Topology

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2. Lecture 10: metric space (continued)
   - Continuity
   - Hausdorff
   - New things

Outline

1. Lecture 10: metric space (continued)
    - Continuity
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    - New things

Theorem 11.1

Let (X, d_X) and (Y, d_Y) be metric spaces. Then $f : X \to Y$ is continuous at $x_0 \in X \iff \forall\, \varepsilon > 0$, $\exists\, \delta > 0$, when $d_X(x, x_0) < \delta$,

$$d_Y(f(x), f(x_0)) < \varepsilon\,.$$

Theorem 11.2

Let X and Y be topology spaces. If f is continuous at x_0, then $x_n \to x_0$ implies $f(x_n) \to f(x_0)$. The converse holds if X is metrizable

Proposition 11.3 (Exercise 3(a), Page 126)

$d : X \times X \to \mathbb{R}$ is continuous.

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Let (X, d_X) *and* (Y, d_Y) *be metric spaces. Then* $f : X \to Y$ *is* *continuous* *at* $x_0 \in X \iff \forall\, \varepsilon > 0,\ \exists\, \delta > 0$, *when* $d_X(x, x_0) < \delta$,

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Let X *and* Y *be topology spaces. If* f *is continuous at* x_0, *then* $x_n \to x_0$ *implies* $f(x_n) \to f(x_0)$. *The converse holds if* X *is metrizable*

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Proposition 11.4

Let X be a metric space. Then X is a Hausdorff space.

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Definition 11.5

Let (X, d) be a metric space. $A \subset X$ is said to be **bounded** if there is $M > 0$ such that $\sup_{x,y\in A} d(x,y) \le M$.

More rules on continuity in special spaces: See Lemma 21.4 and Theorem 21.5 on Page 131.

Question (Assignment)

Is $+ : \mathbb{R}_\ell \times \mathbb{R}_\ell \to \mathbb{R}_\ell$ continuous at $(0,0)$?

Uniform continuity (used on Page 176); uniform convergence (Page 131).

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Definition 11.6

Let X be a topological space.

- $\{U_\alpha\}_{\alpha\in\Lambda} \subset \mathscr{N}(x)$ is said to be a (local) basis at x if

$$\forall\, U \in \mathscr{N}(x),\ \exists\ \alpha \in \Lambda\ \ni: U_\alpha \subset U.$$

- If the index set Λ is countable, we say that x has a countable basis.
- If every $x \in X$ has a countable basis, then X is said to satisfy the first countability axiom.

Proposition 11.7

Every metric space satisfies the first countability axiom.

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