

# 第5章 线性规划的what-if分析

继续研究伟恩德公司案例	5.2
只有一个目标函数系数变动	5.3–5.9
目标函数系数同时变动的的影响	5.10–5.17
单个约束条件变化的影响	5.18–5.23
约束右端值同时变动的情形	5.24–5.26

# 伟恩德公司案例（what-if分析之前）

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			\$3,600

# 用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$200	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			\$3,400

门的利润从300美元增加到200美元，而最优解不变

# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$500	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			\$4,000

门的利润从300美元增加到500美元，而最优解不变

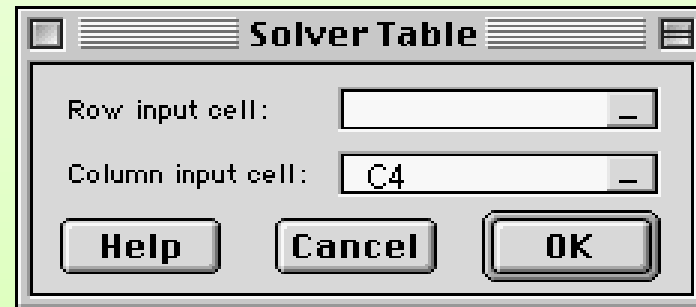
# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$1,000	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	4	<=	4
8	Plant 2	0	2	6	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	4	3			\$5,500

门的利润从300美元增加到1000美元，而最优解变了

# 运用Solver Table 进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit P	Used			Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Pro
12	Units Produced	2	6			\$3,600
13						
14						
15						
16	Unit Profit	Optimal Units Produ	Total			
17	for Doors	Doors	Windows	Profit		
18		2	6	\$3,600		
19	\$100					
20	\$200					
21	\$300					
22	\$400					
23	\$500					
24	\$600					
25	\$700					
26	\$800					
27	\$900					
28	\$1,000					



Select these cell (B18:E28) before choosing the Solve Table.

	C	D	E
16	Optimal Units Produced		Total
17	Doors	Windows	Profit
18	=DoorsProduced	=WindowsProduced	=TotalProfit

# 运用Solver Table 进行敏感性分析

	B	C	D	E
16	Unit Profit	Optimal Units Produced		Total
17	for Doors	Doors	Windows	Profit
18		2	6	\$3,600
19	\$100	2	6	\$3,200
20	\$200	2	6	\$3,400
21	\$300	2	6	\$3,600
22	\$400	2	6	\$3,800
23	\$500	2	6	\$4,000
24	\$600	2	6	\$4,200
25	\$700	2	6	\$4,400
26	\$800	4	3	\$4,700
27	\$900	4	3	\$5,100
28	\$1,000	4	3	\$5,500

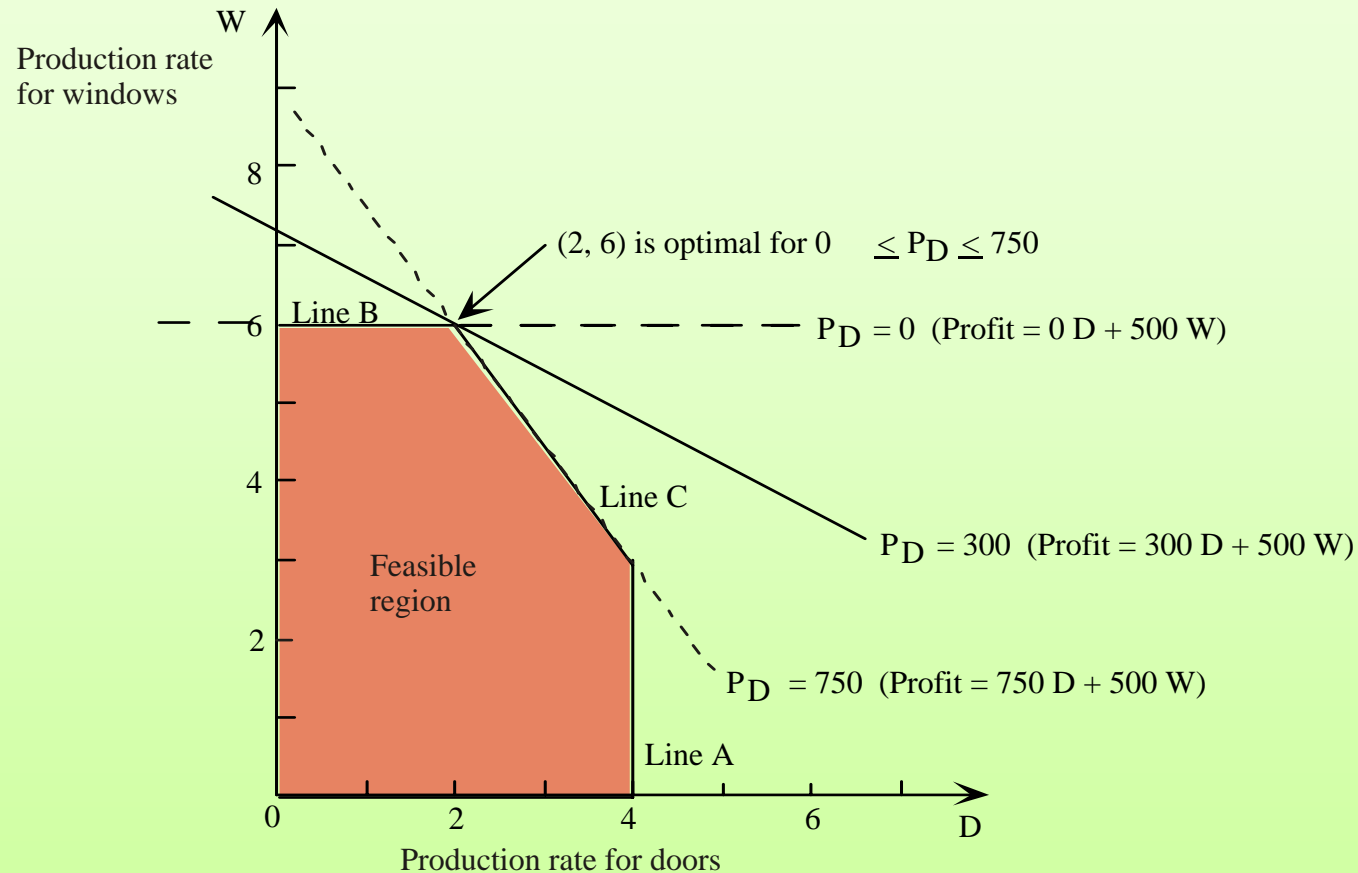
# 运用敏感度报告来寻找允许变化范围

Adjustable Cells

<b>Cell</b>	<b>Name</b>	<b>Final Value</b>	<b>Reduced Cost</b>	<b>Objective Coefficient</b>	<b>Allowable Increase</b>	<b>Allowable Decrease</b>
\$C\$12	Units Produced Doors	2	0	300	450	300
\$D\$12	Units Produced Windows	6	0	500	1E+30	300



# 运用图形来洞察可变范围



通过约束边界的两条虚线分别表示PD在允许取值范围的上下限处的目标函数， $0 \leq PD \leq 750$ 。

# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$450	\$400			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			\$3,300

门、窗的单位利润分别被改为450美元、400美元，但是最优解不变

# 应用电子表格进行敏感性分析

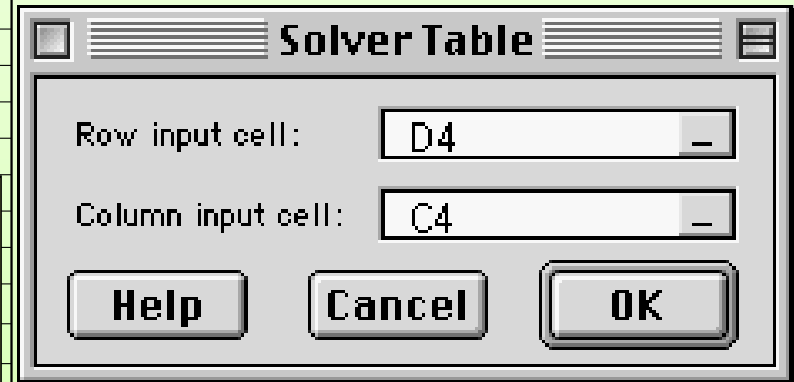
	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$600	\$300			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	4	<=	4
8	Plant 2	0	2	6	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	4	3			\$3,300

门、窗的单位利润分别被改为600美元、300美元，从而最优解改变

# 运用Solver Table 进行敏感性分析

	B	C	D	E	F	G	H	I	
3		Doors	Windows						
4	Unit Profit	\$300	\$500						
5				Hours		Hours			
6		Hours Used Per U		Used		Available			
7	Plant 1	1	0	2	<=	4			
8	Plant 2	0	2	12	<=	12			
9	Plant 3	3	2	18	<=	18			
10									
11		Doors	Windows			Total Profit			
12	Units Produced	2	6			\$3,600			
13									
14									
15									
16	<b>Total Profit</b>	Unit Profit for Windows							
17		\$3,600	\$100	\$200	\$300	\$400	\$500		
18		\$300							
19	Unit Profit	\$400							
20	for Doors	\$500							
21		\$600							

Select these cells (C17:H21), before choosing the Solver Table.



	C
17	=TotalProfit

# 运用Solver Table 进行敏感性分析

	B	C	D	E	F	G	H
16	<b>Total Profit</b>			Unit Profit for Windows			
17		\$3,600	\$100	\$200	\$300	\$400	\$500
18		\$300	\$1,500	\$1,800	\$2,400	\$3,000	\$3,600
19	Unit Profit	\$400	\$1,900	\$2,200	\$2,600	\$3,200	\$3,800
20	for Doors	\$500	\$2,300	\$2,600	\$2,900	\$3,400	\$4,000
21		\$600	\$2,700	\$3,000	\$3,300	\$3,600	\$4,200

# 运用Solver Table 进行敏感性分析

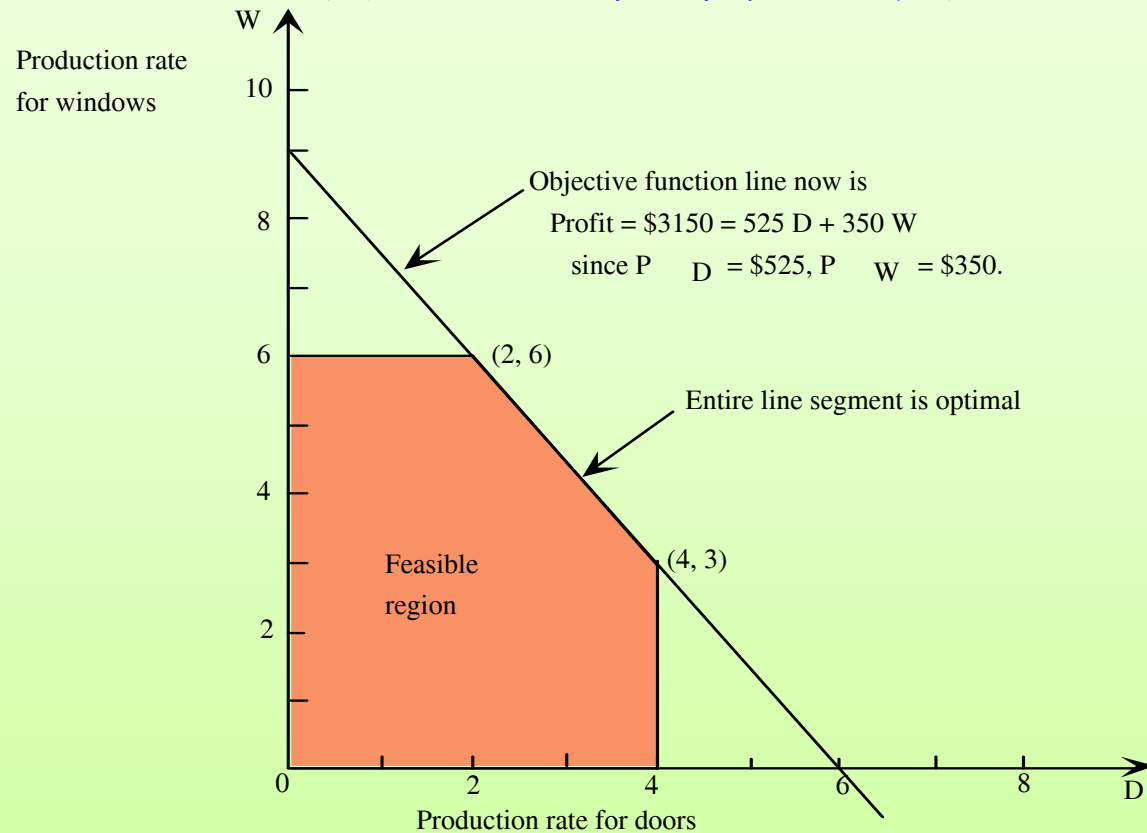
	B	C	D	E	F	G	H
24	<b>Units Produced (Doors, Windows)</b>		Unit Profit for Windows				
25		(2, 6)	\$100	\$200	\$300	\$400	\$500
26		\$300	(4, 3)	(4, 3)	(2, 6)	(2, 6)	(2, 6)
27	Unit Profit	\$400	(4, 3)	(4, 3)	(2, 6)	(2, 6)	(2, 6)
28	for Doors	\$500	(4, 3)	(4, 3)	(4, 3)	(2, 6)	(2, 6)
29		\$600	(4, 3)	(4, 3)	(4, 3)	(4, 3)	(2, 6)

	C
25	= "(" & DoorsProduced & ", " & WindowsProduced & ")"

# 百分之百法则

如果目标函数的系数同时变动，计算出每一系数变动量占该系数允许变化范围允许变动量的百分比，而后，将各个系数的变动百分比相加，如果所得的和不超过100%，最优解不会改变，如果超过100%，则不能确定最优解是否改变。

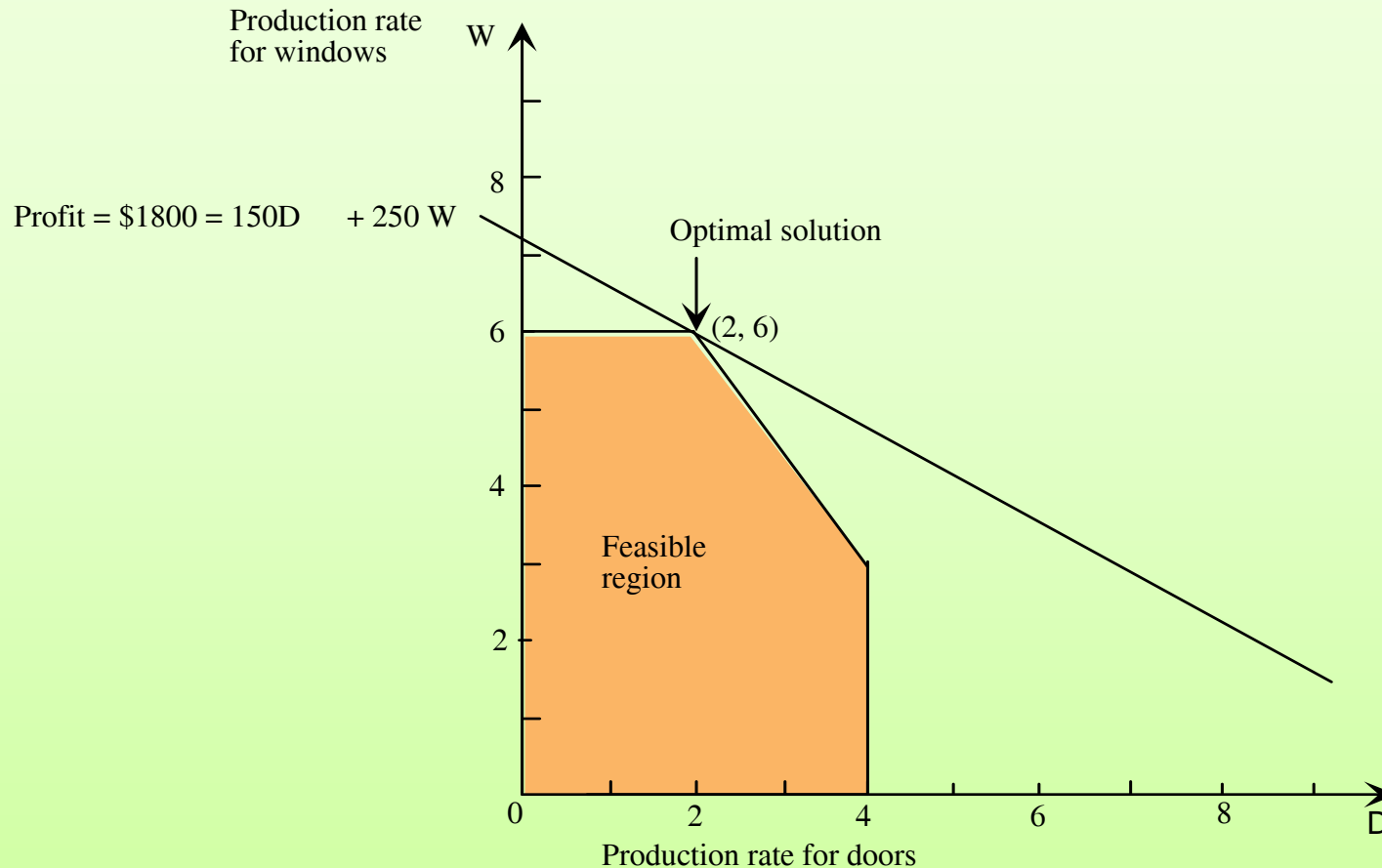
# 用图形来看百分比法则



当门和窗的单位利润的估计值分别改为525美元和350美元，刚好处在百分比法则所允许的临界点上， $(D, W) = (2, 6)$  还是最优解。



# 用图形来看百分比法则



当门和窗的单位利润的估计值分别改为150美元、250美元（最初解的一半时），用作图法可知 $(D, W) = (2, 6)$ 还是最优解，尽管百分比法则表示最优解有可能变动。

# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	1.667	<=	4
8	Plant 2	0	2	13	<=	13
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	1.667	6.5			\$3,750

- ◆ 将工厂 2 的可变时从12小时增加到 13 小时，总利润增长150美元

# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	0	<=	4
8	Plant 2	0	2	18	<=	18
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	0	9			\$4,500

将工厂 2 的可用工时从13增加到18小时，总利润将增加750美元  
（也就是每小时增加150美元）

# 应用电子表格进行敏感性分析

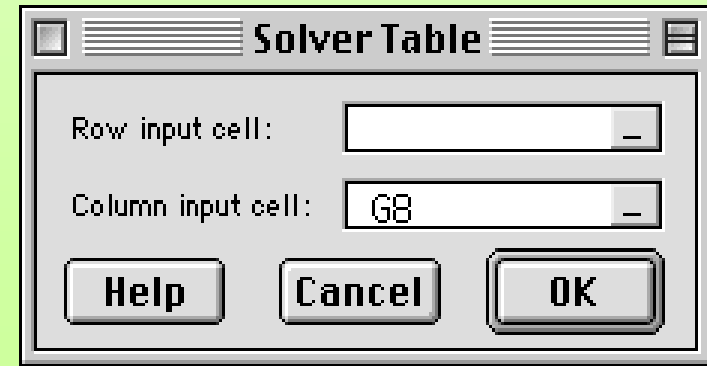
	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	0	<=	4
8	Plant 2	0	2	18	<=	20
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	0	9			\$4,500

再增加 2 个工时，从18小时增长为20小时，总利润没有变化，因为最优解没有用到这些多余工时。

# 运用Solver Table 进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	4
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			\$3,600
13						
14						
15						
16	Time Available	Optimal Units Produced		Total	Incremental	
17	in Plant 2 (hours)	Doors	Windows	Profit	Profit	
18		2	6	\$3,600		
19	4	4	2	\$2,200	\$250	
20	5	4	2.5	\$2,450	\$250	
21	6	4	3	\$2,700	\$150	
22	7	3.667	3.5	\$2,850	\$150	
23	8	3.333	4	\$3,000	\$150	
24	9	3	4.5	\$3,150	\$150	
25	10	2.667	5	\$3,300	\$150	
26	11	2.333	5.5	\$3,450	\$150	
27	12	2	6	\$3,600	\$150	
28	13	1.667	6.5	\$3,750	\$150	
29	14	1.333	7	\$3,900	\$150	
30	15	1	7.5	\$4,050	\$150	
31	16	0.667	8	\$4,200	\$150	
32	17	0.333	8.5	\$4,350	\$150	
33	18	0	9	\$4,500	\$150	
34	19	0	9	\$4,500	\$0	
35	20	0	9	\$4,500	\$0	

Select these cells (B18:E35), before choosing the Solver Table.



# 运用敏感性报告

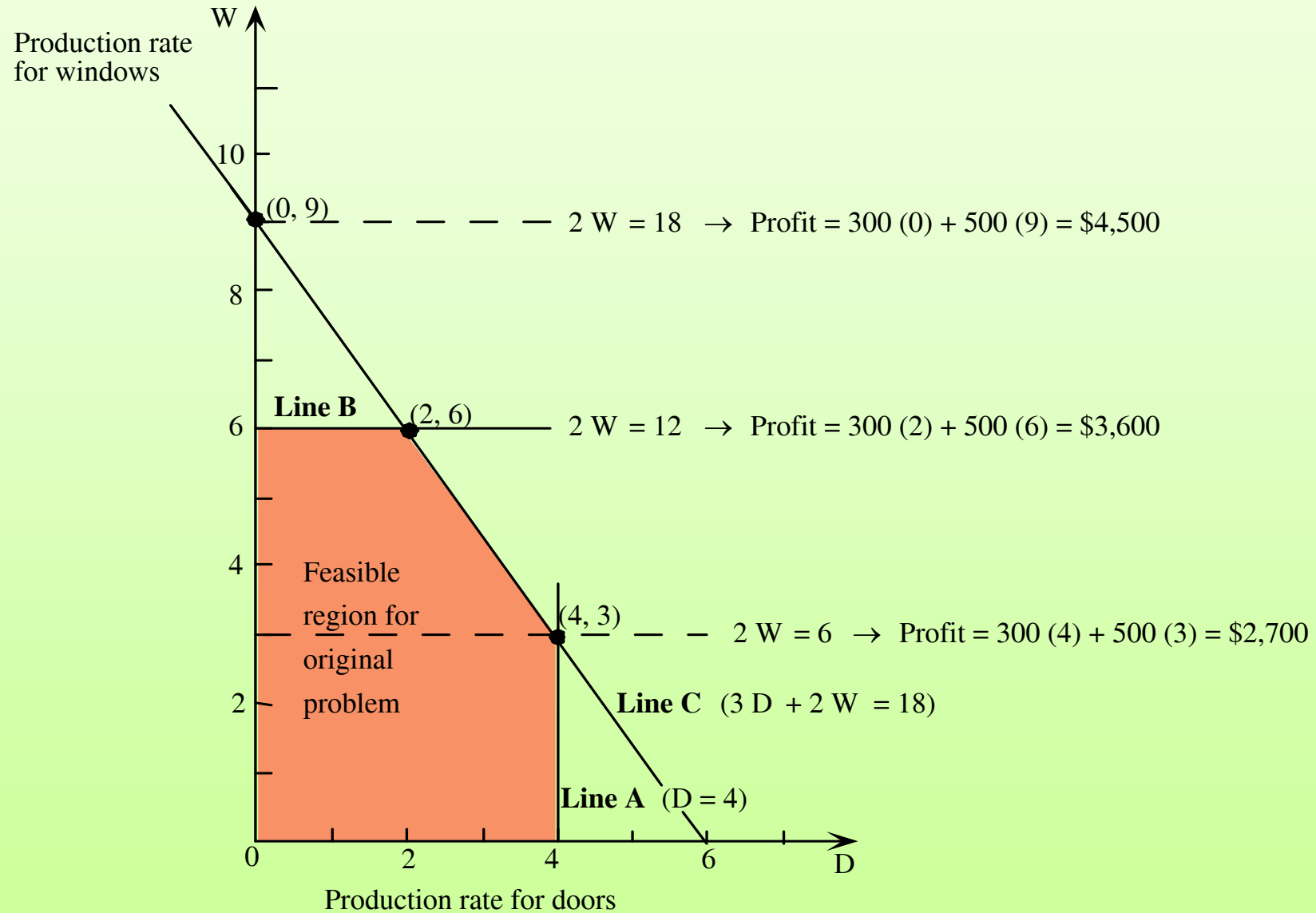
## Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$12	Units Produced Doors	2	0	300	450	300
\$D\$12	Units Produced Windows	6	0	500	1E+30	300

## Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$7	Plant 1 Used	2	0	4	1E+30	2
\$E\$8	Plant 2 Used	12	150	12	6	6
\$E\$9	Plant 3 Used	18	100	18	6	6

# 图形对可行域的解释



# 应用电子表格进行敏感性分析

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	\$300	\$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	1.333	<=	4
8	Plant 2	0	2	13	<=	13
9	Plant 3	3	2	17	<=	17
10						
11		Doors	Windows			Total Profit
12	Units Produced	1.333	6.5			\$3,650

- ◆ 其中一小时的工作时间从工厂 3 移到工厂 2，总利润每星期增加 50 美元



# 运用Solver Table 进行敏感性分析

	B	C	D	E	F	G	H
3		Doors	Windows				
4	Unit Profit	\$300	\$500				
5				Hours		Hours	
6		Hours Used Per Unit Produced		Used		Available	
7	Plant 1	1	0	2	<=	4	Total (Plants 2 & 3)
8	Plant 2	0	2	12	<=	12	30
9	Plant 3	3	2	18	<=	18	
10							
11		Doors	Windows			Total Profit	
12	Units Produced	2,000	6			\$3,600	
13							
14							
15							
16							
17	Time Available	Time Available	Optimal Units Produced		Total	Incremental	
18	in Plant 2 (hours)	in Plant 3 (hours)	Doors	Windows	Profit	Profit	
19			2	6	\$3,600		Select these cells (C19:F26), before choosing the Solver Table.
20	12	18	2	6	\$3,600	\$50	
21	13	17	1.333	6.5	\$3,650	\$50	
22	14	16	0.667	7	\$3,700	\$50	
23	15	15	0	7.5	\$3,750	\$50	
24	16	14	0	7	\$3,500	-\$250	
25	17	13	0	6.5	\$3,250	-\$250	
26	18	12	0	6	\$3,000	-\$250	

# 百分百法则

- ◆ 约束右端同时变动的百分百法则：同时改变几个或所有函数约束的约束右端值，如果这些变动的幅度不大，那么可以用影子价格预测变动产生的影响。为了判断这些变动的幅度是否允许，计算每一变动占可行域所允许的增加值或减少值的百分比，如果所有的百分比和不超过 100%，那么，影子价格还是有效的，如果所有的百分比之和超过 100%，那就无法确定影子价格是否有效。