Topology
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2 Lecture 8

- Continuous function: other equivalent statements
- Continuous function: rules
- Continuous$\leftrightarrow$subspace

Outline

1. Lecture 8
   - Continuous function: other equivalent statements
   - Continuous function: rules
   - Continuous↔subspace

Let $\mathscr{B}_Y$ and $\mathscr{A}_Y$ be the basis and subbasis for the topology of Y, respectively.

Theorem 1.1

The following are equivalent:

1. f *is continuous on* X.
2. *For every* $B \in \mathscr{B}_Y$, $f^{-1}(B)$ *is open in* X.
3. *For every* $S \in \mathscr{A}_Y$, $f^{-1}(S)$ *is open in* X.

Theorem 1.2

The following are equivalent:

1. f *is continuous on* X.
2. *For every subset* $A \subset X$, $f(\bar{A}) \subset \overline{f(A)}$.
3. *For every closed set* $B \subset Y$, $f^{-1}(B)$ *is closed in* X.

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Theorem 1.3

Let X, Y, and Z be topological spaces.

1. *Constant function is continuous.*
2. *(Composite) If $f: X \to Y$ and $g: Y \to Z$ are continuous, then $g \circ f: X \to Z$ is continuous.*

Outline

Theorem 1.4

Let A be a subspace of X. If $f : X \to Y$ is continuous at every point of A, then $f|_A : A \to Y$ is continuous.

Corollary 1.5

Let A be a subspace of X. If $f : X \to Y$ is continuous on X, then $f|_A : A \to Y$ is continuous.

Hint: $(f|_A)^{-1}(V) = \{x \in A : f(x) \in V\} = A \cap f^{-1}(V)$. The converse of Theorem 1.4 and hence of Corollary 1.5 does not hold.

Example 1.6

Let $X := \mathbb{R}$, $A := (-\infty, 0]$, and $f(x) := \begin{cases} 0 & x \leq 0, \\ 1 & x > 0. \end{cases}$

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1. Let A be a subspace of X. Then $j: A \to X$ defined by $j(x) = x$ is continuous.
2. Let Y be a subspace of a topological space Z. If $f: X \to Y$ is continuous, then the function $h: X \to Z$ defined by $h(x) = f(x)$ is continuous.
3. Let Y be a subspace of a topological space Z. If $f: X \to Z$ is continuous and $f(X) \subset Y$, then the function $g: X \to Y$ defined by $g(x) = f(x)$ for $x \in X$ is continuous.

你明白它们在说什么吗?