Preparing Teachers to Foster Algebraic Thinking

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Abstract: The purpose of this article is to share the conceptual framework and beginning analyses of data from a teacher professional development program that focuses on cultivating teachers' understanding of algebraic thinking, learning, and teaching. Specifically, in this paper we share: (1) the conceptual framework that has guided the structure of the professional development program and research agenda, and (2) an initial set of findings from the first component of the program. These findings illustrate strategies for developing community among teachers, as well as the potential of using a professional learning community as a context for fostering teacher learning.

Kurzreferat: Ziel dieses Beitrags ist, den konzeptionellen Rahmen und erste Analysen von Daten aus einem Fortbildungsprogramm für Lehrpersonen darzustellen, das darauf ausgerichtet ist, das Verständnis von Lehrpersonen für algebraisches Denken, Lernen und Lehren auszubilden. Insbesondere stellen wir vor: Erstens den konzeptionellen Rahmen, der die Struktur des Fortbildungsprogramms für Lehrpersonen geleitet hat sowie den Forschungsablauf und zweitens erste Erkenntnisse aus dem ersten Programmteil. Diese ersten Erkenntnisse zeigen zum einen Strategien für die Entwicklung von Lehrergemeinschaften auf und zum anderen das Potential der Verwendung von Lehrergemeinschaften für Lehrerfortbildung.

ZDM-Classification: B50, H20

1. Introduction

Much can be said about ways in which the mathematics education community has narrowed the distance between the vision, and the actual practice, of reform-based teaching and learning in our schools. Yet, recent writings in our field continue to indicate how difficult it can be for many teachers, and in particular American teachers, to embrace, understand, and implement these pedagogical and curricular reforms (Heaton 2000; Ma 1999; Stigler & Hiebert 1999). These reports suggest that there is much work to be done regarding the professional development of mathematics teachers (Remillard & Geist 2002). As Cooney (1988) suggested 15 years ago, "reform is not a matter of paper but a matter of people". (p. 355) This statement still rings true today, and is a reminder of the importance of carefully considering how our community can continue to support the development of teachers' mathematical and pedagogical content knowledge.

The purpose of this article is to share the conceptual framework and beginning analyses of data from a teacher professional development program[1] that focuses on cultivating teachers' understanding of algebraic thinking, teaching, and learning. Specifically, this paper outlines: (1) the theories that have guided the structure of the professional development program, and (2) initial findings from the first cycle of the program that illustrate the promise this design holds for impacting teachers' content knowledge of algebra, as well as their knowledge about the teaching of algebra.

2. Conceptual framework: Designing professional development for the teaching of algebra

Our professional development program and research are framed by a *situative* perspective on teacher learning. This framework connects two constructs that are central components of the program. The first of these constructs—knowledge for teaching—has long been cited as paramount to teacher change. The second construct—teacher learning communities—has enjoyed considerable attention in recent years as researchers and teacher educators alike have acknowledged the impact of sociocultural factors upon teacher learning.

2.1 A situative view of teacher learning

Two views of knowing and learning have captivated the interests of researchers and teacher educators in mathematics education throughout the past decade (Cobb 1994). The first of these trends is the widely accepted notion that learners actively construct ways of knowing as they strive to reconcile present experiences with already existing knowledge structures. In recent years, numerous scholars have advanced arguments, both theoretical and empirical, to promote *constructivist* theories of learning.

This wide acceptance of constructivism can be juxtaposed with a second trend in mathematics education that emphasizes the socially and culturally situated nature of mental activity (Cobb 1994). An equally large body of research supports the notion that participation in social and cultural settings is the catalyst for cognitive development (Nunes 1992). This perspective views learning as changes in participation in socially organized activity (Lave & Wenger 1991), and individuals' use of knowledge as an aspect of their participation in social practices (Greeno 2003). Several theorists have referred to the learning process as one of *enculturation* (Cobb 1994; Driver, Asoko, Leach, Mortimer, & Scott 1994).

Cobb (1994) addressed the perceived "forced choice" (p. 13) between constructivist and sociocultural theories of learning. Along with Driver and colleagues (1994), Cobb argued that learning must be viewed, at least in part, as a process of enculturation *and* construction. "The critical issue", Cobb stated, "is not whether students are constructing, but the nature or quality of those socially and culturally situated constructions…. Learning should be viewed as both a process of active individual construction and a process of enculturation into the …

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practices of wider society". (p. 13) He described the "extremely strong" relationship between social and psychological elements of learning by noting that the relationship "does not merely mean that the two perspectives are interdependent. Instead, it implies that neither perspective exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective". (p. 64)

It is our premise that, just as *children* learn as a process of both construction and enculturation, *teachers* learn in a similar fashion. Cooney (1994) challenged teacher educators to begin thinking in this way a decade ago when he remarked that:

"Teacher development consists of teachers developing a deeper knowledge of children's mathematical thinking, but in the context of the 'community'…. The realization that teachers, as well as their students, are cognizing subjects leads to research questions that focus on how and under what conditions teachers become adaptive agents as well as cognizing agents". (p. 613)

Fennema and Franke (1992) similarly discussed the implications of a *situated* view of teacher learning (i.e., a view that recognizes the contextual influences on knowledge construction) for mathematics professional development. As they suggested:

"The entire construct of situated knowledge is so new that a research paradigm to substantiate it has yet to develop or to be applied to the study of teachers. However, it holds great promise for increasing our understanding of learners' knowledge, and perhaps even greater potential for increasing our understanding of teachers' knowledge. This model has many implications for teacher education, both pre- and in-service". (p. 160)

Since these visionary statements of a decade ago, a growing number of theorists and researchers have promoted *situative* views of teacher learning and professional development. Adler (2000), for example, characterized teacher learning as "a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching". (p. 37) Putnam and Borko (2000) noted that how a teacher learns a particular set of knowledge and skills, and the situations in which a teacher learns, are fundamental to what is learned. Thus, in order to understand teacher learning, we must study it within multiple contexts, and consider both teachers as individual learners (i.e., construction of knowledge) and the social contexts within which they participate in their own professional growth and development.

Both the professional development program that is the subject of this article and the research initiatives that were designed to study it, are firmly rooted in a situative perspective on teacher learning. It is from this perspective that we now move to discuss the two primary constructs that framed the design and research of our program.

2.2 Construct one: The central role of community in teacher learning

A century ago, Dewey (1904) noted the tendency of teachers to "accept without inquiry or criticism any method or device which seems to promise good results. Teachers, actual and intending, flock to those persons who give them clear-cut and definite instructions as to just how to teach this or that". (p. 321) As Frykholm (1998) has suggested, perhaps the most powerful antidote to this form of teacher education and development is to situate teacher learning within communal contexts.

"Only when teachers continually find themselves in discussions about learners, pedagogy, mathematics and reform—when they, out of habit, develop a critical consciousness about teaching—only then will they be able to interrupt the traditional expositional model that has been perpetuated for decades in mathematics classrooms". (p.320)

A number of scholars who ascribe to situative theories of learning have identified *community* as an important ingredient for teacher learning. Cooney's (1994) challenge to mathematics teacher educators was to recognize that "teacher development consists of teachers developing a deeper knowledge of children's mathematical thinking, but *in the context of the 'community'"*. (italics added; p. 613) More recently, Little (2002) noted that "strong professional development communities are important contributors to instructional improvement and school reform". (p. 936) Several notable professional development projects (e.g., QUASAR; Community of Teacher Learners Project) based on notions of community have emerged in the last decade, providing incentive for the work described in this article (Grossman, Wineburg, & Woolworth 2001; Stein, Silver, & Smith 1998; Stein, Smith, Henningsen, & Silver 2000; Wineburg & Grossman 1998).

This research suggests that participation in a community might be a prerequisite for meaningful professional development. At the same time, these studies reveal that the development of teacher communities is difficult and time-consuming work. Norms that promote challenging yet supportive conversations about teaching are some of the most important features of successful learning communities. However, although conversations in professional development settings are easily fostered, discussions that support critical examination of teaching are relatively rare. Such conversations must occur if teachers are to collectively explore ways of improving their teaching and support one another as they work to transform their practices (Ball 1990, 1996; Frykholm 1998; Wilson & Berne 1999).

2.3 Construct two: Teachers' mathematical and pedagogical knowledge

The National Council of Teachers of Mathematics (2000) *Principles and Standards* document states that "teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks". (p. 17) Over 25 years of research have indicated that teachers do not typically possess this rich and connected knowledge of mathematics (Ball 1990; Brown & Borko 1992; Brown, Cooney, & Jones 1990; Lloyd & Frykholm 2000; Mewborn 2003). This may be particularly true within the domain of algebra with its many related and layered constructs (Knuth 2002; Nathan & Koedinger 2000; van Dooren, Verschaffel, & Onghena 2002).

Ma's (1999) comparison of American and Chinese teachers revealed significant weaknesses in the mathematical understandings of typical American teachers. She also articulated a framework from which to evaluate the depth and richness of teachers' content knowledge by examining their "profound understanding of fundamental mathematics … awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instill those basic attitudes in students". (p. 120) As Ma illustrated powerfully throughout her book, the "knowledge package" (p. 113) of most American teachers is fragmented, shallow, and an inadequate base from which to see (and illuminate) connections between procedures and their underlying concepts. Other researchers have linked these limited conceptions of mathematics to classroom *practice*. There is substantial evidence to suggest that teachers' understandings and conceptions of mathematics, mathematics teaching, and students' learning influence their pedagogical decisions and impact teaching practices (Brophy 1991; Fennema et al. 1992; Frykholm 1996; Putnam & Borko 1997; Stigler et al. 1999; Thompson 1992; Weiss 1995). Taken together, these findings point to the significant amount of work yet to be done to enhance the quality and depth of professional development for American teachers.

3. The STAAR Summer Algebra Institute

The pilot professional development component of the STAAR Project began with a two-week institute entitled "Facing the Unknown", designed to strengthen teachers' understanding of central algebraic concepts and to help them begin to explore ways of fostering their students' algebraic thinking. The institute, offered through the Education and Applied Mathematics departments at a large state university, included 60 contact hours of meeting time. It was designed to address four major goals directly related to the guiding principles of the project:

(1) Support the development of teachers' understanding of key algebraic concepts (e.g., representational fluency, equality, functional reasoning);
(2) Support the development of teachers' knowledge about the teaching of algebra (e.g., innovations in curricula, pedagogical strategies, research on student thinking);
(3) Create a professional learning community;
(4) Provide an opportunity for teachers to learn mathematics in a reform-oriented setting.

This article focuses primarily on two of these goals. Specifically, we highlight the ways in which the instructors created a professional learning community with the teachers, and how this community contributed to the development of participants' knowledge of algebra.

3.1 Participants

Sixteen teachers from three different school districts participated in the institute. Most of them were teaching at the middle school level, although three were elementary school teachers. Their classroom teaching experience ranged from 0 to 27 years, although the majority had relatively little experience teaching middle school algebra.

The institute was taught collaboratively by two mathematics educators: Mary Ellen Pittman and Mary Nelson. The instructors were advanced doctoral students in the School of Education. Both had mathematics teaching experience at the secondary and tertiary levels, as well as experience teaching mathematics professional development courses. Both had been members of the STAAR Project team for two years prior to teaching the institute, and are co-authors of this article.

3.2 Institute structure and activities

The institute was structured around four major types of activities: solving mathematical problems; examining children's thinking; reading and discussing current literature; and reflecting on one's own learning. The teachers worked collaboratively with their colleagues throughout the institute as they addressed a wide range of algebra problems (often from contemporary curricular programs). The professional developers selected these problems because they addressed the key algebraic concepts and central mathematical ideas of the program, were open-ended, were situated in real-life contexts, and allowed for multiple solution strategies (and sometimes multiple solutions). The class frequently focused on a single problem for 30 to 60 minutes, first working in small groups and then sharing their solution strategies with the whole class.

The instructors typically provided only enough information about the problems to get the participants started, thereby encouraging them to take ownership of the problems, discuss solution strategies without perceived constraints or "preferenced" approaches, and "reinvent" significant mathematics as the contexts allowed. Several activities focused explicitly on examinations of students' thinking, including samples of work from middle school students, and videos of students engaged in problem-solving tasks.

The teachers read a number of journal articles and book chapters throughout the institute, wrote brief reactions to the readings, and participated in class discussions. These readings were posted on the institute Web site. Some of the discussions occurred in online threaded discussions. On three occasions, teachers discussed the readings in class—each time using a somewhat different format.

3.3 Data collection and analysis

A wide range of data was collected to document teachers' learning experiences in the summer institute, and to trace its impact on their knowledge, beliefs, and practices. Every institute session was videotaped with two cameras. During whole-class activities, one camera followed the lead instructor for the activity while the other maintained a wider shot of the classroom. When teachers worked in small groups, each camera was trained on one group. The two videographers kept extensive daily notes, using a spreadsheet developed specifically for the project. Videotaped interviews with the instructors at the end of each day focused on their plans for, and reflections on, the day's activities. Copies of the participants' written work, including problem solutions, daily reflections, and initial and final papers, were collected throughout the institute. Before and after the institute the teachers completed a mathematics assessment and an interview about their beliefs regarding algebra teaching and learning.

As a first step in data analysis, a member of the research team developed a chronological summary of all activities that occurred during the institute, based on the videotaped record and field notes. This summary included information such as a description of the activity, duration, location on the videotape(s), and codes to indicate the existence of evidence relevant to our initial research questions. This catalog was then used as a tool to coordinate further data analysis. For example, we used it to help identify episodes that occurred during the institute, which we could analyze with respect to the goals of creating community and developing algebra content knowledge. The two vignettes shared in this article are products of that process, reflecting our agreement with Miles and Huberman (1994) that vignette analyses are appropriate for examining in-depth specific features of an event, while simultaneously preserving the complexity and richness of the context they reflect.

4. Teacher learning within community

We developed the vignettes and analyses that follow with the two primary goals for this paper in mind: to illustrate the ways in which the instructors created a professional learning community with the teachers, and to show how this community contributed to the development of participants' knowledge of algebra.

4.1 Creation of a professional learning community

The instructors in the STAAR summer institute worked carefully and systematically to create a professional learning community with the teachers. Our discussion of their efforts focuses on four features of classroom life that are fundamental to establishing and maintaining a successful learning community: safe environments, rich tasks, students' explanations and justifications, and shared processing of ideas (Cobb, Boufi, McClain, & Whitenack 1997; Silver & Smith 1997). These features are as relevant to learning communities for teachers as they are to K-12 mathematics classrooms (Sherin 2002; Silver et al. 1997).

All four features were evident in the ways that the instructors structured activities throughout the institute. They were especially prominent during the initial activities, when creating discourse norms and establishing trust were central to the instructors' goals and intentions. The first vignette, derived from an activity that occurred on the first day of the institute, is illustrative of these goals. The activity featured the Cutting Sidewalks problem, an open-ended task that Mary Ellen adapted from *The I Hate Mathematics! Book* (Burns 1975). Because it was the first mathematics problem assigned during the summer institute, the goals of establishing a safe environment and creating a culture to support the sharing of ideas were uppermost in Mary Ellen's mind as she selected the task and planned the activity.

4.2 Day One: The Cutting Sidewalks problem

It was only the first day of the institute, but Mary Ellen and Mary were already engaged in what would be a well-worn routine by the end of the course: the use of simple questions to prod teachers' mathematical thinking. The first problem of the first day—"Cutting Sidewalks"—required the teachers to consider how they could cut a square or rectangle segment of an imaginary sidewalk into pieces using just one straight line. The teachers had been working for the last five minutes in small groups, drawing lots of lines, crumpling up lots of paper and chatting amongst themselves. Now it was time to talk about their work with the whole class.

"Will you always get two pieces, no matter how you draw the line?" Mary Ellen asked as she placed a transparency of the problem on the overhead. The teachers looked around the room at one another, and one teacher, Marissa, stood up and strode to the front of the room.

"Get used to coming to the overhead", Mary Ellen continued as she took a seat at the back of the classroom, "because I don't talk up here very often". She gestured toward the overhead, "Now—this is my sidewalk—is it always in two pieces? If not, show me". Marissa—who was comfortable in front of the class and eager to share her group's perspective—turned to face the class and began to share how their discussion had unfolded.

"It depends on how you determine it", she said. "You have to define it, because there's really no definition". She took the overhead marker in her hand and began drawing lines on the transparency to demonstrate the group's ideas, looking up occasionally to glance at her classmates. "You could just draw your line right on top of one of the edges. She drew one line along the top edge of the rectangle and paused, reconsidering. "Or, if you just draw it in one place you still get one piece. If you define it that way you have to go from one end to the other end, then you would always get two parts". She had drawn two lines inside the rectangle that did not touch any edge, and a diagonal line extending from the top-right corner to the bottom left. Satisfied with her explanation, she put down the marker, smiled at her group and bounced back to her seat. The room was quiet for a moment. Maybe that was all there was to it, after all.

"Kirsten's got a puzzled look on her face", said Mary Ellen from her roost at the back of the room. "Kirsten, what were you thinking?" Kirsten was not counting on her puzzled look giving her away—but with some reluctance she entered the conversation. She explained that her group, unlike Marissa's, had not considered the possibility of a line segment that did not touch the edges of the rectangle. Mary Ellen prodded her to say more with yet another simple question: "So, what did you guys talk about?"

This wasn't the only answer, then. Karla, another group member, chimed in. "We talked about if it was right on the line—but we didn't think about those segments—if it doesn't reach from end to end. If it doesn't reach from end to end it doesn't necessarily cut it into two pieces". Mary Ellen nodded her head and looked around the room. She continued probing, "Does anyone else have thoughts on this?"

After some additional discussion, the class agreed upon rules and definitions that would enable them to continue working on the problem, namely that the lines should be drawn in such a way that a single line will always cut a rectangle into two pieces. Mary Ellen watched this decision sink in—and then posed another simple question: "Suppose you were to add another straight line. How many pieces will you get?" The teachers looked at one another. "Remember, try to find different ways to do this. Can you make a different number of pieces?"

The room erupted in a buzz of conversation as the teachers dove into solving the new problem. After five minutes of independent work, more teachers went up to the overhead to share how their groups had cut the rectangle. Everyone agreed that with two lines they could make three or four pieces. "So", said Mary Ellen, "you can think about these numbers as the minimum and the maximum". She had saved the actual mathematical terminology until the teachers had experienced the problem and worked it out together. But that wasn't the end of the problem. After a few seconds of silence, Mary Ellen posed the final part: "How many pieces will you get if you cut the sidewalk with three, four, or five—or more—lines? Can you cut the sidewalk using four lines and get every number of pieces between the minimum and the maximum? Does it always work?"

This time the teachers worked for almost 30 minutes, with varying degrees of success, progress and frustration. Mary Ellen visited each group and asked the teachers to write their ideas—correct and incorrect—on a transparency to share with the whole class. In the discussion that ensued, representatives from all four groups presented their findings at the overhead. Mary Ellen first called on a group that had gone down an incorrect path and, at her prompting, had written that incorrect idea on their transparency. The group's representative explained their thinking by saying that initially they did not realize that any rectangle can be divided into a minimum and maximum number of pieces, as well as all numbers of pieces between minimum and maximum. Mary Ellen interjected that, while they felt certain that they had found a pattern, when they repeatedly tested this "theory" they discovered their error. This was a valuable lesson for the entire class: "As teachers, you can help your students move forward if they find themselves in a similar type of situation".

The representative from the next group began her presentation. "My first idea was totally wrong", she said. The teachers in the class nodded in sympathy. For many of them, the experience of being totally wrong was familiar. "See?" said Mary Ellen, "we're going to get comfortable about going down dead-end roads".

The teacher continued. "I thought there was a pattern such that the difference between the minimum and the maximum number of pieces is always twice the previous difference—but in conversation with the rest of the group I saw that this wasn't right after all; the correct pattern is that the current maximum is equal to the previous maximum plus the current number of lines". Again, the teachers nodded. They had similar conversations in their own groups.

The next representative got up and shared that group's method of drawing physical representations to prove that they had found the maximum number of pieces. The last group developed the idea further, showing their formula for the maximum number of pieces based on a pattern for the maximum number of line intersection points. Mary Ellen ended the problem by engaging the class in discussion on the terms "recursive" and "direct".

The pedagogical decisions that Mary Ellen made while implementing the Cutting Sidewalks problem, as well as the ways in which she had the students vary their participation on the task (e.g., large-group discussion, small-group discussion, individual presentations), were seen repeatedly throughout the summer institute. We highlight these pedagogical patterns in this discussion as central features of the instructors' approach to molding the professional community.

4.3 Fostering mathematical conversations

Having taught the problem previously to both teachers and young students, Mary Ellen knew that it was open to various interpretations, and that its multiple entry points would allow individuals with varying mathematical backgrounds to begin working on it on their own. Given these qualities, collaborative groups would uncover more solution strategies than any individual teacher was likely to discover on his or her own.

One of Mary Ellen's intentions for this problem was to begin the task of creating an environment in which the teachers would feel safe to explore unknown mathematical terrain—complete with "dead-end paths" as well as successful strategies. Thus, she highlighted semantic issues (e.g., what does "cut" mean) in the first whole-class discussion in order to foster debate while de-emphasizing the "correctness" of ideas. She initiated the final class discussion by calling on various groups to share ideas and strategies—both correct *and* incorrect. She used their errors both to foster deeper exploration of the mathematics and to encourage them to consider what they would do *as teachers* if their students made similar errors. These techniques allowed Mary Ellen to navigate the slippery slope inherent in discussing challenging mathematical content while at the same time building trust and creating norms for supportive, yet critical, conversations.

4.4 Fostering mathematical justifications

This initial problem-solving activity was also designed to establish the expectation that the teachers explain and justify their solution strategies. During small-group work, Mary Ellen moved from table to table, encouraging the teachers to write down their "working theories", thus ensuring that they would have material to share during the whole-class discussions. She then built on this preparation, calling on group representatives to explain and justify their work for the class.

Mary Ellen used a variety of techniques to encourage the teachers to actively process one another's ideas. She typically asked the teachers to restate, clarify, or elaborate on their responses to make sure that other members of the class understood. She also purposefully sequenced the order of group presentations so that the teachers could make connections and build on one another's ideas.

As this analysis of the Cutting Sidewalks activity indicates, Mary Ellen designed and orchestrated the activity to incorporate all four features of classroom life that are fundamental to creating a professional learning community. She selected a rich task with multiple solution strategies, created a safe environment within which teachers could explore this task, and called upon the teachers to share their explanations and justifications and to process and build upon one another's ideas.

4.5 Teachers' algebraic knowledge and reasoning

The Graphing Geometric Patterns activity depicted in the second vignette occurred on the fourth day of the summer institute. This activity followed close upon the heels of a day in which the teachers conducted "experiments", collecting data of various sorts and displaying their findings in tables and graphs. The conversations about these findings and the graphs the teachers made led to discussions about features of functions such as linearity, slope, continuity, scale, *y*-intercept, and representational fluency. The vignette provides a snapshot of how linearity and slope were addressed through the Graphing Geometric Patterns activity.

4.6 Day four: Graphing geometric patterns

Mary and Mary Ellen distributed a handout that contained five geometric pattern problems, three of which the teachers had worked on during the second day of the institute. Although the teachers had created tables and derived direct and recursive formulas for those three problems, they had not yet graphed them. The first problem, the X-Dot problem, presents a dot pattern in which the first figure has one dot, the second figure makes an "x" with five dots, and the third figure makes an "x" with nine dots. It asks how many dots would be in the hundredth figure.

Mary Ellen introduced the activity. "I want you to begin by individually creating a table and graph for each of these patterns, and then work in groups as you move toward the new patterns". Sensing the teachers' uncertainty, she asked, "What type of information would you need in order to make these graphs?" The teachers looked at her quizzically, as if they weren't sure what she was asking. "This is a strange question, I know", Mary Ellen said. Marissa responded that two pieces of related information were needed for the first problem: the figure number and the number of dots. Satisfied with this response, Mary Ellen sent the teachers off to work on the activity in small groups for approximately 45 minutes. Mary requested that each group draw one of their solutions (a table and accompanying graph) on a large sheet of paper and post it on a wall in the classroom for all their peers to see.

Mary began a discussion by asking which of the graphs were linear and which were nonlinear. By a show of hands, the teachers agreed about the first four graphs, but there appeared to be some lingering confusion regarding the fifth graph. Mary noticed that the scale used on the fifth graph was misleading and may have caused the teachers to erroneously assume it was linear, but she chose not to tackle that graph just yet. Instead she asked if any of the posted graphs looked different than the ones the teachers made on their own. This question led the teachers to consider the issue of scale, noting, for example, that unequal intervals on the x- *and* y-*axes can skew the visual image of a graph. Mary next asked the teachers to make a prediction about the slope of the first graph (the graph of the X-Dot problem) and then to find the slope. She pushed them, "Tell me what you think the slope is and tell me why you think that's the slope".*

Marci replied, "I thought the slope on [graph] number one was going to be 4 because that's how much it changes for each number on the table". Mary paused for a minute and restated Marci's answer: "Marci's prediction is 4. She thinks if you go from here to here [pointing to the y *column of the table]… the* y *is increasing by 4, so the slope is 4. Anybody else?"*

Marissa agreed with Marci that the slope was 4 but then emphasized the need to look at the relationship between x *and* y. *"I'd agree with her that the slope is 4", she paused, "but I'd also have to say that because there is a relationship between the figure number and the number of dots, you need to look at the 4 for the number of dots, but you also have to look at what's going on with the figure number, which is going up a positive 1… What's important is the relationship, the rise over the run". Marissa went on to describe how one might determine the slope using both the graph and the table. "There is my [understanding of the] relationship of slope with the table as well as the graph".*

"Okay", said Mary. "Class—can you think of any other methods for determining the slope of a line? Think about the proportional concept of slope—does it matter which two data points you use or which direction on the line you travel?"

After an extended discussion, Mary asked the class to consider whether these same methods and rules apply to a nonlinear graph. Focusing their attention on one graph that showed a nonlinear set of data points, she drew a line between two of the points and noted that the slope of that line was 2. Several teachers observed that the slope was 2 only for that particular section of the graph. Building on this idea, Mary connected two different points on the same graph, noting that now the slope was 4. She asked, "So, what is that telling me about the graph?" A teacher piped up excitedly, "That it's not linear". Mary Ellen continued, "Why is it not linear?"

Another teacher explained, "Because the slope changed, and you cannot have three points [with different slopes] in a line. Your line is bending, and that's not a line". Nodding, Mary pointed to one of the linear graphs. "In this case, did it matter what points I chose?" The class replied in unison, "No".

Mary summarized this new realization of the relationship between slope and linearity: "A line has a constant slope that does not vary, no matter which two points you examine". She concluded, "That is why we say that two points determine a line".

Returning to the fifth graph, Mary Ellen asked the class if they could now determine whether or not it was linear and then provide evidence for their answer. The teachers discussed this question in small groups for a few minutes and came to quick agreement that it was nonlinear. In the *whole-class discussion that ensued, they provided evidence that the slope was not constant. The class reexamined a graph of data from one of the experiments conducted the previous day on the basis of their newfound understanding of linearity and slope.*

Mary and Mary Ellen chose this progression of graphing activities based on their assessment of the teachers' prior understanding of functional relationships (as evidenced in their work with the experimental data the teachers had previously collected). Although the vignette only highlights the conversations about slope and linearity, the discussion of the geometric patterns also examined issues such as the y-intercept, discrete versus continuous, scale, and representational fluency. These topics were key themes throughout the institute. In this particular episode the instructors pushed the teachers to draw connections among these constructs, as well as to other mathematical topics.

As Ma's (1999) analysis of elementary school mathematics reminds us, concepts such as slope and linearity should be viewed as fundamental components of algebra. When the teachers began the institute, they knew that the slope of a line could be found by taking any two points and using the formula $m = (y_1 - y_2) / (x_1 - x_2)$. As they began to analyze the experimental data during the day prior to the vignette, it became apparent that they did not have the underlying conceptual understanding to match their comfort with the procedure. The vignette illustrates how Mary and Mary Ellen skillfully guided them toward developing this understanding.

Mary Ellen and Mary chose problems for group work that could be represented and solved in various ways. In the discussion of the X-Dot problem, Marci pointed out how she used the table to determine the slope. Mary clarified this approach without necessarily validating it as the "right" method, which led others to examine and build upon Marci's ideas. For example, Marissa brought up the importance of examining the relationship between the figure number and the number of dots. Mary then used Marissa's ideas to introduce the importance of considering slope as a ratio, and to extend to other ideas involving functional relationships. This discussion ultimately returned to the initial question of the teaching episode, "Which of these graphs are linear?" Through their exploration of the graphs the teachers came to understand that a constant slope is the critical factor in determining linearity.

5. Teacher learning and teacher change: Some initial observations

Analyses of pre- and post-institute algebra content tests and interviews, teachers' daily reflections, and their final papers provide initial evidence that the summer institute had an impact on participating teachers. We expect that ongoing analysis of observations of the teachers' classes and of interviews about their beliefs and instructional practices conducted throughout the ensuing school year will provide additional support for our assertions, detailed below, based on these data sources.

5.1 The algebra content knowledge assessment

All 16 teachers were given an assessment of content knowledge on the first and last days of the institute. The assessment consisted of 27 contextually based problems designed to evaluate their understanding of several foundational topics in algebra including variable, equality, pattern recognition, representational fluency, and systems of equations. There was a modest difference in the scores on these identical tests between the pretest (average score: 21.25) and posttest (average score: 25.48).

A second analysis of the content knowledge assessment reflected the institute's emphasis on using multiple methods and representations to solve problems. On the pretest, only one problem elicited multiple strategies from the teachers. On the posttest, however, teachers presented multiple solution strategies on nine problems. A scoring rubric to calculate the number of strategies a teacher used to solve a given problem revealed 350 strategies on the pretest (an average of 21.9 strategies per teacher) and 483.5 strategies on the posttest (an average of 30.2 strategies per teacher), thus indicating the teachers' growing ability to think of problem solutions in multiple ways.

5.2 Teachers' self-reports of course impact

The three self-report data sources (reflections, final course papers, interviews) revealed teachers' impressions about the institute's impact on their content knowledge, mathematics-specific pedagogical knowledge, and recognition of the importance of community.

5.2.1 Content knowledge

The teachers' self-reports provide additional evidence that the summer institute had a positive impact on their knowledge of algebra for teaching. Most teachers commented about their new understanding of specific algebraic topics. Some reported that they learned new techniques for solving particular types of mathematics problems, and that they noticed mathematical connections of which they had previously been unaware. For example, Carmen wrote in her final paper, "I can now easily move between representations and use one representation to find another. Also, when presented with an algebraic relationship, I can choose which representation I want to create and use to solve for an unknown or discover a function rather than having no choice but to revert back to old rules or algorithms that I never fully understood in the first place". Another student reflected, "By putting together the pieces of algebra that were sitting as separate entities in my head, I will be much more able to show my students the interrelationships that exist".

Several teachers commented that they had gained confidence as doers of mathematics, in part because they had confronted gaps in their own understandings. As Linda stated in her final paper, "Now that I have taken this class, which was relearning algebra and its components, I feel empowered. I also know that to be a better teacher for my students I need to learn why rules work and not just 'This is the rule'". Carmen commented similarly, "I can't believe that in my entire math career I never understood why $y = mx + b$ existed.… However, now that we had several days of looking at relationships between two variables, then using tables to find a formula to describe the relationship, I finally get it. The formula describes the relationship. I finally get it".

5.2.2 Mathematics-specific content knowledge

Several teachers reflected on the value of specific instructional strategies, representations, and curricular materials that they experienced as students in the institute, and on their intentions to use these tools with their own students. For example, Kim wrote, "I will use many of the activities we did in this class when I begin the teaching of functions…. I will use problems like the bridge problem so that they can tie their symbolic strategies, which they have a fairly good grasp of, to the graphical and pictorial, just like I was able to [do]". Some, like Karla, commented about the power of multiple strategies: "I liked seeing all the different methods for helping kids understand exponents. It makes it much clearer to me…. I have a couple of ways to help explain why we multiply by the reciprocal when we divide, instead of just telling the kids".

Several teachers reflected on their increased understanding of how students think about, experience, and come to understand mathematics. They attributed this understanding to an awareness of their own learning throughout the course. One teacher noted, "My thinking about how students learn algebra has been altered. Now, I see that elaborating on the informal methods greatly impacts the understanding of the formal process…. Being one of those people that totally 'gets' formal algebraic concepts, during this course, struggling with some of the informal methods was a new experience. In actuality, I'm glad that I had trouble with some of those ideas; it helped me gain perspective on what some kids go through when they struggle with math". This teacher, like others, developed a deeper appreciation of the importance of informal strategies as a stepping-stone to more formal strategies.

5.2.3 Importance of community

Many of the teachers commented that the strong community within the summer institute facilitated their learning and gave them skills to establish similar communities in their own classrooms. They described specific techniques that seemed to foster a strong sense of community, such as peer collaboration and facilitation of whole-class discussions. Celia wrote in her final paper, "I think the most valuable thing from this class has been the small and large group interaction. I have learned the most from my peers. This is also how children learn. When students interact in a learning environment, all participants benefit. Sharing solutions with small and large groups invited us to see problems in a different way". Similarly, Katie wrote, "From my experience in this class, it reflects how people, younger and older, learn. Students learn in a 'safe' environment".

The frequent use of group work in the summer institute seemed to be particularly powerful, and many teachers expressed an interest in experimenting with similar instructional formats. Peter wrote in one of his daily reflections, "I started thinking about how group dynamics affect math learning. I will spend more time this coming school year helping students figure out how to work with one another in groups…. Giving students a chance to speak up (in small groups and whole-class work) also validates a student and gives them confidence". Ken wrote about the value of working in groups with mixed ability levels: "[This class] reinforces the idea that we need to heterogeneously group our math students because there needs to be someone with these strengths in groups with other kids that think on a different path".

6. Concluding thoughts

In this paper we highlighted some positive, albeit preliminary, findings from our professional development program. As stated at the onset, our intent was to use a portion of our data to point to what we believe are two key ingredients for any successful professional development endeavor. Specifically, this paper shared our efforts to develop (and research) a community-centered model for teacher learning and, within this collaborative setting, to enhance teachers' knowledge of algebra. We conclude the paper with some thoughts about strengths of the model as well as some questions that we continue to debate as we further develop the professional development program.

First, we acknowledge the concern that this professional development model, like any similar two-week experience, is subject to questions about long-term impact. The research literature has, for years, indicated that short-term professional development models are ineffective in the long run (Putnam & Borko 1997, Wilson & Berne 1999). What makes us confident, then, that this model is worth pursuing? To begin, we note that this paper reports only on the first phase of the professional development we provided our teachers. Following this two-week experience, the teachers were invited to continue the program into the following school year. Yet, even without considering the follow-up components of this program, we remain confident that the institute had meaningful impact.

It is our contention that the success of any professional development relies on the willingness and ability of teachers to experience change—changes in beliefs, in knowledge, and in attitude. Under what circumstances, then, might we expect these kinds of changes to occur? The literature tells us a few things already. Teachers' resistance to change is legendary. Change is unlikely to take place in isolation. Middle-grade teachers' discomfort with mathematics may be one of the more significant and identifiable impediments to change (Frykholm 2004). With these insights in mind, we attempted to design an experience for teachers that would allow them to gently embrace changes in their content and pedagogical knowledge of algebra with the security of knowing that they were not doing it alone.

Our review of the literature suggested several features of classroom life that are fundamental to establishing a successful learning community: safe environments, rich tasks, students' explanations and justifications, and shared processing of ideas. As the Cutting Sidewalks vignette indicated, by consciously structuring activities to incorporate these features, Mary Ellen and Mary established a vibrant community. Further, their selection of mathematics tasks and teaching strategies reflected important criteria for addressing content knowledge identified in the literature, such as contextually based problems, multiple entry points, multiple solution strategies, and discussions about informal, pre-formal, and formal algebraic strategies. The Graphing Geometric Patterns vignette illustrates the power of these tasks and strategies for helping teachers deepen their mathematical knowledge and reasoning.

We believe that an additional, and perhaps unique, strength of our model is the symbiotic relationship between the two primary goals—community and mathematics understanding. We know that for a community to develop true character and vitality, it must do so around something of value and meaning. We believe that the challenge we offered participants in the summer institute—to improve their understanding of algebra for teaching—provided such value and meaning. We also know that when teachers face their own limitations in content knowledge, it can be intimidating. What makes this kind of personal work and development possible is the support that comes from others on similar trajectories. Hence, as the mathematical challenges helped to foster the integrity of the community of teacher-learners, it was the support of peers that enabled many of our teachers to reach new plateaus in their understanding of mathematics.

We have several questions about this program as we consider how it might be improved and expanded. One challenge has been to imagine how the model might be brought to some larger scale. Is it possible to foster the same sort of intimacy among a larger group of teachers? Among a group of teachers working from remote locations? How dependent is this model on the pedagogical strategies used? On the skills and temperaments of the instructors? That is, might this model ultimately be constrained by the ability of the instructor to navigate the professional development terrain, establishing a vibrant community of teacher-learners while drawing upon a deep understanding of the nuances in mathematics to lead participants to greater insight about algebra, teaching, and learning? Finally, what kinds of supports must be in place for teachers over time, enabling them to build upon the growth they experienced in this professional development program?

We pursue answers to these questions as we continue to refine the professional development model. We invite others in the mathematics education community who are similarly interested in improving the teaching and learning of algebra to engage with us in the important study of these and other issues pertaining to teacher development in mathematics education.

7. References

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