56$	$4,0,24,32,24$	$32,24,24,0,4$	$52,32$	84
$i=4$	$120,6$	$86,40$	$54,24,48$	$14,0,48,32,32$	$0^2,24,80,0,6,0,16$	$8,0^2,32,72,0,8,0,6$	$22,0,48,24,32$	$38,32,48,0,8$	$96,30$	$118,8$
$i=5$	$118,8$	$96,30$	$38,32,48,0,8$	$22,0,48,24,32$	$8,0^2,32,72,0,8,0,6$	$0^2,24,80,0,6,0,16$	$14,0,48,32,32$	$54,24,48$	$86,40$	$120,6$
$i=6$	84	$52,32$	$32,24,24,0,4$	$4,0,24,32,24$	$4,0^2,24,56$	$0^3,32,48,0^3,4$	$0,24,24,36$	$28,32,24$	$60,24$	84
$i=7$	36	$28,8$	$12,0,24$	$0^2,24,8,4$	$0^4,32,0^3,4$	$4,0^2,8,24$	$4,0,24,0,8$	$8,24,0,4$	36	36
$i=8$	$8,1$	9	$1,8$	$1,0^3,8$	$0^3,8,0^2,1$	$0^4,8,0^3,1$	$1,0^2,8$	9	9	9
$i=9$	1	$0,1$	1	1	$0^8,1$	$0^6,1$	1	1	1	1
${}^{\#}_{i}C_j^{(k)}(d_2)$	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
$i=0$	1	1	1	1	$0^7,1$	$0^7,1$	1	1	1	$0,1$
$i=1$	9	9	9	$2,0^3,7$	$0^3,7,0^3,2$	$0^3,7,0^3,2$	$2,0^3,7$	9	$0,9$	9
$i=2$	36	36	$15,0,21$	$0,21,0,14,0^3,1$	$1,0^3,35$	$1,0^3,35$	$0,21,0,14,0^3,1$	$0,15,0,21$	36	36
$i=3$	84	$56,28$	$28,49,0,7$	$7,0,42,28,0^2,7$	$7,0^2,28,49$	$7,0^2,28,49$	$0,7,0,42,28,0^2,7$	$28,49,0,7$	$56,28$	84
$i=4$	$119,7$	$91,35$	$42,56,0,28$	$21,28,0,56,21$	$0^2,21,84,0^2,21$	$0^3,21,84,0^2,21$	$21,28,0,56,21$	$42,56,0,28$	$91,35$	$119,7$
$i=5$	$119,7$	$91,35$	$42,56,0,28$	$21,28,0,56,21$	$0^3,21,84,0^2,21$	$0^2,21,84,0^2,21$	$21,28,0,56,21$	$42,56,0,28$	$91,35$	$119,7$
$i=6$	84	$56,28$	$28,49,0,7$	$0,7,0,42,28,0^2,7$	$7,0^2,28,49$	$7,0^2,28,49$	$7,0,42,28,0^2,7$	$28,49,0,7$	$56,28$	84
$i=7$	36	36	$0,15,0,21$	$0,21,0,14,0^3,1$	$1,0^3,35$	$1,0^3,35$	$0,21,0,14,0^3,1$	$15,0,21$	36	36
$i=8$	9	$0,9$	9	$2,0^3,7$	$0^3,7,0^3,2$	$0^3,7,0^3,2$	$2,0^3,7$	9	9	9
$i=9$	$0,1$	1	1	1	$0^7,1$	$0^7,1$	1	1	1	1
${}^{\#}_{i}C_j^{(k)}(d_{10})$	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$
$i=0$	1	1	1	1	$0^{14},1$	1	1	1	$0,1$	1
$i=1$	9	9	9	$1,0^6,8$	9	$0^7,8,0^6,1$	9	$1,8$	9	$8,1$
$i=2$	36	36	$8,0^2,28$	36	$0^7,8,28$	36	$0^4,28,0^2,8$	36	$28,8$	36
$i=3$	84	$28,56$	84	$0^3,28,0^2,56$	84	$0^7,56,28$	84	$0,56,0^2,28$	84	84
$i=4$	$112,14$	126	$14,56,0^2,56$	126	$0^6,56,56,0^5,14$	126	$14,0^3,56,0^2,56$	126	$56,70$	126
$i=5$	126	$56,70$	126	$14,0^3,56,0^2,56$	126	$0^6,56,56,0^5,14$	126	$14,56,0^2,56$	126	$112,14$
$i=6$	84	84	$0,56,0^2,28$	84	$0^7,56,28$	84	$0^3,28,0^2,56$	84	$28,56$	84
$i=7$	36	$28,8$	36	$0^4,28,0^2,8$	36	$0^7,8,28$	36	$8,0^2,28$	36	36
$i=8$	$8,1$	9	$1,8$	9	$0^7,8,0^6,1$	9	$1,0^6,8$	9	9	9
$i=9$	1	$0,1$	1	1	1	$0^{14},1$	1	1	1	1

References

Ai, M.Y., Zhang, R.C., 2004. Theory of minimum aberration blocked regular mixed factorial designs. J. Statist. Plann. Inference 126, 305–323.

Ai, M.Y., Zhang, R.C., 2004. s^{n-m} designs containing clear main effects or two-factor interactions, Statist. Probab. letters 69, 151–160.

Ai, M.Y., Zhang, R.C., 2004. Multistratum fractional factorial split-plot designs with minimum aberration and maximum estimation capacity, Statist. Probab. letters, 69, 161–170.

Box, G.E.P., Hunter, J.S., 1961. The 2^{k-p} fractional factorial designs. Technometrics 3, 311–351 and 449–458.

Chen, J., 1992. Some results on 2^{n-k} fractional factorial designs and search for minimum aberration designs. Ann. Statist. 20, 2124–2141.

Chen, H., Cheng, C.S., 2006. Doubling and projection: A method of constructing two-level designs of resolution IV. Ann. Statist. 34, 546–558.

Chen, B.J., Li, P.F., Liu, M.Q., Zhang, R.C., 2005. Some results on blocked regular 2-level fractional factorial designs with clear effects. J. Statist. Plann. Inference, in press.

Chen, H., Hedayat, A.S., 1996. 2^{n-l} designs with weak minimum aberration. Ann. Statist. 24, 2536–2548.

Chen, H., Hedayat, A.S., 1998. 2^{n-m} designs with resolution III and IV containing clear two-factor interactions. J. Statist. Plann. Inference 75, 147–158.

Chen, J., Wu, C.F.J., 1991. Some results on s^{n-k} fractional factorial designs with minimum aberration or optimal moments. Ann. Statist. 19, 1028–1041.

Cheng, C.S., Mukerjee, R., 1998. Regular fractional factorial designs with minimum aberration and maximum estimation capacity. Ann. Statist. 26, 2289–2300.

Cheng, C.S., Steinberg, D.M., Sun, D.X., 1999. Minimum aberration and model robustness for two-level factorial designs. J. Roy. Statist. Soc. Ser. B 61, 85–93.

Cheng C.S., Tang B., 2005. A general theory of minimum aberration and its applications. Ann. Statist. 33, 944–958.

Franklin, M.F., 1984. Constructing tables of minimum aberration p^{n-m} designs. Technometrics 26, 225–232.

Fries, A., Hunter, W.G., 1980. Minimum aberration 2^{k-p} designs. Technometrics 22, 601–608.

Mukerjee, R., Wu, C.F.J., 2001. Minimum aberration designs for mixed Factorials in terms of complementary sets. Statist. Sinica 11, 225–239.

Suen, C.Y., Chen, H., Wu, C.F.J., 1997. Some identities on q^{n-m} designs with application to minimum aberrations. Ann. Statist. 25, 1176–1188.

Sun, D.X., 1993. Estimation capacity and related topics in experimental designs. PhD dissertation. University of Waterloo, Waterloo.

Tang, B., Ma, F., Ingram, D., Wang, H., 2002. Bounds on the maximum number of clear two-factor interactions for 2^{m-p} designs of resolution III and IV. Canad. J. Statist. 30, 127–136.

Tang, B., Wu, C.F.J., 1996. Characterization of minimum aberration 2^{n-k} designs in terms of their complementary designs. Ann. Statist. 25, 1176–1188.

Wu, C.F.J., Chen, Y., 1992. A graph-aided method for planning two-level experiments when certain interactions are important. Technometrics 34, 162–175.

Wu , C.F.J., Hamada, M., 2000. Experiments: Planning, Analysis, and Parameter Design Optimization. Wiley, Now York.

Wu, H.Q., Wu, C.F.J., 2002. Clear two-factor interaction and minimum aberration. Ann. Statist. 30, 1496–1511.

Yang, G.J., Liu, M.Q., Zhang, R.C., 2005. Weak minimum aberration and maximum number of clear two-factor interactions in $2_{\rm IV}^{m-p}$ designs. Sci. China Ser. A 48, in press.

Yang, J.F., Li, P.F., Liu, M.Q., Zhang, R.C., 2005. $2^{(n_1-n_2)-(k_1-k_2)}$ fractional factorial split-plot designs containing clear effects. J. Statist. Plann. Inference, in press.

Zhang, R.C., Park, D.K., 2000. Optimal blocking of two-level fractional factorial designs. J. Statist. Plann. Inference 91, 107–121.

Zhang, R.C., Shao, Q., 2001. Minimum aberration $(s^2)s^{n-k}$ designs. Statist. Sinica 11, 213–223.

Zhao, S.L., Zhang, R.C., 2005. 4^m2^n designs with resolution III or IV containing clear two-factor interaction components. Proceedings of the Fifth Eastern Asia Symposium on Statistics and Its Applications, 187–196.

Zhu, Y., Zeng P., 2005. On the coset pattern matrices and minimum M-aberration of 2^{n-p} designs. Statist. Sinica 15, 717–730.