DOMINIC PERESSINI[1], HILDA BORKO[1,*], LEW ROMAGNANO[2], ERIC KNUTH[3]
and CHRISTINE WILLIS[4]

A CONCEPTUAL FRAMEWORK FOR LEARNING TO TEACH SECONDARY MATHEMATICS: A SITUATIVE PERSPECTIVE

ABSTRACT. This paper offers for discussion and critique a conceptual framework that applies a situative perspective on learning to the study of learning to teach mathematics. From this perspective, such learning occurs in many different situations – mathematics and teacher preparation courses, pre-service field experiences, and schools of employment. By participating over time in these varied contexts, mathematics teachers refine their conceptions about their craft – the big ideas of mathematics, mathematics-specific pedagogy, and sense of self as a mathematics teacher. This framework guides a research project that traces the learning trajectories of teachers from two reform-based teacher preparation programs into their early teaching careers. We provide two examples from this research to illustrate how this framework has helped us understand the process of learning to teach.

INTRODUCTION

Calls for reform of mathematics education in the United States pose great challenges for the preparation and continuing education of mathematics teachers. In mathematics classrooms aligned with the vision of reform portrayed in the *Standards* documents of the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 1995, 2000), teachers engage students in rich, meaningful tasks as part of a coherent curriculum; students' thinking, shared orally and in writing, is used by teachers to guide the classroom community's exploration of important mathematical ideas; and teachers gather information from many sources as they assess their students' understanding of these ideas. To support this vision of the mathematics classroom, teacher education programs in colleges and universities and professional development programs for practicing teachers across the United States are being called upon to model good mathematics teaching, to help teachers develop their knowledge of the content and discourse of mathematics and of mathematics pedagogy, to offer perspectives on students as learners of mathematics that have a sound research base, and to provide opportunities for teachers to develop their own identities as teachers of mathematics (NCTM, 1991).

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How do college and university teacher education programs oriented toward this vision of reform influence the teacher-learning trajectory? How does this trajectory continue through the early years of full-time teaching? Fundamentally, how do mathematics teachers learn their craft? Despite the fairly extensive literature on teacher education and learning to teach, there continues to be much disagreement about the nature and extent of influence that teacher education, in general, has on teacher learning (Boaler, 2000; Frykholm, 1998, 1999; Glickman and Bey, 1990; Grossman et al., 2000; Lampert, 2001; Zeichner and Tabachnick, 1981; Zeichner et al., 1987). Some critics have questioned whether teachers learn anything of value in their teacher education programs (e.g., Conant, 1963, Kramer, 1991). Others have claimed that the effects of teacher education are "washed out" once teachers enter the more conservative setting of the school (e.g., Zeichner and Tabachnick, 1981). And, several scholars have cautioned that teacher education experiences can have negative as well as positive consequences for prospective teachers (Feiman-Nemser and Buchmann, 1985; Zeichner, 1985). The project that is the focus of this article – Learning to Teach Secondary Mathematics (LTSM) – peers through a situative lens at the learning-to-teach process.[1]

In this article we articulate the conceptual framework that has guided the design, data collection, analysis, and interpretation of findings for the LTSM project. We offer this framework to stimulate discussion with colleagues in the mathematics education research community about ways in which it might be refined and extended, and contribute to building a shared understanding of the process of learning to teach.

Our framework derives from two related assertions. First, learning is situated; that is, how a person learns a particular set of knowledge and skills, and the situation in which a person learns, are a fundamental part of what is learned (Greeno et al., 1996). Second, we assert that teachers' knowledge and beliefs interact with historical, social and political contexts to create the situations in which learning to teach occurs. Thus, our framework adapts an existing perspective on learning to explore the process of learning to teach. From this perspective, teacher learning "is usefully understood as a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching" (Adler, 2000, p. 37).

We organize our discussion into three sections. We begin by elaborating on our situative perspective, including a discussion of the three domains of professional knowledge we have chosen for our empirical and analytical focus: mathematics, mathematics-specific pedagogy, and conception of self as teacher. Next, we use two brief vignettes drawn from case studies of teachers in the LTSM project to illustrate how this framework has guided our data collection and analysis. We conclude with a discussion of the value of a situative perspective for understanding the process of learning to teach secondary mathematics.

1. A SITUATIVE PERSPECTIVE ON TEACHER COGNITION

For teachers, learning occurs in many situations of practice. These include university mathematics and teacher-preparation courses, preparatory field experiences, and schools of employment. Situative perspectives argue that, to understand teacher learning, we must study it within these multiple contexts, taking into account both the individual teacher-learners and the physical and social systems in which they are participants (Putnam and Borko, 2000).

Traditional cognitive perspectives typically treat knowing as the manipulation of symbols inside the mind of the individual. Learning is typically described as an individual's acquisition of knowledge, change in knowledge structures, or growth in conceptual understanding. Cognitive theorists argue that, while some learning takes place in a social context (e.g., on-the-job training), what is learned can also be independent of the context in which it is learned (Anderson et al., 1997).

The term 'situative' refers to a broad set of theoretical ideas and lines of research, with roots in various disciplines including anthropology, sociology and psychology (Greeno, 2003). These perspectives have in common a conception of the learning process as changes in participation in socially organized activity (Lave, 1988; Lave and Wenger, 1991). Whereas cognitive perspectives focus on knowledge that individuals acquire, situative perspectives focus on practices in which individuals have learned to participate and "consider individuals' acquisition and use of knowledge as aspects of their participation in social practices" (Greeno, 2003, p. 315). (In our work, we use 'practices' to refer to the patterns of thought and action that have been established by participants in particular contexts or settings; see Cobb and Bowers, 1999).

One construct that brings to the fore differences between cognitive and situative perspectives is 'transfer.' Although a detailed consideration of transfer is beyond the scope of this article, we address it briefly, as it is central to understanding the roles of multiple contexts in learning to teach (Adler, 2000). Viewed from cognitive perspectives, knowledge is an entity that is acquired in one setting and then transported to other settings. Research on transfer addresses questions such as characteristics of tasks and contexts that affect the extent to which knowledge learned in one situation will transfer to other situations (Anderson, Reder, and Simon, 1996). Greeno (1997) and Adler (2000; see also Adler and Reed, 2002) have suggested that, from a situative perspective, transfer may be an inappropriate construct for considering questions about the generality of learning. Rather than asking whether or how knowledge transfers across situations, researchers within a situative tradition ask questions about the consistency of patterns of participation across situations, conditions under which successful participation in activity in one type of situation facilitates successful participation in other types of situations, and the process of *recontextualizing* resources and discourses in new situations. (Recontextualizing refers to the transformation of resources and discourses as they are disembedded from one social context and embedded into another [Adler and Reed, 2002; Ensor, 2001].) With respect to teacher education, these issues are particularly relevant for understanding how practices learned in university courses can be recontextualized in elementary and secondary school classrooms.

Another distinction between cognitive and situative perspectives concerns the appropriate unit of analysis in studies of learning. With its focus on individuals' knowledge acquisition, research in the cognitive tradition has the individual as its primary unit of analysis. Some scholars contend that a situative perspective necessarily implies the social collective or activity system as the principal unit of analysis, and that this is a key distinction between cognitive and situative perspectives on knowledge and learning (Anderson et al., 1997; Lave, 1988). Cobb and his colleagues (Bowers et al., 1999; Cobb and Bowers, 1999) disagree with this characterization, arguing instead that "the situated perspective admits a range of units of analysis, the choice in any particular case being a pragmatic one that depends on the purposes at hand" (Cobb and Bowers, 1999, p. 6). As they explained, "an analysis of classroom mathematical practices documents the evolving social context of the students' mathematical development, and an accompanying psychological analysis of the students' activities as individuals documents the diverse ways in which they participated in those practices" (Bowers et al., 1999, pp. 28–29). Also addressing the role of the individual and the interpersonal, Greeno (2003, p. 327) suggested that "The research agenda for the situative perspective will include more detailed studies that combine analyses of the informational and interpersonal aspects of students' participation in learning and that identify, in detail, how students' mathematical knowledge and understanding grow through their sustained participation in learning activities."

Perhaps the most impressive consideration of teaching from a cognitive perspective is the work of Alan Schoenfeld and his Teacher Model Group to develop a theory of teaching-in-context (Schoenfeld, 1998, 1999). Schoenfeld and colleagues are attempting, through the use of cognitive modeling strategies, to explain teachers' decisions at each point of instruction by identifying the goals, beliefs, knowledge, and action plans on which those decisions are based.

In an invited critique of this work, Greeno (1998) suggested that a situative perspective would add significantly to this understanding of teacher decisions by focusing on classroom social practices and examining such features as the patterns of discourse, the kinds of participation that are afforded to the teacher and students by the classroom practices that are in place, and the personal identities developed by the teacher and students through participation in these practices.

The research by Cobb and colleagues provides a clear example of the theoretical and practical contributions that a situative perspective – one that avoids the false dichotomy of individual cognition versus participation in social context – offers to the study of classroom practice. In a teaching experiment designed to facilitate and investigate students' mathematical development within the social context of a third grade classroom, these researchers documented both the development of individual students' place value conceptions and the evolution of the communal mathematical practices in which they participated. Furthermore, they demonstrated that the relationship between classroom practices and individual students' reasoning is reflexive. That is, students contribute to the development of practices within the classroom community; these practices, in turn, constitute the immediate context for their learning.

Recently, Cobb (2000) argued emphatically for the importance of a situative perspective in understanding classroom learning, explaining that the reflexive relationship between social and psychological perspectives:

> is an extremely strong relationship that does not merely mean that the two perspectives are interdependent. Instead, it implies that neither perspective exists without the other in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective. (p. 64)

Similarly, we find a situative perspective to be compelling as a framework for the study of teacher learning across time and across the multiple contexts of early-career teaching. Within the LTSM project, we conceptualize the novice teacher's learning-to-teach experiences as a trajectory through the multiple contexts of teacher education. A situative perspective guides our decisions about data to collect and offers a way of disentangling – without isolating – the complex contributions of these various contexts to novice teachers' development.

Grossman, Valencia and colleagues (Grossman et al., 2000) provide insight into the roles of multiple contexts in teacher learning in their longitudinal study of learning to teach literacy/language arts. Their research, which followed 10 beginning teachers from their final year of university preparation through their first two years of full time teaching, describes how the novice teachers adopted concepts and practices for teaching writing in the various settings of teacher education and then modified and used these conceptual and practical tools in their first years of teaching. For example, during their first year of teaching, despite trying out practices that were, in some ways, antithetical to ideas about modeling and scaffolding, adopted during their teacher education experiences, the teachers were able to hold on to these conceptual tools. Several of the concepts and practices learned in teacher education but not evident in their first year of teaching began to resurface in their second year, as they tried to approximate their goal of good language arts instruction.

In a similar investigation, Ensor (2001) tracked 7 beginning teachers through their one-year university-based secondary mathematics methods course and first year of teaching. These novice teachers drew from the methods course in two ways during their first year of teaching. They reproduced a small number of discrete tasks that were introduced in the course, and they appropriated a way of talking about teaching and learning mathematics. Ensor argued that because the mathematics methods course was offered exclusively in the university setting, the teachers had limited access to recognition rules (which enable student teachers to describe and evaluate best pedagogical practices) and realization rules (which enable teachers to implement best practices in their mathematics classrooms). She suggested that the beginning teachers' limited recontextualizing of pedagogical practices was shaped by their educational biographies and school contexts, and especially by their access to recognition and realization rules. Findings from these two research projects reveal the importance of considering context in the study of teacher learning, and of studying learning in context over a number of years.

In the LTSM project, we also focus our attention on the social nature of prospective teachers' experiences within their teacher education programs and as beginning teachers in their respective schools. Like Schoenfeld (1998) we embrace the idea that teaching and learning to teach occur in real time, and that it is essential to concentrate on the present if our goal is to understand the act of teaching – and the process of learning to teach – as they unfold in particular contexts. Thus, as we study the process of learning to teach, we examine novice teachers' participation in a variety of activities within university and public school settings. Using data from each of these settings of classroom practice, we infer the complex, reflexive relationship between teaching practices and teachers' developing knowledge and beliefs about mathematics, mathematics-specific pedagogy and professional identity. We turn next to an elaboration of these three domains of professional expertise.

2. Domains of professional expertise: A situative perspective

A second component of our conceptual framework is teachers' professional knowledge. From a cognitive perspective, knowledge and beliefs are major determinants of what teachers do in the classroom, and a central goal of teacher education is to help prospective teachers acquire new knowledge and beliefs. A situative perspective suggests that knowledge and beliefs, the practices they influence, and the influences themselves, are inseparable from the situations in which they are embedded (Borko and Putnam, 1996). Knowledge grows more complex, and becomes 'useful' in a variety of contexts, through the learner's participation in these different contexts.

We focus our attention on three domains of knowledge that are particularly relevant to teachers' instructional practices: mathematics content (in particular, the central strands of function, rate, and proof); mathematics-specific pedagogy (specifically, the uses of mathematical tasks and orchestration of classroom discourse); and professional identity (conceptions of self as teacher).

2.1. *Mathematics content*

Prospective teachers come to teacher education without the subject matter knowledge necessary to enact reform-based images of teaching. This conclusion is supported by studies of undergraduate mathematics majors (Schoenfeld, 1992) and prospective mathematics teachers (Cooney, 1985; Ball, 1990; Even, 1993; Thompson and Thompson, 1996). Participants in these studies, who had taken many mathematics courses, had incomplete, rule-based knowledge of mathematical concepts as advanced as the concept of function and as 'elementary' as the concept of division. They believed that doing mathematics means finding correct answers, quickly, using the (one, correct) standard procedure, and that learning mathematics means mastering these procedures. Taken together, these studies support the assertion that prospective secondary teachers who have taken many traditional courses are unlikely to have "clear images of understanding a mathematical idea conceptually" (Thompson and Thompson, 1996, p. 3).

The two teacher education programs that are the focus of our research attempted to address prospective teachers' content knowledge in their mathematics courses and, more substantially, in their mathematics methods courses. We selected several mathematics content areas – function, rate, and proof – that allow us both to trace prospective teachers' belief and knowledge growth over time, and to examine how these beliefs and knowledge about mathematics play out in practice. We refer to these three areas of high school mathematics content as 'mathematics domain slices.'

These domain slices meet several criteria for identifying important mathematics content to examine in research on reform-based mathematics education (Ball, personal communication, 1997). Function, rate, and proof are: likely to be taught by beginning teachers; focal to all secondary mathematics curricula; integral to conceptions of mathematical literacy; important to advanced work in mathematics; prominent in any school of thought about good teaching of mathematics; traditionally difficult for students to learn and for teachers to teach effectively; comprised of a complex interaction between skills and concepts; and a target of reformers.

Our situative perspective requires that we collect data from many sources. This is not simply a good methodological approach. To fully understand the growth of teachers' knowledge of functions, rate, and proof, we have had to examine that knowledge in each of the situations in which our participants use it. Therefore, we have constructed research tasks and have observed and interviewed our participants in their roles as students and as teachers throughout the 4 years of our study.

2.1.1. *Function*

The concept of function is among the most important unifying ideas in mathematics (Romberg et al., 1993). It provides the mathematical foundation for arithmetic in elementary school, algebra in both middle school and high school, and transformational geometry in high school (Harel and Dubinsky, 1992).

There is an extensive body of research focusing on student understanding and learning about functions (reviewed in Leinhardt et al., 1990). However, few studies have investigated teachers' conceptions of functions, and none have treated the situations in which teachers develop their knowledge of functions as explicit features of that knowledge. As Lloyd and Wilson (1998) highlight, "Given the importance of functions in mathematics and the curriculum, it is crucial for researchers to explore the nature of teachers' conceptions of functions and the impact of these conceptions on classroom practice" (Lloyd and Wilson, 1998, p. 250). We are examining prospective teachers' conceptions of functions along several dimensions: (1) univalence and arbitrariness, (2) covariation and correspondence, (3) function as action, process, and object, and (4) multiple representations. The arbitrary nature of a function means that the relationship does not have to exhibit some regularity, or be described by any specific expression or particular shaped graph (Even, 1993). The arbitrary nature of functions is implicit in the definition of a function, but the univalence requirement, that for each element in the domain there be only one element in the range, is stated explicitly. Traditionally, functions have been treated using a correspondence model that considers a function to be a mapping from x to a corresponding y. Confrey and Doerr's (1996) research uncovered student uses of a covariation model that relates the change in x to the change in y. In addition Dubinsky and Harel (1992) outlined a progression of student understanding of the function concept in terms of developing from 'action', to 'process', and finally to 'object.' Multiple representations of functions (e.g., equations, graphs, tables) play an important role in students' mathematical development. In particular, the introduction of different representations of functions can be seen as one of the critical moments in mathematics learning and represents "one of the earliest points in mathematics at which a student uses one symbolic system to expand and understand another" (Leinhardt et al., 1990, p. 2).

2.1.2. *Rate*

Elementary students first encounter the idea of rates when they are introduced to whole-number ratios. Middle school students encounter speed and a variety of other rates that establish a constant ratio between quantities of different units (Vergnaud, 1988). High school and college students build on the fundamental concepts of rates as they learn about slope and the derivative. The concept of rate is central to mathematical models of change.

Rate is also closely connected to the larger concept of rational number. The majority of research in this area has been done at the elementary and middle grade levels (Behr et al., 1992; Carpenter et al., 1993). There is a noticeable void with respect to examining secondary students' and teachers' knowledge of rational number in general, and rates in particular.

In developing our framework we turned to Thompson (1991), who diverged from much of the literature on rates with his elaboration of the unique relationship between ratio and rate. In his view, a ratio is a multiplicative comparison between two static quantities, whereas a rate is a ratio generalized by the learner and used in other contexts. To use an algebra example, a learner compares (by dividing) the change in *y*-coordinates to the change in *x*-coordinates of two points on a line to find the ratio called slope. The slope of a line, however, becomes for the learner the rate of change of *y* with respect to *x* anywhere on a line, a property of the line itself. We posit that these conceptions of ratio and rate develop as individuals use them to make sense of mathematical situations.

2.1.3. *Proof*

Proof is central to the discipline of mathematics and the practices of mathematicians. Yet, its role in secondary school mathematics in the United States has traditionally been peripheral, usually limited to the domain of Euclidean geometry. However, current reform efforts in this country call upon secondary teachers to provide all students with rich opportunities and experiences with proof throughout the curriculum.

Are secondary school mathematics teachers prepared to weave proof into their instructional practices? To date, little research has focused on secondary mathematics teachers' conceptions of proof. Researchers have focused primarily on the conceptions of proof held by students (Balacheff, 1991; Maher and Martino, 1996), prospective elementary school teachers (Martin and Harel, 1989; Simon and Blume, 1996), and undergraduate mathematics majors (Harel and Sowder, 1998). Our framework builds on the work of Knuth (2002), who investigated individual teachers' situated knowledge of the *social nature* of proof, and of the *role* of proof in establishing and explaining mathematical truths across a variety of contexts.

Many mathematicians and mathematics educators view proof as a *social process* engaged in by members of a community of mathematical practice. They describe proof as "a debating forum" (Davis, 1986, p. 352), "a form of discourse" (Wheeler, 1990, p. 3), "a social construct and a product of mathematical discourse" (Richards, 1991, p. 23), "a justification arising from social interactions" (Balacheff, 1991, p. 93).

The primary *role* of proof in mathematics is to demonstrate the correctness of a result or truth of a statement (Hanna, 1991). Yet, mathematicians expect the role of proof to include more than justification and verification of results: "mathematicians routinely distinguish proofs that merely demonstrate from proofs which explain" (Steiner, 1978, p. 135). Hanna (1990) agrees, distinguishing between proofs that establish *that* a result is true, from proofs that illustrate *why* a result holds.

In addition, proof is a primary mechanism for the growth of the discipline. Lakatos (1976) asserts that "mathematics does not grow through a monotonous increase in the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations" (p. 5).

Secondary mathematics students have traditionally conceived of proof as a formal, and often meaningless, exercise to be done for the teacher (Alibert, 1988). "In most instructional contexts proof has no personal meaning or explanatory power for students" (Schoenfeld, 1994, p. 75). However, mathematicians value a proof as much for its explanatory power as for its deductive mechanism. These two features of proof – explanatory power and deductive mechanism – address aspects of teachers' conceptions of proof that are essential for understanding their implementation (or lack thereof) of reform recommendations concerning proof in school mathematics. Accordingly, the overarching concept of proof, by which students and teachers – in the social context of the classroom – explain, justify, critique, and thereby build a body of mathematical ideas, connects all of these domain slices.

2.2. *Mathematics-specific pedagogy*

Our conception of mathematics-specific pedagogy, a refinement of Shulman's (1986) broader construct of pedagogical content knowledge, has its roots in our view that secondary mathematics teachers' pedagogical knowledge is embedded in the specific context of the *mathematics* classroom (McLaughlin and Talbert, 1993; Wineburg and Grossman, 1998). We have focused on two aspects of mathematics-specific pedagogy that are central to reform-based teaching: selection and use of mathematical tasks and classroom mathematical discourse (Clarke, 1994; NCTM, 1991).

2.2.1. *Mathematical tasks*

The questions, problems, exercises, constructions, applications, projects and investigations in which students engage constitute the "intellectual contexts for [their] mathematical development" (NCTM, 1991, p. 20). Tasks provide the stimulus for students' work in classrooms, and they "convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24).

One of the primary responsibilities of teachers is to select and develop worthwhile tasks – tasks that are rich with mathematical possibility and opportunity, and contain hooks that connect the child's world with particular mathematical ideas and ways of thinking (Ball, 1993). Worthwhile tasks contain important mathematical ideas, represent concepts and procedures, foster skill development as well as problem solving and reasoning, and help students to make connections among mathematical ideas and with real-world applications (NCTM, 1991). Further, they "lend themselves to multiple solution methods, frequently involve multiple representations, and usually require students to justify, conjecture, and interpret" (Silver and Smith, 1996, p. 24). It is through engaging in such tasks that students gain access to the phenomena of mathematics (Nespor, 1994) and come to understand and appreciate what doing mathematics entails.

The selection of tasks is situated in particular classrooms filled with students who bring with them different experiences and backgrounds. Teachers must take into account their own particular students' knowledge and interests, what is known about the ways in which students learn particular mathematical ideas, and common student confusions and misconceptions about those ideas.

2.2.2. *Mathematical discourse*

Mathematics reform initiatives in the United States underscore the importance of teachers and students engaging in oral and written discourse that fosters students' understanding of mathematics (NCTM, 1991, 2000). These calls for more meaningful discourse in mathematics classrooms are grounded in research demonstrating the social nature of learning mathematics (Cobb et al., 1997) and a vision of school mathematical practices that reflects the essence of mathematical practices within the discipline (Lampert, 1990; Lampert and Blunk, 1998). It is through participation in classroom discourse that students negotiate with the teacher what counts as knowledge (Cazden, 1986) and become initiated into the community of mathematical practice (Lo et al., 1994). Mathematical discourse in the classroom provides an arena in which the students learn how to represent mathematics through thinking, talking, agreeing, and disagreeing about mathematics, rather than learning from the talk (Lave and Wenger, 1991; Pimm, 1987).

Distinct differences in patterns of classroom discourse are features of what Cobb and his colleagues have described as two distinct mathematics classroom traditions (Cobb et al., 1993). Discourse within the "school mathematics tradition" often follows an elicitation pattern in which the solution is the driving force (Voigt, 1995). Typical interactions can be characterized by a three-part process comprised of teacher initiation, student reply, and teacher evaluation (IRE; Mehan, 1979). The process begins with the teacher posing a known-information question. A student responds, attempting to provide the expected answer, and the teacher follows by evaluating the response. If the student response is incorrect, the teacher continues to call on other students until the desired response is given. This type of interaction promotes "dialogues that typically degenerate into social guessing games when teachers attempt to steer or funnel students to a procedure or answer they have in mind all along" (Cobb et al., 1993, p. 93).

In contrast, discourse in the "inquiry mathematics tradition" resembles a discussion pattern in which student explanations are the driving force (Voigt, 1995). Interaction begins with the teacher asking information-seeking questions that require students to explain how they interpreted and solved tasks, and that expect students' original contributions (Cobb et al., 1993). Students' responses become the object of discussion and are evaluated in terms of established sociomathematical norms, such as what counts as an acceptable mathematical explanation and justification (Yackel and Cobb, 1996). To initiate and guide this type of discourse, a teacher must be skillful at posing questions that challenge student thinking, listening carefully to students' ideas, rephrasing students' explanations in terms that are mathematically more sophisticated, deciding when to provide information, and orchestrating class discussions to ensure participation by all students (Cobb et al., 1991; Peressini and Knuth, 1998).

2.3. *The interdependence of discourse, task and mathematical content*

In sum, we envision the act of teaching mathematics to consist, in part, of: (1) selecting and developing worthwhile tasks which have the potential to immerse students in significant mathematics content, and (2) orchestrating classroom discourse focused on mathematical thinking, reasoning, and communication. These two aspects of teachers' work are clearly interdependent – it is around worthwhile tasks that discourse in the inquiry mathematics tradition will be centered. As Driver, Asoko, Leach, Mortimer, and Scott (1994) noted, "a social perspective on learning in classrooms recognizes that an important way in which novices are introduced to a community of knowledge is through discourse in the context of relevant tasks" (p. 9). Further, classroom discourse that focuses on what counts as an acceptable mathematical explanation and justification engages students in the crucial mathematical notion of proof in the context of discussing important mathematics content (Bauersfeld, 1988).

2.4. *Professional identity*

Situative perspectives coordinate cognitive and sociocultural aspects of identity and identity construction. Cognitive aspects of a teacher's professional identity encompass a complex constellation of goals, values, commitments, knowledge, beliefs, and other personal characteristics, drawn together to create a sense of "who I am" as a teacher. Sociocultural aspects include the ways in which teachers participate in the activities of their professional communities and present themselves to others in the context of professional relationships – the patterns of social interaction that are recognized, expected, embedded in, and shaped by their social and cultural worlds (Luttrell, 1996; Eisenhart, 1996).

Professional identity shapes the ways in which a teacher frames and addresses problems of practice. It serves as a lens through which teaching is analyzed, understood and experienced. Teachers draw upon the ideas and interactions that make up their professional identities as they make instructional decisions and manage classroom dilemmas (Bullough, 1992; Lampert, 1985). Lampert (1985) argued that one of the central challenges in addressing problems of teaching is working to simultaneously satisfy contradictory aims (e.g., honoring both a child orientation and a subject-matter orientation, pushing students to achieve and creating a comfortable learning environment). She claimed that teachers are able to manage dilemmas – to juggle conflicting goals without choosing between them – by using identity as a tool for analyzing problems of practice. Who one is as a teacher – one's professional identity – is a resource for managing problems of educational practice (Lampert, 2001).

Professional identity also affects what the novice teacher learns in her or his teacher education experience. Teachers' knowledge and beliefs are critical in shaping what and how they learn during teacher education (Borko and Putnam, 1996). In much the same way, a teacher's concept of 'self-as-teacher' has a profound impact on learning, decision-making, and knowledge and beliefs about teaching. Professional identity serves as a filter through which learning takes place. It thus has important effects on the process of becoming a teacher, and on a teacher's evolving professional practices (Bullough and Gitlin, 1995; Knowles, 1992).

Identity is not static, but is fluid and constantly changing. Persons' identities develop in and through social practices. They evolve in response to the demands of social and cultural circumstances, as well as personal experiences and histories (Holland et al., 1998). Our identities are forged out of a balancing act between our capacity for individual agency and our dependence on the options that our societies and cultures allow us – the opportunities and constraints we experience as a function of social relationships and cultural norms (Boaler and Greeno, 2000; Eisenhart, 1996; Luttrell, 1996). Thus, at the same time as teachers' professional identities play a central role in determining how they juggle priorities and manage classroom dilemmas, their professional identities must accommodate the ambivalence and tension these conflicts generate. Through this process of accommodation, teachers create and recreate their professional identities.

One key feature of novice teachers' identity construction is the multiple contexts within which the process unfolds. Much has been written about the different, and sometimes incompatible, images of teaching promoted by the university and public school settings within which teacher education takes place (e.g., Ensor, 2001; Grossman et al., 2000; Zeichner and Tabachnick, 1981; Zeichner et al., 1987). These mixed messages can, at times, present competing sets of structural constraints that the novice teacher must negotiate in the process of identity construction. A situative perspective focuses our attention on the impact of these various settings on novice teachers' identity construction. It leads us to view constructing a professional identity, like other aspects of learning to teach, as a single trajectory through multiple communities of practice (Lampert, 2001).

Our framework is designed to allow us to trace teachers' evolving senses of themselves as teachers by focusing on both individual and sociocultural factors that support and constrain the construction of identity. When novice teachers seem to enact inconsistent conceptions of self as teacher in different settings, our framework enables us to consider whether the inconsistencies may be due to incompatible sets of goals, values, and commitments or conflicting requirements for successful participation in these settings.

2.5. *Situativity of knowledge and belief domains*

Our situative lens also brings into focus the connections among the three knowledge and belief domains outlined above. Consider, for example, the context of a mathematics classroom lesson. The teacher's choice of tasks is influenced by knowledge of the mathematical goals of the lesson and the mathematics embedded in the tasks themselves. The extent to which mathematical ideas such as proof and justification appear in classroom discourse will be influenced by both the teacher's choice of task and the questions and comments she makes during class. These pedagogical decisions and actions are, in turn, influenced by the teacher's knowledge of proof and by her sense-of-self as mathematician and as teacher. Furthermore, each of these aspects of the teacher's knowledge and beliefs has its antecedents in experiences during her teacher preparation program (Boaler, 2000). Our conceptual framework for studying the learning-to-teach trajectory is, therefore, one of multiple domains of knowledge and beliefs situated in, and intertwined through, teaching practices enacted over time.

3. Through a situative lens: Insights from two cases of learning to teach

To illustrate the utility of our conceptual framework, we share two brief vignettes drawn from the research project called Learning To Teach Secondary Mathematics (LTSM), in which researchers followed 6 secondary mathematics teachers who began as students in two reform-based teacher preparation programs in the United States, and who have now taught for at least 2 years. The project's data collection scheme was guided by the situative perspective outlined above. Data collected during the teacher preparation programs included initial interviews with participants about mathematics content; interviews throughout the programs about their conceptions of mathematics, mathematics-specific pedagogy, and selves as teachers; a complete videotape archive – with accompanying field notes and interviews with participants and instructors – of mathematics and mathematics methods courses; and videotaped lessons, observation field notes, and interviews with participants, cooperating teachers, and university supervisors during practicum experiences and three 3-day data collection cycles during student teaching. Data collected during participants' first two years of full-time teaching included videotaped lessons, observation field notes, and interviews with participants during five 3- to 5- day data collection cycles. We also conducted extensive end-of year interviews with each of the participants, and interviews with other important players in their professional lives. We collected documents about the schools and districts, as well as other artifacts that helped us understand these teaching situations.

Analyses of this large and diverse data corpus provide the basis for our two examples. The first vignette illustrates the ways in which one beginning teacher, Ms. Audrey Savant, drew upon different conceptions of proof on several occasions during her teacher preparation program and first year of teaching, in order to participate successfully in the multiple contexts of teacher education and early-career teaching. The vignette demonstrates how a situative perspective enables us to interpret seemingly inconsistent instances of mathematical understanding by focusing on the practices and participation structures of the situations in which knowledge is used. The second vignette addresses what one of the reviewers of this article referred to as "the age old problem teachers confront, which is that the institutional environment may or may not support novice teachers in their preferred way of teaching." Our situative perspective enables us to interpret a novice teacher's, Adam Hanson, very different instructional practices during student teaching and first year teaching as an interaction between his developing professional identity and the affordances and constraints of these two settings, rather than a more simplistic explanation focused on either the individual or the context. (For more extensive discussions of the case studies from which these vignettes were drawn, see Borko et al., 2000, and Peressini et al., 2001).

3.1. *Audrey Savant: What is a proof?*[2]

In fall 1995, Audrey Savant put on hold her professional life as a musician and teacher of private flute lessons to obtain a secondary mathematics teaching license. Armed with two degrees in music, she completed a mathematics major (including reform-based courses in the foundations of geometry and mathematics teaching methods) and the teacher preparation program at a local commuter college. In spring 1999 she student-taught at Cumulus High School, a three-year-old school of 1,600 students in a rapidly-growing upper-middle class and predominately white suburb. Ms. Rockford, her mentor teacher (an amateur musician herself), is a National Board for Professional Teaching Standards certified teacher and an instructional leader in the school district.

Our observations of Ms. Savant in several research contexts prior to student teaching revealed that she was quite adept at proof. We present three brief examples here.

In one research task, part of an extended interview designed to elicit mathematics content knowledge, Ms. Savant was asked to prove the following:

Theorem: $1 + 2 + 3 + \ldots + n = n(n + 1)/2$.

She produced a valid proof using the method of Mathematical Induction. When asked to assess the validity of several different purported 'proofs' of this theorem, including one that was almost identical to hers, she commented that Mathematical Induction produces a valid proof "if you follow all of the steps correctly."

Our second example comes from Ms. Savant's Foundations of Geometry class, an upper-division requirement for her mathematics major. She and her classmates were continuing their semester-long project of building a set of axioms for Euclidean Geometry, what the professor described as "a minimal set of descriptors of what we think is good geometry." On this evening, the following question was on the table: As conditions for establishing triangle congruence, are both SAS and ASA correspondences[3] needed as axioms, or does the second follow as a consequence of the first? The professor stated the theorem: SAS implies ASA. He then said to the class that he would "get them started" on the proof. He wrote on the board, in very careful language and using a clearly labeled diagram, the first few steps of a proof by contradiction, in which two triangles were in

ASA correspondence but were not congruent. Ms. Savant completed the proof at the board for the class, carefully writing the steps that led to a contradiction.

Our final example occurred in Ms. Savant's mathematics methods class, which she completed in the semester immediately prior to student-teaching. One evening, this class began with the following task on the overhead projector:

> Place a point A on a sheet of paper. Draw a line through A, and draw a point B about an inch from the line. Use the line you drew as a line of reflection, and use the MIRA to find the image of the point B. Use at least five other lines of reflection through A to find image points for B. If you were to find the set of images of B across all possible lines of reflection through A, what figure would this set of points form? Why?

After about 8 minutes of work in small groups, the instructor asked the class, "All right, so what kind of figure do you get?" Students quickly concluded that the figure is a circle centered at A with radius AB. The instructor asked, "How do you know?" Ms. Savant stepped to the board and offered a proof, in which she showed that all of the image points of B are the same distance from A, to justify the class' conclusion.

Given this emerging picture of Ms. Savant's knowledge of proof, we were drawn to a particular incident observed during her student teaching. In this class, Ms. Savant conducted an activity that she hoped would make an important connection for her students. In this activity, students cut paper circles into sectors and rearranged the sectors to form a rectangle-like figure. Ms. Savant then posed a series of questions about the figure, to help students connect the formula for the area of a rectangle to the formula for the area of a circle. She concluded by telling her students that they had just proved the formula for the area of a circle: "You cut up the area, you rearranged it, and you proved that the area [of a circle] is 'pi-r-squared.' That's great!"

We were interested in Ms. Savant's use of the word 'proved' in this context. In the interview conducted later that day, the researcher asked Ms. Savant whether she thought the sectors-of-a-circle activity was a proof. In her response, Ms. Savant described proof as an informal sense-making process that shows why something is true.

> Yes I do [think this is a proof]. It's not a formal proof in that we don't sit down and we write it all out. But certainly it's a way to prove to yourself, "Hey look, this actually really does work. I can use things that I know and this really does work."

She commented further, "My idea of a formal proof is when you actually use symbolic language with English language, and you give either a paragraph or a step-by-step proof that's very logical." Ms. Savant contrasted this notion of a formal proof with that of an informal proof, or convincing argument, which is acceptable and appropriate in a high school classroom.

Ms. Savant had made this distinction between formal and informal proofs often in interviews prior to this incident. She had repeatedly described formal proof as the structured symbolic presentations acceptable to mathematicians: "A proof is a logical and clear series of steps that take you from point A to point B." In contrast, for Ms. Savant informal proofs are ways to explain why something is true, particularly to students: "I think convincing [informal proof] is different from formal proof. But, in the vernacular English, it would prove to the kids that it works."

Through our situative lens, we see that Audrey Savant developed two somewhat different conceptions of proof as she progressed through her teacher education program – formal proofs that prove, and informal proofs that explain. She learned how to draw upon these conceptions in order to participate successfully in two different types of mathematical communities. In her content courses and on our research tasks, where the standards of proof were seen to be more related to structure and form than to explanatory power – proofs that prove, not necessarily explain – Ms. Savant drew upon a conception of proof that supported her successful participation as a student. In her mathematics methods course and field placements – where the emphasis for her was on fostering student learning – she drew upon a very different conception of proof that contributed to her ways of participating as a teacher.

Ms. Savant's reliance on different conceptions of proof in different situations, viewed through a situative lens, has also helped us understand why researchers have found little relationship between subject matter knowledge, as measured by the number of courses completed successfully, and teaching competence (Grossman et al., 1989). The situations of content preparation – their norms and expectations and the roles played by individuals – are fundamentally different from the situations of teaching practice, and the patterns of participation learned in these two types of situations differ fundamentally as well. Successful participation as a student in undergraduate mathematics courses often requires use of several familiar proof *structures*, such as induction and proof by contradiction. On the other hand, participation as a teacher in a school mathematics classroom often consists of *explaining* and *justifying*, and providing students with opportunities to do the same. Successful participation in one of these situations does not necessarily support successful participation in the other.

Finally, our situative stance allowed us to interpret Ms. Savant's use of different conceptions of proof as evidence of the *strength* of her knowledge of this slice of the mathematics domain. An observer in her class might conclude that Ms. Savant's 'pizza slice' activity is not a proof in the mathematical sense, and that Ms. Savant's knowledge of proof is weak. (In fact, some mathematicians with whom we have shared this vignette have concluded just that.) However, drawing on our multiple data sources, including Ms. Savant's own comments about what she did in the situations presented in this vignette and why, we were able to conclude that Ms. Savant's complex knowledge of proof as a social construct allowed her to use it successfully both as a student in her university courses and as a teacher in her own classroom.

3.2. *Adam Hanson: "They can't handle activities"*

Rose Tall Middle School, in a diverse, mostly working class suburban school district, has over 600 students, a fourth of whom are Hispanic. Adam Hanson, one of two 8th grade mathematics teachers, came to this school after completing his undergraduate studies in applied mathematics and the teacher preparation program at a large research university. As a student teacher, he taught 6th grade mathematics in the wealthier and less diverse district surrounding the university. His mentor teacher, Ms. Largent, is experienced, well-known for her inquiry-based mathematics teaching, and well-respected as an innovator. She was a participant in a mathematics teacher mentoring program run at the university by Mr. Hanson's mathematics methods instructor.

Our observations revealed quite different patterns of instruction during Mr. Hanson's student teaching and first year of teaching. Throughout his student teaching, we observed classes in which students actively explored important mathematical ideas, in groups and as a whole class. Mr. Hansen typically used a variety of activities – some chosen by his mentor, some developed collaboratively, some selected by Mr. Hanson – to explore the same mathematical concept. Students discussed these ideas readily with Mr. Hanson and with each other; they were comfortable expressing their positions and jumping into the conversation to share their justifications or conclusions. Mr. Hanson sometimes relied on teacher-directed, fill-in-the-blank questions in his interactions with students. However, because Ms. Largent had created a discourse community in which students expected to initiate discussions and build on each other's contributions, they often pushed to make sure that their voices were heard, and Mr. Hanson responded by letting their conversations go on without interrupting. When viewed across consecutive days, there was a clear narrative flow to his classes.

In contrast, throughout Mr. Hanson's first year of teaching we found classes that followed a very different, but just as consistent, pattern. Each class session began with a warm-up consisting of one or several tasks that students worked on individually and then went over as a whole class, led by Mr. Hanson. The lesson, which followed the warm-up, typically entailed Mr. Hanson presenting new information or guiding students through solutions to a set of example problems. Homework review occurred either as part of the warm-up or during the instructional portion of the lesson. During the final 15–25 minutes of the class period, students worked on worksheets, individually or in informal groups of their own choosing. Some tasks from these worksheets would comprise the homework assignment for that night. Most of the discussions in the classes we observed were initiated by Mr. Hanson. He asked many questions, but most of these were attempts to elicit particular correct responses from students. The mathematical focus in these classes was on finding solutions to tasks using specific sets of procedures.

One possible explanation for the differences in practices we observed centers on the relationship between Mr. Hanson's evolving identity as a teacher and two very different teaching situations. During student teaching, Mr. Hanson described his ideal mathematics teacher – the teacher he was striving to become – as "somebody who has a grasp of mathematics and who can explain things in multiple ways," who "cares about each student," and has a goal of building students' conceptual understanding of mathematics. Ms. Largent shared a similar image of teaching and learning and had created a classroom environment in which active student learning and rich mathematical discourse were expected. Thus, Mr. Hanson found himself in a situation that supported his efforts to construct his identity as a teacher and learner of teaching. One of his goals during student teaching was to maintain patterns of participation and discourse that were already in place. Ms. Largent and her students encouraged his experimentation, appreciated his successes, and forgave his unsuccessful efforts.

Mr. Hanson's developing identity was not a good match for the sociocultural context of Rose Tall Middle School. He was attempting to create opportunities in his classes for students to "either transfer something that we've learned or discover it on their own, because I really believe that you have a much better understanding when you come to it on your own, as opposed to me trying to tell you over and over again." In contrast, his mathematics department chair believed that "The more you repeat stuff, the more you show them examples, the more you go over it, the more it's going to stick in their heads. Thus I am constantly reviewing with them all year, on everything we do." The school's principal held similar beliefs about the role of the teacher, "What are teachers in front of kids for? They're the show."

Mr. Hanson consistently noted that his students at Rose Tall Middle School appeared uninterested in understanding the mathematics they were being asked to learn; "They don't care about what the heck I am really doing. They just want to be able to get that answer." Also, "They don't handle the activities very well. They can handle working together so they'll be working together with worksheets." On one occasion, he recounted his decision not to use an activity in his first-year class that he had used in student teaching, even though it was a good one for fostering understanding of the topic of the day. He explained that he was sure his students would see this activity as 'stupid' – as they had with other activities he had tried that used manipulatives. Thus, although one of his goals as a teacher was to support students' understanding of mathematics through tasks that use manipulatives, group work, and student presentations, students' reactions to these features of activities led him to make different types of instructional decisions.

Mr. Hanson attributed his students' desire to be told how to do tasks to their 7th grade teacher. According to him, she gave students clear procedures to follow and lots of practice. Students liked this approach, and they liked her for it. They reacted negatively to his attempts to break this pattern, and therefore to him as well. As he explained, "Because of the way the 7th grade teacher did it last year, we still haven't established that they can trust me. . . I think that they keep thinking, 'Last year's teacher knew everything, and you don't know anything. She taught it so much better.' " Considering the students' experiences in their 7th grade mathematics classes and the image of teaching pervasive among faculty and administration at Rose Tall Middle School, it is not surprising that Mr. Hanson's students complained, "He doesn't give us notes; he doesn't give us enough examples; and he won't explain step-by-step."

Mr. Hanson's concerns about district and state testing, and the time required for test preparation, also impacted his instructional decisions during his first year of teaching. He consistently asserted that the pressure to cover each of the district's curriculum standards, in preparation for district- and state-level standardized tests, made it impossible for him to take the time to help students develop conceptual understanding. This attitude is very different from what we observed in his student teaching, where Mr. Hanson typically planned several activities to explore a single concept and often changed lesson plans to take additional time if he thought it was needed.

Through the lens of our conceptual framework, we see that time constraints, the emphasis on covering district standards in preparation for district and state tests, the expectations brought to his class by students, and the norms for mathematical tasks and discourse that were fostered by other teachers and administrators at the school all contributed to the creation of a situation that placed new and different demands on Mr. Hanson – demands that were at odds with his developing professional identity. In contrast, the norms and participation structures for rich mathematical discussions had been established in Mr. Hanson's student teaching classroom prior to his arrival, and he had to do little to facilitate them. Ironically, he may have learned less about establishing and participating as a teacher in a middle-school discourse community in this carefully chosen student teaching placement than he would have in a class with less successful discourse practices, and may have been less prepared for the resistance he encountered during his first year of teaching. However, what he did learn about fostering discourse during student teaching was the foundation upon which Mr. Hanson built, as he learned about the very different situation of his own classroom and developed strategies to modify that situation and to foster discourse during his second year of teaching. From this perspective, our observations of Mr. Hanson's very different patterns of practice during student teaching and his first year of teaching, and the frustration he expressed about his first year experience, make sense.

4. Concluding comments

In this article we described a conceptual framework for studying teacher learning. This framework focuses a situative lens on teachers' developing knowledge and beliefs about mathematics and mathematics-specific pedagogy, and professional identities.

Through this lens we have been able to trace Ms. Savant's developing understandings of the social nature and role of proof across the settings of university and high school mathematics classes. For her, successful participation in mathematics courses, mathematics education courses, and field placements – communities in which she played different roles (student in her coursework, teacher in her field placements) and with different sets of norms and expectations – led to the development of two different conceptions of the fundamental notion of proof. And, she learned to draw upon these different conceptions in order to participate successfully in different types of mathematical communities.

Through the situative lens, we saw that for Mr. Hanson, a carefully selected and nurtured student-teaching experience provided a sense of what could be. He successfully played the role of teacher in this mathematical sense-making community. However, this experience did not prepare him well for the very different situation of his first teaching job. As a result, his first-year teaching practices looked markedly different, and he expressed dissatisfaction with the role he found himself enacting at the school. Yet, despite the difficulties he experienced, Mr. Hanson's image of the teacher he was striving to become remained essentially unchanged during his first year of teaching. Further, his vision of the ideal mathematics classroom, an abstraction given form during student teaching, was stronger than ever at the end of that year. His struggles to create such a classroom, combined with his frustrations during his first year, led him to develop a series of specific strategies for beginning his second year. Rather than "washing out" the influence of his teacher preparation experiences (Zeichner and Tabachnick, 1981), Mr. Hanson's participation as a full-time teacher enhanced his understanding of how to create discourse in his classroom.

As these brief examples illustrate, the framework described in this paper has helped us better understand the ways in which the various contexts of teacher education and early career teaching made a difference in the professional development of the mathematics teachers in our project. Some of the differences in knowledge and practices that we observed across these settings could easily have seemed confusing and even contradictory. The framework enabled us to see these differences as coherent and sensible – as being integrally connected to norms and expectations specific to the different contexts, and to the novice teachers' evolving professional identities. Thus, the situative perspective helped to bring the numerous and varied contexts of teacher education into focus and supported our characterization of teacher learning as a single trajectory through these multiple contexts.

This conclusion is similar to ones reached by Adler (2000) and Ensor (2001) in their studies of mathematics teacher development. In her analysis of teachers' 'take-up' from an in-service professional development program, Adler concluded that social practice theory helped to explain one teacher's changing knowledge-in-practice as a function of her participation in multiple and sometimes contradictory contexts of practice, rather than as located either in her head or in her involvement in the in-service program. Ensor interpreted another teacher's pedagogical practices during her first year of teaching as a recontextualization of practices learned in her mathematics methods course, and concluded that the effects of teacher education were transformed rather than washed out. Thus, both researchers found theoretical lenses that take into account both the individual and the social context to be helpful in accounting for teacher development.

We began this paper by noting that our use of situativity is an attempt to extend this theoretical framework for studying learning to the study of learning to teach (secondary mathematics). Our work thus far indicates that such an extension of the range of applicability of this theory holds great promise for conducting research on, and building an understanding of, the learning to teach enterprise. The editors of this journal have called for researchers to mine existing theories by testing their applicability in ever-wider settings, and to make connections between different existing theories before proposing new ones (*Educational Studies in Mathematics*, 2002). The framework we have outlined above has led us to conceive of the complexity of the knowledge base for teaching as stemming from its situated nature. It has dictated that we collect a large and varied corpus of data, not to be methodologically sound, but to enable us to span the situations in which teachers learn their craft. It has forced us to rethink cognitive ideas, such as the notion of transfer. We offer it here so that others in the mathematics education research community will further refine and extend this approach and consider its connections other theories that focus on both individuals and their contexts of practice. In this way, our relatively young discipline might consolidate a core theory of learning to teach.

NOTES

1. The Learning to Teach Secondary Mathematics project (LTSM) is a 5-year research project funded in part by the United States National Science Foundation (NSF Grants REC-9605030 and REC-0087653).
2. The names of participants and their schools are pseudonyms.
3. The class had already *defined* two congruent triangles as having all sides and angles of one be congruent to corresponding sides and angles of the other. The SAS axiom, for example, assumes that showing this relationship for only two sides and the included angle is sufficient to establish congruence.

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[1]*University of Colorado at Boulder,*
[2]*The Metropolitan State College of Denver,*
[3]*University of Wisconsin – Madison,*
[4]*University of Colorado at Boulder,*
[*]*for correspondence:*
School of Education, C.B. 249,
University of Colorado at Boulder,
Boulder, CO 80309-0249,
Telephone (303) 492-8399, Fax (303) 492-7090,
E-mail: Hilda.Borko@Colorado.edu