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Strategies for Building Mathematical Communication in the Middle School Classroom: Modeled in Professional Development, Implemented in the Classroom

Karen K. Clark
University of Colorado at Denver and Health Sciences Center

Jennifer Jacobs
University of Colorado at Boulder

Mary Ellen Pittman
University of Colorado at Boulder

Hilda Borko
University of Colorado at Boulder

Abstract

The mathematics education literature stresses the importance of establishing mathematics discourse communities in mathematics classrooms and suggests a number of specific strategies teachers can draw on to foster student communication (Chazan & Ball, 1999; NCTM, 2000; Silver & Smith, 1997). We present four of these strategies in detail: (1) posing rich tasks, (2) creating a safe environment, (3) asking students to explain and justify solutions, and (4) actively processing one another's ideas. We describe how these strategies were addressed in a professional development program for middle school mathematics teachers, and we offer a vignette that illustrates how one of the participating teachers implemented them in her eighth-grade classroom. Finally, we consider the potential impact that modeling communication strategies in professional development can have on teachers' classroom practice.

Educational researchers and business leaders in the United States underscore the need for students to learn how to effectively communicate their thinking both orally and in writing (National Middle School Association [NMSA], 2004; Secretary's Commission on Achieving Necessary Skills [SCANS], 1991; National Council of Teachers of Mathematics [NCTM], 1989, 2000; Cobb, Boufi, McClain, & Whitenack, 1997). For example, the National Middle School position report on middle level education advocates "learning experiences which use the full range of communications in purposeful contexts" (NMSA, 2004,). In addition, a report issued by the U.S. Department of Labor, entitled What Work Requires of Schools, specifies that the workforce will need to be able to "communicate thoughts, ideas, information, and messages in writing; and create documents such as letters ... and organize ideas and communicate orally" (SCANS, 1991, p. xviii). The report urges schools to emphasize the development of communication skills for America's youth.

Effective communication is now seen as a skill that middle school students should demonstrate in all subject areas, not just language arts and social science courses (Kist, 2003). Indeed, mathematics is increasingly seen as a field in which effective communication is essential as both a learning process and an outcome. Principles and Standards for School Mathematics (PSSM), a guide published by the National Council of Teachers of Mathematics outlining essential components for improving the quality of school mathematics programs, lists communication as one of the five process standards that students will need to function effectively in the twenty-first century. The PSSM document elaborates that communication is an essential part of mathematics and mathematics education because it is a "way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process helps build meaning and permanence for ideas and makes them public" (NCTM, 2000, p. 60).

In keeping with this emphasis on communication skills, recent educational research has stressed the importance of establishing mathematical discourse communities in mathematics classes. Discourse communities are those in which students feel free to express their thinking, and take responsibility for listening, paraphrasing, questioning, and interpreting one another's ideas in whole-class and small-group discussions. A number of teachers and researchers have offered suggestions about how to establish and maintain such communities (e.g., Chazan & Ball, 1999; Grouws & Cebulla, 2000; Kazemi, 1998; Silver & Smith, 1997). Cobb, Boufi, McClain, and Whitenack (1997) noted that it is critical for teachers to foster children's emerging abilities to participate in "reflective" and "collective" discourse, and to become skilled at supporting such conversations. They argued that "children actively construct their mathematical understandings as they participate in classroom social processes" (p. 264) and suggested that teachers guide conversations such that students play a prominent role in "stepping back" and making sense of the mathematical work that has taken place.

Unfortunately, classrooms that are characterized by mathematical discourse communities are not yet the norm in the United States (Ball, 1991; Stigler & Hiebert, 1999). For example, the TIMSS 1999 Video Study, an international survey of eighth-grade mathematics lessons, revealed that on average in the United States, the ratio of teacher-to-student words was 8:1, and 71% of student utterances were fewer than 5 words (Hiebert et al., 2003).

The STAAR Project

A central goal of the professional development component of the "Supporting the Transition from Arithmetic to Algebraic Reasoning" (STAAR) Project—the project featured in this article—was to facilitate teachers' learning of strategies for fostering mathematical communication in their middle school classrooms. In particular we focused on four strategies that appear to be fundamental to creating a mathematical discourse community: (1) rich tasks, (2) safe environments, (3) students' explanations and justifications, and (4) processing of ideas. Facilitators modeled these strategies in a content-focused professional development summer institute.

The next section of the article discusses some of the literature on these four strategies for building a discourse community. We describe the STAAR professional development program and briefly discuss how the four strategies were modeled during the summer institute. We then use a vignette to illustrate and analyze how one teacher carried out these strategies in her classroom.

We recognize that these four strategies are inextricably intertwined. Furthermore, teachers' instructional moves and communication decisions are naturally driven by the demands of specific contexts and cannot be prescribed or scripted. However, our intention is to promote awareness of particular communication strategies and provide images and interpretations of their enactment in a middle school classroom. Thus, for analytical purposes, we address each strategy separately.

Strategies for Establishing and Maintaining Mathematical Discourse Communities

Strategy 1: Posing rich tasks that promote discussion. Rich mathematical tasks are key ingredients in classrooms that have communication as a central goal (NCTM, 2000). Open-ended and challenging tasks that build on students' prior knowledge are conducive to discussions because they encourage students to think collaboratively and build upon one another's ideas (Stein, Smith, Henningsen, & Silver, 2000). Tasks should have multiple levels of access to enable students with different levels of background knowledge and mathematical abilities to work on them and to collaborate as they move through the solution process (Cohen, 1984). It is also desirable for tasks to have multiple exit points, so that students can complete the problem with varying degrees of sophistication (Fosnot & Dolk, 2001). Such tasks enable students, guided by the teacher, to make connections between various solutions and solution strategies, and to learn both important mathematical content and valuable communication skills.

The manner in which mathematical tasks are posed and problem-solving activities are structured also impacts how students solve the tasks and how they communicate their ideas about the solution (Stein et al., 2000). Teachers sometimes turn rich, complex problems into simpler ones for their students, and thereby remove opportunities for the students to discover mathematical solutions on their own (Stein, Grover, & Henningsen, 1996). One strategy that encourages students to work on and discuss challenging problems is to break them into smaller pieces. For example, students can work individually or in groups to tackle one component of a complex task, and then convene as a whole class to discuss that specific component before moving back to individual or group work to continue the problem. This process of dividing the task into manageable chunks enables students to be responsible for much of the mathematical work on challenging problems for which a larger amount of teacher guidance might otherwise be needed.

Strategy 2: Establishing and maintaining a safe environment. A safe environment for communication is vital to a successful mathematical discourse community (Lampert, 2001). An environment that is conducive to the sharing of ideas will enhance the quality and quantity of discussion, debate, and ideas that are publicly exchanged in a classroom (Brown & Campione, 1994). Of particular importance is establishing student talk as a classroom norm, both in small groups and during public sharing of ideas (Silver & Smith, 1997). Communication in small groups can be stimulated by purposeful grouping of students, continual encouragement to work and talk together, and reinforcement of the importance of each student's contributions (Brophy, 1999). During whole-class work, calling on struggling students or eliciting incorrect ideas can help to promote a feeling of safety in the classroom, as students come to understand that the teacher is not just looking for the correct answer but for students to justify and explain their methods for solving the task (McClain & Cobb, 2001). Incorrect ideas often can be particularly instructive because they offer the opportunity to explicitly discuss misconceptions and build on intuitive understandings.

Strategy 3: Asking students to explain and justify their thinking. The PSSM document calls on teachers to support their students' learning by encouraging students to explain and justify their mathematical thinking to their peers and teachers in a coherent and clear manner (NCTM, 2000). Establishing this type of inquiry environment in the mathematics classroom involves inviting students to share their strategies, pose questions, and "think out loud" (Cobb, Wood, Yackel, & McNeal, 1992; Grouws & Cebulla, 2000). By making their thinking public, students may have to negotiate the meaning of mathematical ideas with others, and to defend and justify their reasoning so that they can convince others of the legitimacy of their ideas. Through this process of negotiation and justification, students are often motivated to think more deeply about their own ideas and the ideas of their classmates (Bauersfeld, 1995; Yackel & Cobb, 1996).

Strategy 4: Encouraging students to actively process one another's ideas. Effective and meaningful discourse requires that students listen closely to the thinking of others, and that they process and understand one another's ideas (Brown & Campione, 1994). As Davis (1992) noted, "If we invite students to think, we have the obligation to take their ideas seriously" (p. 349). One aspect of taking students' ideas seriously is ensuring that their classmates attend to the ideas and work to understand them. Classroom activities should be structured to ensure that students have ample time and encouragement to process others' ideas, for example, by discussing them with the whole class or considering them in small groups (Grouws & Cebulla, 2000).

Modeling Communication Strategies in Professional Development

The STAAR Professional Development Program

Professional development programs that model and engage middle school teachers in thinking about effective communication strategies can play a central role in helping teachers learn to establish and maintain mathematical discourse communities in their classrooms. Our own experience with the STAAR professional development program leads us to conjecture that when teachers learn mathematical content effectively in a professional development context, and when they identify the mathematical discourse community within the program as a major factor in their learning, they are eager to establish similar communication practices in their own classrooms.

As one component of the STAAR Project, we conducted a professional development program for middle school mathematics teachers during the 2003–2004 academic year. Our goals for this professional development program included (a) supporting the development of teachers' knowledge of algebra, (b) supporting the development of teachers' knowledge about the teaching of algebra, (c) creating a professional learning community, and (d) providing an opportunity for teachers to learn mathematics in a reform-oriented setting (Borko et al., 2005). We designed the STAAR professional development program to include three complementary components: a summer algebra institute, ongoing monthly workshops, and monthly observations in each teacher's classroom.

Sixteen teachers enrolled in the two-week summer algebra institute entitled "Facing the Unknown," which met for a total of 60 hours during July 2003. The institute was jointly delivered by the School of Education and the Applied Mathematics Department at a large state university. Throughout the institute the instructors encouraged the teachers to reflect on their learning experiences and on the types of instructional strategies and discourse patterns that were being modeled. Eight of the 16 participants attended seven professional development workshops during the 2003–2004 school year.[1] The primary focus of the workshops shifted during the school year from algebraic content to mathematics-specific pedagogy in middle school classrooms. In both components the facilitators modeled strategies for fostering communication and promoting mathematical reasoning, and the teachers were expected to share their mathematical ideas in whole-group and small-group discussions.

The facilitators started the first day of the institute with a problem that had multiple entry points and was challenging enough to encourage participants to work collaboratively. They chunked the problem into several; sections in order to ensure access for learners with different levels of mathematical expertise and to scaffold the learning experience. The facilitators established a safe environment for discourse and set expectations for communication patterns. For instance, they had the teachers work in small groups from the outset, intentionally establishing a climate in which teachers were expected to look to their colleagues for assistance rather than to the facilitator.

Throughout the problem-solving activity, the facilitators demonstrated that the teachers' explanations and justifications could serve as key entry points for more extensive communication. They used openings such as "Let's talk about this" and "What do you think?" to initiate whole-group conversations that focused on the mathematical ideas and provided an invitation for wide participation. They asked increasingly specific questions to highlight the importance of mathematical reasoning. The facilitators also used several strategies to encourage teachers to listen to and process one another's ideas. They called upon teachers to share both correct and incorrect mathematical conceptions, and they encouraged them to explore, question, and clarify one another's ideas.. These characteristics were carefully documented through the collection and analysis of data in the STAAR professional development activities and in the teachers' own classrooms.

Data Collection and Analysis

An extensive set of data was collected during all three components of the STAAR professional development pilot program. We videotaped the entire summer institute and each monthly workshop, using at least two cameras at all times. Multiple cameras enabled us to focus simultaneously on the facilitator and on the participating teachers during whole-group activities. During small-group work, one camera followed the lead facilitator as she or he moved from group to group; additional cameras were trained on each of the small groups. This approach enabled us to capture the facilitators' use of strategies to promote discourse, and participants' communication patterns during small-group and whole-class activities.

The teachers were observed and videotaped in their mathematics classrooms on a monthly basis throughout the year. Data collected during classroom observations included videotapes, instructional materials, and copies of students' work. We used two cameras when videotaping in the classrooms, documenting both the teachers' instructional moves and the student interactions during whole class and small-group activities. In addition to providing rich sources of data about the teachers' learning, these records of practice formed a basis for discussion during the monthly workshops, and for teachers' individual reflections on their teaching and their personal goal(s) for improving classroom practice. The classroom activity featured in this article occurred during one of these observations.

In addition to observational data, we collected written reflections from the teachers throughout the summer institute and monthly workshops. Teachers used these reflections to describe their experiences during the professional development program and to consider the impact of these experiences on their instructional practices. We also conducted interviews with the participating teachers several times during the program. Interview questions addressed various aspects of the participants' experiences during the professional development program, including their thoughts and practices regarding mathematical discourse.

We chose to use vignette analysis for this paper in order to carefully examine and portray the ways in which a mathematical discourse community was established in one middle school teacher's classroom. To identify a teaching episode that was representative of the teachers' efforts to enact the discourse strategies modeled in the professional development program, the STAAR research team met and discussed the video records from the teachers' monthly observations. We selected a teacher who made a substantial effort to foster a discourse community within her classroom. We then identified a teaching episode that provided evidence of her efforts and wrote a corresponding vignette. The goal of the vignette was to capture specific features of this teacher's pedagogical practice in detail, while preserving the complexity and richness of the teaching episode and the classroom context in which it occurred (Miles & Huberman, 1994). We present the vignette followed by an analysis of the discourse strategies it illustrates. Our analysis draws on the videotaped lesson, as well as interviews conducted with the teacher, her written reflections, and her final paper for the summer institute. It focuses on the four strategies described in the beginning of this article.

One Teacher's Movement Toward Establishing a Discourse Community in a Middle School Mathematics Classroom

Many of the teachers who participated in the STAAR professional development program commented in interviews and written reflections that they hoped to establish discourse norms in their classrooms that were similar to those modeled in the summer institute. During our monthly observations of a subset of these teachers in the school year that followed the institute, we saw many attempts to create and maintain mathematical discourse communities. In this section, we introduce Pam Marsten (a pseudonym), a middle school mathematics teacher who took part in the STAAR professional development program. We describe Pam's teaching and professional development history, her reflections on the STAAR summer institute, and one of her lessons that illustrates her efforts to implement many of the recommended discourse strategies.

At the onset of the 2003–2004 academic year, Pam had taught for 27 years as a secondary school teacher. Pam taught mathematics in a "mountain" school whose student population consisted of just under 800 sixth through eighth graders, mostly Caucasian and middle class. Students' scores on the state's standardized test corresponded roughly to the state average. Pam was the most experienced classroom teacher in our project. She participated in all three components of the STAAR professional development program. She attended the summer algebra institute and all seven monthly workshops, and she was observed seven times in one of her eighth-grade pre-algebra classrooms over the course of the school year.

We do not claim that Pam learned the discourse strategies exhibited in her classroom entirely from the STAAR project. Rather, these are strategies Pam has been developing for a considerable length of time, with assistance from multiple sources. However, Pam was quite articulate about the impact the summer institute had on her, particularly with respect to these efforts. She wrote the following in her final paper:

> After spending many days talking to my peers in class about my ideas and theirs, it is glaring that justifying one's answer and being able to convince someone else of its validity is part of the stuff [of which] good mathematics is made….. Conversations about math bring it alive…. As I saw so clearly during my ten days in this class … it is my responsibility as a math facilitator to create an environment where math 'talk' or conversations take place.

In interviews during the school year following the summer institute, Pam continued to discuss its enduring influence and the changes that she experienced both in her beliefs and in her teaching practices. Specifically, Pam commented on the importance of student-driven communication in the classroom which, she believed, could be fostered by creating opportunities for small-group work prior to whole-class discussions. Not only did Pam experience the powerful impact of peer collaboration firsthand in the summer institute, but she thought deeply about how to bring some of these same communication structures into her own middle school classroom. In one interview she commented,

> I really loved the small groups in the summer course…We just talked to someone next to us or across from us…. I found that I was more likely to want to defend what I was thinking because I didn't feel threatened…. It also helped me get unstuck much of the time…. I found that to be really powerful and I thought, you know, that works with kids as well. Just talking about their thinking is a powerful technique.

Pam noted that the STAAR professional development program was instrumental in challenging her to reflect continually about and take risks with respect to the communication style in her classroom. In workshops and interviews throughout the school year, she commented that she used more small-group work and provided more effective facilitation of groups than she had done in previous years. In particular, she believed that she asked more targeted questions of the groups, and gave them more time to think and talk on their own before moving to large-group discussions and formal writing tasks. Statements such as the following capture Pam's general appraisal of the influence of the summer institute: "My classroom teaching has been so positively impacted by everything that I learned and everything that we did in the summer class."

In the vignette below we highlight one of Pam's lessons, in which she successfully put into practice many of the strategies recommended by the literature and modeled in the STAAR professional development program.

The Painted Cubes Problem: A Vignette

In one of the STAAR monthly professional development workshops we asked teachers to solve collaboratively the "Painted Cubes" problem, adapted from Driscoll's (1999) book *Fostering Algebraic Thinking: A Guide for Teachers,* *Grades 6-10*. The "Painted Cubes" problem reads as follows:

> A cube with edges of length 2 centimeters is built from centimeter cubes. If you paint the faces of this cube and then break it into centimeter cubes, how many cubes will be painted on three faces? How many will be painted on two faces? On one face? How many will be unpainted? What if the edge has a length different from 2? What if the length of the large cube is 3 cm? 50 cm? n cm? (p. 108)

During this workshop, held in January 2004, the teachers solved the Painted Cubes problem and discussed how they might teach it to their own students. In the month that followed, all the teachers implemented the problem in their own classrooms, and we videotaped those lessons. By coincidence, shortly after the problem was introduced in our January workshop and before Pam taught the lesson, a version of the Painted Cubes problem was discussed in another professional development workshop Pam attended (a monthly "problem-solving group" composed of mathematics teachers from across the state).

The Painted Cubes problem consumed three mathematics lessons for Pam's eighth-grade pre-algebra class. Two of these lessons were block periods (90 minutes) and one was a 60-minute period. The following vignette is drawn from the first 90-minute lesson, which contained clear examples of each of the discourse strategies advocated and modeled in our professional development program.

Pam anticipated that the Painted Cubes problem was going to be a difficult problem for her students. She also knew that they would be easily frustrated by the day's planned activity. In order for the Painted Cubes problem to work the way she wanted, Pam had to make sure the students understood the type of classroom environment she wanted to see.

"I'm going to ask you to do two things," she said. "I'm going to ask you not to depend on a neighbor to do your listening and your focusing for you. You all need to listen because we've done problem solving and we've done group work a lot this year, but it is even more important today for you to be focused as an individual so that when you're collaborating as a group, you'll be able to give your full attention to that because you have paid attention to what was being said beforehand when I was laying the groundwork. Does everybody understand that part?"

Pam stopped to breathe and looked around the room. The students nodded. "Okay, and then the second thing is I want you to have lots of stick-to-it-ness today." Pam eyed the front row of silent, still-nodding students. "I'd like for you to know that the activity that we're going to go through is going to take a little bit of stick-to-it-ness and a lot of self-talk about 'don't get frustrated,' 'don't give up.' Okay? And everybody's going to help each other."

The students nodded again, looking at one another and at Pam. Just what were they going to be doing?

Pam then presented the first part of the problem by holding up two 3x3x3 cubes, each made from blocks (Unifix® cubes) using four different colors, and noted that the cubes were color coded. She asked the students to work in groups and build a similar 3x3x3 cube that was color coded using four different colors. After briefly reviewing that a cube has six sides of equal length, Pam instructed her students to begin constructing their 3x3x3 cubes. She then walked from group to group, showing (but not discussing) her 3x3x3 cube. Although Pam provided a relatively limited explanation and only minimal guidance, all groups were able to begin building a cube.

After about 10 minutes, Pam directed the students' attention to a "cube patterns" worksheet. She explained that the worksheet asked for a definition of where each color would be placed in any size of cube. She said, "You have to agree as a table or as a group what is going to be color 1, what is going to be color 2, 3, and 4. And what I'd appreciate not hearing when I walk around to the groups is that some people are saying one thing and other people are saying another thing and there's no agreement." The students had stopped working and were listening. "You really have to discuss this before writing. When you reach consensus at your table, then and only then should you write up a definition of color placement."

Pam cautioned that the groups' definitions must be precise and clear enough such that they could *explain to someone over the phone how to construct the cube. Students again worked in groups for approximately 10 minutes, with Pam walking around the room providing encouragement and guidance. One student, Joe, announced his rather colorful realization that a 2x2x2 cube is composed of one color: "stinkin'" red. Pam encouraged Joe to explain his observation. "Why do you say that a 2x2x2 is going to be all stinkin' red?" she asked. Joe laughed and responded, "Because every corner is red."*

Pam then decided to stop the class for a minute and share this idea. At her prompting, Joe held up his cube and told his classmates, "The 2x2x2 is all stinkin' reds." Pam continued, "Now, what I'm challenging Joe to discuss with his group is, why would a 2x2x2 have nothing but 'stinkin' red' as its color." Mouths opened and hands shot up, but Pam said, "Now, don't say anything out loud, but why would that be?" The students quieted.

Pam advised the class to build 2x2x2 and 4x4x4 cubes in their groups. After another 5 minutes, she brought the class together to discuss one group's 4x4x4 cube. Pam held up their cube for everyone to see, and pointed out that it was appropriately color coded. However, she noted that many groups were having difficulty coming up with the vocabulary to accurately describe the placement of their four colors and were using words like "center of the cube" that were too vague. Pam could see that several of the students who had been laughing earlier were starting to disengage in frustration. This wasn't as easy as it looked, and Pam had to keep them interested. She encouraged the students to stay focused despite their frustration, and gave them a short break before explaining the next part of the worksheet.

The Painted Cubes Problem: Analysis of Discourse Strategies

Pam's emphasis on effective communication as both a process and an outcome was evident throughout her lesson. As students developed the definitions of their color codes, Pam emphasized the importance of creating a consensus. This consensus led to a greater shared understanding of the color placements on a 3x3x3 cube. As the students extended these understandings to a variety of different-sized cubes they were able to identify the changing patterns and communicate about them, building on their discussions about the color codes. These conversations about patterns eventually led to the development of formulas of varying levels of sophistication. Throughout this process Pam continued to help her students communicate mathematically as they justified and explained their formulas to one another and explored their connections.

The Painted Cubes problem was not a part of Pam's normal curriculum, nor had she taught it before. She was enthusiastic about trying it, however, believing that it would provide a good opportunity for her students to work on their discourse skills, particularly within their small groups. As the following discussion indicates, Pam's lesson incorporated all four of the discourse strategies described in this paper as recommended by the literature and modeled in the STAAR summer institute.

Posing rich tasks that promote discussion. The Painted Cubes problem was described in both of the professional development programs that Pam participated in as rich and challenging, and she was encouraged by both programs to experiment with it in her classroom. Pam was concerned that the problem was so complex that her students might disengage. In particular, she worried that the complexity in the wording of the problem might throw students off track. She used several strategies to mitigate this complexity while maintaining the richness of the problem and providing an appropriate level of challenge for her students. These strategies included breaking the problem down into more manageable components and continuously shifting between small-group and whole-group discussions as they worked on the different components. To further ensure that students with different ability levels would be able to progress successfully through the task, Pam allowed for multiple exit points. For example, she worked to ensure that by the end of the lesson all students understood at least one color pattern related to the corners of a cube, and that they could relate this pattern to a more abstract generalization (i.e., cubes always have eight corners). She encouraged groups that were more advanced to find additional patterns.

Another strategy Pam used was to have students begin solving the problem by using concrete objects (Unifix® cubes). She believed that it was critical for students to become adept at building cubes, and that familiarity with the physical representation would enable them to shift to more abstract thinking. Pam also ensured that the students would write down their thoughts in an organized fashion by providing specific questions for them to answer and encouraging them to create tables to display their results.

Establishing and maintaining a safe environment. The students in Pam's mathematics class were seated in small groups and were accustomed to frequent small-group work. However, throughout the Painted Cubes lessons, Pam continually reestablished classroom norms regarding collaborative group work. In her introduction to the lesson, she clearly articulated her expectations for how students should work on the problem, including what they needed to do as individuals and what they needed to talk through and agree upon as a group. Pam reminded her students that they were part of a community, but that each student was also individually responsible and accountable. She warned the students they might get frustrated because this was a challenging problem, and she encouraged them to look to their peers for assistance.

On numerous occasions, Pam provided encouragement to individual students and groups that were struggling. She used these occasions as opportunities not only to create an environment in which it was acceptable to make mistakes, but also to show students how their struggling could be helpful and informative. In one case, a student said his group "screwed up" when they were making their cube. Pam sat down with the group and told them, "I like when people screw things up, and I'll tell you why.... That's the way we learn. You're going to pick that up now and you're going to try and decide as a group how you can solve that.... That might be a really good mistake that you made 'cause you're gonna learn from it."

Asking students to explain and justify their thinking. Pam frequently asked her students to explain and justify their thinking, both to their small group and to the whole class. She pressed students to talk through their ideas, not only when they were fully developed but also while they were still in the formative stage. For example, as she looked over a student's worksheet during small-group work, she challenged him, "Math is communication. You have to be able to communicate the concepts. You have to be able to communicate your thinking.... Numbers aren't enough for me. Numbers aren't enough for any good mathematician. You have to prove it. You have to convince me." A few moments later, Pam described this conversation to the whole class in an attempt to encourage other students to similarly think through and explain their answers.

Toward the end of the lesson Pam provided an extended opportunity for students to share, describe, and prove their ideas with one another. During the final whole-class discussion of the problem, Pam pushed students to reason about the corners of a cube. With her prompting, they were able to provide relatively elaborate responses and justifications. For example, they noticed that (a) a 2x2x2 cube was all one color, owing to the fact that it was composed entirely of corners; (b) the corners of a 3x3x3 cube were the same color as the 2x2x2 cube; and (c) there were always eight corners in a cube. All these ideas came directly from the students. One of Pam's main strategies for eliciting the ideas was to call on multiple students to clarify and extend the comments of others. Even after one student espoused a mathematically correct idea, Pam prompted the rest of the class to continue talking and thinking, and in doing so she was able to push the group further along mathematically without stepping in and telling them herself.

Encouraging students to actively process one another's ideas. Pam not only made students' ideas a prominent part of the Painted Cubes lesson, but she also provided the time and structure for students to actively process these ideas. One way she did this was by highlighting ideas and solution methods she observed during small-group work. For example, after one group realized that their 2x2x2 cube was built from blocks that were all the same color, Pam shared this realization with the entire class. Yet even after she did so, another group still struggled to build a 3x3x3 cube with a clear color pattern. Pam continued to push that group to process their classmates' ideas in order to further their own thinking. She encouraged them to build a 2x2x2 cube and reminded them that all the blocks would be the same color. Using the same language as the group that originally made this discovery, Pam told the struggling group, "What Joe said is true. They all have to be 'stinkin' red' in a 2x2x2 [cube]. Figure out why." The students soon noticed that their 2x2x2 cube was composed entirely of corners, and this understanding prompted them to redesign their 3x3x3 cube using a designated color pattern. By attributing a key idea to a classmate, Pam used that student's authority, rather than her own, to motivate these struggling students to delve deeper into the task and ultimately take ownership of the ideas for themselves.

Discussion

Participation in a professional development program with an emphasis on the creation of a discourse community can be a powerful learning experience for teachers (Putnam & Borko, 2000). By reflecting on their own learning and the strategies that support this learning, teachers can gain valuable new pedagogical insights (Barnett, 1998; Farmer, Gerretson, & Lassak, 2003). The STAAR professional development program's summer institute provided rich opportunities for participating teachers to learn algebraic content within a mathematical discourse community. Teachers had the experience of being both mathematics learners and reflective practitioners, and many became inspired to implement strategies similar to those modeled in the summer institute in their own classrooms.. During the following school year, as the teachers attended monthly professional development workshops and had their classrooms videotaped, they continued to refine their pedagogical practices and become more versed in the art of reflection,

The STAAR professional development program demonstrates the powerful impact that facilitators can have when they are viewed as role models by the participating teachers. The pedagogical strategies that facilitators use may have an especially strong influence if the facilitators explicitly point out these strategies and the teachers have an opportunity to reflect carefully on their learning. In daily reflections written during the summer institute, teachers commented on the mathematics they were learning and specific features of the environment within which this learning took place. Many, including Pam, commented on the ways in which the facilitators' use of communication strategies contributed to their learning (Borko et al., 2005). If the implementation of such strategies is an explicit goal of the professional development, our experiences suggest that critical ingredients include active involvement, adequate reflection time, and the continued support of peers as well as professional development providers.

As the vignette from Pam's "Painted Cubes" lesson illustrates, it is possible to put into practice many of the recommended strategies for establishing a discourse community in a middle school mathematics classroom. Pam worked hard to pose a rich task, create a safe environment, have her students explain and justify their solutions, and actively process one another's ideas. Pam's success with this lesson was evidenced by the learning her students demonstrated with respect to the mathematical properties of cubes. Students' written reflections included statements such as "We initially thought that it was only possible to make our cube pattern on cubes with odd numbered measurements. We learned that we needed to think outside the box and be more open to new ideas." As this quote suggests, Pam's lesson did challenge her students "to think outside the box" and ultimately they were able to communicate complex mathematical ideas. In worksheets completed by the end of the three days spent on this problem, almost all of Pam's students were able to successfully write generalizable patterns for cubes with 3, 2, 1, or no faces painted.

References

Ball, D. (1991). What's all this talk about discourse? Arithmetic Teacher, 39(3), 44-49.

Barnett, C. (1998). Mathematics teaching cases as a catalyst for informed strategic inquiry. Teaching and Teacher Education, 14(1), 81-93.

Bauersfeld, H. (1995). Structuring the structures. In L. P. Steffe & J. Gale (Eds.), Constructivism and education (pp. 137-158). Hillsdale, NJ: Lawrence Erlbaum Associates.

Borko, H., Frykholm, J., Pittman, M., Eiteljorg, E., Nelson, M., Jacobs, J., Clark, K. K., & Schneider, C. (2005). Preparing teachers to foster algebraic thinking. Zentralblatt für Didaktik der Mathematik: International Reviews on Mathematical Education, 37(1), 43-52.

Brophy, J. (1999). Teaching. In H. Walberg (Ed.), Educational Practices Series-1 (pp. 1-36). Geneva: International Academy of Education. Available online at http://www.ibe.unesco.org.

Brown, A. L., & Campione, J. C. (1994). Guided discovery in a community of learners. In K. McGilly (Ed.), Classroom lessons integrating cognitive theory and classroom practice (pp.229-279). Cambridge, MA: MIT Press.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. For the Learning of Mathematics, 19(2), 2-10.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. Journal of Research in Mathematics Education, 28(3), 258-277.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Interaction and learning in mathematics classroom situations. Educational Studies in Mathematics, 23, 99-122.

Cohen, E. G. (1984). Talking and working together: Status interaction and learning. In P. Peterson, L. S. Wilkinson, & M. Hallinan (Eds.), Instructional groups in the classroom: Organization and processes. Orlando, FL: Academic Press.

Davis, R. B. (1992). Reflections on where mathematics education now stands and where it might be going. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 724-734). New York: Macmillan.

Driscoll, M. (1999). Fostering algebraic thinking: A guide for teachers, grades 6-10. Portsmouth, NH: Heinemann.

Farmer, J. D., Gerretson, H., & Lassak, M. (2003). What teachers take from professional development: Cases and implications. Journal of Mathematics Teacher Education, 6, 331-360.

Fosnot, C. T., & Dolk, M. (2001). Young mathematicians at work: Constructing multiplication and division. Portsmouth, NH: Heinemann.

Grouws, D. A., & Cebulla, K. J. (2000). Improving student achievement in mathematics: Recommendations for the classroom (Report No. SE 064 318. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. (ERIC Document Reproduction Service No. EDO-SE-00-10).

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study (NCES Publication No. 2003-013). Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. Teaching Children Mathematics, 4(7), 410-414.

Kist, W. (2003). Student achievement in new literacies for the 21st century. Middle School Journal, 35(1), 6-13.

Lampert, M. (2001). Teaching problems and the problems of teaching. New Haven, CT: Yale University Press.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. Journal for Research in Mathematics Education, 32(3), 236-266.

Miles, M. B., & Huberman, A. M. (1994). An expanded sourcebook: Qualitative data analysis. Thousand Oaks, CA: Sage Publications.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

National Middle School Association (2004). Position paper of the National Middle School Association middle level curriculum: A work in progress. Westerville, OH: NMSA.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? Educational Researcher, 29(1), 4-15.

Secretary's Commission on Achieving Necessary Skills (1991) What work requires of schools: A SCANS report for America 2000. Washington, DC: U.S. Department of Labor. (NTIS No. PB92-146711INZ)

Silver, E. A., & Smith, M. S. (1997). Implementing reform in the mathematics classroom: Creating mathematical discourse communities. In Reform in math and science education: Issues for teachers. Columbus, OH: Eisenhower National Clearinghouse.

Stein, M. K., Grover, B. W., & Henningsen, M. A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33(2), 455-488.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). Implementing standards-based mathematics instruction. New York: Teachers College Press.

Stigler, J. W., & Hiebert, J. (1999). The teaching gap. New York: Free Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27(4), 458-477.

Author Note

Correspondence concerning this article should be addressed to Karen K. Clark at the University of Colorado at Denver and Health Sciences Center, North Classroom 4001G, Campus Box 106, P.O. Box 173364, Denver, Colorado 80217. E-mail: karen.clark@cudenver.edu. Telephone: 303-556-8893.

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Footnotes

[1] Although all but one participant indicated interest in attending the extended professional development, seven were not able to continue due to travel time to the university or teaching assignments not in middle school mathematics.