Topology

by Munkres James

何诣然

7-405, Sichuan Normal University

2nd semester, 2010-2011

2 Lecture 7

Definition of continuous function

$\diamondsuit$ How to extend the $\varepsilon - \delta$ definition to topological spaces?
$f : X \to Y$, where X and Y are topological spaces.

Definition 1.1

$f : X \to Y$ is said to be continuous at $x \in X$ if for every $V \in \mathscr{N}_Y(f(x))$, there is $U \in \mathscr{N}_X(x)$ such that $f(U) \subset V$.

Link with $\varepsilon - \delta$ definition

Proposition 1.2

Let $f : \mathbb{R} \to \mathbb{R}$. Then f is continuous at a point x_0 (in the topological sense) $\iff$ $\forall\ \varepsilon > 0$, $\exists\ \delta > 0$ such that

$$|f(x) - f(x_0| < \varepsilon, \text{ whenever } |x - x_0| < \delta.$$

Definition 1.3

$f: X \to Y$ is said to be continuous on X if f is continuous at every $x \in X$.

Theorem 1.4

$f: X \to Y$ is continuous on X $\iff$ For every open subset $V \subset Y$, $f^{-1}(V)$ is open in X.

Remark 1.5

The continuity of a function depends on the specified topologies. For example, in the real line, $f(x) = x$ is continuous in the standard topology, but not continuous when the range space is assumed with the lower limit topology $\mathbb{R}_\ell$.

Definition 1.3

$f: X \to Y$ is said to be continuous on X if f is continuous at every $x \in X$.

Theorem 1.4

$f: X \to Y$ is continuous on X $\iff$ For every open subset $V \subset Y$, $f^{-1}(V)$ is open in X.

Remark 1.5

The continuity of a function depends on the specified topologies. For example, in the real line, $f(x) = x$ is continuous in the standard topology, but not continuous when the range space is assumed with the lower limit topology $\mathbb{R}_\ell$.

Definition 1.3

$f: X \to Y$ is said to be continuous on X if f is continuous at every $x \in X$.

Theorem 1.4

$f: X \to Y$ is continuous on X $\iff$ For every open subset $V \subset Y$, $f^{-1}(V)$ is open in X.

Remark 1.5

The continuity of a function depends on the specified topologies. For example, in the real line, $f(x) = x$ is continuous in the standard topology, but not continuous when the range space is assumed with the lower limit topology $\mathbb{R}_\ell$.