第四章 拟稳定渗流与拟稳定试井

第一节 达西拟稳定渗流与拟稳定试井

法定单位制中，油藏达西渗流的数学模型为：

$$\frac{1}{r}\frac{\partial p}{\partial r}+\frac{\partial^2 p}{\partial r^2}=\frac{1}{\eta}\frac{\partial p}{\partial t}$$

$$\eta=\frac{10^{-3}K}{\phi\mu C_t}$$

初始条件：

$$p(r,t=0)=p_i$$

井边界条件：

$$q_t=2\pi\times 86.4\delta_o\frac{kh}{\mu}\left(r\frac{\partial p}{\partial r}\right)\Bigg|_{r=r_w}$$

外边界条件：无穷大地层，

$$p(r\to\infty,t)=p_i$$

外边界定压，

$$p(r=r_e,t)=p_e$$

外边界封闭，

$$\frac{\partial p}{\partial r}\Bigg|_{r=r_e}=0$$

拟稳定流动，　$\partial p/\partial t = 常数$

由简单的物质平衡得：　$C_t V(p_i - p) = \frac{1}{86400\delta_o} q_t t$

$$C_t V \frac{dp}{dt} = -\frac{1}{86400\delta_o} q_t \qquad V = \pi r_e^2 h\phi$$

$$\frac{dp}{dt} = -\frac{q_t}{86400 C_t \pi r_e^2 h\phi\delta_o}$$

$$\frac{1}{\eta}\frac{\partial p}{\partial t} = -\frac{\phi\mu C_t}{86.4K}\frac{q_t}{C_t \pi r_e^2 h\phi\delta_o} = -\frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = -\frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o} = -C \qquad \frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = -Cr$$

积分　$r\frac{\partial p}{\partial r} = -0.5Cr^2 + C_1$　　$\frac{\partial p}{\partial r} = -0.5Cr + C_1\frac{1}{r}$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_e} = -0.5Cr_e + C_1\frac{1}{r_e} = 0 \qquad C_1 = 0.5Cr_e^2 = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}$$

$$\frac{\partial p}{\partial r} = -0.5Cr + C_1\frac{1}{r} = -\frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\frac{r}{r_e^2} + \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\frac{1}{r}$$

$$\frac{\partial p}{\partial r} = -0.5Cr + C_1\frac{1}{r} = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\frac{1}{r} - \frac{r}{r_e^2}\right)$$

再积分，

$$p(r) - p_{wf} = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln r - \frac{r^2}{2r_e^2}\right)\Bigg|_{r_w}^{r}$$

$$p(r) - p_{wf} = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2} + \frac{r_w^2}{2r_e^2}\right)$$

$$p(r) - p_{wf} \approx \frac{q_t\mu}{2\pi\times 10^{-3}\delta_o Kh}\left(\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2}\right)$$

$$p_e - p_{wf} = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln\frac{r_e}{r_w} - \frac{r_e^2}{2r_e^2}\right) = \frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln\frac{r_e}{r_w} - \frac{1}{2}\right)$$

$$\overline{p} = \frac{\int_{r_w}^{r_e} p\,dV}{\int_{r_w}^{r_e} dV} = \frac{2\pi h \phi \int_{r_w}^{r_e} p \cdot r\,dr}{\int_{r_w}^{r_e} 2\pi r h \phi\,dr}$$

$$\overline{p} = \frac{2\pi h\phi \int_{r_w}^{r_e} \left[p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left(\ln \frac{r}{r_w} - \frac{r^2}{2 r_e^2} \right) \right] r\,dr}{\pi \left(r_e^2 - r_w^2 \right) h \phi}$$

$$\int x \ln(ax)\,dx = 0.5 x^2 \ln(ax) - 0.25 x^2$$

$$\overline{p} = 2 \frac{\left\{ p_{wf} \frac{r^2}{2} + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left(0.5 r^2 \ln \frac{r}{r_w} - 0.25 r^2 - \frac{r^4}{8 r_e^2} \right) \right\} \Big|_{r_w}^{r_e}}{\left(r_e^2 - r_w^2 \right)}$$

$$\overline{p} = \frac{p_{wf}\left(r_e^2 - r_w^2 \right) + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left[r_e^2 \ln \frac{r_e}{r_w} - \frac{r_e^2}{2} - \frac{r_e^4}{4 r_e^2} - \left(r_w^2 \ln \frac{r_w}{r_w} - \frac{r_w^2}{2} - \frac{r_w^4}{4 r_e^2} \right) \right]}{\left(r_e^2 - r_w^2 \right)}$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left[\ln \frac{r_e}{r_w} - \frac{1}{2} - \frac{r_e^2}{4 r_e^2} \right]$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left[\ln \frac{r_e}{r_w} - \frac{3}{4} \right]$$

考虑污染时

$$p_e - p_{wf} = \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left(\ln \frac{r_e}{r_w} - \frac{1}{2} + S_{skin} \right)$$

$$q_t = \frac{2\pi \times 86.4 \delta_o K h}{\mu} \frac{p_e - p_{wf}}{\ln \frac{r_e}{r_w} - \frac{1}{2} + S_{skin}}$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 K h \delta_o} \left[\ln \frac{r_e}{r_w} - \frac{3}{4} + S_{skin} \right]$$

由(3.2.5式)不考虑表皮效应时稳定流产量公式为

$$q_{t稳定流} = 2\pi \times 86.4\delta_o \frac{Kh}{\mu} \frac{p_e - p_{wf}}{\ln \frac{r_e}{r_w}}$$

由(4.1.23式)不考虑表皮效应时拟稳定流封闭边界产量公式为

$$q_{t拟稳定封闭边界} = \frac{2\pi \times 86.4\delta_o Kh}{\mu} \frac{p_e - p_{wf}}{\ln \frac{r_e}{r_w} - \frac{1}{2}}$$

$$\mathrm{R}_{拟稳定封闭边界} = \frac{\ln \frac{r_e}{r_w}}{\ln \frac{r_e}{r_w} - \frac{1}{2}}$$

拟稳定流封闭边界产量与稳定流产量的比

re/rw	10	100	1000	10000	100000	1000000
R拟稳定流封闭边界	1.2774	1.1218	1.078	1.0574	1.0454	1.0376

拟稳定流封闭边界产量比稳定流产量大；当边界距离趋于无穷大时，拟稳定流封闭边界产量等于稳定流产量。

拟稳定流定压边界产量与稳定流产量的比:

$$R_{拟稳定封闭边界}=\frac{\ln\frac{r_e}{r_w}}{\left[\left(1+\frac{r_w^2}{r_e^2}\right)\ln\frac{r_e}{r_w}-\frac{1}{2}\left(1-\frac{r_w^2}{r_e^2}\right)\right]}$$

r_e/r_w	10	100	1000	10000	100000	1000000
$R_{拟稳定封闭边界}$	1.2578	1.1216	1.078	1.0574	1.0454	1.0376

拟稳定流定压边界产量比稳定流产量大；当边界距离趋于无穷大时，拟稳定流定压边界产量等于稳定流产量；拟稳定流定压边界产量与拟稳定流封闭边界产量一样。

由(3.2.5式)不考虑表皮效应时稳定流产量公式为

$$q_{t稳定流} = 2\pi \times 86.4\delta_o \frac{Kh}{\mu} \frac{p_e - p_{wf}}{\ln \frac{r_e}{r_w}}$$

由(4.1.23式)不考虑表皮效应时拟稳定流封闭边界产量公式为

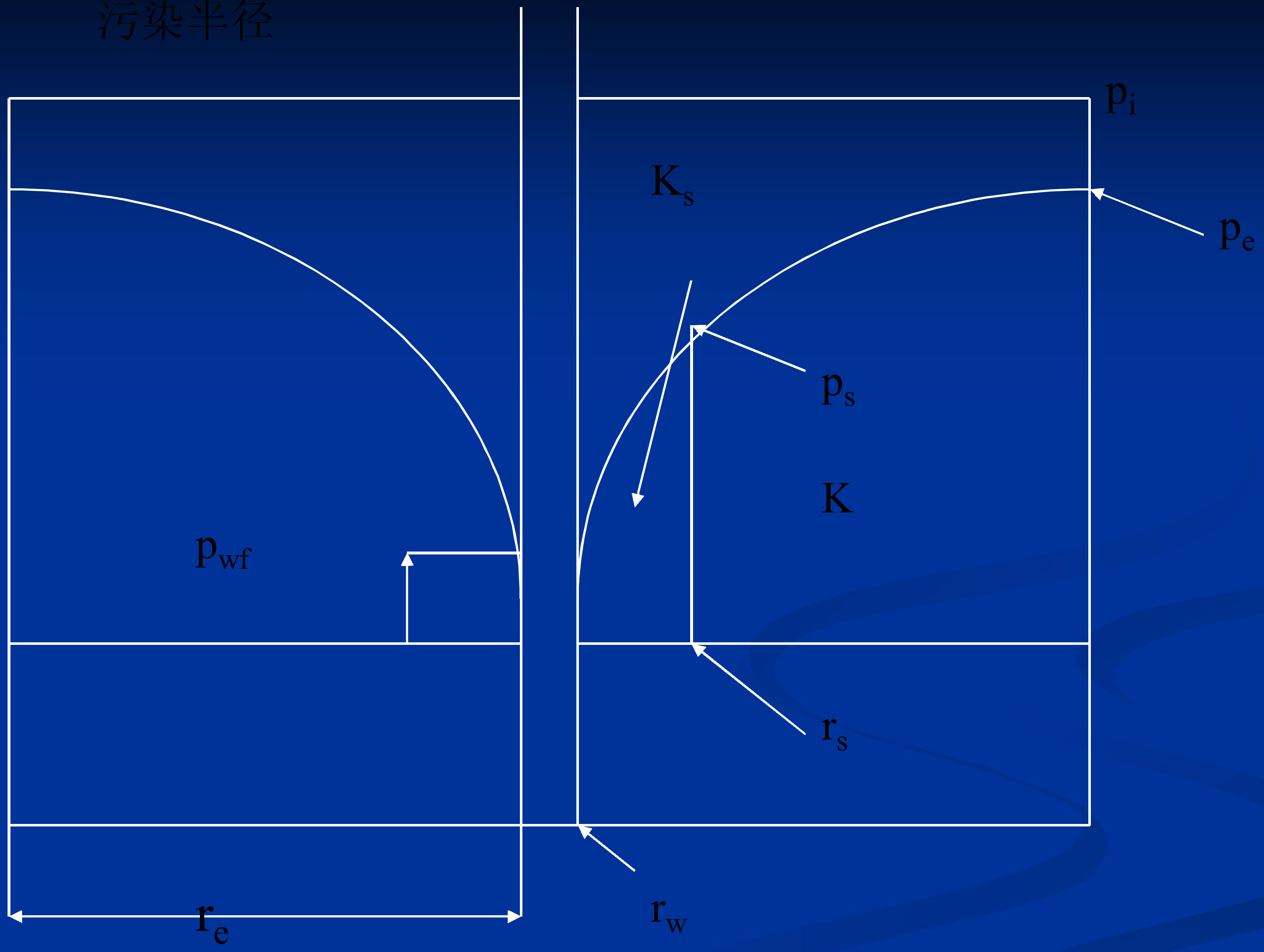
$$q_{t拟稳定封闭边界} = \frac{2\pi \times 86.4\delta_o Kh}{\mu} \frac{p_e - p_{wf}}{\ln \frac{r_e}{r_w} - \frac{1}{2}}$$

$$\mathrm{R}_{拟稳定封闭边界} = \frac{\ln \frac{r_e}{r_w}}{\ln \frac{r_e}{r_w} - \frac{1}{2}}$$

第四章　拟稳定渗流与拟稳定试井

第二节　污染半径

污染半径



污染半径

法定单位制中,考虑近井区稳定流动:

$$p(r)=p_{wf}+\frac{q\mu}{2\pi\times 86.4 B_o h K_s}\ln\frac{r}{r_w}\qquad r_w<r\le r_s$$

远井区拟稳定流动:

$$p(r)-p_s=\frac{q\mu}{2\pi\times 86.4 B_o K h}\left(\ln\frac{r}{r_s}-\frac{r^2}{2r_e^2}\right)\qquad r_s<r\le r_e$$

在r_s上,

$$p_s=p_{wf}+\frac{q\mu}{2\pi\times 86.4 B_o h K_s}\ln\frac{r_s}{r_w}$$

在r_e上,

$$p_e-p_s=\frac{q\mu}{2\pi\times 86.4 B_o K h}\left(\ln\frac{r_e}{r_s}-\frac{1}{2}\right)$$

两式相加:

$$p_e-p_{wf}=\frac{q\mu}{2\pi\times 86.4 B_o K h}\left(\ln\frac{r_e}{r_s}-\frac{1}{2}\right)+\frac{q\mu}{2\pi\times 86.4 B_o h K_s}\ln\frac{r_s}{r_w}$$

$$p_e - p_{wf} = \frac{q\mu}{2\pi \times 86.4\delta_o Kh}\left(\ln\frac{r_e}{r_s} - \frac{1}{2} + \frac{K}{K_s}\ln\frac{r_s}{r_w}\right)$$

$$p_e - p_{wf} = \frac{q_t\mu}{2\pi \times 86.4\delta_o Kh}\left(\ln\frac{r_e}{r_s} - \frac{1}{2} + \ln\frac{r_e}{r_w} - \ln\frac{r_e}{r_w} + \frac{K}{K_s}\ln\frac{r_s}{r_w}\right)$$

$$p_e - p_{wf} = \frac{q_t\mu}{2\pi \times 86.4\delta_o Kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + \ln\frac{r_e}{r_s} - \ln\frac{r_e}{r_w} + \frac{K}{K_s}\ln\frac{r_s}{r_w}\right)$$

$$p_e - p_{wf} = \frac{q_t\mu}{2\pi \times 86.4\delta_o Kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} - \ln\frac{r_s}{r_w} + \frac{K}{K_s}\ln\frac{r_s}{r_w}\right)$$

$$p_e - p_{wf} = \frac{q_t\mu}{2\pi \times 86.4\delta_o Kh}\left[\ln\frac{r_e}{r_w} - \frac{1}{2} + \left(\frac{K}{K_s} - 1\right)\ln\frac{r_s}{r_w}\right]$$

与下式比较：

$$p_e - p_{wf} = \frac{q_t \mu}{2\pi \times 86.4 \delta_o K h}\left[\ln\frac{r_e}{r_w} - \frac{1}{2} + S_{skin}\right]$$

表皮系数与污染半径的关系为:

$$S_{skin} = \left(\frac{K}{K_s} - 1\right)\ln\frac{r_s}{r_w}$$

污染半径为:

$$r_s = r_w \exp\left(\frac{S_{skin}}{K / K_s - 1}\right)$$

第四章　拟稳定渗流与拟稳定试井

第三节　非达西低速拟稳定渗流与拟稳定试井

法定单位制中，油藏非达西低速渗流的数学模型为：

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial P}{\partial r}-\lambda_B\right)\right]=\frac{1}{\eta}\frac{\partial p}{\partial t}\qquad \eta=\frac{10^{-3}K}{\phi\mu C_t}$$

初始条件： $p\left(r,t=0\right)=p_i$

井边界条件： $q_t=2\pi\times 86.4\delta_o\frac{kh}{\mu}r\left(\frac{\partial P}{\partial r}-\lambda_B\right)\Big|_{r=r_w}$

外边界条件： 无穷大地层， $p(r\to\infty,t)=p_i$

外边界定压， $p(r=r_e,t)=p_e$

外边界封闭， $\left.\frac{\partial p}{\partial r}\right|_{r=r_e}=0$

拟稳定流动， $\partial p/\partial t=$常数

由简单的物质平衡得： $C_t V(p_i - p) = \frac{1}{86400\delta_o} q_t t$

$$C_t V \frac{dp}{dt} = -\frac{1}{86400\delta_o} q_t \qquad V = \pi r_e^2 h\phi$$

$$\frac{dp}{dt} = -\frac{q_t}{86400\pi C_t r_e^2 h\phi\delta_o}$$

$$\frac{1}{\eta}\frac{\partial p}{\partial t} = -\frac{\phi\mu C_t}{86.4K}\frac{q_t}{C_t \pi r_e^2 h\phi\delta_o} = -\frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial P}{\partial r} - \lambda_B\right)\right] = -\frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o}$$

$$\frac{\partial}{\partial r}\left[r\left(\frac{\partial P}{\partial r} - \lambda_B\right)\right] = -\frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o} r = -Cr$$

$$C = \frac{q_t\mu}{86.4\pi r_e^2 Kh\delta_o}$$

积分 $r\left(\frac{\partial P}{\partial r}-\lambda_B\right) = -0.5Cr^2 + C_1$ $\qquad \frac{\partial P}{\partial r}-\lambda_B = -0.5Cr + C_1\frac{1}{r}$

$$\frac{\partial P}{\partial r} = -0.5Cr + C_1\frac{1}{r} + \lambda_B$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_e} = -0.5Cr_e + C_1\frac{1}{r_e} + \lambda_B = 0$$

$$C_1 = 0.5Cr_e^{\ 2} - \lambda_B r_e$$

$$\frac{\partial P}{\partial r} = -0.5Cr + \left(0.5Cr_e^2 - \lambda_B r_e\right)\frac{1}{r} + \lambda_B$$

$$\frac{\partial P}{\partial r} = 0.5Cr_e^2\left(\frac{1}{r}-\frac{r}{r_e^2}\right) - \lambda_B\left(\frac{r_e}{r}-1\right)$$

$$\frac{\partial P}{\partial r} = 0.5Cr_e^2\left(\frac{1}{r}-\frac{r}{r_e^2}\right) - \lambda_B\left(\frac{r_e}{r}-1\right)$$

再积分:

$$p(r)-p_{wf}=0.5Cr_e^{\,2}\left(\ln r-\frac{r^2}{2r_e^2}\right)\Bigg|_{r_w}^{r}-\lambda_B\left(r_e\ln r-r\right)\Big|_{r_w}^{r}$$

$$p(r)-p_{wf}=0.5Cr_e^{\,2}\left(\ln\frac{r}{r_w}-\frac{r^2}{2r_e^2}+\frac{r_w^{\,2}}{2r_e^2}\right)-\lambda_B\left(r_e\ln\frac{r}{r_w}-r+r_w\right)$$

$$p_e-p_{wf}=0.5Cr_e^{\,2}\left(\ln\frac{r_e}{r_w}-\frac{r^{\,2}}{2r_e^2}\right)-\lambda_B\left(r_e\ln\frac{r_e}{r_w}-r_e\right)$$

$$p_e-p_{wf}=\frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln\frac{r_e}{r_w}-\frac{r_e^{\,2}}{2r_e^2}\right)-\lambda_B r_e\left(\ln\frac{r_e}{r_w}-1\right)$$

$$p(r)-p_{wf}=0.5Cr_e^{\,2}\left(\ln\frac{r}{r_w}-\frac{r^2}{2r_e^2}\right)-\lambda_B\left(r_e\ln\frac{r}{r_w}-r\right)$$

$$p(r)-p_{wf}=\frac{q_t\mu}{2\pi\times 86.4Kh\delta_o}\left(\ln\frac{r}{r_w}-\frac{r^2}{2r_e^2}\right)-\lambda_B\left(r_e\ln\frac{r}{r_w}-r\right)$$

$$\bar{p} = \frac{\int_{r_w}^{r_e} p\,dV}{\int_{r_w}^{r_e} dV} = \frac{2\pi h\phi \int_{r_w}^{r_e} p \cdot r\,dr}{\int_{r_w}^{r_e} 2\pi r h\phi\,dr}$$

$$\bar{p} = \frac{2\pi h\phi \int_{r_w}^{r_e} \left[p_{wf} + 0.5 C r_e^2 \left(\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2} \right) - \lambda_B \left(r_e \ln\frac{r}{r_w} - r \right) \right] r\,dr}{\pi \left(r_e^2 - r_w^2 \right) h\phi}$$

$$\int x \ln(ax)\,dx = 0.5x^2 \ln(ax) - 0.25x^2$$

$$\bar{p} = 2\frac{\left. \left\{ \begin{array}{l} p_{wf}\dfrac{r^2}{2} + 0.5 C r_e^2 \left(0.5r^2 \ln\dfrac{r}{r_w} - 0.25r^2 - \dfrac{r^4}{8r_e^2} \right) \\ -\lambda_B \left[r_e \left(0.5r^2 \ln\dfrac{r}{r_w} - 0.25r^2 \right) - \dfrac{r^3}{3} \right] \end{array} \right\} \right|_{r_w}^{r_e}}{\left(r_e^2 - r_w^2 \right)}$$

$$\bar{p} = \frac{\begin{aligned} & p_{wf}\left(r_e^2 - r_w^2\right) + 0.5Cr_e^{\,2}\left(0.5r_e^{\,2}\ln\frac{r_e}{r_w} - 0.25r_e^{\,2} - \frac{r_e^{\,4}}{8r_e^2}\right) \\ & - 0.5Cr_e^{\,2}\left(0.5r_w^{\,2}\ln\frac{r_w}{r_w} - 0.25r_w^{\,2} - \frac{r_w^{\,4}}{8r_e^2}\right) \\ & - \lambda_B\left[r_e\left(0.5r_e^{\,2}\ln\frac{r_e}{r_w} - 0.25r_e^{\,2}\right) - \frac{r_e^{\,3}}{3}\right] \\ & + \lambda_B\left[r_e\left(0.5r_w^{\,2}\ln\frac{r_w}{r_w} - 0.25r_w^{\,2}\right) - \frac{r_w^{\,3}}{3}\right] \end{aligned}}{\left(r_e^2 - r_w^2\right)}$$

$$\bar{p} = p_{wf} + 0.5Cr_e^{\,2}\left(0.5\ln\frac{r_e}{r_w} - 0.25 - \frac{r_e^{\,2}}{8r_e^2}\right) - \lambda_B\left[r_e\left(0.5\ln\frac{r_e}{r_w} - 0.25\right) - \frac{r_e}{3}\right]$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 \delta_o K h}\left(\ln\frac{r_e}{r_w} - 0.5 - \frac{r_e^2}{4r_e^2}\right) - \lambda_B r_e\left[0.5\ln\frac{r_e}{r_w} - 0.25 - \frac{1}{3}\right]$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 \delta_o K h}\left(\ln\frac{r_e}{r_w} - \frac{3}{4}\right) - \frac{1}{2}\lambda_B r_e\left[\ln\frac{r_e}{r_w} - 0.5 - \frac{2}{3}\right]$$

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 \delta_o K h}\left(\ln\frac{r_e}{r_w} - \frac{3}{4}\right) - \frac{1}{2}\lambda_B r_e\left[\ln\frac{r_e}{r_w} - 1\frac{1}{6}\right]$$

考虑污染时

$$\bar{p} = p_{wf} + \frac{q_t \mu}{2\pi \times 86.4 \delta_o K h}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S_{skin}\right) - \frac{1}{2}\lambda_B r_e\left[\ln\frac{r_e}{r_w} - 1\frac{1}{6}\right]$$

$$q_t = \frac{2\pi \times 86.4 \delta_o K h}{\mu} \frac{p_e - p_{wf} + \lambda_B r_e\left(\ln\frac{r_e}{r_w} - 1\right)}{\ln\frac{r_e}{r_w} - \frac{1}{2} + S_{skin}}$$

第四章　拟稳定渗流与拟稳定试井

第四节　非达西高速拟稳定渗流与拟稳定试井

法定单位制中，油藏非达西高速渗流的数学模型为：

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right)\right]=\phi\rho C_t\frac{2\beta K}{\mu}\frac{\partial p}{\partial t}$$

初始条件： $p(r,t=0)=p_i$

井边界条件：

$$q_t=2\pi\times86.4\delta_o\frac{10}{2\beta\rho}\frac{\mu h}{K}r\left[1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right]\Bigg|_{r=r_w}$$

外边界条件：无穷大地层， $p(r\to\infty,t)=p_i$

外边界定压， $p(r=r_e,t)=p_e$

外边界封闭， $\left.\frac{\partial p}{\partial r}\right|_{r=r_e}=0$

拟稳定流动， $\partial p/\partial t=$常数

由简单的物质平衡得： $C_tV(p_i-p)=\dfrac{1}{86400\delta_o}q_tt$

$$C_tV\frac{dp}{dt}=-\frac{1}{86400\delta_o}q_t \qquad V=\pi r_e^2h\phi$$

$$\frac{dp}{dt}=-\frac{q_t}{86400\pi C_tr_e^2h\phi\delta_o}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right)\right]$$

$$=\phi\rho C_t\frac{2\beta K}{\mu}\frac{\partial p}{\partial t}=-\phi\rho C_t\frac{2\beta K}{\mu}\frac{q_t}{86400\pi C_tr_e^2h\phi\delta_o}$$

$$=-\rho\frac{2\beta K}{\mu}\frac{q_t}{86400\pi r_e^2h\delta_o}=-C$$

$$\frac{\partial}{\partial r}\left[r\left(1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right)\right]=-Cr$$

积分

$$r\left(1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right)=-0.5Cr^2+C_1$$

$$\frac{q_t\beta\rho K}{864\pi\delta_o\mu h}=r\left[1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}\right]\Bigg|_{r=r_w}=-0.5Cr_w^{\ 2}+C_1$$

$$C_1=\frac{q_t\beta\rho K}{864\pi\delta_o\mu h}+0.5Cr_w^{\ 2}$$

$$1+\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}=-0.5Cr+C_1\frac{1}{r}$$

$$\sqrt{1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}}=-0.5Cr+C_1\frac{1}{r}-1$$

$$1+4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}=\left(-0.5Cr+C_1\frac{1}{r}-1\right)^2$$

$$4\times10^{-4}\frac{K^2\beta\rho}{\mu^2}\frac{\partial p}{\partial r}=\left(-0.5Cr+C_1\frac{1}{r}-1\right)^2-1$$

$$\frac{\partial p}{\partial r}=A\left(0.25C^2r^2+C_1^{\ 2}\frac{1}{r^2}+1-CC_1+Cr-2C_1\frac{1}{r}-1\right)$$

$$A=\frac{\mu^2}{4\times10^{-4}K^2\beta\rho}$$

再积分

$$p(r)=A\left(0.25C^2\frac{r^3}{3}-C_1^{\ 2}\frac{1}{r}-CC_1r+0.5Cr^2-2C_1\ln r\right)+C_2$$

$$p_{wf}=A\left(0.25C^2\frac{r_w^{\ 3}}{3}-C_1^{\ 2}\frac{1}{r_w}-CC_1r_w+0.5Cr_w^{\ 2}-2C_1\ln r_w\right)+C_2$$

$$B = A\left(0.25C^2\frac{r_w^{\ 3}}{3} - C_1^{\ 2}\frac{1}{r_w} - CC_1 r_w + 0.5Cr_w^{\ 2} - 2C_1 \ln r_w\right)$$

$$p_{wf} = B + C_2$$

$$p(r) - p_{wf} = A\left(C^2\frac{r^3}{12} - C_1^{\ 2}\frac{1}{r} - CC_1 r + 0.5Cr^2 - 2C_1 \ln r\right) - B$$

$$p_e - p_{wf} = A\left(C^2\frac{r_e^{\ 3}}{12} - C_1^{\ 2}\frac{1}{r_e} - CC_1 r_e + 0.5Cr_e^{\ 2} - 2C_1 \ln r_e\right) - B$$

$$\bar{p} = \frac{\int_{r_w}^{r_e} p\,dV}{\int_{r_w}^{r_e} dV} = \frac{2\pi h\phi\int_{r_w}^{r_e} p\cdot r\,dr}{\int_{r_w}^{r_e} 2\pi r h\phi\,dr}$$

$$\bar{p} = \frac{2\pi h\phi\int_{r_w}^{r_e}\left[p_{wf} + A\left(\begin{array}{l} C^2\frac{r^3}{12} - C_1^{\,2}\frac{1}{r} - CC_1 r \\ + 0.5Cr^2 - 2C_1\ln r \end{array}\right) - B\right] r\,dr}{\pi\left(r_e^2 - r_w^2\right)h\phi}$$

$$\int x\ln(ax)\,dx = 0.5x^2\ln(ax) - 0.25x^2$$

$$\bar{p} = 2\frac{\left\{p_{wf}\frac{r^2}{2} + A\left[\begin{array}{l} C^2\frac{r^5}{60} - C_1^{\,2}r - CC_1\frac{r^3}{3} \\ + 0.5C\frac{r^4}{4} - 2C_1\left(0.5r^2\ln r - 0.25r^2\right) \end{array}\right] - 0.5Br^2\right\}\Bigg|_{r_w}^{r_e}}{\left(r_e^2 - r_w^2\right)}$$

$$\bar{p} = \frac{p_{wf}\left(r_e^2 - r_w^2\right) + A\left[\begin{array}{l} C^2 \dfrac{1}{60}\left(r_e^{\ 5} - r_w^{\ 5}\right) - C_1^{\ 2}\left(r_e - r_w\right) - CC_1 \dfrac{r_e^{\ 3} - r_w^{\ 3}}{3} \\ +0.5C \dfrac{r_e^{\ 4} - r_w^{\ 4}}{4} - 2C_1\left(0.5 r_e^{\ 2} \ln r_e - 0.25 r_e^{\ 2}\right) \\ +2C_1\left(0.5 r_w^{\ 2} \ln r_w - 0.25 r_w^{\ 2}\right) \end{array}\right] - 0.5B\left(r_e^{\ 2} - r_w^{\ 2}\right)}{\left(r_e^2 - r_w^2\right)}$$

$$\bar{p} = p_{wf} + A\left[C^2 \frac{1}{60} r_e^{\ 3} - C_1^{\ 2} \frac{1}{r_e} - \frac{1}{3} CC_1 r_e + C \frac{r_e^{\ 2}}{8} - 2C_1\left(0.5 \ln r_e - 0.25\right)\right] - 0.25B$$

第四章　拟稳定渗流与拟稳定试井

第五节 分型油气藏拟稳定渗流与拟稳定试井

法定单位制中，分型油藏渗流的数学模型为：

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^{d_f-d-\theta+1}\frac{\partial p}{\partial r}\right]=\frac{1}{\eta}\frac{\partial p}{\partial t}$$

$$\eta=\frac{10^{-3}K_{well}}{\phi_{well}\mu C_t r_{well}^{-\theta}r^{d_f-d}}$$

初始条件：$p(r,t=0)=p_i$

井边界条件：$q_t=2\pi\times 86.4\delta_o\frac{k_{well}h}{\mu}\left(r\frac{\partial p}{\partial r}\right)\Big|_{r=r_w}$

外边界条件：

无穷大地层，$p(r\to\infty,t)=p_i$

外边界定压，$p(r=r_e,t)=p_e$

外边界封闭，$\frac{\partial p}{\partial r}\Big|_{r=r_e}=0$

$$\phi_f=\phi_{well}\left(\frac{r}{r_{well}}\right)^{d_f-d}\qquad k=k_{well}\left(\frac{r}{r_{well}}\right)^{d_f-d-\theta}$$

拟稳定流动，　$\partial p/\partial t = $ 常数

由简单的物质平衡得：　$C_t V\left(p_i - p\right) = \frac{1}{86400\delta_o} q_t t$

$$C_t V \frac{dp}{dt} = -\frac{1}{86400\delta_o} q_t \qquad V = \pi r_e^2 h\phi$$

$$\frac{dp}{dt} = -\frac{q_t}{86400\pi C_t r_e^2 h\phi\delta_o} = -\frac{q_t}{86400\pi C_t r_e^2 h\phi_{well}\left(\frac{r}{r_{well}}\right)^{d_f-d}\delta_o}$$

$$= -\frac{q_t r_{well}^{d_f-d}}{86400\pi C_t r_e^2 h\phi_{well} r^{d_f-d}\delta_o}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^{d_f-d-\theta+1}\frac{\partial p}{\partial r}\right] = \frac{1}{\eta}\frac{\partial p}{\partial t} = -\frac{\phi_{well}\mu C_t r_{well}^{-\theta} r^{d_f-d}}{10^{-3}K_{well}}\frac{q_t r_{well}^{d_f-d}}{86400\pi C_t r_e^2 h\phi_{well} r^{d_f-d}\delta_o}$$

$$= -\frac{q_t r_{well}^{d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well} h\delta_o}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r^{d_f-d-\theta+1}\frac{\partial p}{\partial r}\right]=-\frac{q_t r_{well}^{\,d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well} h\delta_o}=-C \qquad C=\frac{q_t r_{well}^{\,d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well} h\delta_o}$$

$$\frac{\partial}{\partial r}\left[r^{d_f-d-\theta+1}\frac{\partial p}{\partial r}\right]==-Cr$$

积分 $$r^{d_f-d-\theta+1}\frac{\partial p}{\partial r}=-0.5Cr^2+C_1$$

由井底定产 $$q_t=2\pi\times 86.4\delta_o\frac{k_{well}h}{\mu}\left(r\frac{\partial p}{\partial r}\right)\Bigg|_{r=r_w}$$

$$r\frac{\partial p}{\partial r}\Bigg|_{r_{well}}=-0.5Cr_{well}^{\,2-d_f+d+\theta}+C_1\frac{1}{r_{well}^{\,d_f-d-\theta}}=\frac{q_t\mu}{2\pi\times 86.4k_{well}h\delta_o}$$

$$C_1=\frac{q_t\mu r_{well}^{\,d_f-d-\theta}}{2\pi\times 86.4k_{well}h\delta_o}+0.5Cr_{well}^{\,2}$$

$$\frac{\partial p}{\partial r} = -0.5Cr^{1-d_f+d+\theta} + C_1 \frac{1}{r^{d_f-d-\theta+1}}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

再积分，

$$p(r) = -0.5C\frac{1}{1-d_f+d+\theta} r^{2-d_f+d+\theta} + C_1 \frac{1}{2-d_f+d+\theta-1} r^{2-d_f+d+\theta-1} + C_2$$

$$p(r) = -0.5C\frac{1}{2-d_f+d+\theta} r^{2-d_f+d+\theta} + C_1 \frac{1}{1-d_f+d+\theta} r^{1-d_f+d+\theta} + C_2$$

$$p(r) = \left[\begin{matrix} -0.5\dfrac{q_t r_{well}{}^{d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well} h\delta_o}\dfrac{1}{2-d_f+d+\theta} r + \\ \left(\dfrac{q_t \mu r_{well}{}^{d_f-d-\theta}}{2\pi \times 86.4 k_{well} h\delta_o} + 0.5\dfrac{q_t r_{well}{}^{d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well} h\delta_o} r_{well}{}^2 \right) \dfrac{1}{1-d_f+d+\theta} \end{matrix} \right] r^{1-d_f+d+\theta} + C_2$$

$$p(r)=\left[\begin{array}{l}-0.5\dfrac{r_{well}^{\;d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well}h\delta_o}\dfrac{1}{2-d_f+d+\theta}r+\\ \left(\dfrac{\mu r_{well}^{\;d_f-d-\theta}}{2\pi\times 86.4k_{well}h\delta_o}+0.5\dfrac{r_{well}^{\;d_f-d-\theta+2}\mu}{86.4\pi r_e^2 K_{well}h\delta_o}\right)\dfrac{1}{1-d_f+d+\theta}\end{array}\right]q_t r^{1-d_f+d+\theta}+C_2$$

$$B_1=0.5\frac{r_{well}^{\;d_f-d-\theta}\mu}{86.4\pi r_e^2 K_{well}h\delta_o}\frac{1}{2-d_f+d+\theta}$$

$$p(r)=\left(A_1-B_1 r\right)q_t r^{1-d_f+d+\theta}+C_2$$

$$p_e=\left(A_1-B_1 r_e\right)q_t r_e^{1-d_f+d+\theta}+C_2$$

$$C_2=p_e-\left(A_1-B_1 r_e\right)q_t r_e^{1-d_f+d+\theta}$$

$$p(r)=\left(A_1-B_1 r\right)q_t r^{1-d_f+d+\theta}+p_e-\left(A_1-B_1 r_e\right)q_t r_e^{1-d_f+d+\theta}$$

$$p_{wf} = (A_1 - B_1 r_{well}) q_t r_{well}^{1-d_f+d+\theta} + p_e - (A_1 - B_1 r_e) q_t r_e^{1-d_f+d+\theta}$$

$$p_e - p_{wf} = (A_1 - B_1 r_e) q_t r_e^{1-d_f+d+\theta} - (A_1 - B_1 r_{well}) q_t r_{well}^{1-d_f+d+\theta}$$

$$p_e - p_{wf} = \left[(A_1 - B_1 r_e) r_e^{1-d_f+d+\theta} - (A_1 - B_1 r_{well}) r_{well}^{1-d_f+d+\theta} \right] q_t$$

$$p_e - p_{wf} = A q_t$$

$$A = (A_1 - B_1 r_e) r_e^{1-d_f+d+\theta} - (A_1 - B_1 r_{well}) r_{well}^{1-d_f+d+\theta}$$

$$\bar{p} = \frac{\int_{r_{well}}^{r_e} p dV}{\int_{r_{well}}^{r_e} dV} = \frac{2\pi h \phi \int_{r_{well}}^{r_e} p \cdot r dr}{\int_{r_{well}}^{r_e} 2\pi r h \phi dr}$$

$$\bar{p} = \frac{2\pi h \phi \int_{r_{well}}^{r_e} \left[\left(A_1 - B_1 r \right) q_t r^{1-d_f+d+\theta} + C_2 \right] r dr}{\pi \left(r_e^2 - r_{well}^2 \right) h \phi}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\bar{p} = 2 \frac{\left\{ \left(\begin{array}{l} A_1 \dfrac{1}{2 - d_f + d + \theta} r^{2-d_f+d+\theta} \\ - B_1 \dfrac{1}{3 - d_f + d + \theta} r^{3-d_f+d+\theta} \end{array} \right) q_t + C_2 \frac{r^2}{2} \right\} \Bigg|_{r_{well}}^{r_e}}{\left(r_e^2 - r_w^2 \right)}$$

$$\bar{p}=2\frac{\left\{\left[\begin{array}{l}A_1\dfrac{1}{2-d_f+d+\theta}\left(r_e^{\,2-d_f+d+\theta}-r_{well}^{\,2-d_f+d+\theta}\right)\\-B_1\dfrac{1}{3-d_f+d+\theta}\left(r_e^{\,3-d_f+d+\theta}-r_{well}^{\,3-d_f+d+\theta}\right)\end{array}\right]q_t+C_2\dfrac{1}{2}\left(r_e^{\,2}-r_{well}^{\,2}\right)\right\}}{\left(r_e^2-r_{well}^2\right)}$$

$$\bar{p}=2\left(A_1\frac{1}{2-d_f+d+\theta}r_e^{\,-d_f+d+\theta}-B_1\frac{1}{3-d_f+d+\theta}r_e^{\,1-d_f+d+\theta}\right)q_t+C_2$$

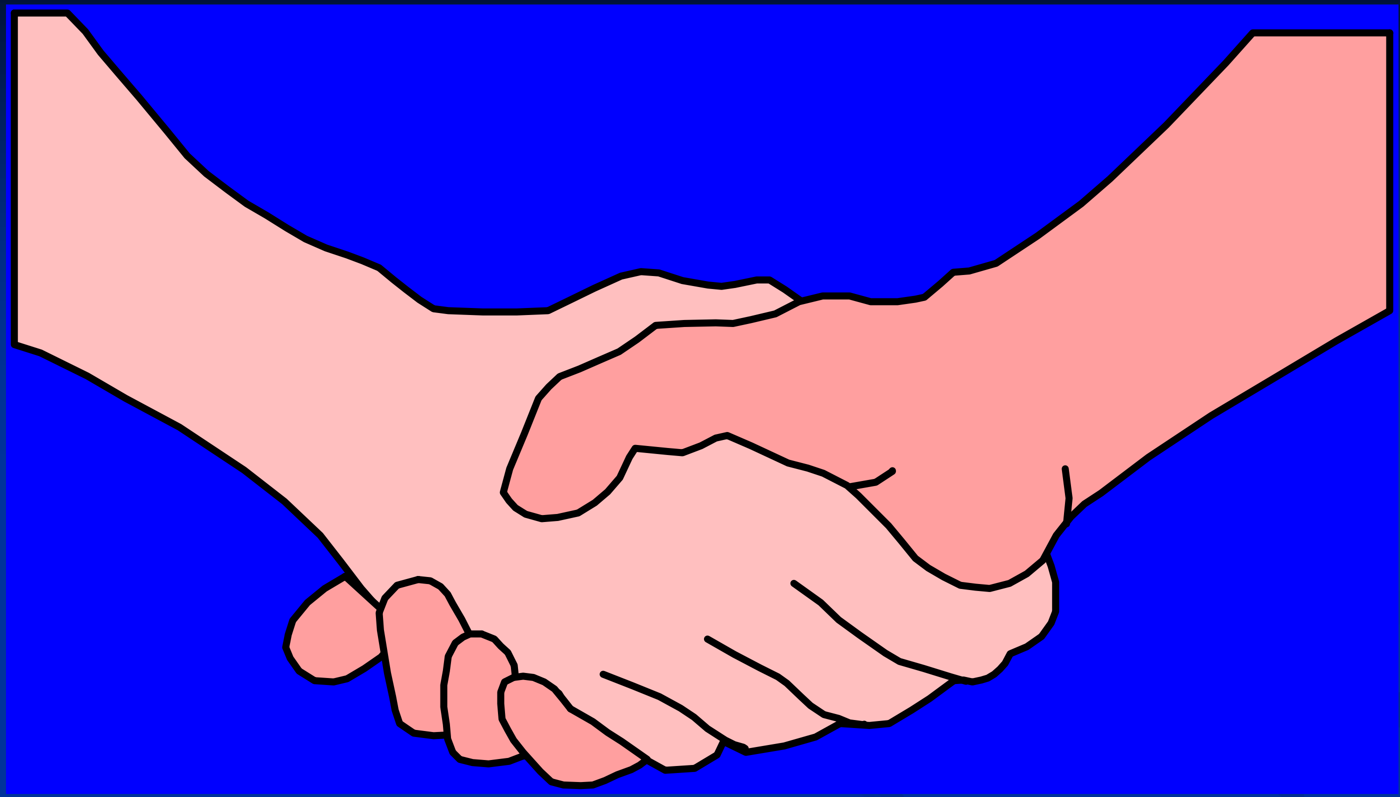
$$\bar{p}=2\left(A_1\frac{1}{2-d_f+d+\theta}r_e^{\,-d_f+d+\theta}-B_1\frac{1}{3-d_f+d+\theta}r_e^{\,1-d_f+d+\theta}\right)q_t$$
$$+p_e-\left(A_1-B_1r_e\right)q_t r_e^{\,1-d_f+d+\theta}$$

$$\bar{p} = 2\left(A_1 \frac{1}{2-d_f+d+\theta} - B_1 \frac{1}{3-d_f+d+\theta} r_e \right) r_e^{-d_f+d+\theta} q_t$$

$$+ p_e - \left(A_1 - B_1 r_e\right) q_t r_e^{1-d_f+d+\theta}$$

$$\bar{p} = p_e - \left[-A_1 \frac{2}{2-d_f+d+\theta} + B_1 \frac{2}{3-d_f+d+\theta} r_e + \left(A_1 - B_1 r_e\right) r_e \right] r_e^{-d_f+d+\theta} q_t$$

$$\bar{p} = p_e - \left[A_1 \left(r_e - \frac{2}{2-d_f+d+\theta} \right) - B_1 \left(r_e - \frac{2}{3-d_f+d+\theta} \right) r_e \right] r_e^{-d_f+d+\theta} q_t$$



谢 谢