Topology

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7-405, Sichuan Normal University

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2. Lecture 12
   - Compact
   - Compact↔subspace
   - Compact↔Hausdorff
   - Compact↔continuous
   - Compact↔product
   - Compact↔connected
   - Finite intersection property
   - Compact sets in $\mathbb{R}^n$
   - Extreme value theorem

Outline

Lemma 13.8

Let X be a Hausdorff space and Y be compact in X. If $x \notin Y$, then there exist open subsets U and V of X such that

$$U \in \mathscr{N}_X(x), Y \subset V, \text{ and } U \cap V = \emptyset.$$

Question

If Y is a singleton, who do you find? Could two disjoint compact sets be separated? (Ex.3.)

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Theorem 13.9

Let X be a Hausdorff space and Y be compact in X. Then Y is closed.

Revisit Item 6 of Example 13.3.

Example 13.10 ("Hausdorff" cannot be removed)

$X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. $\{a\}$ is compact, but not closed.

小结：理清关系越来越不容易．"剪不断，理还乱"．

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Theorem 13.11

Assume that $f : X \longrightarrow Y$ is continuous on X. If X is compact, then so is $f(X)$.

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Let $f : X \longrightarrow Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then f is a homeomorphism.

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Theorem 13.13

If X and Y are compact, then so is $X \times Y$.

Lemma 13.14

Let X be a topological space and Y be a compact space. If N is open in $X \times Y$ and $\{x\} \times Y \subset N$, then there is $W \in \mathcal{N}_X(x)$ such that $W \times Y \subset N$.

Example 13.15 ("Compact" cannot be removed)

Let $N := \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq \frac{1}{x}\}$, $\{0\} \times \mathbb{R}_+ \subset N$.

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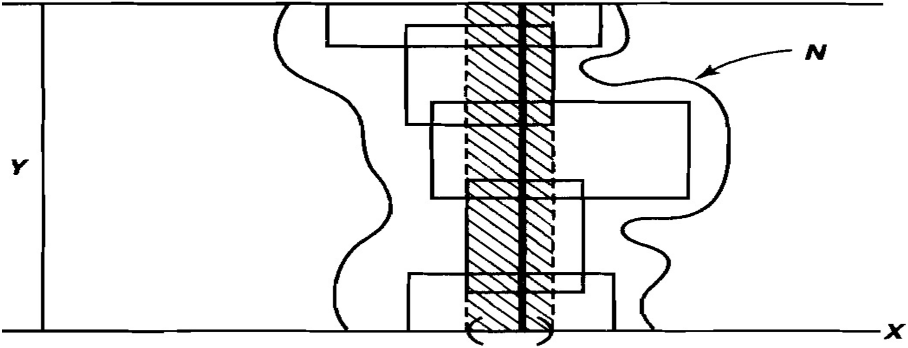

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